Exploring the Development of Thinking in Senior Secondary Mathematics: A Focus on Probability

Heather Joy Ernst

Bachelor of Science Education (Melbourne College of Advanced Education)

Master of Numeracy (The University of Melbourne)

A thesis submitted in fulfilment of the requirements for the degree of

Doctor of Philosophy

Federation University Australia

2021

School of Education

Statement of Authorship and Originality

I, **Heather Ernst,** hereby certify that the thesis entitled: **Exploring the Development of Thinking in Senior Secondary Mathematics: A Focus on Probability** submitted for the degree of **Doctor of Philosophy** contains no material which has been submitted for examination in any other course or accepted for award of any degree or diploma in any university and, to the best of my knowledge or belief, no material previously published or written by another person, except where due references are made in the text of the thesis. No editorial assistance has been received in the production of the thesis without due acknowledgement. Except where duly referred to, the thesis does not include material with copyright provisions or requiring copyright approvals.

Signed:	Signed:
Date: 26/02/2021	Date: 26/02/2021
Name: Heather Ernst	Name: Assoc Prof Margaret Plunkett
Candidate	Principal Supervisor

This dissertation contains 86,462 words

Acknowledgements

I have been very fortunate to have been surrounded by the support of many people during this PhD journey.

Firstly, I would like to thank my supervisors, Associate Professor Margaret Plunkett and Dr Peter Sellings who provided theoretical, practical, ethical, professional and motivational support. My special thanks also for the student and teacher participants in this study, who spent their time participating in the interviews and trusting in me to share their views. They have made a significant contribution to this thesis, as without their insights, this study would not have been possible.

I am grateful to my university friends and colleagues, especially Sue, Michael, Clare, Anna, Cheryl, Hazel, Jo-ann, Mary, Alison, Nhi, Nicola, Monica, Wendy, Paula, Martin, Alice, Kate, Sneh… I could go on. I would like to acknowledge all your support in so many ways. We are a small but influential university community which is strengthened by its diversity and passion.

Many thanks to the extended university networks, my MAV and MERGA colleagues, especially Hazel, Nicole and Robyn, but to the whole range of networks who were so welcoming to early career researchers. The online groups of OWLs, Wrens, and I Should be Writing, were always there with advice and moral support. Dr Sue Plowright provided professional editing services in accordance with the Institute of Professional Editors' *Guidelines for editing research theses*.

I would not have been able to complete this project without the support of my lovely husband, John. He carried the load of my stresses and continued to motivate me both emotionally and practically. To the rest of my family, Joy, Jim, Anthony, Milo, Eliza, Jess, Adrian, Shaun, Carmen, Tim, Tahluah, and my extended family Ang, Lou, Geoff, Marie, Gary, and Michelle, who all helped in spiritual and practical ways, I sincerely thank you.

Finally, I would finally like to acknowledge the support of the Department of Education and Training Victoria, and the Australian Government Research Training Program (RTP) Fee-Offset Scholarship through Federation University Australia.

Abstract

Higher order thinking skills have been identified as desirable although elusive outcomes of many educational curricula. Through a qualitative case study, the alignment between the three levels of the curriculum: *intended, implemented,* and *attained,* was examined to determine the tensions and possibilities in the development of mathematical and thinking skills in senior secondary students in Gippsland, a large regional area of Victoria, Australia. Probability was the mathematical content area of focus. Data from document analysis of the intended curriculum, textbooks as the implemented curriculum, and assessments as the attained curriculum, was combined with qualitative data from semi-structured interviews with twenty students and fourteen senior secondary mathematics teachers. These diverse data sources scaffolded each other to identify tensions and possibilities influencing development of student thinking in senior secondary mathematics.

This research demonstrated that the flow of content via the intended-implemented-attained curriculum was not adequate to describe all the influences on student learning. The lens of Activity Theory (Engeström, 2001) came closer to capturing the related complexities whereby the textbooks, calculators, bound reference books and assessments, combined with the balance of agency demonstrated by the teachers and students, were found to both support and cause tensions within the activity system.

Probability was found to be a valuable topic to study in relation to the development of thinking skills due to its relevance in decision making, how it linked many areas of mathematics and the uniqueness of the classic, experimental, and subjective views of probability.

This study is significant in the contribution it makes to understanding the tensions and possibilities associated with the development of mathematical thinking relating to probability through the lens of Activity Theory. While the intended curriculum encouraged a range of thinking skills, this intended curriculum could be implemented in a way that promotes memorisation rather than the intended higher order thinking. This study concludes with recommendations for the curriculum designers, textbook publishers, teachers, and students which may support the development of mathematical and thinking skills.

Table of Contents

List of Tables

List of Figures

Abbreviations

AAMT	Australian Association of Mathematics Teachers
ACARA	Australian Curriculum Assessment and Reporting Authority
ACER	Australian Council for Educational Research
ATAR	Australian Tertiary Admission Rank
CAS	Computer Algebra System
FM34	Further Mathematics in Year 12, Units 3 and 4
GM12	General Mathematics in Year 11, Units 1 and 2
HOT	Higher order thinking
LOT	Lower order thinking
MAV	Mathematical Association of Victoria
MM12	Mathematical Methods in Year 11, Units 1 and 2
MM34	Mathematical Methods in Year 12, Units 3 and 4
PISA	Programme for International Student Assessment
SAC	School Assessed Coursework (SACs are plural)
SM12	Specialist Mathematics in Year 11, Units 1 and 2
SM34	Specialist Mathematics Methods in Year 12, Units 3 and 4
SOLO	Structure of the Observed Learning Outcome
TIMSS	Trends in International Mathematics and Science Study
VCAA	Victorian Curriculum Assessment Authority
VCAL	Victorian Certificate of Applied Learning
VCE	Victorian Certificate of Education

Chapter 1: Introduction and Background

1.1 The Research Problem

Is the development of thinking skills inherent in Victorian Certificate of Education (VCE) mathematics, or is encouragement of memorisation and examination techniques the main focus? Similarly, does VCE mathematics help students develop a range of thinking skills to assist them in becoming lifelong learners? These are questions which concerned this teacher, who became a researcher, and could also interest other researchers, teachers, students, parents, and community members.

Australian students are displaying lower levels of essential deep-thinking skills according to results from the 2018 Programme for International Student Assessment (PISA) data (Masters, 2019). Mathematics levels in Australia have fallen by the equivalent of one-and-a-quarter years of schooling since 2003 (Masters, 2019). The PISA assessment considers the ability to transfer and apply knowledge, as well as the ability to think, rather than just the demonstration of basic numeracy skills. Masters (2019) emphasises that these deeper thinking skills are required for future employees, as machines will perform low-level skills and tasks. He proposes that every student will need deep thinking capabilities to be able to maintain employability in the future world of work, arguing that future curricula will need to focus on a smaller range of concepts covered in much greater depth.

Assessment processes, especially in the final years of senior school, will also need to reflect the changing demands. According to Masters (2019), “Broader approaches to assessment that include opportunities for students to apply their knowledge and skills to meaningful problems and to think critically and creatively are required to align assessments with new curriculum priorities” (para 8). I argue in this thesis that to implement these curriculum priorities effectively, the multiple aspects and objectives of the curriculum from conception to implementation and assessment need to align.

Australian mathematics educators have considered the issue of incorporating deeper thinking, also called higher order thinking, within the aligned curriculum for many years (Hogan, 2012; Stanley, 2008). The crowded curriculum and the dependence on textbooks which present mathematics as a series of rules and procedures has also been an issue for many years (Mousley, 2008). Hogan (2012) recommended a greater emphasis on problem-solving and reasoning proficiencies, which is also more recently supported by Callingham et al. (2021), with a particular focus on the statistics and probability curriculum.

This study investigates how the mathematics component of the Victorian Curriculum currently supports (or hinders) the development of higher order thinking skills. Skills which senior secondary students require in our current modern society, and which are of particular importance in regional areas where many students experience disadvantage. A focus on probability was selected due to the revisions outlined in the 2016–2022 VCE Mathematics Study Design where the content in relation to the topic of probability was increased. The reasoning behind the focus on probability content occurred as a result of advice from industry bodies, and findings and advice from worldwide research (ACARA, 2012). Thinking requirements within the content strand of probability involve a range of thinking skills including deep/shallow, higher order/lower order, cognitive, critical, creative, and mathematical thinking, but also probabilistic thinking. This makes probability an appropriate case study context for exploring the development of a range of thinking skills in senior secondary mathematics.

1.2 Australian and Victorian Curriculum: Background information

Within Australia, curriculum frameworks operate at both a national and state level, this can cause a level of confusion. For example, within Victoria, curriculum for primary school and the first four years of secondary school is covered by the Victorian Curriculum for Years Foundation[1](F) to Year 10 (VCAA, 2010). The final two years of secondary education falls under the jurisdiction of the Victorian Certificate of Education (VCE). Both the F-10 and VCE curriculum frameworks are based on the Australian Curriculum (ACARA, 2010), which underpins the more specifically nuanced curriculum frameworks of each state and territory.

In terms of mathematics within the Victorian Curriculum F–10, there are three content strands: Number & Algebra, Measurement & Geometry, and Statistics & Probability. Although pedagogy is not prescribed, the proficiencies of Understanding, Fluency, Problem-solving and Reasoning are applied to all the content strands (VCAA, 2010). The cross-curricula capabilities include critical and creative thinking, ethical, intercultural, and personal and social. Literacy, Numeracy and Information and Communications Technology are also embedded into all student learning within all areas (VCAA, 2010).

Senior secondary students aged 16–18 years, complete the VCE as administrated by the Victorian Curriculum Assessment Authority (VCAA). In VCE, the study scores for each subject studied combine to form an Australian Tertiary Admission Rank (ATAR), which is used as the university entrance requirement (VCAA, 2019b). Each subject gains a study score out of 50, which are scaled

[1] Level Foundation is the first year of school, for five-year-old children in Victoria.

and combined to get an ATAR up to 100. There are a range of mathematics subjects available within the VCE, which are described in the Study Design (VCAA, 2015a) as outlined in Table 1.1.

Table 1.1

Victorian Certificate of Education Mathematics Subjects and their Areas of Study

Units 1 and 2	Units 3 and 4
Foundation Mathematics	
Space, shape, and design	
Patterns and number	
Data	
Measurement	
General Mathematics GM12	**Further Mathematics FM34**
Algebra and structure	Data analysis
Arithmetic and number	Recursion and financial modelling
Discrete mathematics	Matrices and modelling
Geometry, measurement, and trigonometry	Networks and decision mathematics
Graphs of linear and non-linear relations	Geometry and measurement
Statistics	Graphs and relations
Mathematical Methods MM12	**Mathematical Methods MM34**
Functions and graphs	Functions and graphs
Algebra	Algebra
Calculus	Calculus
Probability and Statistics	Probability and Statistics
Specialist Mathematics SM12	**Specialist Mathematics SM34**
Algebra and structure	Functions and graphs
Arithmetic and number	Algebra
Discrete mathematics	Calculus
Geometry, measurement, and trigonometry	Vectors
Graphs of linear and non-linear relations	Mechanics
Statistics	Probability and Statistics

Note. Adapted from the VCAA Victorian Certificate of Education: Mathematics Study Design 2016-2022 (VCAA, 2015a). The focus area of Probability within Mathematical Methods is shaded.

The VCE mathematics units include Units 1 and 2 in Foundation Mathematics, General Mathematics, Mathematical Methods and Specialist Mathematics; and Units 3 and 4 in Further Mathematics, Mathematical Methods and Specialist Mathematics. Students typically complete Units 1 and 2 in Year 11 prior to Units 3 and 4 in Year 12 and can study as many mathematics subjects as they like in sequence. VCE Mathematical Methods includes functions and graphs, algebra, calculus, and probability and statistics. This study focuses on the topic of probability, which is highlighted in

Table 1.1. Satisfactory completion of Mathematical Methods Units 3 and 4 is a prerequisite for selected university courses, such as Computing, Commerce, Information Technology, Science and Engineering. The participation rate for Mathematical Methods Units 3 and 4, or similar forms of Intermediate mathematics, has been around 30% of all Year 12 students in both Victoria and Australia over the last 10 years (VCAA, 2019c).

The current configuration of mathematics subjects in Victoria has been in place since 2006 with minor modifications introduced in 2016. Modifications to the VCE Mathematics Study Design in 2016 included an increase in the probability and statistics content in Mathematical Methods. The scope of study of VCE mathematics is described as:

> … the study of function and pattern in number, logic, space, and structure, and randomness, chance, variability and uncertainty in data and events. It is both a framework for thinking and a means of symbolic communication that is powerful, logical, concise and precise. (VCAA, 2015a, p. 6)

This scope of the Mathematics Study Design indicates the importance placed on the topic of probability, chance, and randomness. Probability is a particular area of interest in the current study due to its importance in the modern world (Kahneman, 2012), linking mathematics to science, business, psychology, and decision making. Probability entered the Victorian curriculum over 30 years ago (Callingham et al., 2021), however moved from the popular General Mathematics in 1978 to the higher-level Mathematical Methods in 1999-2016 and beyond (Ernst, 2018), reducing the proportion of students studying this useful topic.

The aims of the VCE mathematics units are for students to:

- develop mathematical concepts, knowledge, and skills,
- apply mathematics to analyse, investigate and model a variety of contexts and solve practical and theoretical problems in situations that range from well-defined and familiar to open-ended and unfamiliar,
- use technology effectively as a tool for working mathematically. (VCAA, 2015a, p. 6).

This extract from the VCE Mathematics Study Design illustrates the aim of including a wide range of thinking, application and problem-solving into the curriculum and avoiding shallow memorisation of mathematics formula or blindly following of procedures. Probability, and the development of thinking skills, were obviously considered to be of importance in the VCE Mathematics Study Design, which ultimately underpinned my interest in the current research project. As such, an important consideration for this research is to examine whether the development of thinking skills was followed through from the intended curriculum, into the implemented and assessed curriculum.

In addition to the changes in 2016 to the content of the Victorian senior mathematics curriculum, changes also occurred in relation to the internal assessment. The internal assessments called School Assessed Coursework (SACs) were worth 34% of the final grade, and had changed to increase the focus on modelling, problem-solving and application tasks, with a move away from the option of tests as internal assessments. In Mathematical Methods Unit 4, one of the School Assessed Coursework tasks must be from the probability and statistics area of study (VCAA, 2015a). The senior secondary mathematics units continue to be assessed partially through two external examinations, which combine multiple-choice, short-answer problems and extended-answer problems, both with and without the support of a bound reference book[2] (see Appendix A) and Computer Algebra System (CAS) calculators[3]. They are worth a total of 66% of the final grade (VCAA, 2015).

1.2.1 VCE Mathematics Study Design

The VCE Mathematics Study Design does not explicitly cover the teaching and learning methods, however the expected outcomes of the assessments suggest the objective of the development of a variety of thinking skills as well as mathematical content knowledge. The three required outcomes of each of the mathematics subjects described in the Study Design (VCAA, 2015a) explain the expectations of knowledge of mathematical skills and procedures, as well as applications, analysis and investigation. They are:

> Outcome 1. Concepts – define and explain key concepts and apply a range of related mathematical routines and procedures.
>
> Outcome 2. Applications – apply mathematical processes in non-routine contexts, including situations requiring problem-solving, modelling, or investigative techniques or approaches, and analyse and discuss these applications of mathematics.
>
> Outcome 3. Technology – select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches. (VCAA, 2015a)

---

[2] One bound reference book, student or commercially created, can be used in the SACs and one of the two examinations

[3] CAS calculators were used by all students, teachers, and schools in this study. The VCE Mathematics rules allow for CAS computer software instead, but none of the participants in this study used them.

These three outcomes imply expectations of knowing, understanding, and applying mathematical concepts, with and without the aid of technology, in routine and non-routine contexts.

VCE students can achieve a pass in the mathematics units, without sitting or passing the SACs or examinations, through demonstrating the three outcomes to their teachers, as described by policies of individual schools (VCAA, 2019b). However, not completing the SACs and examinations makes university entrance difficult as a study score and ATAR cannot be calculated. This system of passing a subject based on demonstrating the outcomes, while separately earning a study score based on the assessments, can cause tensions. In theory, students could gain high scores on their examinations and still fail the subject. Students could also pass the VCE subjects and gain a VCE certificate without sitting or passing their assessments and examinations. This is not the same for all of Australia, as each state and territory has a different system of assessments in the final year of secondary school.

1.2.2 Rules regarding Calculators, Bound Reference Books and Assessment

The VCE Study Design (VCAA, 2015a) describes the content of the curriculum for each unit, and also the rules relating to each of the subjects. In the SACs, and one of the two final examinations, the assumption is made that students will make use of the CAS calculators and a bound reference book. These rules support both the students and teachers in the learning and teaching of these subjects and aim to assist students to develop and demonstrate a range of skills under a variety of conditions.

Mathematical Methods Units 3 and 4 involve three SACs worth 34% of the overall score, one an application task and the other two focusing on either problem-solving or modelling. One must relate to the area of study Probability and Statistics (VCAA, 2015a). All SACs assess the attainment of all three of the previously described outcomes but are weighted more heavily towards Outcome 2: applying mathematical processes, problem-solving, modelling, and investigations. Other rules relate to the timing, marking and authentication of the SACs. Marking is scaffolded by rubrics or assigning the marks between the three outcomes in the manner prescribed by the VCE Study Design (VCAA, 2015a). The aim of these application, modelling and problem-solving SACs, is to supplement the examinations with a broader range of assessment types to allow students a variety of ways to develop the mathematical knowledge and skills, and to demonstrate what they know. In previous years, SACs may have included “a broad range of problems in a given context; a set of application questions…, or item response analysis for a collection of multiple-choice questions” (VCAA, 2010, p. 16), but since 2016 these are no longer acceptable styles of SACs. SAC scores are moderated by the examination results to consider the range of cohorts and marking variation between schools. Sample

SACs were available both on the VCAA website (VCAA, n.d.-b, n.d.-c) and commercially, yet they are expected to be modified by schools to ensure they cater for their individual needs. For example, schools with a large English as an Additional Language (EAL) cohort might scaffold the language of the SACs, or the SACs might be broken into sub-tasks to cater to the school timetable. The governing body, VCAA, occasionally audit schools regarding the implementation of their SACs.

The VCE senior secondary mathematics subject Mathematical Methods, which is the focus within the current study, is formally assessed with two examinations worth 66% of the overall score. The first examination is one hour long, with no calculator or reference books allowed. The second examination is two hours long, with use of a CAS calculator, scientific calculator and one bound reference book assumed (VCAA, 2015a). The same mathematical content areas are covered in the two examinations, but the style of the problems differ. The one-hour non-calculator examination has short-answer problems while the longer two-hour examination, involving the use of a calculator and reference book, consists of multiple-choice problems and longer multi-part analysis problems.

The VCE Mathematics Study Design aims to cater for a wide range of students, as quoted in the rationale, “This study is designed to provide access to worthwhile and challenging mathematical learning in a way which takes into account the interests, needs, dispositions and aspirations of a wide range of students” (VCAA, 2015a, p. 6). This aim is demonstrated by a variety of SAC types and the two types of examinations which are intended to encourage a range of teaching, learning and assessment tasks. This combined with the option for students to pass the mathematics units by demonstrating the outcomes in class and not attempting the examinations, suggests the VCE studies are attempting to cater for a broad range of students. The current Mathematics Study Design was reaccredited in 2016 and is due for reaccreditation again in 2022. Submissions from stakeholders including universities, schools, teachers, business groups and international research (ACARA, 2012) were invited in 2019.

This background section on the intended senior secondary mathematics curriculum provides information about how the topic of probability fits into the system, but also explains how the rules of the intended curriculum support and influence students in their development of mathematical and thinking skills. The intended curriculum leaves room for influence by teachers and local communities.

1.3 Mathematics Education in Gippsland

This section provides a contemporary overview of the situation for mathematics students and teachers in the regional area of Gippsland, which is the geographical focus of the current study. The performance of students in regional areas in comparison to their urban counterparts is examined, and enablers and blockers for mathematics teachers in regional areas identified. This section also examines some of the unique influences of regional education.

Geographically, Gippsland is a regional district in south-eastern Australia within Victoria, extending from Melbourne's eastern suburbs to the New South Wales border in the east, encompassing beaches, farmland, mountains, lakes, and national parks. With 18% of the land mass in Victoria, but only 5% of the population, Gippsland is a diverse region, more recently moving away from its dependence on coal mines, agriculture and timber, towards recreation and service industries (State Government of Victoria, 2014). Over the past fifty years, Gippsland has undergone significant change, with the privatisation of the power stations and coal mines in the 1980s resulting in high levels of unemployment. More recently, the bush fires of 2009 and 2020 have left the region struggling in relation to both physical and social resources (Bowman, 2020).

These changes and challenges are particularly apparent in education within the Gippsland region. Approximately 35% of the population have Year 12 equivalent qualifications, compared to 52% Australia wide (ABS, 2016). Like many regional teachers, those in Gippsland tend to be professionally isolated and more likely to be teaching mathematics while not qualified in the mathematical method of teaching, with unmet needs concerning professional development, material resources and the provision of appropriate student learning opportunities (Lyons et al., 2006). Students in Gippsland have been found to be less likely to study the higher-level mathematics courses in VCE, and less likely to aspire to attend university (Fray et al., 2019; Sheehan & Mosse, 2013; The Good Schools Guide, 2019). The participation rate in Mathematical Methods of Gippsland students was just 70% of the state average, however the lower level Further Mathematics was 115% of the state average in 2010 (Sheehan & Mosse, 2013). Gippsland is thus an important setting for a case study to research the teaching and learning of mathematics in senior secondary education, and some of the factors of regional living that may influence teachers, students, and communities.

Australian mathematics students in regional areas have been reported as demonstrating disadvantage in all international, national, and state assessments. For instance, Australian regional students' mathematical literacy at Year 9 is three quarters of a year behind metropolitan students at age 15, according to the PISA international assessment (Thomson et al., 2019). Thomson et al. (2016) also reported lower scores trending with increased distance from metropolitan centres in the TIMSS international study of Year 4 and 8 students. The annual Australian National Assessment Program – Literacy and Numeracy (NAPLAN) assessments in Years 3, 5, 7 and 9, also demonstrate lower mean scores for regional students (ACARA, 2018). The average number of students gaining their Year 12 certificate in Australian major cities is 78%, but only 63% for regional areas (Mitchell Institite, 2015).

