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Hydraulic modelling of interfacial processes for two-layer maximal exchange

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ABSTRACT

The paper deals with the hydraulic modelling of two-layer maximal exchange; where two control sections are required for the stratified, bi-directional flow to be fully controlled. A novel mass flux transfer model is considered in the two-layer hydraulic exchange that includes a solution for the reversed-flow conditions of the two-layer system. This stratified-flow effect is associated with an internally-generated net-exchange barotropic flow components, which may be associated with the interfacial mixing processes. Similar recirculation-type effect in the stratified flow is present in salt-wedge estuaries. Predictions from the hydraulic model incorporating mass flux transfer between the counterflowing layers is compared to experimental data of exchange flows with and without net-barotropic forcing.

1. Introduction

Stratified-flow dynamics in estuaries and sea straits, connecting water masses of different origin and properties, can be driven by variable internal forcing conditions due to topography and external forcing under different hydrodynamic and atmospheric conditions (e.g. river flows, tides, wind stresses). Internal-flow dynamics are dependent on interfacial mixing and turbulence due to friction at the channel boundary. A salt-wedge estuary, for example, may include exchange flow through a freshwater river channel outflow in the upper layer experiencing entrainment of denser, counter-flowing marine waters of different density in the lower layer (Arita and Jirka, 1987).

Despite internal-flow hydraulics being of considerable importance to a wide range of environmental fluid flows in estuaries, the internal-flow hydraulics for more complex topographies has typically found less attention. An aim of the present study is to demonstrate how non-rectangular cross-section channels shapes, that have variable constriction topographies, can be analysed by adopting the hydraulic functions of two-layer flow (Dalziel, 1991). The present study focuses specifically on channel cases with quadratic-shape cross sections, introduced for the two-layer hydraulic exchange in Laanearu and Davies (2007). This model is an extension of the two-layer hydraulic model study by Dalziel (1992), who introduced a functional approach defining real-valued roots of the internal-flow head function; a prerequisite for the two-layer hydraulic exchange flow to be realizable. The internal-flow head function for a rectangular channel cross-section had been introduced by Armi (1986). However, previous experimental studies (e.g. Zhu and Lawrence, 2000) of two-layer exchange flows through geometrically-determined opening have been developed to make use of internal-flow hydraulic theory and also consider internal energy losses. The presence of external forcing in the two-layer hydraulic exchange is also accounted for by imposing a net-exchange barotropic flow component either in the upper fresh or lower saline fluid layer. Furthermore, the hydraulic modelling of two-layer system is also modified by parameters to deal with boundary and interfacial friction and entrainment effects between layers, all of which are needed to account for turbulent stresses and buoyancy fluxes, respectively, within the internal flows.

2. Internal-flow hydraulics of two-layer maximal exchange

According to Laanearu and Davies (2007), the internal-flow head for buoyancy-driven flow through the quadratic-type channel can be defined as the following function:

$$H = \left(\frac{\xi\, h(x)^{(\xi-1)}}{w(x)}\right)^2 K\left(\left(\frac{1}{h_2(x)^{\xi}}\right)^2 - \frac{q^2}{\left(h(x)^{\xi} - h_2(x)^{\xi}\right)^2}\right) + h_2(x) + h_s(x) \quad (1)$$

where $h(x) = h_1(x) + h_2(x)$ is the cross-sectional maximum of two-layer fluid height (i.e. water depth at its deepest point), with h_1 and h_2 being the upper and lower-layer fluid heights, $w(x)$ is the cross-sectional maximum of two-layer fluid width (i.e. water-surface width), and ξ is a channel shape factor that represents the inverse ratio of the cross-sectional flow area of a specific channel geometry to the equivalent rectangular cross-sectional area having identical $w(x)$ at the surface and $h(x)$ at the axis of cross-sectional symmetry. For instance, if $\xi = 1.0$, the channel has a rectangular cross section (Armi, 1986), while $\xi = 3/2$ and $\xi = 2.0$ correspond to parabolic and triangular cross section, respectively, (Dalziel, 1992). The lower-layer volume-flux parameter $K = Q_2^2/(2\,g')$, the upper and lower layer flow rates ratio squared $q^2 = Q_1^2/\,Q_2^2$ are defined by the volumetric fluxes $Q_1(x)$ and $Q_2(x)$ in the upper and lower layers, respectively. For the case of non-mixing flow, the hydraulic model parameters K and q^2 are constants at any along-channel location x.

3. Internal-flow head loss and interfacial displacement of two-layer exchange

The two-layer hydraulic modelling can be used to investigate the "mixing" characteristics of the internal-flow dynamics of the stratified bi-directional flow that is generated in a channel with a sill obstruction. Thus, the critical flow at the sill crest, corresponding to a control section $\boldsymbol{sill}$ $(\boldsymbol{s})$, and the critical flow at the end of the channel within the denser-fluid reservoir, corresponding to the second control section $\boldsymbol{exit}$ $(\boldsymbol{e})$, are both present for maximal exchange. In the hydraulic modelling theoretical solutions, presented above, the "globally" determine parameter squared ratio $\boldsymbol{q^2 = Q_1^2/\,Q_2^2}$ of the source fresh water and salt water volume fluxes squared across the sill is considered to be constant along the channel in the non-mixing case, i.e. $\boldsymbol{q^2 = q_e^2 = q_s^2}$. However, if the interface displacement (e.g. due to interfacial processes) between the superimposed layers of stratified flow occurs, the key non-dimensional parameter can be introduced in the internal-flow hydraulic model, which is defined as

$$M = \frac{\Delta Q_2}{Q_{2e}} \qquad (2)$$

where per definition, the interfacial-displacement parameter $M > 0$. Thus the loss of volumetric flow rate in the lower layer $\Delta Q_2 = Q_{2e} - Q_{2s} > 0$ corresponds to "entrainment" of the saline water layer between channel two control sections $exit$ and $sill$, and the increase of volumetric flow rate in the upper layer $\Delta Q_1 = Q_{1s} - Q_{1e} < 0$ corresponds to "detrainment" of the fresh water layer between channel two control sections $exit$ and $sill$. The ratio of source fresh and saline volume fluxes in the internal-flow hydraulic model formulae at the channel control section $exit$ can be corrected as

$$q_e = q + M \qquad (3)$$

and at the control section $sill$ can be corrected as

$$q_s = \frac{q}{1-M} \qquad (3)$$

4. Concluding remarks

It should be underlined here that the introduction of the "mixing" parameter M in the two-layer hydraulic model for the maximal exchange without the net-exchange barotropic flow component ($q = 1.0$) actually includes the net-exchange barotropic flow component in the upper layer, i.e. $q > 1$ due to the interface displacement. It should be mentioned here that the maximal exchange of the two-layer "mixing" flow corresponds to the comparatively small changes in the upper- and lower-layer volumetric flow rates between the channel control sections $exit$ and $sill$. The two-layer maximal exchange "mixing" effect can be expressed with the difference of magnitude for the flow-rates ratio parameters at two controls, i.e. $q_e - q_s$. However, in the case of more intensive interfacial mixing, the two control sections $exit$ and $sill$, may be dynamically decoupled by the internal-flow hydraulic jump, and the two-layer hydraulic exchange should be classified as the sub-maximal in nature (De Falco et al, 2021).

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