Research into the challenges faced within regional education has been the focus of several academic studies. For example, an investigation by the *Independent Review into Regional, Rural and Remote Education* (Halsey, 2018) reviewed the literature and accepted submissions from government, public and private educational and other agencies around Australia. Hudson and Hudson (2019) conducted a case study on the graduates of a university where regional and remote education placements were encouraged. Plunkett and Dyson (2011) also surveyed new graduates to uncover their experiences of teaching in regional areas. The National Centre of Science, Information and Communication Technology and Mathematics Education for Rural and Regional Australia (SiMERR Australia) focused specifically on challenges related to mathematics and science education (Lyons et al., 2006). These studies found that an issue for regional areas is the high level of teaching staff turnover, which is twice that of metropolitan areas (Halsey, 2018; Lyons et al., 2006), making it very difficult to fill teaching positions (Hudson & Hudson, 2019). Teachers tend to gain employment in locations similar to where they studied at university (Hudson & Hudson, 2019; Lyons et al., 2006) and once teachers were living in a regional area, they stayed due to quality of life, community spirit and relationships. Some teachers who leave regional areas do so due to social and professional isolation, as well as limited opportunities for their partners and children (Lyons et al., 2006).

An important issue raised in the aforementioned research was that teachers in regional areas were twice as likely to be teaching in an area in which they were not formally qualified (Lyons et al., 2006). Plunkett and Dyson (2011) also uncovered that the lack of ongoing positions and security of employment discouraged teachers from staying in regional areas, specifically Gippsland. Mathematics and science teachers were interviewed by Handal et al. (2013) to investigate their experiences in regional parts of the Australian state of New South Wales (NSW). In NSW, teachers were encouraged to work in regional areas in order to gain permanent teaching positions, but Handal et al. (2013) suggest that the inexperienced mathematics and science teachers found the demands of teaching with the lack of professional development, greater demands of teaching diverse students and lack of experienced staff support, made regional teaching very challenging. Lack of professional development, isolation, lack of medical and other support for their families, and travel costs were also factors that encouraged teachers to move back to metropolitan areas as soon as they could (Lyons et al., 2006).

Studies focussing on students, such as the examination of Year 12 results of students by location and socioeconomic status by Murphy (2019), found that regional students were disadvantaged in all areas of education, but particularly in mathematics education. Collating the results of the international, national and state assessment results, Murphy (2019) found the mathematics results of students in all age groups decreased with socioeconomic status (SES), and also decreased with distance from cities. He did find isolated examples of non-metropolitan government schools with low SES that performed better than expected. All these studies indicate why Gippsland has been described as an educationally disadvantaged or challenged regional community in need of understanding and support, which makes it an ideal location for examining development of thinking skills as a means of tackling the disadvantages and challenges for both students and teachers.

1.4 Motivation for the Study

My personal experience of teaching mathematics over the past 30 years demonstrated that while students appeared to be happy enough to attend school and complete mathematical tasks, they were often reluctant to think deeply and persistently regarding those tasks. Anecdotally, along with my secondary school teaching colleagues, I also found that students often had difficulty remembering mathematical skills from one year to the next, which made us collectively wonder whether the students had fully understood the concepts in the first place. For these reasons, I am interested in researching the nature and role that higher order thinking has in the learning of mathematics. During the twelve-year period in which I taught VCE senior secondary mathematics, I found that many students, despite excelling in algebra, graphs, and calculus, still found the topic of probability quite challenging. Probability is a vital area of study as it impacts on many aspects of life and as such underpins decision-making in areas as diverse as choosing suitable clothing for different weather conditions to making considered investment choices. The value associated with understanding probability underpinned my motivation to focus on VCE probability teaching and learning, which coincided with the increased focus on probability content that occurred as part of the changes made in 2016 to the Mathematical Methods VCE curriculum.

Over my teaching career, I witnessed the introduction of scientific and graphic calculators, and then CAS calculators as support tools for learning in senior secondary mathematics. I also observed assessment alter from internally assessed modules, to Common Assessment Tasks, which comprised large projects students could do at home and were also internally assessed, to the current system of SACs. I noted how both teachers and students struggled with the desire to fully understand the mathematical concepts, while covering the large amount of content required for the examinations. Examinations changed from one long examination at the end of the year with short-answer problems, to two shorter examinations with a variety of problem types, with and without the allowance of support material.

Other factors influencing the focus and aims of this research project relate to accessibility, as not all students in regional Gippsland have access to senior secondary mathematics due to limited staff resources. Some students and members of the community assume senior mathematics is too demanding, irrelevant, or uncreative. These assumptions may exclude students from learning mathematics and hence limit their future study and career opportunities. My own primary and secondary education occurred in metropolitan Melbourne, but I taught in a variety of schools in the regional district of Gippsland, Victoria for 25 years. My own children attended local government schools in Gippsland, and I felt they had many advantages growing up in Gippsland. However, once they moved to the city to further their education and careers, they acknowledged the difference in educational opportunities available to metropolitan students that had not been available to them. As such, I felt it was important to try to understand the issues associated with the development of thinking skills in mathematics within a regional context, in the hope that some specifically relevant insights might aid regional students and teachers, while also being more broadly relevant and applicable.

1.5 Research Questions and Research Design

A qualitative case study methodology was selected as the framework through which to investigate the tensions and possibilities for thinking skills development in the context of the VCE probability curriculum. A range of curriculum documents were analysed, and a sample of regional students and teachers were interviewed to uncover their perceptions of the opportunities for the development of mathematical and thinking skills. Several theoretical lenses were used to frame the study including Activity Theory, which originated through the ideas of Vygotsky (1978) and later Leont’ev (1981), but specifically utilising Engeström’s (2001) representation of activity systems. These will be explained in Chapter 2 and 3. To that end, the main research question underpinning the study was:

Research question

How does the study of probability impact on the development of thinking skills?

To answer the main research question, a regional case study investigating senior secondary mathematics curriculum in relation to probability, through the lens of Activity Theory, was conducted to specifically answer the following sub-questions:

Sub-questions

1. *In what ways are thinking skills developed through the senior secondary Victorian mathematics probability curriculum?*
2. *What factors impact on the development of thinking skills through the teaching and learning of the senior secondary Victorian mathematics probability curriculum?*

A summary of the research design will now be described.

A literature review was conducted focusing on curriculum, probability and thinking skills development corresponding to the issues identified in international assessments, and the changes to the VCE senior secondary mathematics curriculum. The theoretical frameworks of Activity Theory (Engeström, 2001) and the TIMSS curriculum model (Mullis & Martin, 2013) were selected, and the Two-Tiered Mathematical Thinking Framework (Chapter 3, Section 3.3.2) was designed.

The probability sections of the VCE Mathematical Methods Unit 3 and 4 curriculum documents were collected for the purposes of document analysis. The Study Design, sample SACs and examinations were located online. Other SACs were collected from participating teachers and professional development sessions run in the Gippsland region. Altogether, nine SACs were analysed. The textbook recommended by the VCAA was purchased from a local school. The curriculum statements from the Victorian Curriculum Years 7-10 and the VCE Mathematical Methods Study Design were classified according to the Two-Tiered Mathematical Thinking Framework. All the probability problems from the recommended textbook and three years of examinations were answered to determine the level of thinking required. The results of the curriculum analysis were compared and contrasted to answer the first research sub-question.

As this is a regional case study, university students who had recently completed VCE Mathematical Methods (n = 20) and were currently studying first-year mathematics and statistics subjects were recruited from a Gippsland Regional University. Current VCE Mathematical Methods teachers (n = 14) who were teaching in the Gippsland region were recruited from professional development programs. Students and teachers were interviewed to uncover current teaching and learning practices and issues. The interviews were transcribed and analysed to uncover the tensions and possibilities on the development of thinking skills, using the activity system framework provided by Engeström's (2001) interpretation of Activity Theory to structure the arguments. These interviews informed the answer to the second research sub-question.

The results of the curriculum document analysis and the student and teacher interviews were then combined and compared to respond to the overarching research question, to better understand how the development of thinking skills in senior secondary students could be supported by the mathematics probability curriculum.

1.6 Significance of the Study

The purpose of this study was to examine the probability sections of the VCE Mathematical Methods Units 3 and 4 curricula to discover the tensions and possibilities in the development of mathematical and thinking skills. Higher order thinking skills are important in our changing world, where flexibility and creativity are vital. The ability of students to think deeply is a topical issue, with critical and creative thinking being a strand in the Victorian and Australian Curriculum, and thinking being part of all 21st century skills (for example; Dede, 2010; GPE, 2020; Rotherham & Willingham, 2010). International have reported on the decline in the use of deeper thinking skills by Australian student results in recent years, and the widening disparity in results widening across a range of criteria including between urban and regional geographic areas (Echazarra & Schwabe, 2019; Thomson et al., 2017). As such, a main aim of the current study was to investigate thinking skill development in a specific mathematical context to discover and identify tensions and possibilities which may be influencing these results.

Within Australia, the VCE mathematics curriculum involves a number of unique components with the combination of internal and external assessments, two examinations completed both with and without support of the powerful CAS calculators and the option of utilising bound reference books. This context makes for an interesting and important case study. Research with senior secondary students is comparatively rare, with many schools hesitant to support research with these students during the stressful high stakes' year. Although Year 12 students were not targeted in the current study, the recent VCE student participants were asked to reflect on that year of their schooling.

This study is significant as it compares all three aspects of the curriculum—the intended, implemented and attained curriculum. It also included the perspectives of recent VCE Mathematical Methods students and current teachers, considering their views and experiences as well as the analysis of curriculum documents. The use of an activity system to understand the influence of curriculum is an original way to investigate student learning and enabled consideration of both interpersonal and physical influences. Activity theory, the TIMSS Curriculum Model and a combined thinking framework were all used to analyse the data collected in this case study, but also to expand the discussion on the important issues of curriculum alignment, encouraging higher order thinking and improving the development of mathematical and thinking skills.

1.7 Structure of the Thesis

This introductory chapter outlined the context and background of the study. The aims were explained, research questions that emerged from the review of the literature were outlined, and the structure of the thesis was explained. Chapter 2 presents a review of the various bodies of literature that informed the study, highlighting the identified gaps and resulting emergence of the research questions that directed this research. Chapter 3 provides a description of the methodology, including the theoretical frameworks underpinning this study, as well as the research design and research method. Chapters 4 and 5 report the findings from the analysis of the data with Chapter 4 reporting on the analysis of the curriculum documents and Chapter 5 presenting the findings from the interviews with the students and teachers. Chapter 6 provides a discussion of these findings considering the research questions, the underpinning theoretical framework(s) and the literature. The final chapter, Chapter 7, concludes with a discussion of the overall findings, the significance of this study, limitations, and recommendations.

Chapter 2: Literature Review

This chapter presents a review of the various bodies of literature that have informed the current research project. It is structured around three broad intertwining areas relating to mathematics curriculum and more specifically the topic of probability, and how curriculum influences the development of thinking skills. The chapter begins with a review of literature surrounding the concept of curriculum, specifically mathematics curriculum, with a focus on the tools and rules used to scaffold student learning. Literature relating to recent research around the mathematical content area of probability is then reviewed. The definition and classification methods associated with the development of thinking skills is then explored, focusing on investigating the development of thinking skills within probability curriculum. The final section considers Activity Theory as a theorical framework or lens through which to view the type of multi-facetted issues associated with the development of mathematical and thinking skills.

2.1 Mathematics Curriculum

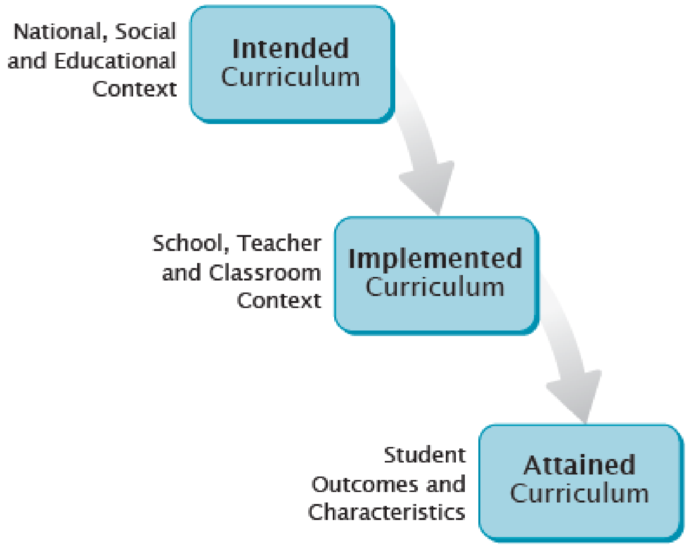
This section examines definitions of curriculum and introduces the TIMSS curriculum model, with a focus on senior secondary mathematics curriculum. The research literature, both national and international around the associated tools and rules which support students in their learning is reviewed, including textbooks, bound reference books, calculators and the interaction between teachers and students in the mathematics learning process. In later sections, the interactions between various components that make up activity systems within the framework of Activity Theory are also considered, including the subject, object, tools, rules, community, and division of labour.

2.1.1 What is Curriculum?

Curriculum has been defined in many ways. In a narrow sense, curriculum refers to the content to be taught or learnt, the syllabus. It can also include what is often referred to as the hidden curriculum, the assumed social and attitudinal values of the people involved (Gobby, 2017). In mathematics education circles, curriculum is defined as the planned and actual learning experiences designed to support the mathematics objectives that students will encounter (Remillard & Heck, 2014; Way et al., 2016). As the needs of the workforce change, the curriculum might change in response to increased requirements for technologies and the need for critical skills such as thinking and reasoning, problem-solving, estimation, flexibility and initiative (Anderson et al., 2012; Atweh & Goos, 2011). Curriculum has also been described as comprising three aspects: the intended, implemented and attained curriculum, by the international assessment Trends in International Mathematics and Science Study (TIMSS) (Mullis et al., 2009), see Figure 2.1. TIMSS is a large-scale assessment involving approximately 60 countries and conducted every four years in mathematics and science at school year levels four and eight. It is independently administered by the International Association for the Evaluation of Educational Achievement based at Lynch School of Education, Boston College, USA.

Figure 2.1

TIMSS Curriculum Model: Three Aspects of the Curriculum



Note. From "*TIMSS 2015 Assessment Frameworks*" by I. Mullis & M. Martin (2013) p. 5. Copyright 2013 by International Association for the Evaluation of Educational Achievement

The intended curriculum refers to the planned learning about mathematical content, including the syllabus (Marsh & Stafford, 1984), support material, and the rules around how these are executed (Anderson et al., 2012; Clements, 2007). The implemented, enacted or operational curriculum is the curriculum as interpreted and delivered by teachers (Atweh, Miller, et al., 2012; Remillard & Heck, 2014), which can include pedagogy, local contexts, instructional materials such as textbooks, and learning tasks. The attained curriculum involves what students have learnt and what they think about the subject (Mullis et al., 2009). The attained curriculum is measured by assessment of the skills, values and attitudes which are used in future work, study, and adult life. The attained curriculum might include mathematical reasoning and strategic use of technology (Gravemeijer et al., 2017). The three aspects of the curriculum, as represented in Figure 2.1, should align in order for the aims of the curriculum to be realised (Biggs & Tang, 2011; Hattie et al., 2017). All elements of the school system should provide a consistent message about what is valued by that system (Barnes et al., 2000). While there are other curriculum models, such as Remillard and Heck (2014) and Webb (1997), which were used in classroom analysis (Ross, 2017), the TIMSS model was considered most appropriate for the current research as it enabled comparison of the three levels of curriculum.

2.1.2 Intended Curriculum

The literature includes a range of definitions for the intended curriculum. The intended curriculum can be defined as the theoretical overarching curriculum policy (Webb, 1997), or the practical list of content standards for the subject, grade level and instructional content targets (Porter et al., 2011). Mathematical intended curriculum can include the content, organisation and culture of the school (Appelbaum & Stathopoulou, 2016).

Webb (1997) proposes that the intended curriculum is influenced by the community, higher education, public opinion, employers, and government departments. This leads to the question, what mathematical knowledge is important, and who gets to decide? (Appelbaum & Stathopoulou, 2016; Ernest, 2018b). For example, probability was only introduced into the Chinese Mathematics curriculum in 2001 (Jun, 2000; Wang et al., 2017) under the influence of the TIMSS international assessments. Many factors influence the intended curriculum. Curriculum can be biased toward particular careers, social groups, or cultures (Appelbaum & Stathopoulou, 2016). Curriculum should be inclusive and cater for a range of students (Masters, 2020).

Mathematics curriculum should be used to support students' developments of 21st century skills relating to the use of technology (Gravemeijer et al., 2017; Masters, 2020). Context is important within a curriculum. Mathematics related to the workplace and realistic situations is important, rather than isolated abstract mathematics (Gravemeijer et al., 2017). Values are implied in the intended curriculum, with objectism, rationalism and openness found to be implied in the Australian curriculum, but not necessarily implemented (Seah et al., 2016). Rationalism includes argument, reasoning, analysis, and explanations, all indicators of higher order thinking (Krathwohl & Anderson, 2002; Seah et al., 2016). Students also valued realism within their mathematical problems (Barkatsas & Seah, 2015).

The definition of intended curriculum used for this study is that of TIMSS (Mullis & Martin, 2013), namely the mathematical content which students are expected to learn, as defined in curriculum policies and publications. As such, the intended curriculum includes the mathematical content, but also the rules surrounding the implementation of the subject. The intended curriculum can also include assumptions of pedagogy, such as a focus on test results or students creating and solving their own problems (Appelbaum & Stathopoulou, 2016). For example, Gravemeijer et al. (2017) emphasise the importance of "high-level conceptual understanding" (p. S120) as part of the intended curriculum, as well as modelling and interpreting mathematics, especially as technology is used to support the mathematics.

2.1.3 Implemented Curriculum

The second aspect of curriculum depicted in Figure 2.1, relates to the implemented curriculum. In Australia, learning and teaching activities within the classroom are discretionary for individual teachers rather than prescribed (VCAA, 2015a). This may be due to a high level of contention in research and media regarding implementation of the curriculum, particularly the pedagogy of teaching and learning mathematics. In popular media, the *Math Wars* (Allen, 2011) describe a clash between those who argue students should construct their own learning (constructivism or discovery learning) and those arguing for memorising the basic facts and procedures within the back-to-basics movement (Chernoff, 2019). Mathematics teaching methods have been used as a political tool, with the back-to-basics versus discovery dichotomy being a part of the conservative versus progressive political debate (Ernest, 2018a). Academic research around mathematics teaching approaches was summarised by Schwarz (2020), where he identifies three progressive theoretical and pedagogical approaches:

- Constructivism: students construct their own knowledge though discovery, based on Dewey.
- Conceptual change: teachers support student growth through conceptual change using manipulatives, identifying misconceptions, and building on prior knowledge, based on Piaget.
- Sociocultural approaches: learning is influenced by teachers, peers, and socio-norms, based on Vygotsky.

Schwarz (2020) did not describe behaviouristic approaches as his intention was to focus on learning and teaching pedagogies, which, he argued, would improve education.

In Victoria, the Department of Education and Training (DET, 2020a) published the *High Impact Teaching Strategies* (HITS), which sets out ten recommended evidence-based teaching strategies. These include setting goals, explicit teaching, worked examples, collaborative learning, multiple exposures, and metacognitive strategies, a mix that could be interpreted as a combination of conservative and progressive teaching. This document was part of the overall *Pedagogical Model* (DET, 2020b) for Victorian schools, which also includes a focus on a positive climate for learning, professional leadership, and community engagement. In senior secondary mathematics, several tools assist with the implementation of the intended curriculum. For example, textbooks are a key teaching tool to support teachers' implementation of the curriculum (Fan et al., 2018), as are the bound reference books and technology, all of which are focal points of this research study.

2.1.3.1 Textbooks in the Curriculum.

Textbooks have also been described as part of the intended curriculum through their role in helping to interpret the curriculum documents for teachers and students (Schoenfeld, 2004; Son & Kim, 2015). Textbooks were also found to be part of the implemented curriculum as they support teachers in their planning and teaching (Remillard & Kim, 2017), and are also a tool which scaffolds students as they learn, practice, and demonstrate their understanding of the content (Hadar, 2017; Valverde et al., 2002). This section will outline the role of textbooks as implemented curriculum and as instructional material and the associated research in this area.

Research on textbooks as a component of the implemented curriculum has taken several perspectives. For example, research has investigated textbooks around the language used (O'Keeffe & O'Donoghue, 2015); page layout (Valverde et al., 2002); content (Porter, 2002); procedural complexity and repetition (Vincent & Stacey, 2008); and cognitive demand (Son & Diletti, 2017). Textbooks have been analysed in several ways within the research literature, as there does not appear to be a uniformly accepted analytical approach (Son & Diletti, 2017). Many research studies begin with the TIMSS 1999 framework as described by Valverde et al. (2002) and then are adapted to the needs of the country, level of education or requirements of the project. Other priorities of textbook research include investigating the extent to which textbooks are socio-culturally inclusive. Findings have supported the need for textbooks to relate to real-life (OECD, 2019); be free from bias including gender-bias; and be suitable for use in multi-cultural, urban and rural settings, with students from non-English speaking backgrounds and of all ability levels (Clarkson, 1993; Gough, 2012; Mikk, 2000). They should include a variety of learning tasks, including open-ended tasks and tasks with hands-on materials (Campbell, 2006; Gough, 2012).

Vincent and Stacey (2008) investigated nine Australian Year 8 textbooks, using the TIMSS framework. They examined whether Australia had the 'shallow teaching syndrome' implied by the TIMSS Video study (Vincent & Stacey, 2008, p. 82) and found that the Year 8 textbooks included between 25-60% repetition of problems and 56-83% of problems involved low procedural complexity. Vincent and Stacey's (2008) analysis suggested that a balance of repetitive and original problems was required to support concept development and reasoning. As such, they suggested that on balance, there were too many repetitive problems of low procedural complexity in the textbooks and hence determined textbooks were still contributing to *shallow teaching*.

A literature review of mathematics textbook analysis research in the USA and five Asian countries was conducted by Son and Diletti (2017). They considered content, the physical size of the textbooks, use of pictures, mathematical presentations, concepts and procedures, repetition, procedural complexity, response type, cognitive demand, and depth of knowledge. Of the 31 articles they reviewed, just three articles covered senior secondary mathematics in Years 11 or 12, while only two articles included the topic of probability. Chang and Silalahi (2017) also conducted a literature review of 44 mathematics textbooks analyses worldwide. They considered the type of thinking required for the textbook tasks, logical and creative thinking, communication (language, use of diagrams), problem-solving and mathematical content. Just two articles investigated data analysis and probability was not explicitly identified in any article. In summary, the literature indicates a gap in the research of probability curriculum in senior secondary mathematics teaching, especially regarding the depth of teaching.

2.1.3.1.1 Textbooks – Senior Secondary Textbooks.

Textbook publishers promote the idea that textbooks are an indication of the mathematical content to cover, which hopefully aligns with the intended curriculum. Textbooks aim to reflect the style of problems and the depth of thinking required as expected by the associated community and education authorities. Several examples of how textbooks have been analysed in research will now be described, including some of the research identified in the previously mentioned literature reviews.

Two studies (Brehmer et al., 2016; Rafiepour-Gatabi et al., 2012) that examined senior high school mathematics textbooks investigated the context and problem types. The first study involved a comparison of Australian and Iranian textbooks (Rafiepour-Gatabi et al., 2012), and found that Australian texts included more problems but also a higher degree of repetition. The Australian textbooks represented a wider variety of contexts, including abstract contexts and problems relating to real-world situations. The Iranian textbooks included more problems of the multi-step problem-solving type. None of the textbooks included mathematical modelling as recommended by PISA (OECD, 2019), and all real-life problems had been “cleaned” or overly simplified for student use (Rafiepour-Gatabi et al., 2012, p. 419). The second study analysed a series of three mathematics textbooks for senior secondary mathematics in Sweden (Brehmer et al., 2016). The authors found an emphasis on procedural skills and operations, with few problem-solving tasks, which if included, were all concentrated at the end of the chapters. Brehmer et al.’s (2016) study highlighted that there was one Swedish mathematical subject and accompanying textbook which was completely problem-solving based, due to the syllabus of that subject, supporting the idea that learning and applying mathematics are seen as separate skills.

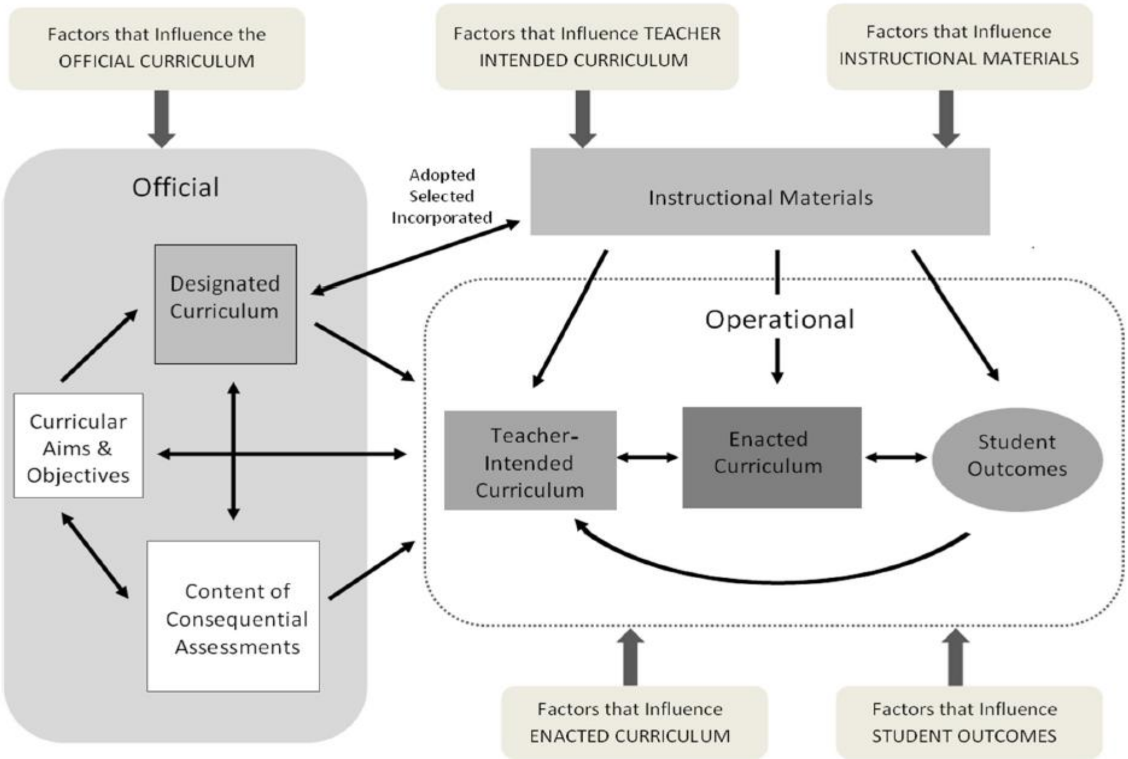
In a review of a Victorian senior secondary mathematics textbook, Polster et al. (2012), noted minor mathematical irregularities and reported the textbook’s failure to take advantage of solving problems in several ways and of linking ideas together. Their review focused on the examples and explanations with minimal comment regarding the tasks for students. Polster et al. (2012) perceived the senior secondary Mathematical Methods textbook as a disconnected set of ritual techniques. Their findings were followed by Gough (2012) who reviewed several secondary textbooks for the Australian Curriculum who advocated, "We should hope that a textbook will present **mathematics**, and **mathematical thinking**, clearly and engagingly" (Gough, 2012, p. 12, original emphasis). He suggested that when looking for a textbook, teachers should consider criteria including curriculum coverage; a variety of learning activities; links to real-life; links between the mathematical concepts; and sound mathematical concepts. He advised it was important to emphasise "problem-solving, modelling, and logical (deductive, and inferential) reasoning as ways of thinking mathematically, as opposed to 'use the formula, and hang the explanation'" (Gough, 2012, p. 13).

2.1.3.1.2 Textbook Influences on Teachers and Students.

Textbooks also provide a source for teacher professional learning (Remillard & Heck, 2014) and are considered to play an active role in supporting and developing teachers' pedagogy (Hadar, 2017). Textbooks influence the way mathematics is taught (Remillard, 2005; Son & Kim, 2015). One model of curriculum implementation by Remillard and Heck (2014), includes the textbooks as part of the instructional material. In this model, the intended curriculum is called the official curriculum, which leads to the teacher-intended curriculum, which leads to the enacted curriculum mirroring the TIMSS model. Here the instructional materials form part of the operational curriculum as one of the factors under the teachers' influence. Remillard conducted several studies (1999, 2000, 2005) investigating how teachers used textbooks and found that while textbooks could support the implementation of a new curriculum, support through professional development was also required. This finding was reinforced in Ross' (2017) study of five teachers in one school in Queensland as they implemented the new Australian Curriculum, which also found that while textbooks influenced teachers in their implementation of the curriculum, professional development was required. Remillard and Heck's (2014) model outlining where instructional materials such as textbooks fit into the learning and teaching of mathematics is displayed in Figure 2.2.

Figure 2.2

Factors Influencing the Official and Implemented Curriculum and Instructional Materials



Note. From "Conceptualizing the curriculum enactment process in mathematics education" by Remillard, J and Heck, D. (2014). *ZDM*, 46:5, p. 709. Copyright 2014 by Springer FIZ Karlsruhe 2014.

In the previously discussed examples from the USA and Australia, textbooks played a supporting role in the interpretation of the intended or official curriculum (Remillard & Heck, 2014; Ross, 2017), while research conducted in Estonia, Finland and Norway, suggested that textbooks were actually treated as part of the intended curriculum (Lepik et al., 2015). The National Boards of Education of these three countries pre-inspected textbooks up until recently, consequently teachers and students treated the textbooks as the curriculum. Lepik et al. (2015) surveyed four hundred mathematics teachers across the three countries and found that textbooks were used as the primary (or only) tool to plan lessons. Textbooks were designed to be used as a page per lesson, and included theory, examples, and exercises, as well as challenging tasks for the more able students. As such, teachers implemented the curriculum as designed by the textbooks (Lepik et al., 2015).

Son and Kim (2015) from the USA, investigated use by three teachers of different textbooks, particularly regarding cognitive demand. They utilised a modified version of Bloom's revised taxonomy (Krathwohl & Anderson, 2002), divided into two levels of tasks, high and low order thinking including low-level questions (i.e., *remembering* and *knowing procedures*) and high-level questions (i.e., *understanding, applying, reasoning,* and *evaluating*) (Son & Kim, 2015). How teachers selected and modified the tasks impacted the associated cognitive demand. In some instances, the cognitive demands were lowered through teachers breaking tasks down "into subtasks… or by focusing only on correct answers to the exclusion of reasoning and explanation" (Son & Kim, 2015, p. 492). The TIMSS video study 1999 conducted by Hollingsworth et al. (2003), also found teachers modified tasks to decrease the cognitive demand by scaffolding the tasks.

While highly regarded textbooks support teachers in their planning and teaching, they also support students in their learning, practice and revision (Lowe, 2009) by providing examples, practice problems and solutions. However, there are few studies on the influence of textbooks on student outcomes, and studies are difficult to design due to the many other confounding factors influencing student mathematical outcomes (Son & Diletti, 2017). In Israel, Hadar (2017) analysed two popular Year 8 textbooks through a large-scale study of 4040 students, finding one textbook had a higher level of cognitive demand and that students using it gained higher scores in standardised tests. Gender and socio-economic status (SES) also had an impact but overall Hadar (2017) argued that the textbook a student uses influences what they learn, how they learn, and the cognitive level at which they learn. Rezat (2009, 2013) investigated student use of textbooks in both a Year 6 and Year 12 class in Germany. To help interpret the multiple influences of students' mathematical learning, he asked students to mark on their textbooks exactly which parts they used and why. He also observed classes for three weeks and found students used the textbooks in four main ways: solving tasks and problems, consolidation, acquiring mathematical knowledge, activities associated with interest in mathematics, and combinations of these. Students also used textbooks independently of the teacher, referring back to examples, summary pages and highlighting rules. Rezat (2009) concurred with Valverde et al. (2002), who argued that the structure and content of textbooks can have a large impact on student learning.

Worldwide reform movements in school mathematics have called for an emphasis on investigation, problem-solving and mathematical reasoning, reported Valverde et al. (2002) as they reviewed the TIMSS 1999 study. Most of the textbooks contained 10% or less of these tasks, although problem-solving and proving was slightly higher for the Year 12 textbook. Valverde et al. (2002) reported that "the most common expectation for student performance was that they read, understand, recognise or recall, or that they use individual mathematical notation, facts or objects" (p. 128). The use of textbooks in the implementation of probability curriculum is a research area of interest in the current study. Textbooks are not the only resources used by teachers and students to support the implementation of the intended curriculum. Bound reference books and technology are also used, which will be explored in the next two sections.

2.1.3.2 Bound Reference Books.

To support students in their learning and assessment, both in class assessments and some examinations, students in senior secondary mathematics in Victoria are expected to use a 'bound reference book' (VCAA, n.d.), which are also sometimes referred to as a s*ummary book* (see Appendix A). Bound reference books can be student created or commercially made and although there is no limit on the number of pages, they must be securely attached and A4 size or smaller. This rule is unique to Victoria, although similar rules worldwide include the use of one page of notes called cheat-sheets or crib notes, or open-book assessments, although much of the literature on these examination aides relates to undergraduate courses in the USA (Larwin et al., 2013). Bound reference books provide a support for students, but their creation can also be a learning experience, and the books themselves can be an indication of the attained curriculum (Block, 2012). The underpinning reasoning for this form of aid is to reduce anxiety, improve study techniques (Erbe, 2007; Settlage & Wollscheid, 2019) and encourage deeper thinking (Settlage & Wollscheid, 2019).

When cheat-sheets were used in a USA study with university students studying Economics, examination results improved, especially with problems involving higher order thinking according to Bloom's taxonomy, and students displayed increased focus in their studies (Settlage & Wollscheid, 2019). Another study by Erbe (2007) involving USA secondary mathematics students found the use of cheat-sheets decreased cheating, leading to improved and creative studying. In contrast, students who copied from textbooks did not have improved examination results, suggesting the preparation of cheat-sheets is of value to improve learning, and some students require support in preparing their support material (Erbe, 2007). Hamouda and Shaffer (2016) also found a positive relationship between the content of the cheat-sheet and examination results. They analysed content of cheat-sheets of Egyptian Engineering students and found results on the comprehension questions improved, but that the quality of results on the higher order thinking questions according to Bloom's taxonomy were not related to the quality of the cheat-sheets. In contrast, Burns (2014) asked students to note when they used their cheat-sheets and found high achieving secondary statistics students rarely used their cheat-sheets, and students who used them often came to depend on them and demonstrated lower-level skills.

Open-book examinations also aim to reduce stress and rote memorisation (Agarwal, Karpicke, Kang, Roediger Iii, & McDermott, 2008; Block, 2012) and can also reduce anxiety, and increase student satisfaction (Block, 2012). Agarwal et al. (2008) compared the results of 26 university students in the USA between open and closed book examinations and found the open book increased examination performance, but long term retention was equivalent because feedback on the assessment enhanced long term learning more than any other factor (Agarwal et al., 2008;

Pelech, 2016). Another USA study on a comparison of open and closed book assessments in a first-year undergraduate statistics course found the effect on examination results and deeper learning was unclear, with students spending more time reviewing their books during the examination and less time writing their responses (Block, 2012).

A meta-literature review by Larwin et al. (2013) on the impact of cheat-sheets and open-book examinations found conflicting results. On average, cheat-sheets improved student results more than open-book examinations. While student engagement and long-term learning improved, time on task during assessments decreased due to consulting the support documents (Agarwal & Roediger, 2011). Anxiety decreased with open-book examinations compared to cheat-sheet examinations in one first-year psychology university course in the USA, although examination results only slightly improved (Gharib, Phillips, & Mathew, 2012). Block (2012) recommended closed book examinations with handwritten notes, suggesting “Handwritten notecards and closed-book exams help students develop good preparation skills for conceptual understanding of course material and foster deeper learning” (p. 236). The preparation of the support material can be a learning experience, with the focus on higher order thinking rather than memorisation.

Overall, the use of a bound reference book in an assessment task provides a physical aid which can be described as between a cheat-sheet and an open-book assessment, and is unique to Victoria, Australia. While cheat-sheets and open-book exams can decrease stress, anxiety and improve engagement, the effect on long term retention and deeper thinking are unclear. If one of these scaffolding resources are used, assessments can be designed to involve deeper thinking, with less reliance on short term memory. However, students need to be taught how to use these tools to their best advantage, to support their study techniques and save time. While there is limited research on the use of open-book examinations and cheat-sheets, very little of this research focuses on student views of these study aids, and there is no research on the use of a bound reference book in mathematics assessment tasks. The rules of the Victorian senior secondary mathematics assessments allow for the use of bound reference books and calculators for certain assessment tasks, but no support aids in others, making this a unique situation.

2.1.3.3 Technology in Senior Secondary Mathematics.

Technology can be used to support mathematics teaching and learning at all stages of education and its use is continuing to be developed (Attard et al., 2020). The current generation of students and young teachers have been called “digital natives” (Prensky, 2001, p. 1), however this does not mean they are all comfortable using technology in all aspects of their lives (Attard & Holmes, 2019). Similarly, older teachers, or “digital immigrants” (Prensky, 2001, p. 1), can be very comfortable using technology in their teaching and learning (Attard & Holmes, 2019). In a study of early career teachers in New South Wales (NSW), Orlando and Attard (2016) found teachers may understand how technologies such as iPads and Interactive Whiteboards work, but still not know how to incorporate them into their teaching. Technology can have both positive and negative effects on teaching and learning, for example iPads can encourage learning to be more student centred, whereas Interactive Whiteboards can assist classes to be more teacher centred (Orlando & Attard, 2016). In Victoria, there is an expectation that CAS calculators are used in several of the assessment tasks including examinations in the Victorian senior secondary Mathematics subjects. Other possible technological aids include quizzes, spreadsheets; graphing programs such as *Mathematica*; dynamic geometry programs; eBooks; and videos (Goos & Bennison, 2008; McCulloch et al., 2018). The following sub-sections describe how technology has recently been used in mathematics education; examples of frameworks for explaining its use; tensions associated with learning with technology; and how technology can assist with a variety of thinking skills.

2.1.3.3.1 Frameworks.

Incorporation of technology has been acknowledged as having diverse influences on teaching and learning (Goos, 2012). Activity systems (Engeström, 2001) have been used to explain the integrated impact of technology in teaching mathematics. Examples include studies on how the introduction of a new tools such as Interactive Whiteboards (Abbott, 2016), Notebook computers (Larkin, 2011), or whole school plans (Razak et al., 2018), impacts on the community of students and teachers, their roles, and the academic outcomes. In a Queensland study conducted by Larkin (2011), the use of Notebook computers changed the teaching practices of Year 7 mathematics teachers to make the learning more student centred. However, taking into account the need for collaborative learning, variety of learning activities and limited resources of the school, having one Notebook computer to two students was found to be more effective. Professional development of teachers and financial costs associated with the required hardware and software were some of the predictable tensions noted in ICT whole school implementation in Malaysia (Razak et al., 2018). Trust between teachers, students, school leaders and technology systems, was a tension noted in a Victorian whole school Community of Practice inquiry (Abbott, 2016). Abbott (2016) found that when the students trusted the teachers, and the teachers trusted the school leaders and technology system, the implementation of new technologies and changed pedagogies occurred more readily.

2.1.3.3.2 CAS Calculators.

Any technology can be used in the teaching and learning of senior mathematics in Victoria, but only Computer Algebra System (CAS) calculators can be used in school-based assessments and some end of year examinations (VCAA, 2010). However, the two higher-level mathematics subjects of Mathematical Methods and Specialist Mathematics, continue to have both a technology and technology-free examination at the end of the year. CAS calculators incorporate spreadsheets, dynamic geometry, and graphing; can solve and calculate multiple solutions; and have statistical analysis tools (Zelkowski, 2011). Research surrounding such calculators includes a study by Leigh-Lancaster (2010) during the trial and early implementation of CAS calculators in Victoria. Findings showed that examination results were similar for both the technology and technology-free exams. Research into the use of graphics and CAS calculators in senior secondary mathematics found that calculator use did not compromise traditional 'by-hand' skills, and actually improved mathematics understanding, according to the Victorian study by Leigh-Lancaster (2010).

A number of studies found that students use CAS calculators for various reasons including carrying out procedures, checking answers after calculations were completed by hand, and to support deeper learning of mathematical concepts (Goos & Bennison, 2008; Özgün-Koca, 2010; Pierce & Bardini, 2015). In Turkey, Özgün-Koca (2010) followed 40 prospective teachers as they studied with CAS calculators, finding the majority initially thought the calculators were not helpful in learning algebra, but most changed their minds with experience. By the end of the study, most participants favoured the use of calculators for learning algebra, graphing and conceptual development (Özgün-Koca, 2010). Australian teachers and students were found to seldom use the CAS calculators other than for checking answers and graphing (Pierce & Bardini, 2015). In another Australian study by Pierce and Stacey (2013), students were found to use CAS calculators more often than teachers, but also tended to follow the teacher's example in the method of use. One limiting factor of calculators is the time and skills required to learn how to effectively use them, especially the alternative algebraic syntax (Pierce & Bardini, 2015).

CAS calculators also support equipment to enable the gathering and sharing of data. For example, the Navigator system can allow teachers to share the calculator screens of the whole class, or selected students' calculators on a classroom projector, for the purposes of demonstration and discussion (Pape & Prosser, 2018; Zelkowski, 2011). This system has the limitation of expense, and the time required to learn the system. The Navigator system supports student learning of both calculator use and mathematics. Students are supported by sharing the demonstration of the calculation processes, thereby increasing student agency as reported in a recent USA study of community college teachers (Pape & Prosser, 2018). Engagement and feedback also increased according to another USA study with pre-service teachers conducted by (Zelkowski, 2011). In Australia, Barkatsas et al. (2016) similarly found CAS calculators increased engagement, confidence and performance in Year 7-12 students. CAS calculators, combined with professional development in their use, also supported a decrease in teacher instruction and an increase in student autonomy

(Barkatsas et al., 2016). Teacher access to professional development can be a limiting factor in their development of TPACK, which is knowledge of how calculators can improve the teaching of mathematics (Goos & Bennison, 2008; Zelkowski, 2011).

2.1.3.3.3 Videos.

Videos that support the learning of senior secondary mathematics include freely available videos, videos from specialist websites, and videos created by teachers themselves (Muir, 2018). Online videos can be used to demonstrate mathematical processes and relate the mathematical concepts to real-life. For example, Khan Academy (Khan, 2019) and Wootube (Woo, 2019) are popular free websites with thousands of videos, animations, worksheets and quizzes to teach mathematical concepts. Online video series have been used both in and out of class time. Cargile and Harkness (2015) interviewed Year 8 and 9 students in the USA, where some students watched mathematics videos in class time, at their own pace, with individual practice making up the rest of the class time and homework. Other students watched the videos at home, which encouraged passive learning, and completed individual practice sets in class time. Zengin (2017) investigated calculus college students in Turkey on their use of Khan Academy online videos and *GeoGebra*, a free dynamic graphing software. Students reported the two programs used together increased their understanding, motivation and enjoyment, and helped move away from memorisation, hence increasing the level of thinking. In New England, over a thousand students used Khan Academy during their developmental mathematics courses at college in various ways, with 77% of students reported improved confidence in mathematics (Chan et al., 2016). Khan Academy (2019) also testified that teachers need time and support in using the program, which could be personalised, and suggested the application should be used in conjunction with good teaching and quality resources.

Criticism of the use of the video series in learning mathematics included the need to learn how the system works, the lack of professional development for teachers in the implementation of the programs, and the passive learning model used (Cargile & Harkness, 2015; Kelly, 2018). Students found the Khan Academy online program supported teachers and textbooks teaching; however, it did not replace them as their primary learning supports (Cargile & Harkness, 2015).

Another strategy that is sometimes utilised to incorporate broader technology use is flipped classrooms, where new mathematical concepts are explained in videos or written material that students watch or read before class, freeing class time for individual and collaborative practice, inquiry or real-life investigations (Muir & Geiger, 2016; Stillman, 2017). Muir and Geiger (2016) investigated the use of a flipped classroom in their mixed-methods study of Year 10 students and teachers in a large secondary school in Tasmania, enabling a differentiated learning experience for the students. Some positive outcomes of flipped classrooms include increased student engagement and motivation (Attard & Holmes, 2019; Clark, 2015; Muir & Geiger, 2016), improved use of class time, positive relationships between students and teachers, and increased student agency (Muir, 2018). Videos for flipped classrooms have been found to be more effective when class time is used for hands-on learning and when teachers produce their own videos, rather than use videos produced by others, as this supports the relationship between teachers and students (Clark, 2015; Muir & Geiger, 2016). Stillman (2017), in her study of Queensland senior secondary teachers, highlighted the need to use the flipped classroom to broaden and deepen the mathematics content covered in class time, and increase critical thinking through mathematical modelling, rather than just move through the curriculum more quickly.

2.1.3.3.4 Technology and Probability.

Several studies in Australia and internationally have supported the use of technology in mathematics as assisting teachers with incorporating higher levels of creativity within their lessons; including more problem-solving, modelling and focusing on higher order levels of Bloom's Taxonomy problems; and interpreting of answers (Gürbüz et al., 2018; López-Martín et al., 2016; Porter et al., 2011; Tan & Tan, 2014). The use of calculators for teaching the topic of probability in Malaysia (Tan & Tan, 2014) were shown to be beneficial for students of high, average and low achievement levels, particularly with the learning of probability distributions. Graphics calculators were used by Zimmermann and Jones (2002) in the USA, where students who were in Advanced Placement classes (college classes taken in high school) could use calculators to model and simulate two step probability situations, however few students could use the calculators for the higher order thinking tasks of evaluating or constructing a simulation for a real-world situation. Computers also were found to assist instruction for the teaching of probability in secondary schools in Turkey (Gürbüz et al., 2018), by directly confronting misconceptions with simulated experiments and games. The explicit teaching of links between experimental, theoretical and intuitive probability were influential in improving students' probabilistic thinking. Porter et al. (2011) similarly found graphics calculators not only enhanced the pre-university students' probabilistic thinking, but also improved their attitude towards probability and decreased misconceptions. Technology used for the learning of probability supported discussion between students, increased engagement and reduced tedious calculations (Gürbüz et al., 2018; Tan & Tan, 2014).

The computer program *Fathom* was used in the USA (Conant, 2013), in targeted tutorials with university students to improve students' probabilistic reasoning, which was demonstrated in pre and post-tests, although it did not affect their examination results compared to a control group.

In contrast, an Australian study by Reaburn (2011) found that computers did improve university students' probability reasoning but writing about their work and carefully chosen sample tasks were the most influential factor. A statistics and probability simulator that has been found to be easy to use for instruction is *TinkerPlots* (Konold & Miller, 2011). Watson and colleagues have illustrated how *Tinkerplots* can be used to develop secondary students' probabilistic reasoning, from dice simulators to demonstrating informal inference (Watson & Chance, 2012; Watson & English, 2015a; Watson & Fitzallen, 2016). *Tinkerplots* has also been used in primary schools to assist with probability concept development (Bakker et al., 2006; Fitzallen, 2012; Watson & Fitzallen, 2016). Context is important to support understanding in mathematics, as found by the PISA international assessment (OECD, 2019), so authentic data from the internet can provide real world context. For example, the birthdays of friends on Facebook, *CensusAtSchools* data, statistics from games like Minecraft, have all been used to teach probability concepts, to engage and develop reasoning (Braham & Ben-Zvi, 2019; Ernst & Morton, 2020; Mills, 2016; Watson & English, 2015b).

2.1.3.3.5 Technology and Thinking.

Technology can increase the complexity of mathematics attempted by relieving the necessity for conducting mundane calculations and solving algebraic equations (Stillman & Brown, 2011; Zelkowski, 2011), and allowing time for investigations of real-life problems. Stillman and Brown's (2011) study of twenty-three Queensland teachers on their technology use found technology could move the focus from rote memorisation to visualising conceptual understanding, modelling, relating to real-world contexts and increase thinking and reasoning. This was also found in a study of three Irish secondary schools which were involved in Realistic Mathematics Education, where technology was used to relate mathematics to real-life situations (Bray & Tangney, 2016) in an all-girls school in a low socio-economic area. This study described mathematical enquiries around bungy jumping, catapults, and probability and used simulations, spreadsheets, and graphing programs. Mathematical confidence, conceptual understanding, behavioural engagement, the students' sense of meaningfulness, all increased, with students acknowledging that their outlook on mathematics had become more positive and relevant after the open-ended technology-based inquiry lessons (Bray & Tangney, 2016).

In contrast, in New Zealand, Nicholas and Fletcher (2017) investigated a variety of six disadvantaged schools through interviewing principals, teachers and observing classes. Technology was predominantly used for whole-class activities, quizzes, and videos, rather than the higher order thinking the principals had envisaged. Teachers felt the disadvantaged schools had adequate hardware, iPads, and computers, but they were lacking in professional development, technical support, and reliable internet connections. Technology tasks were not set as homework as not all students had access to the internet (Nicholas & Fletcher, 2017). Technology can also be confined to quizzes, constructing graphs and "number crunching" (Stillman & Brown, 2011, p. 715), thus a pedagogical change is needed on the part of teachers in order to be able to increase the quality of technology use in mathematics education (Muir & Geiger, 2016; Stillman & Brown, 2011).

In summary, the literature reviewed around technology use in mathematics curriculum implementation indicated that higher order thinking could be developed through use of technology, but this outcome could not be taken for granted. The use of technology in mathematics education can lead to increased engagement, effective use of class time, and can cater to a range of students. This potential for the use of technology to improve learning, particularly in relation to probability, suggests an opportunity for further research.

2.1.3.4 Student and Teacher Agency within the Implemented Curriculum.

While the use of textbooks and technology mediates and supports curriculum implementation, so does the role of teachers and students and the division of labour within the classroom. Teacher and student agency, as well as influences from parents and peers, can simultaneously scaffold student learning and create tensions. Agency relates to the ability to act independently and make informed choices (Bandura, 2006) and affects engagement with, and learning of, content. Student agency includes the characteristics of self-organisation, self-regulation, self-reflection and being proactive (Bandura, 2006), all desirable characteristics of senior secondary students. The responsibility for learning in senior secondary schools is ideally shared by both students and teachers.

To best support their agency, Reeve (2009) argues that "Students function more positively when teachers support their autonomy rather than control and pressure them toward a specific way of thinking, feeling, or behaving" (Reeve, 2009, p. 163). Students with agency and autonomy, who feel they influence their own learning, have improved engagement and outcomes (Assor, 2012; Boggiano et al., 1993). For example, in a study conducted by Jang et al. (2016) inquiring into the preferred ways of learning of students in first year at a Korean university, teachers taught the same content in several ways to different groups of students. When the material was taught via the student's preferred method, the students perceived they had enhanced autonomy, reported increased engagement, considered the teacher to be of higher expertise, and demonstrated higher conceptual understanding (Jang et al., 2016). Thus, by providing choices and explanatory rationales using informative language in learning activities, welcoming and respecting students' diverse opinions, and displaying patience and minimising controls (Assor, 2012), teachers can encourage and support student agency.

While self-motivated students might be valued by teachers, methods of controlling motivation are still used for several reasons including: assumed interpersonal power differential; burdens of responsibility and accountability; desire for structure; lack of time; assessment requirements; desire for particular attitudes of students; and teacher's personal dispositions (Liu et al., 2019; Reeve, 2009). However, Biesta et al. (2015) argue that teacher agency, including how teachers implement the curriculum in their classes, is an area which needs more research. There is ongoing debate as to the appropriate balance between evidence-based, data-driven curriculum delivery and adaptive implementation based on teacher professional judgement (Biesta et al., 2015).

In Victoria, teachers have little influence on the senior secondary mathematics intended curriculum, with the content and final assessments controlled by the VCAA (2010). However, teachers do have autonomy over pedagogy and curriculum implementation. The balance between teachers as curriculum makers and deliverers, and their implementation of the curriculum, varies in terms of culture, resources, schedules, teachers' personal histories and goals (Priestley & Philippou, 2018). While Priestley et al. (2015) discuss issues relating to "teacher proofing" (p. 188) the curriculum, others consider curriculum alignments or implementation gaps (Biesta et al., 2015; Priestley et al., 2015). Ways to support implementation of the curriculum which encouraged teacher agency found in the literature includes dialogue (Wallen & Tormey, 2019), professional learning communities (Philpott & Oates, 2017), professional development (Calvert, 2016) and involvement in curriculum design (Severance et al., 2016).

Professional development and professional learning communities can support teachers' changing roles from 'teachers as knowers' to 'teachers as learners', which alters the agency, roles and responsibilities of teachers and students (Bobis et al., 2019; Philpott & Oates, 2017). Bobis et al. (2019) conducted a study with thirty-nine Australian mathematics teachers from Years 5 to 7 who participated in professional development on engagement and motivation in mathematics over one year. The teachers read educational theory, support materials, held peer discussions, planned team action research projects and trialled a variety of learning tasks with their students and as a result, reported shifts in their own identities, describing themselves as facilitators, learners and co-creators of knowledge. The changing identity of teachers improved student engagement and autonomy for learning mathematics, but importantly for the current research, the teachers reported increased student thinking. Following the research intervention, teachers expanded the nature of tasks to include real-life contexts, open-ended tasks, collaborative tasks and tasks which required reflection (Bobis et al., 2019). A Scottish learning community experience with *Learning Rounds* (Philpott & Oates, 2017), had similar results to Bobis et al. (2019), also finding teachers needed more information on academic learning theory, and to be risk-takers in order to exercise agency. Similarly, a study involving two hundred secondary mathematics and science teachers in Singapore (Liu et al., 2019) found that personal motivation of teachers increased when students demonstrated self-motivation, with teachers then supporting student autonomy. Generally, the Singaporean education system allows for a high degree of autonomy, leading to teachers supporting strong autonomy orientations in their students, but pressure from local school authorities was found to increase the likelihood of a more controlling attitude to students being displayed by teachers (Liu et al., 2019).

Issues around teacher and student agency, and the influence on student outcomes are complex. While teachers need autonomy to cater for their students, they also need support in interpreting the curriculum, and implementing it in a way to realise the objectives. Senior secondary students also benefit from control of their own learning, but also require support and guidance. This study aims to investigate these issues.

This concludes the review of literature focussing on curriculum implementation, particularly in mathematics and senior secondary situations. In summary, the literature suggests several points between the educational, social, and cultural desirability for encouraging higher order thinking. Whether or not this is achieved through curriculum implementation could be better understood through further research, which could include research into the design and use of textbooks and aids, including technology and reference books in the senior secondary years. Research around the attainment of the curriculum will be discussed next.

2.1.4 Attained Curriculum

In the TIMSS model of curriculum alignment, once the intended curriculum is implemented, the degree of attainment needs to be considered. Thought needs to be given to how students demonstrate their learning and how the curriculum is assessed. The examinations and School Assessed Coursework (SACs) used to assess VCE mathematics are considered, with a focus on the tools and rules which support or cause tensions within these assessments.

2.1.4.1 Rules of VCE Mathematics Assessments

The Australian Curriculum for senior secondary mathematics was implemented in all Australian states and territories in 2015–2016, to align the content and structure of the subjects across Australia (Stephens, 2014), although assessment continues to be administrated by the individual states and territories. Assessments for the Victorian Certificate of Education (VCE) mathematics subjects of Mathematical Methods and Specialist Mathematics have internal assessments including SACs and two external examinations. The details of the specific mathematics curriculum for this research project are outlined in Chapter 1, Section 1.2, while examinations and internal assessment in the wider research are now reviewed.

2.1.4.2 Research around Examinations and SACs as Attained Curriculum

Examinations are a common traditional assessment method which have been used in mathematics for many years. Their value has been constantly debated according to Husbands (1976), who conducted a literature review on the use of examinations in the final year of schooling. Summative assessment is however, currently an 'unfashionable topic for research' with the rise of interest in formative assessment (Iannone & Jones, 2017). Issues around examinations include the use of technology (Ball, 2014; Forgasz & Tan, 2010) and disadvantaging some student groups (Kyei et al., 2011). Another issue in the use of examinations is the predominance of procedural items which creates a negative view of mathematics teaching and learning (Iannone & Jones, 2017). In Germany, where only some states have mathematics examinations, Jürges et al. (2012) found that centrally assigned examinations led to improved learning of the mathematical content but decreased intrinsic motivation and there was no increase in mathematical literacy as assessed by PISA (OECD, 2003). German secondary students from states with central examinations demonstrated more negative emotions, including anger, anxiety and despair (Jürges et al., 2012). A similar result was found in Peru where mathematics examinations also caused mathematics anxiety in pre-university students, resulting in a negative effect on academic performance (Iraola-Real & Gonzales-Macavilca, 2020).

SACs involving problem-solving and modelling application tasks provide another way to assess student learning. As technology becomes cheaper and more accessible its use in classrooms increases, which makes using real data feasible instead of simplified, clean data (Stillman & Brown, 2011). Technology can also reduce the tedium associated with performing calculations. A framework to identify the issues in modelling and support the teaching of mathematical modelling and problem-solving, was suggested by Stillman et al. (2007) to scaffold the tasks as the teachers set up real life tasks with key questions and structure. The increased engagement related to the students' lives, adding value to the tasks (Galbraith & Stillman, 2006). Recent research into problem-solving as a form of assessment in the USA demonstrated how it can increase engagement and career aspirations in STEM subjects for gifted students, however these tasks were not used as school summative assessments (Pease et al., 2020). A literature review on alternative assessment by Swedish researcher (Frejd, 2013), reviewed seventy assessment research articles and described the issues around practicalities of alternative assessment consistency and validity. Mathematical modelling was described as an under researched area in an international review (Schukajlow et al., 2018), although they found examples of research into modelling in primary schools and universities. Brown (2015) investigated a trial of a realistic modelling task with Year 11 students which utilised the graphic calculators in Victoria. The students could not visualise the transformations needed by the task, and the teachers lacked the understanding of how the calculators could be used to support students in their cognitive understanding of the modelling task (Brown, 2015). Mathematical problem-solving and modelling application tasks are under researched areas in senior secondary schools, especially their use as summative assessment.

Changes in senior secondary mathematics assessments to include internal assessments, in addition to end of year examinations, had a trickledown effect on Years 7–10 (Clarke et al., 1993). In 1990 in Victoria, the curriculum assessment changed to include an internally assessed challenging problem, to be worked on over 10 hours, and a 20-hour investigation project, with the vision of providing students with the opportunity to engage in mathematical thinking in a variety of forms (Clarke et al., 1993). The changes in senior assessment types affected Years 7–10 through changes in middle school assessment, reporting, terminology, and changes to the classroom teaching activities, including the explicit teaching of problem-solving and project report writing skills. This was followed up by a comparison of curriculum in Victoria and NSW (Barnes et al., 2000). Both states had undergone changes to their curriculum and included internal assessment, with Victoria implementing the externally set challenging problems and investigations. In NSW the teachers designed the internal assessments themselves, heavily relying on examination style problems (Barnes et al., 2000). This demonstrates the focus on examinations in Year 12, and the difficulty of implementing alternative assessments.

Curriculum and assessment need to be changed together for any reform to be effective. The Netherlands changed the curriculum in some pilot schools in 2011, while keeping the examinations the same (Drijvers et al., 2019). The students in the schools which were trialling the higher thinking curriculum increased their results in the common exams by 4–5% and demonstrated more diverse problem-solving skills. However, the problem-solving and thinking had decreased in the examination between 2011–2017, as a result of conservative pressure from teachers and personal changes on the examination panel (Drijvers et al., 2019).

2.1.4.3 Summary of Attained Curriculum.

The assessment of the VCE senior secondary mathematics subjects has been outlined in terms of how the attained curriculum can be measured. The research literature around the assessment techniques of examinations and problem-solving, modelling and application tasks was also presented. While examinations are common assessment tasks in the senior secondary years, they have the disadvantage of causing stress which can disadvantage students. Examinations can be designed to assess higher order thinking, but many factors influence this. Any changes to curriculum and assessments need to be designed and implemented at the same time to maximise efficacy, and it is important to acknowledge that changes to senior subjects can influence junior subjects.

Problem-solving and modelling tasks can be used as assessment tasks, especially with the support of technology, but they come with their own implementation issues, including inexperience on the part of teachers. Curriculum alignment will now be considered.

2.1.5 Research on Curriculum Alignment

This section describes the research on curriculum alignment focusing on mathematics education. Curriculum alignment has been a recent focus of research worldwide as many countries aim for a national curriculum, for example the Australian Curriculum (Atweh, Goos, et al., 2012), the National Curriculum for England (2014), and the Common Core Curriculum in the USA (Porter et al., 2011). This builds on the long-standing international mathematics curriculum research by TIMSS and PISA. Curriculum alignment thus also includes alignment between geographical areas. Research focuses on content alignment, authorship, interpretation, local contexts, assessment, use of instructional materials including textbooks, alignment, levels of thinking, cohesiveness, national testing, educational leadership, equity, professional development and pedagogy (Anderson et al., 2012; Way et al., 2016).

Within the intended curriculum, the various components aim for "*internal cohesiveness*, or the synergy between its stated aims and rationale, on the one hand, and the content and its articulation on the other" (Atweh, Miller, et al., 2012, p. 11). Atweh, Miller, et al. (2012) concluded the intended Australian Curriculum was not internally cohesive as the aims and goals of general capacities and cross curriculum priorities were not included in the curriculum statements. Additionally, the mathematics curriculum statements were lacking in diverse proficiency skills. According to Atweh, Miller, et al. (2012), the four proficiencies of Fluency, Understanding, Problem-solving and Reasoning were unevenly represented. The Year 8 curriculum statements in the Australian Mathematics Curriculum included 53% Understanding, 56% Fluency, 12% Problem-solving and only 7% Reasoning. Some of the curriculum statements were classified into more than one category. Of the problem-solving statements, almost half were in the statistic and probability strand, which indicate these topics are helpful for demonstrating this proficiency.

Alignment between the implemented and the attained curriculum is called *Constructive alignment* (Biggs, 2003). Constructive alignment is where the learning objectives, learning activities and assessments are aligned, and explicitly explained to students (Biggs, 2003, 2012; Hattie, 2012; Hattie et al., 2017). Clarifying the desired learning outcomes is consistent with learning intentions and success criteria as suggested by Hattie et al. (2017), which have been endorsed by the Victorian Department of Education and Training in their policy document *High Impact Teaching Strategies* (DET, 2020a). Biggs (2012) and Hattie et al. (2017) stress the importance of defining intended learning outcomes and matching the learning activities and assessments to these intentions. The content and the performance attributes should aim to be clear to all involved, both teachers and students.

2.1.6 Summary of Literature on Curriculum

The literature regarding the intended, implemented and attained mathematics curriculum and curriculum alignment has been described. Influencing factors included the supporting tools of textbooks, technology, calculators, reference books and assessments including examinations. This literature review highlights that use of curriculum can support teachers and have a positive influence on the mathematical learning of students. The intended curriculum aims to incorporate a variety of thinking skills, but the literature reports that implementation by teachers and via textbooks can skew the implemented curriculum towards lower order thinking or teaching focused on the use of rote learning and memory (Hollingsworth et al., 2003; Son & Kim, 2015). Assessment of the curriculum is a measure of the attained curriculum. Issues around assessment, for example stress associated with examinations, can bias the outcomes. The literature into the mathematics curriculum has been described as the intended, implemented and attained curriculum, investigating the tools, rules and community which support and influence student development of mathematical and thinking skills. The mathematical topic of probability will be specifically covered next.

2.2 Probability Research

This section reviews probability education literature, with a focus on the senior secondary years of schooling. The issues specific to the teaching and learning of this topic within mathematics will be analysed to identify gaps or opportunities for the current research project, with the aim of focusing attention on probability as a vital branch of mathematics. Probability was defined by de Moirve (1756) in the *Doctrine of Chances* as a fraction of the number of favourable cases divided by the number of all possible cases, and this continues as the *classical* definition of probability (Batanero, 2014) today. Probability can be viewed as a sub-section of statistics, especially by curriculum designers (Biehler, 1994; Franklin et al., 2007). Statistics describe current data objectively, while probability predicts what might happen next, including the level of uncertainty. It has both practical and theoretical applications that are particularly relevant to the 21st century uncertainties about climate change and other global issues. It can help explain past events and predict future consequences in a variety of disciplines including Education, Business, Insurance, Sports Science, Health and Research. In their seminal work in the probability literature, Fischbein and Schnarch (1997) argue that:

> Probability does not consist of mere technical information and procedures leading to solutions. Rather, it requires a way of thinking that is genuinely different from that required by most school mathematics. In learning probability, students must create new intuitions. Instruction can lead students to actively experience the conflicts between their primary intuitive schemata and the particular types of reasoning specific to stochastic situations. If students can learn to analyse the causes of these conflicts and mistakes, they may be able to over-come them and attain a genuine probabilistic way of thinking. (p. 104)

Probability can be thought of in three ways, in terms of a *classical, frequentist and subjective* perspective (Batanero, 2014; Sharma, 2015). *Classical probability,* also known as theoretical probability, involves examining the number of desirable outcomes divided by the possible outcomes with *impossible* being 0, and *certain* being 1. Many of the difficulties experienced by students in probability are due to difficulties associated with understandings of fractions, decimals or percentages rather than the probability concepts themselves (Fischbein & Gazit, 1984; Tversky & Kahneman, 1981; Watson & Callingham, 2015). *Frequentist probability,* also known as experimental probability, is related to experiments or simulations, where you do not obtain the same results from one trial to the next. Probability is inconsistent with all other branches of mathematics, where the solution to problems is constant, both practically and theoretically. The Law of Large Numbers (Batanero, 2014; de Moirve, 1756) shows that the frequentist value will tend to the classical value of probability over time. For example, the number of heads from a coin toss tend to 50% over many throws, but each toss is unpredictable. *Subjective probability* relates to a perception that achieving a particular outcome depends on belief, luck and personal experience (Chernoff & Sriraman, 2014). Young children will be convinced that red is lucky for them, while perfectly logical, educated people buy lotto tickets based on their children's birth dates. Some individuals even believe that probability is dependent on a divine being or that luck is an inherent quality (Batanero et al., 2005).

Schools attempt to move students from having a subjective probability view to understanding probability in terms of experimental and classical probability views. Probability can be a confusing concept, so comparing and contrasting the three perspectives can lead to a greater understanding of the concept of probability. For example, USA researcher Ernst (2011), used a loaded dice for teaching his college students. By physically rolling the dice, the difference between subjective, frequentist, classical probability and the concept of randomness were investigated while gaining the students' interest through turning the learning activity into a game. In another example in Turkey, the use of games and simulations when learning probability, improved the test results of prospective teachers (Koparan, 2021).

Probability also involves a form of literacy and reasoning of its own, which adds to the complexity and confusion for students. Garfield et al. (2010) distinguished among statistical literacy, reasoning and thinking, arguing that statistical *literacy* involves tasks associated with describing, reading, computing and identifying, while statistical *reasoning* involves why and how questions, and statistical *thinking* is used in relation to critiquing, evaluating and generalising for statistical purposes. They compared literacy, reasoning and thinking to Bloom's revised taxonomy (Krathwohl & Anderson, 2002) and argued statistical *literacy* is consistent with the first levels of *knowing*, *reasoning* with *comprehending* while statistical thinking encompasses the higher order skills required for *application*, *analysis* and *synthesis* (Garfield et al. (2010). In tertiary courses, statistical and probabilistic literacy, reasoning and thinking have been assessed by the USA Assessment Resource Tools for Improving Statistical Thinking (ARTIST) online tests (Garfield et al., 2010). This team of researchers, who worked mainly in statistics, felt that "traditional approaches to teaching statistics focus on skills, procedures, and computations, which do not lead students to reason or think statistically" (Garfield et al., 2010, p. 75). The popularity and results of the ARTIST test in many universities demonstrate there is a need for this type of assessment to support the aims of the university courses. Importantly for the current project, Jolliffe (2005) proposed that statistical literacy, reasoning and thinking could be adapted to probability teaching and learning, which becomes the focus on the next section.

2.2.1 Teaching and Learning Probability

The formal research into the learning of probability began with the Swiss psychologists Piaget and Inhelder (1975). They observed children perform chance activities and proposed that children go through stages of development from preoperational to concrete (approximately 7 - 11 years old), to the formal stage (roughly over 11 years old). Not all adults however progress to the formal stage. In this example of how Piaget's stages describe a child's development in understanding probability concepts, the activity involves drawing coloured balls from a jar containing three yellow balls, two blue balls and one red ball:

- *Preoperational* - a child will predict a red ball will be chosen next as its red's turn.
- *Concrete* - a child will predict a yellow ball will be drawn next as there are more of them.
- *Formal stage* - a child will explain the expected result using fractions or ratios to reflect the population of the various colours of balls.

Piaget's stages of development were further investigated and supported by the English researcher (Green, 1982), and in Israel by Fischbein and colleagues (Fischbein & Gazit, 1984; Fischbein &

Schnarch, 1997), who reinforced the developmental stages. They progressed to describe factors which enhanced children's understanding of chance and to classify student misconceptions.

In the last forty years considerable further research has been conducted on children and adults' thinking and learning in relation to probability. A wide range of methods have been used to evaluate students' probabilistic thinking and to gauge the effectiveness of various teaching methods and activities. Table 2.1 outlines a sample of the research reviewed and demonstrates the diversity of research studies involving probability learning both in Australia and internationally.

Table 2.1

A Sample of Research in Probability Reasoning and Teaching

Author	Country	N	Level	Key concepts investigated
Tversky & Kahneman (1981)	Canada	150	Tertiary	Decision making
Green (1982)	England	2930	Secondary	Development with age
Fischbein & Gazit (1984)	Israel	600	Middle school	Teaching methods
Watson et al. (1997)	Aus	1014	Primary/Secondary	Development with age
DelMas et al. (1999)	USA	149	Tertiary	Technology
Zimmermann & Jones (2002)	USA	9	Secondary	Technology
Watson et al. (2006)	Aus	5514	Primary/Secondary	Curriculum
Delmas et al. (2007)	USA	1470	Tertiary	Assessment of thinking
Batanero et al. (2010)	Spain	166	Teachers	Teaching methods
Even & Kvatinsky (2010)	Israel	120	Secondary	Teaching methods
Gürbüz (2010)	Turkey	80	Middle school	Teaching methods
Konold et al. (2011)	NZ	28	Secondary	Reasoning with Technology
Conant (2013)	USA	184	Tertiary	Technology
Reaburn (2013)	Aus	75	Tertiary	Technology
Watson & Callingham (2015)	Aus	110	Secondary	Teaching methods
Pfannkuch et al. (2016)	NZ	7	Adults	Prob. modelling careers
Maher (2019)	Aus	38	Senior Secondary	Teaching methods
Peters & Stokes-Levine (2019)	USA	19	Teachers	Content knowledge
Pratt et al. (2019)	UK	7157+	Adults	Online curriculum

The research listed in Table 2.1 covers all age groups and many foci, including the use of technology, teaching methods, content knowledge, and the use of probability in the development of thinking and reasoning. Some inconsistency in findings is evident. For example, research by Reaburn (2013) illustrated that statistical and probability modelling with technology supports student thinking, while Zimmermann and Jones (2002) found that misconceptions remain, even with technological modelling. Overall, the language and wording of probability problems and associated student difficulties with proportion reasoning (fractions, decimals, and percentages) are regularly identified as issues with the teaching and learning of probability concepts (Fischbein & Gazit, 1984;

Tversky & Kahneman, 1981; Watson & Callingham, 2015). Researchers working in Tasmania have established a large body of research on the development of probability skills over Years 3–11, as the curriculum changed in 1993–2003 (for example, Watson et al., 1997; Watson et al., 2006).

In relation to intended probability curriculum, within the probability strand of mathematics, many curricula recommendations in the literature highlight the value of students engaging with probability simulations and investigations (Franklin et al., 2007; Jones et al., 2007), this includes the Australian Curriculum (ACARA, 2010). Probability simulations and investigations are encouraged to complement the theories and procedures, to increase the development of student thinking skills, and applications to real life contexts. Investigations, discussions and projects were recommended for students of all ages when teaching probability, as opposed to memorisation of rules and procedure following (López-Martín et al., 2016). This dichotomy is in line with the wider *Math Wars* beginning in the 1990s when a conflict emerged between active problem-solving, communicating, reasoning style of teaching and learning mathematics, and the previous emphasis on memorisation and practice (Chernoff, 2019; Schoenfeld, 2004, 2016). Examples of the active approach include use of game-based learning. For example, teachers in primary schools have used computer-animated arcade games in Turkey (Gürbüz et al., 2018), culturally diverse games in the USA (McCoy et al., 2007), and card, dice or bingo games in Australia (Gürbüz et al., 2018; McCoy et al., 2007; Nisbet & Williams, 2009). In addition, university statistics courses in the USA have included projects, investigations, and simulations, with multiple representations. For example, a semester-long programming task of a dice game of Yahtzee (Wilson et al., 2011) and the card game of Poker (Wroughton & Nolan, 2012).

Explicitly linking experimental and classical probability and discussing counterintuitive probability problems can support students to develop sound conceptual understanding of the content. Examples of counterintuitive problems in the literature are the “Monty Hall” problem and the “Birthday problem” (Edwards, 2012; Guan, 2011; Klymchuk & Kachapova, 2012; Taylor & Stacey, 2014). The Monty Hall problem is based on a television game show where the chance of winning seems to depend on changing your mind, and the Birthday problem involves the chance of friends having the same birthday appearing to be surprisingly high. These intriguing counterintuitive problems have several ways to explain them and encourage deep thinking about the probability concepts. Prodromou (2016) conducted a study about conditional probability in New South Wales Year 12 classes and asked counterintuitive problems, which students solved using enquiry, discussion, and eventually by two ways tables and tree diagrams (Prodromou, 2016). The problem started with the question: If Mr Jones had two children, and at least one is a boy, what is the probability both children were boys? It was then modified to speculate on what happened to the probability if the boy was born on a Tuesday, born after midday, or born in Autumn. Census data was consulted to relate this problem to real-life. According to Prodromou (2016), the advantages of this teaching method were that all students were challenged, and all could succeed. The use of frequency tables influenced the students' reasoning and supported conceptual understanding.

The literature also suggests that the context for teaching mathematics, particularly probability, is important. For example, learning about current environmental issues such as recycling (Watson & English, 2015b) can involve students in collecting their own data to make learning meaningful. Another example concerns gambling awareness, where a unit of work in Victoria has been designed and trialled with Year 10 and 11 students using simulations of gambling to tackle this social issue (Lowe & Money, 2017; Money, 2015). This government sponsored resource for teachers uses role play, excel spreadsheets and research to link experimental and theoretical probability, and demonstrate the risks of gambling.

Teachers can present the topic of probability in many creative ways to support the learning of senior secondary students. Even within schools, and with the same textbooks and resources, teachers can implement the teaching of probability differently. For example, O'Keeffe and White (2017) found that more experienced teachers relied less on textbooks. Studies on professional development methods tended to focus on shifting teachers away from textbook learning to active-learning approaches. For example, in Brunei, a lesson study approach was used in senior secondary mathematics education, related to curriculum reform shifting to inquiry-based lessons (Chong et al., 2017). Lesson Study was trialled in senior secondary teaching of conditional probability, whereby teachers planned classes together, observed and modified lessons over several trials (Chong et al., 2017). Instead of 'chalk and talk', exploratory activities were integrated, including theoretical methods then experimental trials. Findings demonstrated that Lesson Study did support teachers in their planning, and student results in post-tests demonstrated the probability lessons became more effective (Chong et al., 2017). In another study in Israel, two senior secondary teachers at one school were compared as they taught probability (Even & Kvatinsky, 2010). Both classes used the same textbook, but one teacher focused on individual students repeating teacher-directed processes, while the other encouraged student discussion, collaborations, and solving problems in different ways. The second teacher also used experimental probability. The study found that the students of the innovative teacher were more likely to be able to apply their learning in new situations.

In an Australian study, pedagogy and content knowledge of Tasmanian senior secondary teachers regarding probability and calculus was examined. The emphasis on solving standard text-book problems was broken down to the intricacies of how the experienced teachers broke down their explanations and used examples and knowledge of the high stakes external examinations to support their students (Maher, 2019; Maher et al., 2016). In this study, the teachers demonstrated many skills in their teaching: knowledge of the examination, textbooks, and students, being able to deconstruct the mathematics, choice of examples and knowing when to go into detail and when to skip the proof, using the Mathematical Knowledge for Teaching framework (Ball et al., 2008).

In 2016, the VCE mathematics curriculum for Mathematical Methods was reaccredited with an increased focus on probability and statistics. *The Australian Curriculum: Senior Secondary Mathematics* (ACARA, 2012) was informed by two world-wide research-based sources, Batanero et al. (2011) and Franklin et al. (2007). The wide-ranging ideas of these two sources included a description of the teaching of statistics and probability in several countries, examples of pedagogy ideas, the role of thinking, assessment, technology, teacher beliefs, knowledge, and skills. The six recommendations from the *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) report from the USA (Franklin et al., 2007) were:

1. Emphasize statistical literacy and develop statistical thinking
2. Use real data
3. Stress conceptual understanding, rather than mere knowledge of procedures
4. Foster active learning in the classroom
5. Use technology for developing conceptual understanding and analysing data
6. Use assessments to improve and evaluate student learning. (p. 4)

The Victorian Curriculum modifications were underpinned by understandings arising from these reports and research findings. The VCE senior secondary mathematics key changes included an increased focus in content towards more probability and an increase in modelling and problem-solving, away from the test type internal assessments (VCAA, 2015a).

In summary, probability is a vital mathematical content area, which is increasing in importance as it relates to decision making in the modern world. The literature indicates the topic of probability can be used as a vehicle to develop a variety of thinking skills in students, however, probability knowledge is mixed with misconceptions and can be seen as counterintuitive. Importantly, the literature indicates that probability can support the teaching and learning, not only of statistical and probabilistic literacy, but of reasoning and thinking skills. Discussion on probabilistic thinking frameworks, especially the work of Jones and colleagues (for example Jones et al., 1997), will be discussed in Section 2.3.4 The following section explores the literature on thinking skills in mathematics education.

2.3 Thinking Skills in Mathematics Education

This section covers expectations in association with *thinking* and reasoning in school-based mathematics education. It also introduces how *thinking* is classified and analysed in the mathematics education research literature, specifically within the topic of probability, and closes with a description of the literature around curriculum alignment and how a variety of thinking skills are an integral component of this.

International (Anthony & Walshaw, 2009), Australian (Stanley, 2008) and Victorian (Stephens, 2009) reviews into mathematics education all recommend a focus on a variety of thinking skills and tasks. An assortment of thinking and reasoning skills are essential to encourage long term learning and support the linking of school mathematics to real-world mathematics (Peña-López, 2012), and could include creative, critical, cognitive, analytical, and mathematical thinking. Internationally, Anthony and Walshaw (2009) concluded that one of the guiding principles of the International Academy of Education is that mathematics pedagogy must be “focused on optimising a range of desirable academic outcomes that include conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning” (p. 6). Higher order thinking is important for 21st century skills and should be in the bulk of tasks in senior secondary classes (Collins, 2014). Theoretically, a number of classification systems have been used to categorise or classify thinking, including Bloom’s taxonomy (Collins, 2014; Krathwohl & Anderson, 2002) and the Structure of the Observed Learning Outcome (SOLO) framework (Biggs & Collis, 1982). These classification frameworks (and more) have been used as the basis for measuring thinking skills attainment, as well as international assessments such as the TIMSS (Mullis & Martin, 2015) and PISA (OECD, 2019).

In Victoria, the Victorian Curriculum aims to support the need for a variety of thinking skills in the Foundation to Year 10 (F–10) and senior mathematics curriculum, as well as the cross-curriculum capability of critical and creative thinking, as detailed in Chapter 1. Within the F–10 Mathematics learning area, the four proficiencies include *understanding, fluency, problem-solving* and *reasoning*. This demonstrates the curriculum is expected to develop a range of thinking, with reasoning described as “students developing an increasingly sophisticated capacity for logical, statistical and probabilistic thinking and actions, such as conjecturing, hypothesising, analysing, proving, evaluating, explaining, inferring, justifying, refuting, abstracting and generalising” (VCAA, 2015c, para. 5). The 2016–2022 VCE Mathematics Study Design (VCAA, 2010) describes mathematics as both a thinking framework and means of communication:

> Mathematics is the study of function and pattern in number, logic, space and structure, and of randomness, chance, variability and uncertainty in data and events. It is both a framework

> for thinking and a means of symbolic communication that is powerful, logical, concise and precise. (p. 6)

Throughout the VCE Mathematics Study Design, a range of thinking skills are inferred, with one of the three compulsory outcomes being the aim to apply, analyse and solve problems including open-ended and unfamiliar problems. These statements from all levels of the Victorian curriculum suggest that a range of thinking and reasoning skills are valued, and a review of a range of thinking frameworks follows.

2.3.1 Classifying Thinking

Moseley et al. (2005) described 42 thinking frameworks of various types. Of these Bloom's taxonomy is arguably the most recognised reference (Webb, 2014) and is used by many classroom teachers, researchers and curriculum planners to classify the cognitive demand of curriculum, learning tasks and assessments (Darlington, 2013; Webb, 2014). Another frequently utilised framework is the SOLO taxonomy (Pegg, 2014). These frameworks and their relevance to mathematics and particularly probability are discussed in this section, with an initial focus on critical and creative thinking as discussed in the literature.

Ennis (1985) defined critical thinking as "reflective and reasonable thinking that is focused on deciding what to believe or do" (p. 45). Critical thinking is higher order thinking, which links to the top three or more levels of Bloom's taxonomy (Darlington, 2013; Ennis, 1985), as outlined in the next section. In a mathematical context, critical thinking tasks should use a real-world context that is relevant to the student and the mathematical content, as well as open-ended tasks with multiple possible solutions that can be approached in several ways (Aizikovitsh-Udi et al., 2014). Examples could be around interpreting graphs about medications where no one answer is obvious, or decisions on which computer is best, with a variety of criteria (Kuntze et al., 2017). Finding mathematical solutions is not always enough, as finding solutions that are realistic and reasonable is also important (Su et al., 2016).

Creative thinking is characterised by the ability to formulate or develop hypotheses, and to plan and produce new ideas (Apino & Retnawati, 2016; Collins, 2014). Like critical thinking, creative thinking is also higher order thinking and links to the highest level of Bloom's taxonomy. In mathematics, creative thinking manifests itself in divergent thinking, lateral thinking, and convergent-integrative thinking (Hadar & Ruby, 2019). Hadar and Ruby (2019) analysed textbooks from Israel and found the younger the target age group, the more creative thinking was included. Creative thinking will be a criterion of the next PISA assessments to be held in 2021 (Lucas, 2019).

Critical and creative thinking improve achievement in mathematics (Lucas, 2019; Su et al., 2016). The best learning tasks to encourage critical and creative thinking are open-ended and non-routine tasks (Apino & Retnawati, 2016; Hadar & Ruby, 2019; Sullivan et al., 2005; White et al., 2016). Creative thinking can also be encouraged by linking different areas of mathematics together and with other areas of the curriculum (Hadar & Ruby, 2019). Standard mathematical tasks could be adapted to critical and creative thinking tasks by teachers using questioning techniques and encouraging discussions about validity, reasonableness, and methods, and the topic of probability is an ideal context for this (Aizikovitsh-Udi & Amit, 2011). Including critical and creative thinking in the teaching of mathematics can increase higher order thinking, which is a focus of this study. Probability is an ideal topic to support this concept, where non-routine tasks, links to real-world contexts, and thinking about the reasonability of the solutions are also important.

2.3.2 Bloom's Taxonomy

The original Bloom's taxonomy aimed to develop language and communication between educators about cognitive demand (Bloom, 1956; Webb, 2014) and could be used to increase the range of the cognitive demand of tasks. The original framework included a hierarchy of six levels of cognitive domains beginning with *knowledge, comprehension, application, analysis, synthesis,* and *evaluation* (Bloom, 1956; Webb, 2014). Bloom's taxonomy was revised in 2001 (Krathwohl & Anderson, 2002) which changed the focus of the language of the taxonomy from nouns to verbs, altered wording and added a level to reflect the emphasis on innovation and creativity highlighted in the field of educational psychology. The new categories were thus: *remember, understand, apply, analyse, evaluate* and *create* (Krathwohl, 2002). The first three levels represented lower order thinking processes and the final three represented higher order thinking processes.

In the literature, Bloom's taxonomy (revised) has been used to design teaching strategies (Duron et al., 2006; Sutherland, 2004); evaluate curriculum (Madhuri et al., 2012; Näsström, 2009); evaluate examination items (Darlington, 2013; Fuaad et al.); investigate teacher interpretations of higher order thinking (Thompson, 2008); identify student's cognitive level of thinking (Azizan & Ibrahim, 2012); and for students to self-evaluate their thinking (Aline et al., 2013). Two categories of higher order thinking (HOT) and lower order thinking (LOT) can be used (Thompson, 2008). Son and Kim (2015) also used just two simplified levels of Bloom's revised taxonomy to classify the expected cognitive demand of textbook problems. Criticism of both versions of Bloom's taxonomy within mathematics education includes the difficulty of aligning mathematical problems with just one category. Equally, because Bloom's levels are not hierarchical, the complexity of a question might not be taken into account (Darlington, 2013; Webb, 2014). Webb (2014) suggested Bloom's taxonomy is more useful in classifying tasks, rather than student responses to the tasks, because problems can be completed in different ways as skills develop.

Overall, Bloom's taxonomy is the most recognised (Webb, 2014), long-standing framework used to describe thinking in curriculum, learning tasks and assessment in the mathematics education context (Darlington, 2013). However, Hattie (2012), in his meta-analysis of research studies on student achievement, proposed both Bloom's revised taxonomy and the SOLO framework (Biggs & Collis, 1982) as useful measures of depth of understanding. The SOLO framework is described next.

2.3.3 Structure of the Observed Learning Outcome—SOLO Framework

The Structure of Observed Learning Outcome (SOLO) framework (Biggs & Collis, 1991) cycles through the following level classifications as each new concept is learnt:

1. Prestructural. The task is engaged, but the learner is distracted or misled by an irrelevant aspect belonging to a previous stage or mode.
2. Unistructural. The learner focuses on the relevant domain and picks up one aspect to work with.
3. Multistructural responses. The learner picks up more relevant or correct features but does not integrate them.
4. Relational. The learner now integrates the parts with each other, so that the whole has a coherent structure and meaning.
5. Extended abstract. The learner now generalises the structure to take in new and more abstract features. (Biggs & Collis, 1991, p. 65)

According to Hook et al. (2014), this framework has advantages in mathematics education over Bloom's taxonomy because of ease of use, consistency in internal reliability and for use in a differentiated classroom. By using the SOLO framework, one mathematical problem can encourage answers from several levels which is a sign of differentiated teaching (Hook et al., 2014). According to Hattie and collaborators (Hattie et al., 1996; Hattie et al., 2017), SOLO is particularly useful in mathematics where one way to demonstrate a deep understanding of a concept is using several different methods and being able to link mathematical concepts together. Thus, teachers can use the SOLO framework to plan their lessons and identify how flexible or limiting many problems are. Equally, Pegg and Tall (2005) argue that while Bloom's taxonomy classifies the thinking inherent in learning or assessment tasks, the SOLO framework describes the thinking evident in student work as demonstrated by their responses, making it useful for gaining insight into the degree of student understanding (Goos et al., 2007).

The SOLO framework was used to classify responses of a class of Tasmanian Year 5–6 students in a probability learning activity (Ireland & Watson, 2009), with students demonstrating a wide range of understanding and thinking but also difficulties that were consistent with other research into probability misconceptions (Fischbein & Schnarch, 1997). Students had difficulties with the language surrounding the concepts, and also linking their knowledge of fractions, decimals and percentages with the probability concepts, which was reflected in the SOLO scale. The linking of a variety of mathematical concepts is an indicator of higher order thinking in the SOLO scale, which is not as obvious in Bloom's taxonomy. The complexities of students' thinking as seen in their responses can be described by the SOLO framework. For example, when Year 10 students critiqued statistical investigations (Arnold & Pfannkuch, 2018), the number and variety of reasons for classifying the investigation questions were used within the SOLO framework to measure the students' growth in statistical understanding.

The literature indicates the SOLO framework has been used in a wide variety of situations and for different aged students in mathematics. For example, it has been used to design and assess undergraduate university courses (Boulton-Lewis, 1995); assess pre-service teacher's statistical knowledge (Groth & Bergner, 2006); assess teacher's mathematical knowledge of applied numeracy questions (Forgasz & Ledfer, 2016); examine primary school students' understanding of probability (Lidster et al., 1996); and explore primary students' thinking around the use of technology and probability (Ireland & Watson, 2009).

The SOLO framework has been used to describe probabilistic thinking in the USA by Jones (2006), and in Australia by researchers led by Watson (for example, Watson et al., 1997; Watson & Kelly, 2007; Watson & Moritz, 2003). Mooney et al. (2014) also described the framework as useful in defining mathematical skill development. SOLO can be used to describe the way student thinking moves from concrete to abstract, and the way mathematical concepts can be combined. For example, in probability, mathematical experiments can lead to simulations using technology, which can be modelled by functions and graphs. Higher order thinking can be expressed using the SOLO framework (Hattie, 2012) to analyse the curriculum, lesson plans and student thinking while making the thinking visible by focusing on student responses (Pegg & Tall, 2005).

2.3.4 Probabilistic Frameworks

Frameworks for describing and classifying probability in particular have also been designed. For instance, Jones et al. (1997) built on the work of Piaget and Inhelder (1975), Green (1982), and Fischbein and Gazit (1984), to describe and validate a framework of probabilistic thinking with example tasks and primary school students' responses across 4 levels:

Level 1–associated with subjective thinking.

Level 2–a transitional stage between subjective and naive quantitative (classical) thinking.

Level 3–involved the use of informal (classical) quantitative thinking.

Level 4–incorporated (classical) numerical reasoning. (Jones et al., 1997, p. 102)

The probabilistic thinking frameworks described by Jones et al. (1997) assumes the starting point of subjective probability with the attainment of the classical concept of probability is the ideal aim.

Probability problems within textbooks were analysed by D. Jones and Tarr (2007) in the USA, using a framework of Levels of Cognitive Demand:

Lower-level demands (Memorization)

Lower-level demands (Procedures without Connections)

Higher-level demands (Procedures with Connections)

Higher-level demands (Doing Mathematics). (Jones & Tarr, 2007, p. 9)

Jones and Tarr (2007) conducted probability textbook analyses on two textbook series for Years 6–8 in the USA, one popular textbook and one alternative (innovative) textbook. They also reviewed the books over four phases of curriculum reform, dating from 1957 to 2004. Their aim was to investigate whether the changes to the intended curriculum was reflected in the ways the textbooks implemented the curriculum. Jones and Tarr (2007) found teachers omitted teaching the topic of probability in the past due to lack of time, knowledge, or misconceptions about the topic. They found that 17% of the tasks in the popular probability textbook were described as involving high cognitive demand, while the alternative textbook had 59% of tasks deemed to involve high cognitive demand (Jones & Tarr, 2007). While the newer textbooks included more high demand tasks and investigations, it was possible for teachers to "use all of the probability tasks in an investigation-oriented textbook but present these tasks in a traditional manner by providing students with explicit rules, formulas, and repetitive practice problems" (Jones & Tarr, 2007, p. 55). They point to the issue that while cognitive demand of the textbook is important and could be appropriate, how the textbooks are implemented by teachers and students is also essential to examine. Thus, Jones and Tarr (2007) recommended further research on the alignment between the cognitive demand of the textbooks and assessment tasks.

Wild and Pfannkuch (1999) in New Zealand (NZ) described four statistical frameworks based on the investigation cycle, types of thinking, the interrogative cycle, and dispositions. More recently, other NZ research by Pfannkuch and Ziedins (2014) developed a probability framework where the students designed the model. This incorporated prior knowledge and theoretical models while working towards a real world purpose. Sharma (2015) described a teaching sequence of probability in the social constructivist tradition, which includes:

1. Posing a task
2. Making predictions
3. Playing the game in pairs
4. Planning explorations
5. Data collection and analysis
6. Further exploration including linking to theory. (pp. 81-82)

Probability is unique in mathematics, as it contains randomness, which confuses the classic (theoretical) probability. Explicit links between the experimental practical probability and expected theoretical results need to be made, and they are not always self-evident. Teaching could start with the theory and linking to the real world (Pfannkuch & Ziedins, 2014), or start with the real world and link to the theory (Sharma, 2015).

Education and mathematical thinking frameworks have been examined, complemented by the recommended teaching sequences and probability frameworks. Teaching and learning probability has its own complexities as there are three concepts of probability (*classical, frequentist, and subjectivist)* that can align, but also demonstrate cognitive development. The next section investigates research using thinking frameworks to describe mathematics curriculum alignment.

2.3.5 Thinking and Curriculum Alignment

Thinking frameworks such as Bloom's (Krathwohl & Anderson, 2002) and SOLO (Biggs & Tang, 2011) can be used to describe, categorise and compare the learning and assessment tasks with student responses. This ensures the assessments demonstrate the application of knowledge, rather than a retelling. To support constructive alignment, assessments should be designed to reflect the class objectives, regarding content, but also cognitive demand. When student responses are evaluated, SOLO was found to be a useful framework where the same learning task or problem, can have student responses demonstrating a range of achievement levels. For example, constructive alignment using the SOLO framework was used in two Australian and New Zealand University Mathematics courses (Murphy, 2017). The lecturers made the expectations around the levels of understanding explicit, using the SOLO framework. They measured attitudes to learning compared to the student's mathematics results. Murphy (2017) found the performance in the university mathematics bridging course was directly related to their deeper learning approach in class, and the students performed better in the long run than those who did the prerequisite mathematics at secondary school. Several studies focussing on explicit alignment between intentions and implementation found improved student learning, as well as creating a positive learning environment (Cain et al., 2018; Jaiswal, 2019; Jorgensen & Larkin, 2017). For example, Cain et al. (2018) from a Victorian University, used constructive alignment and SOLO to improve their Engineering students' learning through formative assessment, assessment by portfolios and continuous evaluation of the teaching program using an Action Research model

Seminal research into curriculum alignment between the intended, implemented and assessed curriculum was conducted by Porter et al. (1978) and Webb (1997) with their investigations into methods of evaluation of curriculum alignment in USA mathematics curriculum. Porter (2002) reported the TIMSS results found the USA mathematics curriculum as disappointedly "a mile wide and an inch deep" (p. 3). The mathematical content, consistency across year levels, and depth of knowledge of mathematical concepts were compared with a numerical scale to find an Alignment Index. Porter used contour graphs to display the alignment between content and problem types in the curriculum, textbooks and assessments in the US as the National curriculum of the Common Core was designed and implemented (Porter, 2002; Porter et al., 2007). Porter (2002) found the alignment in eighth-grade mathematics instruction between states was higher than between the instruction and the assessment, implying the assessments were misaligned. Porter (2002) also found the full range of mathematical content was covered by the curriculum standards, but the cognitive demand was low. The focus of the standards was on solving routine problems and communicating, rather than non-routine problems or interpretations. This is consistent with the TIMSS USA results being described as disappointing (Schmidt et al., 2001).

Porter (2002) and Webb (1997) created their own criteria for cognitive demand and depth of knowledge. Porter used memorize, solve routine problems, communicating, understanding, solve non-routine problems and conjecture or prove. In comparison, Webb's categories for Depth of Knowledge criteria were Recall, Skill/Concept, Strategic Thinking and Extended Thinking. The focus of Webb's studies (for example, Webb, 1997; Webb, 2007) was to describe the complications of the process of aligning mathematics curriculum and assessment. The alignment between the curriculum and the assessment was very low overall with a maximum of 55% for Number and Algebra, and an even lower 33% for Statistics and Probability, results which again suggest greater attention is required to encourage research into the implementation of statistics and probability. Webb's instructions to the teachers analysing the curriculum were to compare the depth of knowledge of the curriculum standard to what would be reasonable for a typical student at that level, which relied heavily on the experience of the content experts used for the project's data collection.

More recently, Seitz (2017) conducted investigations with five Canadian teachers into the alignment between the intended, enacted and assessed curriculum with Year 9 Algebra students, using document analysis and teacher observations and interviews. The results demonstrated 97% content alignment but only 7.3% cognitive alignment. Year 9 mathematics classrooms were observed over approximately 10 weeks in five classes during the topic of pattern and relations. This was backed up by teacher interviews and analysis of the written curriculum documents and assessment tasks. Tables comparing the aspects of the curriculum with the revised Bloom's taxonomy (Krathwohl & Anderson, 2002) were created independently, then compared. The teachers followed the content as expected. While the cognitive levels of the intended curriculum were spread throughout the six levels of the Bloom's taxonomy, the enacted and assessed curriculum were predominantly in the lower cognitive levels. Seitz (2017) recommended the cognitive level for learning expectations be identified and included in the intended curriculum documentation, to support the teachers in understanding their expectations.

There are many ways to measure and categorise cognitive demand or depth of thinking. Bloom's, SOLO, Porter and Webb, have been used in the examples above. Näsström and Henriksson (2008) compared nine methods of curriculum alignment on the Swedish chemistry curriculum and the corresponding assessments. They found that Bloom's revised taxonomy was the most appropriate due to its content, range of categories, complexity, and accessibility. The other methods of curriculum alignment methods also had value, especially the Porter framework (Porter et al., 2007). Still, Bloom's came out as the most useful for evaluating the knowledge range and cognitive demands of a curriculum, which is a finding acknowledged in this research design. Within Australia, Ziebell et al. (2017) developed the *Performance type categories for mathematics*, which collates the work of Webb, Porter, TIMSS, PISA and Bloom's revised thinking frameworks as shown in Figure 2.3.

Figure 2.3

Performance Type Categories for Mathematics

Alignment Project: performance types	Bloom's Revised Taxonomy (2002)	Porter (SEC) (2007)	Webb (2005)	PISA (2009)	TIMSS (1997)
Knowing	Remember	Memorise Facts, definitions, formulas	Level 1 Recall and Reproduction	Reproduction, definitions and computations	Knowing
Performing	Apply	Perform procedures	Level 2 Skills & Concepts		Routine procedures
Communicating	Understand	Demonstrate understanding			Communicating
Mathematical Reasoning	Analyse Evaluate	Conjecture, analyse, prove	Level 3 Strategic Thinking	Mathematisation, mathematical thinking, generalisation and insight	Justifying and proving
Non-routine problem-solving	Creating	Solve non-routine problems, make connections	Level 4 Extended Thinking		Solving problems
Making connections				Connections and integration for problem-solving	

Note. From "*Aligning curriculum, instruction and assessment"* by Ziebell, N., Ong, A., and Clarke, D. In T. Bentley & G. C. Savage (Eds.), Educating Australia: Challenges for the decade ahead, pp. 259. Copyright 2017 Melbourne University Press.

Ziebell et al. (2017) used the *Performance type categories for mathematics,* shown in Figure 2.3, to analyse and compare the aspects (intended, implemented, and attained) of the curricula, at two primary and secondary Victorian schools in Mathematics and Science. She found the science learning tasks and assessments consisted of a full range of performance types, while the Mathematics learning tasks and assessment were generally of a low level, even though the official intended curriculum included a variety of performance types. The external state test of NAPLAN and lack of teachers' ownership of the curriculum for mathematics were suggested as contributing factors to the low-level mathematics tasks (Ziebell et al., 2017). This combined framework was the starting point for the thinking framework used for this study in Chapter 3, Section 3.3.2.

2.3.6 Summary of the Literature on Thinking.

Whether considering 21st century skills, mathematics, or the topic of probability, there are many ways to classify thinking skills. This section summarises the research literature around several of the thinking frameworks used in mathematical education, including probability. While Bloom's taxonomy is the most popular, it has limitations when used in mathematics. A combined framework which considers cognitive demand, but also critical and creative thinking would be desirable, with considerations of originality and links to realist contexts. Thinking frameworks also need to be easy to interpret and implement consistently for teachers. It would be desirable if the three levels of the curriculum, intended, implemented, and attained, could be classified by the same thinking framework. There are many factors which influence the development of mathematical and thinking skills by students, including the teachers, curriculum, prior knowledge, content, and context. The next area to consider is Activity Theory and in particular Engestrӧm's (2001) interpretation, utilising a framework involving one or more activity systems, which is a theoretical framework that can help explain how various areas interconnect, influence each other and support student outcomes.

2.4 Activity Theory

Curriculum alignment suggests the formal written curriculum informs teachers, who support students in their learning of the mathematical content, which is then assessed. Ideally these three curriculum aspects would align with common expectations to reach the desired objectives and outcomes. This is the linear model described by the TIMSS Curriculum Model (Mullis & Martin, 2013) and previously detailed in Section 2.1.1. However, this linear model is a very simplified view of the influences on student learning, as many additional factors have an impact including peers, communities, textbooks, and expectations of parents and teachers. Engeström's (2001) framework of an activity system can provide a systematic methodology for explaining the tensions and possibilities between all these elements within such a complex and dynamic system. It can be used as a theoretical framework as well as an effective means of theorising, describing, analysing and designing systems with a multitude of elements (Engeström, 2015; Jonassen & Rohrer-Murphy, 1999; Roth, 2014).

Activity Theory originates from the work of Marx who then influenced Vygotsky, Leont'ev and others (Engeström, 2015). Marx proposed that individual consciousness and activity arise from collective activity, therefore, Engeström (2015) proclaims that Activity Theory was a child of Marxist scholarship. Vygotsky was influenced by Marx (Yamagata-Lynch & Smaldino, 2007) establishing a cultural-historical approach to understanding the relationship between individuals and their social, cultural and historic factors (Daniels, 2018). Vygotsky also considered *mediation* in terms of subjects and their social environment, as represented by an artifact or tool and its influence on an activity. An artifact or tool may be psychological or physical (Daniels, 2018; Yamagata-Lynch & Haudenschild, 2009). Leont'ev (a student of Vygotsky) elaborated on the difference between individual and collective activity, as well stressing the centrality of the object with regard to the motivation for an activity (Leont'ev, 1981). Leont'ev highlighted the importance of activities being differentiated by the components of the activity system, but particularly their objects (Daniels, 2016).

Engeström further developed Activity Theory as a framework to link theory and practice. Engstrom (2001) defined the five principles of Activity Theory. The first principle is that the prime unit of analysis is the collective artifact-mediated and object-oriented activity system. The second principle is that an activity system should involve multi-voicedness, to reflect the division of labour and differing position for the participants. The third principle is that an historical perspective of an activity system is important to understand the system. The fourth principle is that contradictions are the source of change and development. The final principle is the possibility of expansive transformation, through questioning and challenging of the established norms. Expansive transformation, or expansive learning, comes about by asking the four questions of *who, why, what,* and *how* learning occurs (Engeström, 2001). Engeström (2001) proposes that the most intriguing learning comes through challenging the idea that we know what is to be learnt, and that people and organisations are always learning, even if they do not realise it.

Activity Theory provides a theoretical framework for examining different forms of human consciousness and activity, including learning (Yamagata-Lynch & Haudenschild, 2009), where individuals are influenced by social activity, but also mediating artifacts and tools. Engeström's (2001) framework of Activity Theory can also be used as a methodological tool to understand the learning process and to organise the findings of research studies into activity systems (Yamagata-Lynch & Haudenschild, 2009). This current project uses Activity Theory as a methodological tool to describe the current system of education, the activity system, but also as an instrument of transformative change (Avis, 2009; Martin & Pein, 2009). A key concept in transaction is the interdependency and interconnection of the elements within the activity system (Barab et al., 2004).

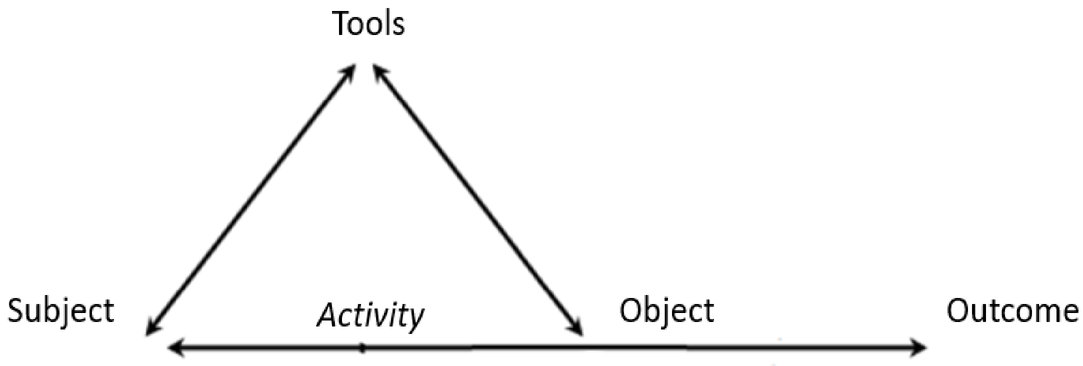
As a framework, Activity Theory has been used extensively in education to investigate pedagogical change resulting from the introduction of technology (for example; Coupland, 2004; Fitzsimons, 2005; Larkin & Richardson, 2013; Nardi, 1996a). It has also been used to uncover issues within established systems in education (Williams et al., 2007) and medical practices (Engeström, 2001), and to propose solutions (Engeström, 2015; Jonassen & Rohrer-Murphy, 1999). The word *activity* in Activity Theory for an educational setting is used to denote social, cultural and historical contributions to the collective needs (Hasan & Kazlauskas, 2014; Roth, 2007), rather than necessarily a learning activity. Within the current study, Activity Theory will be used to investigate the complex influences on students as they learn mathematics and develop associated thinking skills. Firstly though, the several iterations of Activity Theory as it developed over time and complexity are outlined.

2.4.1 Generations of Activity Theory

Activity Theory was founded by Vygotsky (1978) and Leont'ev (1981) when they described the links between a subject and their outcome, which was mediated by *tools*. This model was illustrated by Engeström (2001), as illustrated and adapted in Figure 2.4.

Figure 2.4

First Generation Activity Theory

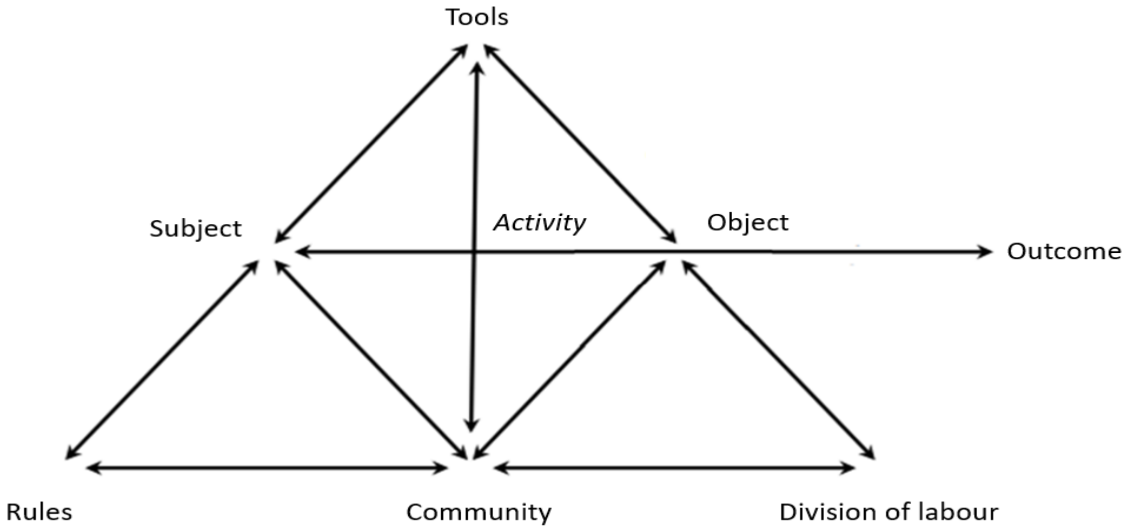


Note: Adapted from "Expansive Learning at Work: Toward an activity theoretical reconceptualization" by Engeström, Y. (2001). *Journal of Education and Work*, 14:1, p. 134. Copyright 2001 by Taylor & Francis Ltd.

In terms of this first generation of Activity Theory, in education this could be translated as the learning (*activity*) of the student (*subject*), influenced or mediated by the learning activities, such as textbooks (*tools*), with an ideal outcome of increased student knowledge. This model is limited as it does not include the many influences that occur within a social learning community, which is bounded by rules, expectations, roles, culture, and history. The second generation of Activity Theory considers that the activity takes place within a socio-cultural context or community which includes numerous influences (Engeström, 2015), which is represented by the activity system (see Figure 2.5).

Figure 2.5

Engeström's Second Generation Activity System Presented in the Engeström Triangle



Note: Adapted from "Expansive Learning at Work: Toward an activity theoretical reconceptualization" by Engeström, Y. (2001). *Journal of Education and Work*, 14:1, p. 135. Copyright 2001 by Taylor & Francis Ltd.

In a school, these *community* influences can include the teachers, family, peers, and the wider community (Williams et al., 2007). The r*ules* include the formal procedures that influence the community practices such as the overseeing education departments, curriculum bodies, school policies and procedures, as well as the unwritten rules of cultures, histories and beliefs (Jonassen & Rohrer-Murphy, 1999). The *division of labour* includes factors like the roles of the student/teacher/school/parents in the learning process. Tensions and contradictions between and within these elements continuously vary. Any change in any one of the elements (*subject, rules, roles, objects*) will have an impact (positive or negative) on the object and outcome. While the *object* of the system can be a physical object, it can also be a goal (Yamagata-Lynch & Smaldino, 2007). The learning emerges from the combined activity (Jonassen & Rohrer-Murphy, 1999; Kaptelinin & Nardi, 2006).

Tensions within and between the elements of the activity system are dynamic but hold the system together. *Contradictions* (tensions, dilemmas, disturbances, and possibilities) can occur when things change, for example, when a new *tool* or technology is introduced or when someone challenges the rules, which can result in an expansion of an activity, known as an "expansive transformation" (Engeström, 2001, p. 137). Contradictions might occur within the elements themselves, between the elements, over time, or between neighbouring activity systems (Roth & Lee, 2004; Yamagata-Lynch & Haudenschild, 2009). Contradictions and tensions are considered to be opportunities for transformation and growth in a cultural sense, in line with a dialectical relationship where each individual can be empowered to contribute towards a change in the system (Engeström, 2001). For example, as the calculators become more powerful, the mathematical problems may change their focus from calculation to interpretation, and the data involved can be realistic.

The Engeström triangle representation of the second-generation Activity Theory (Sweeney, 2010) has been used to describe and analyse information in a multitude of ways. For example, the elements themselves could be explained (Coupland, 2004; Sweeney, 2010), the tensions between the elements (Larkin, 2011; Lloyd & Cronin, 2002), or the relationships between similar activities with different subjects could be analysed (Romeo & Walker, 2002). Second generation Activity Theory as described by the Engeström triangle in Figure 2.5, should not be seen as rigid and simplistic, as it aims to represent the dynamic, evolving relationships within the system (Roth, 2004). The cultural history of the system is also important, as it influences the current system (Yamagata-Lynch & Smaldino, 2007). Activities are not fixed; they can be flexible and undergoing continuous change. It is important to understand history, to understand the current system and why things are the way they are now, and as such an activity system represents an evolving framework (Jacobs et al., 2017) that can explain dynamic and changing systems. An Activity system has been used to represent the influence of technology in Mathematics education, as described earlier this chapter in Section 2.1.3.3.1.

Third generation Activity Theory describes when the system is considered though several viewpoints, represented by two or more Engeström triangles (Engeström, 2001). Using an example from research in education, students could be the subject (as in Davis, 2007), then another diagram could be constructed with teachers, the administration or curriculum as the subject, see Figure 2.6. The common factor of these variations is the overarching desired outcome of student learning, however there may be other subjects to be considered (Engeström, 2001). There may be a model with the teacher as a subject, or the curriculum authority as the subject. All of these diagrams can be grouped around a common outcome, to form a diagram of the third generation of Activity Theory, such as the example with two interacting activity systems as displayed in Figure 2.6 (Lemos et al., 2013; Roth, 2004).

Figure 2.6

Education Example of Third Generation Activity Theory with Two Interacting Activity Systems

Note: Adapted from "Expansive Learning at Work: Toward an activity theoretical reconceptualization" by Engeström, Y. (2001). *Journal of Education and Work*, 14:1, p. 136. Copyright 2001 by Taylor & Francis Ltd.

Essentially, Activity Theory represents a positive, holistic approach, focusing on dialectical relationships. Activities are explained in terms of interactions between the individual and society, environment, culture, and cognitive processes. Rather than accepting circumstances as they are, it encourages us to challenge and contribute to collective learning and transformation.

Activity Theory has been described, as it developed from the initial models of Vygotsky (1978) and Leont'ev (1981) through the first and second generation as designed by Engeström (2001). This approach of explaining and analysing a complex system has proved ideal for many educational projects, especially when considering both physical and social influences on a system, making it ideal for researching the role of curriculum while also considering the views of the students and teachers.

2.5 Identified Gaps in the Literature and Implications for the Current Study

This chapter has covered the literature around the topics of curriculum, thinking and probability. Several gaps have been identified that inform the focus and design of the current research project. Firstly, the research literature surrounding curriculum (intended, implemented, and attained) and curriculum alignment have shown most of the research has been undertaken in either one or two aspects of the curriculum, with very little across all three aspects. Thus, a further examination of curriculum alignment across all three aspects would be useful, especially if the perspectives of students and teachers could be included. Secondly, probability has been researched in mathematical education regarding younger children and university students, but little research has specifically examined probability within the senior secondary education levels. Probability is a unique mathematical topic due to the three ways of interpreting the topic; classical, frequentist or subjective. It also combines real-life situations with several branches of mathematics (algebra, functions, graphs, and calculus) and hence can be used to develop a variety of thinking skills. Finally, thinking skills have been defined and classified through a range of different methods. Each of the thinking frameworks has important unique features, but none are ideal for use with curriculum statements, teaching resources and the expected responses to learning and assessment tasks, in the topic of probability. As such, it is appropriate that further investigation of the development thinking skills in senior secondary years, occur in the topic of probability.

An aim therefore of the current study is to combine the objectives of the development of mathematical and higher order thinking skills, with the content area of probability, and curriculum alignment in senior secondary mathematics, which has not been attempted. The issues affecting the development of mathematical and thinking skills in students are complex, involving the curriculum, content, and teachers, plus a multitude of other factors including community, assessments, culture, historical context, and the specific mathematical content. Second generation activity systems (Engeström, 2001) provide an appropriate platform to explain and analyse these factors and identify the alignment and tensions between these physical and social factors. The research thus aims to discover possible factors which influence student development of mathematical and thinking skills, by answering the following research questions, which have emerged from the background information in the introductory chapter and the identified gaps in the literature.

2.5.1 Main Research Question:

The main research question which will be the focus of this research project is:

How does the study of probability impact the development of thinking skills?

Sub-questions

1. In what ways are thinking skills developed through the senior secondary Victorian mathematics probability curriculum?

2. What factors impact on the development of thinking skills through the teaching and learning of the senior secondary Victorian mathematics probability curriculum?

The lens through which these interconnecting areas will be viewed is Engeström's interpretation of Activity Theory (Engeström, 2001) utilising an activity system, which will help describe the influence of the multitude of factors on the development of mathematical and thinking skills by the past senior secondary mathematics students. While activity systems have been used in education research literature, especially in the study of the effect of technology, this interpretation of Activity Theory has not yet been used to report on the influence of curriculum.

2.6 Chapter Summary

This chapter provided an overview of the existing bodies of literature relating to senior mathematics curriculum more broadly, but also specifically within Victoria and the relationship or links between components of the curriculum, the development of thinking skills, especially in regard to the content area of probability. This literature review described how the mathematical topic of probability can be used to support the development of thinking skills, through the explicit inclusion of consideration for thinking in all aspects of the curriculum. The literature surrounding the use of Activity Theory as a framework to explain the relationship between influencing elements with teaching and learning was also explained. Gaps in the literature were identified as a way of introducing the emergent research questions which underpin the study. The next chapter presents the theoretical perspectives and the research design developed to most appropriately answer the research questions, including the methodological framework and research methods used in this exploratory case study into senior secondary mathematics curriculum, and how it can support the development of thinking.

Chapter 3: Research Design and Methodology

The previous chapter presented literature regarding the role of curriculum, in the content area of probability in particular, and in relation to defining and measuring the development of thinking within mathematics education. This chapter sets out the research design of the exploratory qualitative case study that was conducted into how the senior secondary mathematics curriculum, focusing on the topic of probability, supports the development of student thinking. Case study methodology was selected as an appropriate design for answering the emergent research questions. As such, the study was bounded in several ways. Geographically it is bound to the Gippsland area in the south east of Victoria, home of the researcher. Contextually it is focussed on senior secondary (VCE) mathematics, specifically the strand of probability, and temporally relates to the introduction of the 2016–2022 VCE Mathematics Study Design. The research design methods involved a collective study of the curriculum and support material, and an examination of perceptions of 14 teachers and 20 students regarding how the curriculum influenced their teaching, learning, or thinking. The final section of this chapter deals with the issues of trustworthiness and the ethical considerations required for this study.

This case study was both an exploratory study to discover the current practice, and an explanatory study to explore the enablers and blockers, tensions and possibilities in the flows of influence among the curriculum, teacher and student elements (Thomas, 2011; Yin, 2009). The development of student thinking was the focus. Qualitative research methods were used to consider the three aspects of the curriculum, teachers, and students independently, with the results integrated with the final discussion. An explanation and justification of this case study is included in this chapter, followed by a description of how the research project was carried out.

This research project was based on a constructivist epistemology with an interpretivist perspective (Creswell, 2013; Crotty, 1998), and used a multiple case study methodology (Yin, 2018). All these foundations are explored in depth in this chapter beginning with an examination of ontological, epistemological, and theoretical perspectives underpinning the case study methodology and justifying the selection of the interpretive approach. This is followed by a description and justification of the methods utilised to collect and analyse the data to most appropriately respond to the research questions outlined at the end of the literature review.

3.1 Mathematics Education Research and Theoretical Perspectives

When undertaking educational research, it is essential to understand the nature of the knowledge to be discovered or developed, in this case generated through case study methodology. This section then explores the epistemological underpinnings of case study. While ontological concerns include the nature of existence, and the structure of reality (Crotty, 1998; Wilson, 2013), epistemological concerns include questions such as, what is knowledge? (Bryman, 2012), and how do we know what we know? (Crotty, 1998). Epistemology and ontology are however interrelated (Crotty, 1998; Gray, 2013), with ontologies relating to understanding of *what is*, while epistemologies help us decide the extent to which knowledge is valid and significant (Gray, 2013; Wilson, 2013).

Mathematics education research itself contains internally contradictory stances (Burton, 2002). The epistemology of mathematics with its objectivity, conflicts with the socially negotiated beliefs of education researchers. These two viewpoints can be aligned with the expansion of the definition of mathematics education. Mathematics education in senior secondary schools can treat mathematics as a narrow, fixed body of knowledge that is taught in a constricted manner (Brown et al., 2008; Connor, 2012). The teaching of mathematics at these year levels in this way can encourage the false beliefs that mathematics is difficult, involves an inherited talent, is predominant in the male domain, is abstract, theoretical, universal, value and culture free (Ernest, 2015). However, mathematics education is a science of enquiry, discovery and problem-solving, according to a growing number of teachers and researchers (for example; Anthony & Walshaw, 2009; Boaler, 1998, 2019; Zaslavsky & Sullivan, 2011). These two contrasting views of learning mathematics mirror the theoretical research perspectives of objectivism and constructivism, although theoretical perspectives tend to be divided into three types, objectivist, constructivist, and subjectivist perspectives (Crotty, 1998; Gray, 2013).

The first theoretical perspective relates to objectivism, which holds that reality is fixed, to be measured and observed (Crotty, 1998) so that phenomena are external facts beyond our influence (Bryman, 2012). Objectivist epistemology holds that reality exists independently to the observer, and that research is about discovering the unchanging truths (Gray, 2013) that already exist but just need to be discovered. The objectivist epistemology is closely linked to positivism, which claims that it is the scientific method that can discover the absolute truth. Related to positivism is the post-positivist view, which infers that all observation is imperfect, and while the truth is fixed, it can never be perfectly determined. The post-positivist researcher assigns probabilities to observed findings (Gray, 2013). Positivist and post-positivist research tend to be quantitative, with numerical data collected and analysed. Pure mathematics aligns with a positivist epistemology.

Secondly, constructivist research describes reality in terms of engagement, such that knowledge is constructed by interaction with reality (Crotty, 1998). As such, phenomena are produced though social interaction with observers and are, therefore, in a state of constant revision

(Bryman, 2012). People construct their understanding of knowledge differently, based on culture, time, and experiences. The constructivist epistemology is closely linked to interpretivism (Gray, 2013), where the researcher interprets the actions of the subjects. Meaning might change over time or depend on the observer. Research methods associated with a constructivist epistemology include ethnography, grounded theory and phenomenology, and other new methods being developed and accepted over time. Constructivist research tends to be qualitative (or mixed), where the information collected for research can be gained using a wide variety of methods. Education research often aligns with a constructivist epistemology.

A third epistemological stance is subjectivism. In this view, the object of the research does not contribute to the generation of meaning (Crotty, 1998), instead, the meaning emerges from dreams, unconsciousness or religious beliefs (Gray, 2013). This third epistemological stance is the least used in education research, however it is relevant in that it relates to subjective probability, as described in the literature review. Subjective probability is belief that events are controlled by luck or a higher power (Batanero, 2014).

Of the three main epistemological ways of examining knowledge, an objectivist stance is most often taken in pure mathematics research, where numbers and symbolic concepts are manipulated and measured. Education research tends to use objectivist or constructivist methods, or a combination of several methods (Creswell, 2012; Johnson & Christensen, 2014). An educational researcher needs to understand a range of theoretical perspectives to study the associated multifaceted educational issues. The conflict that often exists in the mathematics verses education basic philosophical view is reflected in my interest in probability, one area of mathematics in which the subjective, experimental, and theoretical solutions are not consistent. As such, the epistemological and theoretical perspectives influence the conceptual framework and methodology used.

Increasingly, combined, mixed methods or pragmatic research methodologies are employed (Teddlie & Tashakkori, 2003), with many research projects also unintentionally having mixed methods (Onwuegbuzie & Leech, 2005). More researchers are finding that combined methodologies provide more detailed conclusions (Creswell, 2012; Teddlie & Tashakkori, 2003). Mixed methods as described by Merriam and Grenier (2019), and Teddlie and Tashakkori (2003), involve the collection and analysis of a combination of qualitative and quantitate data. Whereas the current study involved a combination of qualitative methods of interview and document analysis. The variety of data collected using these two qualitative methods was compared and contrasted in a case study, to increase the validity and reliability of the findings.

The theoretical perspective deemed most appropriate for the current research project was underpinned by a constructivist epistemology with an interpretivist perspective (Creswell, 2013; Crotty, 1998). This was because the study investigated *how, in what ways, what roles,* and *descriptive* questions around the development of mathematical and thinking skills. These questions supported the combined use of qualitative research methodologies to induce (discover) the theories involved and to deduce (evaluate) current theoretical frameworks, to explore their validity. This research project, which investigated thinking skill development by students as influenced by the senior secondary mathematics probability curriculum, used a constructivist epistemology while being aware of the influence of the objectivity of mathematics, and subjectivity of probability.

3.2 Case Study Methodology

Methodology relates to the theoretical framework behind the methods used to conduct a study. A qualitative, embedded case study methodology was used in the current research project. A case study explores in-depth, a program involving one or more individuals, who are bounded by time and activity (Creswell, 2013; Stake, 1995). Stake (1995) defines a case study as the bounded unit to be studied, while Yin (2018) proposes case study as a research process to explain the *how* or *why* of a social phenomenon. By combining these two conceptions, a case study allows investigators to gain a holistic, realistic study of real-life events including organisational and school performance using a variety of data-gathering methods (Merriam & Grenier, 2019; Yin, 2013). Case study represents “an in-depth exploration from multiple perspectives of the complexity and uniqueness of a particular project, policy, institution, program or system in a ‘real-life’ context” (Simons, 2009, p. 21). A research case study is used to explain real-life situations, which are too complex for surveys or experimental means, so is ideal when there is a contemporary set of events over which the researcher has no control (Yin, 2013).

There are several types of case studies. Yin’s (2018) classification of case studies includes single or multiple, holistic or embedded. The current study is an embedded multiple case study, as there are multiple cases and more than one unit of analysis. Students and teachers of the Gippsland region in Victoria were selected from diverse backgrounds, as described in Section 3.4.2.2. While Gippsland is a large geographic area with diverse communities, participants were not intended to be a representative sample of the community but were considered a group that could highlight the tensions and possibilities of the system.

While only one geographic area and one educational system was studied, several aspects of the curriculum were compared and contrasted within the activity system that was devised. The perceptions of a range of student and teacher participants in relation to their experiences were collected in parallel, then combined to describe the situation as a whole (Johnson & Christensen, 2014; Yin, 2018).

Under Yin's (2018) definition of a research case study, the research design is critical and it is important to have the research design planned before hand, which can be supported by a pilot study. The research question, data types, analysis methods and reporting methods need to be cohesive and consistant with each other, and set up before the project begins (Yazan, 2015). Yin (2018) suggests a varitey of data types be used, in a triangulating fashion, including documentation, and interviews, as in this project.

In the current study, there were many potential factors that could influence the development of thinking skills in senior secondary mathematics students. These include the curriculum, teachers, resources, assessments and culture, all factors not able to be influenced by the researcher. As such, case study provided an appropriate framework for examining the identified issues through several qualitative methods. Thus, this research project involved an in-depth exploration from the perspective of several aspects of the curriculum, particularly senior secondary mathematics curriculum, in the bounded regional area of Gippsland. It investigated the real-life context from the experiences of teachers and students.

3.2.1 Case Study in Mathematics Education

This section describes several examples of case study research methodologies previously used in mathematics education investigations. They include examples of case studies involving large geographical regions, comparisons between teachers, and within individual classes. This selection informed the research design of the current project. A large-scale comparison case study of four states in the USA became the seminal work of Porter (2002), as he designed a tool to calculate the alignment between instruction and assessment of mathematic curriculum. He used document analysis and surveys to create topographical maps to display the cognitive demand of the various areas of the Year 7 mathematics curriculum (Porter, 2002). He was particularly interested in the role played by teachers in interpreting curriculum (Porter et al., 2007). Case studies have also been used to compare teachers and their implementation of the curriculum in Australia, USA, and Canada. Another qualitative comparative case study was employed in Australian schools comparing four Grade 5 classrooms, to examine the alignment of the primary curriculum, in particular the intended, enacted and assessed curriculum (Ziebell, 2014). Interviews, questionnaires, document analysis and video-recorded lessons focused on ascertaining the assessment practices, to discover the skills privileged by teachers. Ziebell et al. (2017) combined and trialled a theoretical framework which compared the performance types of various cognitive models. This combined framework was the basis for classifying the thinking skills in the curriculum.

Pierce (2002) utilised a case study of one first-year university class of 21 students to investigate the use of CAS calculators in Australia. Various theoretical frameworks were tested and developed focussing on ways of using calculators, the effect on teacher's pedagogy, and problems of developing algebraic symbol sense. Class teaching materials were collected, along with observation of mathematical learning activities and interviews. This data from seven students made up the embedded case study (Pierce, 2002; Pierce & Stacey, 2001). Similarly, a teacher-researcher in the USA conducted a case study as she implemented a teaching sequence which involved probabilistic simulations (Zimmermann & Jones, 2002). Here a class of 23 students were involved in pre, post and retention assessments, and a small group of students were interviewed to test the theoretical framework about how students develop reasoning about probability. Seven students within the Advanced Placement class were chosen as a representative sample for interviews (Zimmermann, 2002). Misconceptions in probability were discovered and overcome using explicit instruction, technology, and inter-student discussions. This case study also found that graphing calculators were effective for demonstrating probability simulations. Building on this, a quantitative Canadian case study was conducted by Seitz (2017) with five teachers to investigate alignment between the intended, enacted and assessed curriculum with Year 9 Algebra students. They utilized document analysis, teacher observations, and interviews. The results demonstrated 97% content alignment but only 7.3% cognitive alignment. Seitz (2017) recommended the cognitive level for learning expectations be identified and included in the intended curriculum documentation, to support teachers in understanding their expectations.

These examples of research into mathematical curriculum implementation, which used case study methodology, range from comparisons across several states to an individual classroom. A variety of data collection methods were used including surveys, interviews, classroom observations and document analysis. All these case studies involved the comparison of real-life situations to recognised theoretical frameworks or the development of a conceptual framework, and utilised both inductive and deductive links to theory. Collectively these studies indicated the research design of the current project was based on sound and tested case study methodology.

3.2.2 Case Study and Theory Development

Case study, when utilised in the fields of education and social sciences, tends to produce rich descriptive works, but can also be used to test theories (Merriam, 1998; Stake, 1995; Thomas, 2013). In quantitative studies, many subjects are used to test a few variables or theories, while in a case study, one subject (individual or group of people) is investigated with a detailed and holistic approach, to describe and test several theories (Longhofer et al., 2017; Merriam, 1998). Post-positivists argue that large samples are required to prove a theory. Still, only one counterexample is required to disprove a theory, and one rich case study can support a theory (Easton, 2010). Case study can be used to evaluate projects, but also to evaluate the theory behind the project (Koenig, 2009) taking on an inductive focus to find theories, or a deductive one to evaluate a theory (Meyers & Nulty, 2009; Thomas & Myers, 2015). Case study does have a number of limitations that have been outlined by various authors (Flyvbjerg, 2006; Thomas, 2011; Yin, 2009). These include lack of generalisability, bias because of small sample size, interpretation bias, and the often-overwhelming volume of data given the variety of data sources (Flyvbjerg, 2006; Merriam, 2014). All of these were considered in the current study and are discussed further in Section 3.4.4, later in this chapter.

In summary, this research project was underpinned by a constructivist epistemology with an interpretivist perspective (Creswell, 2013; Crotty, 1998), and used a multiple case study methodology (Yin, 2018). While case study was the selected methodology for the research design, Engeström's (2001) framework of activity systems within the broader context of Activity Theory provided the theoretical framework used to describe, analyse and interpret the interconnecting relationships in this case study, which will now be described and discussed.

3.3 Theoretical Frameworks

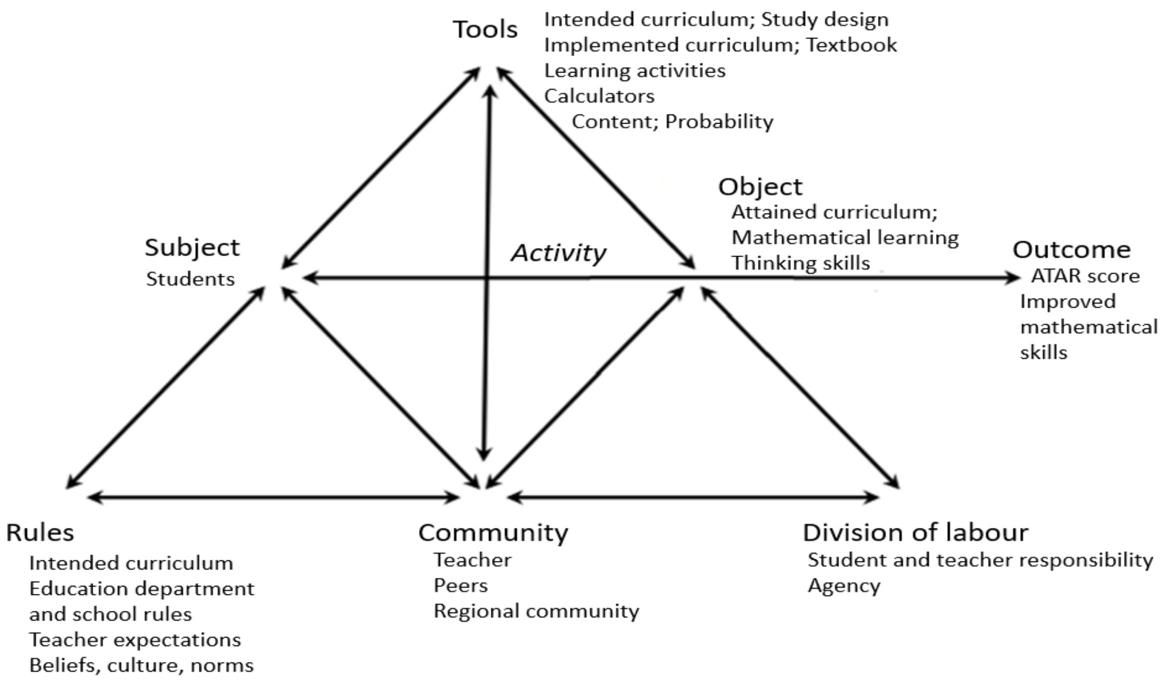
Case study was used to bound this study of the development of thinking skills in senior secondary students. Within the case study, three theoretical frameworks were used to support the analysis and interpretation of the data. The overarching framework was an activity system (Engeström, 2001) as part of Activity Theory, which represented an effective way to mediate the physical and social influences on this complex system. Using Activity Theory, the mathematics curriculum was a key influential factor as a tool, rule, and an indication of the object. The mathematics curriculum is complex, so the three-level TIMSS Curriculum Model (Mullis & Martin, 2013) was used to help classify its aspects, and to structure the discussion around curriculum alignment. The object or goal of the activity was the development of mathematical and thinking skills, so a thinking skills framework also needed to be defined and used.

3.3.1 Activity Theory in Mathematics Education

Activity Theory (Engeström, 2001) was used to interpret the current case study, through examining an activity system. An overview of Activity Theory and its use in the research literature was explained in Sections 2.1.3.3.1 and 2.4. Second generation Activity Theory considers all the elements that are relevant to the current study and the choice of the unit of analysis is central to the implementation of Activity Theory (Nardi, 1996b). The details relating to the current project, were overlaid onto Engeström's triangle, and illustrated in Figure 3.1. In each section of the diagram is an influencing element(s) of the activity system.

Figure 3.1

Possible Activity System Model for Mathematics Education



Note. Adapted from "Expansive Learning at Work: Toward an activity theoretical reconceptualization" by Engeström, Y. (2001). *Journal of Education and Work*, *14*(1), p. 135. Copyright 2001 by Taylor & Francis Ltd.

In the current study, the research question investigated how students developed thinking skills, as supported by the curriculum. The students were the subjects, with the goal or object being the development of mathematical learning and thinking skills. The possible outcomes were ATAR scores to gain entry to university and expertise to support future work and life. The influences on the students included the tools, for examples the Study Design, textbooks, calculators, and the mathematical content. The rules surrounding the use of these tools also influenced their effectiveness. The community of teachers, peers, and other community members supports or creates tension through the division of labour towards the student's objects and outcomes.

An activity system was used to analyse relationships within the elements (for example, the tools, textbooks compared to assessments), and also between the elements (for example the relationship of the use of the assessment tools, contrasted to school rules). Activity systems support a qualitative approach, where the focus progresses through the elements to gain varying viewpoints, to attempt to improve the outcome of the whole (Jonassen & Rohrer-Murphy, 1999). The students as subjects form a starting point, while the object of the study is the development of mathematical and thinking skills. The activity is the mathematical learning activities both in and out of class. The other elements under consideration can be categorised as rules and tools, community, or division of labour. Third generation of Activity Theory would be necessary if the system was to be investigated from the point of view of several actors, for example teachers, schools or educational authorities, but as the focus of this study is the students as subjects, second generation of Activity Theory (Engeström, 2001) would provide an appropriate framework for analysing the data that emerges from the current study. The object of this activity system is the development of thinking skills, so a thinking framework needed to be selected, which is discussed next.

3.3.2 Two-Tiered Mathematical Thinking framework

In the current study, the development of thinking skills is an outcome within the activity system, so requires definition. The Two-Tiered Mathematical Thinking framework derived from several thinking frameworks was used for this study. While most thinking frameworks contain three to six levels of thinking, the general two categories of higher and lower order thinking were used in the final conclusions in several studies (David & Lopes, 2002; Holmes, 2012; Jones & Tarr, 2007; Näsström & Henriksson, 2008; Thompson, 2008). The current study also used the two broad categories, and while identifying this as a limitation, it also provided an appropriate starting point, where the aim was to determine if there was a breadth of thinking types.

The Two-Tiered Mathematical Thinking Framework can be used to describe Higher order thinking (HOT) and Lower order thinking (LOT) curriculum statements, textbook and examination problems. These categories stem from the Ziebell performance types, which is itself a combination of many cognitive demand frameworks (Ziebell et al., 2017). The analysis of the curriculum content also included elements of the SOLO framework (Biggs & Collis, 1982; Pegg, 2014). SOLO focuses on student responses to tasks, and considers responses which use multiple methods, interconnecting or used in new ways to be at a higher level (Boulton-Lewis, 1995). A summary of the Two-Tiered Thinking Framework used in this study is shown in Table 3.1.

Table 3.1

Two-Tiered Mathematical Thinking Framework

	Alignment Project: performance types	Blooms Revised Taxonomy (2002)	Porter (2007)	Webb (2005)	PISA (2009)	TIMSS (1997)	SOLO
Higher order thinking	Making connections				Connections and integration for problem-solving		Extended Abstract
	Non-routine problem-solving	Creating	Solved non-routine problems, make connections	Level 4 Extended thinking		Solving problems.	
	Mathematical Reasoning	Analyse Evaluate	Conjecture, analyse, prove	Level 3 Strategic Thinking	Mathematisation, mathematical thinking, generalisation, and insight	Justifying and proving	Relational
Lower order thinking	Communicat- ing	Understand	Demonstrate understand-ing			Communicat-ing	
	Performing	Apply	Perform procedures	Level 2 Skills & Concepts		Routine procedures	Multi structural
	Knowing	Remember	Memorise facts, definition, formulas	Level 1 Recall and reproduce	Preproduction, definitions, and computations	Knowing	Uni structural

Note. The first six columns are from *"Aligning curriculum, instruction and assessment"* by Ziebell, N., Ong, A., and Clarke, D. In T. Bentley & G. C. Savage (Eds.), Educating Australia: Challenges for the decade ahead. pp. 259. Copyright 2017 Melbourne University Press. The final column has been added for this study, SOLO (Biggs & Collis, 1991)

The Two-Tiered Mathematical Thinking Framework combines several thinking frameworks and was the framework used in the current study. This diverse, flexible framework was used for the curriculum statements, and the textbook and examination problems. The curriculum statements were classified by the key verbs, while the textbook and examinations problems were classified by the procedural complexity, repetition, and connections between concepts, as summarised in Table 3.2.

Table 3.2

Two-Tiered Mathematical Thinking Framework for Mathematics

Higher order thinking
Verbs include analyse, create, evaluate, investigate, interpret, compare
Making connections between different mathematical concepts
High procedural complexity, more than four steps or two decisions
Non-routine problem-solving, unfamiliar problems, original solutions
Lower order thinking
Verbs include calculate, collect, define, list, select, identify,
Low procedural complexity, few steps, formula, or decisions
Performing routine procedures
Repetition of problems

The two-tiered method of classifying textbook problems was trialled and reported on while comparing the probability review sections of Australian Senior Secondary textbooks between 1978 and 2016 (Ernst, 2018). It broadly classifies the mathematical content of all aspects of the curriculum into higher and lower order thinking skills, which can be used to indicate curriculum alignment, and hence support the curriculum aims. If the aspects of the curriculum do not align, or the aims of the curriculum are not included in all aspect of the curriculum, the objectives of the curriculum will not be realised (Biggs, 2012; Ziebell et al., 2017).

3.4 Methods

The purpose of this section is to provide specific details of the research process and methods of data collection and data analysis used in this case study. Firstly, the pilot study will be described, including how it informed the main study. This will be followed by the methods of data collection, and an explanation of the details of the data analysis process.

3.4.1 Pilot Study

A pilot study was conducted with interviews involving two students and one teacher, and the analysis of four textbooks. This was to trial the semi-structured interview questions to confirm their wording, determine how long the interviews would take, and test the voice recording equipment. This also trialled the data analysis methods. A survey was created to help recruit participants, with many of the same questions as the interview questions. Invitations to complete the survey were sent to students and teachers however very few replied. The survey was discontinued, and other recruiting methods were used.

Two post VCE students were recruited from an Education university course for the student component of the pilot study. As these students were in my Education classes, they were not able to be included in the main research project, they were however helpful in responding and commenting on the interview protocol. They answered questions about the teaching and learning experiences they had found most helpful and solved the two probability problems included in the interview protocol—the two-coins problem and the frequency table problem. These two students, one who had completed Specialist Mathematics and Mathematical Methods, and the other who had completed Further Mathematics, both helped with the rewording of the interview questions and provided insight into the interpretations and possible responses of future participants. For example, the students used the term *summary book* rather than *bound reference book*. Subsequently, the question, “Please tell me about your VCE mathematics classes, what was a typical class like?” had the prompts “Summary books, calculators, group work/individual, whiteboard, computers, teacher talk vs student talk…” added.

A recently retired senior secondary mathematics teacher was selected for the pilot study. He was chosen as a convenience participant (Yin, 2013). The pilot interview with this teacher was conducted with an expanded set of guiding questions which included questions about his senior mathematics teaching experience and descriptions of typical classes, learning and teaching activities, professional development and views on resources including calculators and books. These questions remained in the final interview protocol. In the pilot interview, there were many probability questions taken from the research literature. These became less effective as the interview went on, and the answers became brief and superficial. After consulting the literature and reviewing the answers from the pilot, the probability problems were narrowed down to two, the first known as the two-coins problem and the second known as the frequency table problem, which are described later in this chapter, in Section 3.4.2.3. The pilot interviews took two hours, which was too long to ask of participants. During the transcription process, it became obvious that I had interrupted the participants too much. I needed to remember the process was an interview rather than a conversation. This first pilot interview was unstructured and side-tracked onto educational topics unrelated to this study. As a result of this experience, a decision was made to email the questions to the teacher participants prior to the interview to enable them to prepare. The teacher interviews in the main study followed the protocol closely and the focus was on the questions posed and took less than an hour.

Four textbooks were examined to trial the analysis techniques. The textbooks for two Victorian senior secondary mathematics subjects were selected from both 1978 and 2016 and analysed as a comparison study for the Mathematics Education Research Group of Australia peer-reviewed conference (Ernst, 2018). The intended curriculum and four textbooks were analysed by classifying the probability review problems for procedural complexity and context (Vincent & Stacey, 2008), SOLO (Biggs, 1989) and the number of problems which required probability tables or a calculator. This conference paper was presented, and feedback received to inform the curriculum analysis of the final research project of this thesis. The analysis method using the procedural complexity and the SOLO levels of the review problems were similar, so a combined measure was developed for the final study, using a combination of several thinking frameworks. The *Two-Tiered Mathematical Thinking Framework* developed through review of the literature was used as the frame for analysis as outlined in Section 3.3.2.

The pilot study described here, informed the final research project regarding the semi-structured interview questions and the data analysis methods, which is now be explained.

3.4.2 Data Collection

This next section sets out in detail the data collection methods, which included document analysis and semi-structured interviews, as summarised in Table 3.3.

Table 3.3

Data Collection Methods

	Research questions	Method
Curriculum	*In what ways are thinking skills developed through the senior secondary Victorian mathematics probability curriculum?*	Document analysis of curriculum documents, Study Design, textbook and assessments concerning thinking skills, content, and context
Teachers and students	*What factors impact on the development of thinking skills through the teaching and learning of the senior secondary Victorian mathematics probability curriculum?*	Interviews with 14 senior secondary Mathematical Methods teachers recruited from external professional development programs
		Interviews with 20 recent VCE students from first-year university courses

The components of the research, the curriculum document analysis, the teacher, and student interviews, were carried out in parallel, to minimise one component influencing the others. The documents collected as the intended, implemented and attained curriculum were the VCE Mathematics Study Design, textbook and assessments. This study was conducted over a period of several years. The intended curriculum materials covered the 2016–2022 VCE Mathematics Study Design (VCAA, 2015a). The assessments came from the years 2016–2018, while the interviews were carried out in 2017 and 2018.

3.4.2.1 Curriculum Documents—Collection.

The documents collected as the intended, implemented and attained curriculum were the official curriculum material and the support material, textbooks and assessments used in the teaching and learning of the probability strand in Victorian Certificate of Education (VCE) Mathematics. The documents collected were:

- Foundation to 10 mathematics curriculum statements from the Victorian Curriculum website (VCAA, n.d.-f)
- VCE Mathematics Study Design 2016–2022 Mathematical Methods Units 1–4, (VCAA, 2015a)
- Textbook for Mathematical Methods Units 3 and 4.
- School Assessed Coursework (SAC) assessment tasks:
  - Two from the VCE Mathematical Methods support material (VCAA, 2019d).
  - Five commercially available SACs
  - Two supplied by participating teachers.
- Three sets of VCE Mathematical Methods examinations, 2016–2018 (VCAA, n.d.-a)

Many of these documents were available as open access online, for example the Study Design and the examination materials. The textbooks and SACs were purchased and collected from the participating teachers.

3.4.2.2 Interviews—Selection

Semi-structured interviews were conducted with students and teachers, using purposive samples (Merriam, 2014). A purposive sample is when potential participants are explicitly sought—in this case students and teachers— with the relevant qualifications and experience. The semi-structured interview questions, and Plain Language Information Statements for the students and teachers, are attached (see Appendices B to E).

The interview questions were similar for both the student and teacher participants. The interview protocol included background information, questions about teaching and learning practices, use of resources, attitudes, and perceptions of how and why such practices were used, and mathematical problems. Two mathematical probability problems were used to extricate the thinking of the student participants and the participant teachers' knowledge of their students' thinking. The two probability problems used were the two-coin problem and the frequency table problem, both of which came from the research literature, as described in section 3.4.2.3. The prompts were attached (see Appendices D and E). The interview questions were prompts for the discussion, which provided participants an opportunity to expand on their areas of interest. Semi-structured interviews allow the researcher to seek clarification of responses, thoughts and motivations which could not be observed but can be considered in interviews (O'Toole & Beckett, 2013).

Student participants (n = 20) were recruited from the first-year mathematics and statistics tutorials in 2017 and 2018 at the regional university in Gippsland. All the students had previously studied VCE mathematics. This cohort was selected because they would have had the time to reflect on their previous VCE mathematics in comparison to their current university experience. The post-VCE students were from an assortment of university programs including Engineering, Science, Medical Science, Arts and Education. They came from a variety of geographical areas within Gippsland. Current VCE students were not sourced due to the lack of time they would have had to reflect on their mathematics experience and due to the high-stakes nature of VCE. Students in first-year mathematics and statistics courses were approached in their tutorial classes, and interview times were scheduled and held in the university café. Students were offered a coffee or soft drink for their time. The interviews were voice recorded and took on average 30–40 minutes. Students were asked if they would then encourage their peers to be interviewed, using the snowball method (Bryman, 2012). The participating students appeared to be keen to share their thoughts on their Year 12 mathematics studies. Participants were involved in a range of university programs and had studied Year 12 mathematics in a range of Gippsland schools, which increased the degree of representation of the region.

The semi-structured interviews with the students began with general questions to confirm recent completion of VCE Mathematical Methods, followed by questions about their mathematics studies, including the use of calculators, textbooks and their perceptions of support received from their teachers. The students were then presented with the probability problems and asked to recount how they might solve these as explained in Section 3.4.2.3. Time was given for student participants to explain their thinking and link to any other comments about their classes. The voice recordings were transcribed by a combination of an automatic computer program (https://trint.com/) and me checking the recordings against the transcriptions several times. Student participants were asked if they were interested in receiving transcripts for the purposes of accuracy checking, but none indicated an interest.

VCE Mathematical Methods teachers (n = 14) were recruited through mathematics professional development courses. The Mathematics Association of Victoria (MAV) and Texas Instruments (calculator manufacturers) conducted professional development sessions in 2017 and 2018. I attended the professional development sessions where teachers were invited to offer their email contact details. The face-to-face interviews were arranged and conducted outside of school hours, in a café or library. Two interviews were conducted by phone at the request of the participants. Interviews were voice recorded with permission, on a mobile phone. The teacher participants of Mathematical Methods were from a wide variety of government schools in the Gippsland region, although the individual schools were not identified. These conditions were part of the ethical requirements of the Victorian Department of Education and Training approval process. These VCE mathematics teachers were also invited to share the support material they used, including schedules, assignments, and resources. The teachers were then asked to share details of the project with colleagues, and a few more participants were invited through this snowball method (Bryman, 2012). Teacher recruitment continued over two years, interviewing all eligible participants who volunteered. The aim was to have participating students and teachers with a variety of experience and from a wide geographical range within Gippsland. For example, a recent graduate teacher from far East Gippsland was specifically invited.

The semi-structured interview questions (Appendix E) and consent forms were emailed to the participants prior to the interview. The discussions varied in length but were less than one hour. Following a few general introductory questions about the teachers, their education and experience to confirm they had recently taught VCE Mathematical Methods, the interview protocol first asked about their teaching practices and the probability problems followed. The voice recordings were transcribed via a combination of an automatic computer program (https://trint.com/), with myself checking the recordings with the transcriptions several times. The transcripts of the interviews were then emailed to the teacher participants for accuracy checks, resulting in three adding comments to their transcripts or providing support curriculum material. The interviews were transcribed verbatim, but the excerpts included in this thesis were occasionally edited for clarity (Goldberg & Allen, 2015).

3.4.2.3 The Probability Problems

Two probability problems were selected and used in the student and teachers' interviews. The literature around the previous use of these problems and the misconceptions will be described. The relevant connections to the Study Design, textbooks and examinations will be compared. In Chapter 5, the student and teacher interview responses regarding these problems is explored. These problems allowed me to initiate a conversation with the student and teacher participants surrounding their VCE mathematics experience. Both problems were discrete random variable problems, labelled as the *Two-coins problem*, and the *Frequency table problem*. They were selected as they relate to problems which have been well used in the research literature (for example, Batanero et al., 2015; Chick, 2010; Fischbein & Schnarch, 1997; Pfannkuch et al., 2002; Watson & Callingham, 2014), and the probability concepts are developed throughout the intended curriculum across many secondary school years.

The exact wording of the *Two-coins problem* is shown in Figure 3.3. The context of the problem being set as a game played between family members was thought to be a context many students could relate to. Participants were asked to not only answer the problems, but also to explain the approach they used.

Figure 3.2

Mathematical Interview Probability Question 1—The Two-coins Problem

Question 1 - The two-coins problem.

Two fair coins are thrown

Mum wins if there are two heads

Dad wins if there are two tails

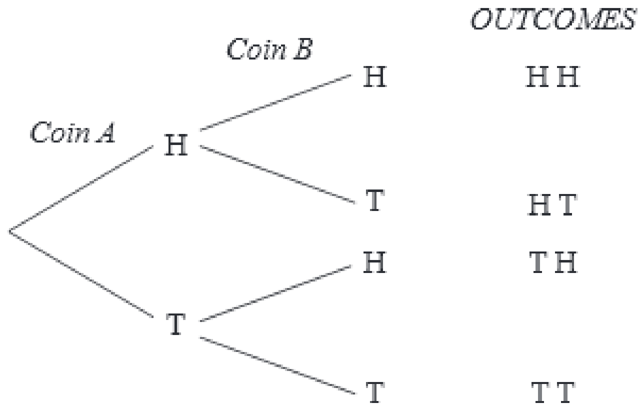
The kid wins if there is a head and a tail

a) What is the probability of mum winning?
b) If this game was played 1000 times, how many times would you expect mum to win?
c) Exactly?
d) Would you use a tree diagram, coins, or simulations?

The possible solutions to the problem will now be explained, compared with the research literature, with an explanation of possible misconceptions. For the first sub-question, the chance of mum winning with two heads when two-coins are thrown is ¼. There are several ways to answer this question, including a tree diagram, as in Figure 3.4, and a table or a list of the sample space (Chick, 2010). For one fair coin, the chance of a head is ½ or 50%. This event is independent of a second throw, so the probability of two heads from two throws is ¼ or 25%. Another way to look at the problem is the possible outcomes from the two-coins is: head-head, head-tail, tail-head, and tail-tail. Each of these four outcomes are equally likely, so the probability of head-head is one quarter.

Figure 3.3

One Possible Solution to the Two-coin Problem



A misconception around this problem is that there are only three outcomes, with the incorrect assumption that a head-tail is the same outcome as a tail-head. These misconceptions are called *equiprobability bias*, or lack of understanding of independence, as described by Garfield (2003). The tail-head and head-tail confusion could also be a language issue (Sharma, 2015). Another misconception commonly found with students about coin tossing is around subjective probability, that coins might take turns i.e., if it is a head on the first throw of the coin, it will be a tail on the second throw of the coin. This misconception is known as the *negative and positive recency effects* (Fischbein & Schnarch, 1997). Fischbein and Schnarch (1997) found a third of five-year-old children had this misconception, but this was reduced to 10% of Year 11 students, and 6% of college students. Additionally, some students might have the misconception that this coin, or player, is 'lucky' and will land with one side more often. These misconceptions are called *belief or human control* (Ang & Shahrill, 2014). In a study of Year 10 and 11 students, 25% had misconceptions around coin tossing, and half of these were around the belief misconception, where the student believed that luck or a higher power influenced the events (Ang & Shahrill, 2014).

The second part of this problem enquired into how many times you would expect mum, with two heads on the thrown coins, to win this game after 1000 plays? This involved multiplying the previous answer of ¼ by 1000. It would be anticipated that senior secondary mathematics students would answer this correctly, with 250. The third part of the coin problem investigated the concept of variation, and the law of large numbers (Batanero & Borovcnik, 2016; Fischbein & Schnarch, 1997). This was a new part of the VCE Mathematical Methods course in 2016. Due to randomness, it would be unlikely that in 1000 games of throwing two dice, exactly 250 heads would occur in any batch of games. The law of large numbers states, that as the number of trials increases, the closer the experimental probability will get to the theoretical probability. While the students were not asked to remember or explain the formula for confidence intervals, it would be anticipated they would understand the concept of the law of large numbers and estimate an appropriate range for the solution. This problem aimed to investigate students' probabilistic reasoning and understanding of randomness. This interview question also aimed to start discussions with the teacher participants about this new part of the Mathematical Methods course.

The final part of the *Two-coins problem*, inquired into how the topic of randomness, expected, experimental and theoretical probability were taught, in particular whether tree diagrams, simulations or experiments were used. The intended curriculum suggests that simulations and tree diagrams could be used in the teaching of these concepts (VCAA, 2015a). Research literature also reports that simulations support understanding and reasoning of probability concepts. For example, Watson and English (2015a) describe students' experiments with die and simulations with the computer program *Tinkerplots* (Konold & Miller, 2011), to investigate the variation of gaining a six on a die. They found that students demonstrated they understood the concepts of variation and expectations after their simulations. Watson and English (2015a) argue that with explicit teaching of probability language and the use of computer simulations, students could understand and explain their reasoning (VCAA, 2015b).

This probability problem was chosen for the interviews as the first part should be familiar to the students as it relates to curriculum statements from Years 8 to VCE Mathematical Methods. The latter part of the problem involved a new part of the subject since the changes in 2016. It had a context the students would relate to. While the questions involved whole numbers, which did not require a calculator, it still could have been confusing as it related to a misconception reported in the literature.

The second probability problem posed in the student and teacher interviews was the frequency table problem outlined in Figure 3.5. The student and teacher participants were asked to explain their thinking behind their answers and were asked if they thought the problem needed to be reworded to make it easier to understand. The problem was multiple-choice, which is common in the learning tasks and examinations for senior secondary mathematics.

Figure 3.4

Mathematical Interview Question 2—The Frequency Table Problem

Question 2 - Frequency table.

Smalltown	Male	Female	Total
Employed	580	645	1225
Unemployed	72	98	170
Total	652	743	1395

Using the information in the table above, which is the correct calculation for finding the percentage of females unemployed?

a) $\frac{98}{743}\times 100 \approx 13\%$ b) $\frac{98}{1395}\times 100 \approx 7\%$ c) $\frac{98}{170}\times 100 \approx 58\%$

Which answer is correct and why? What might be confusing about this question.

The correct answer is a) which is found by finding the number of females who were unemployed, out of the total numbers of females. The incorrect alternative of b) is the percentage of female unemployed people out of the total number of people. The incorrect alternative c) represents the percentage of unemployed people who are female. The difference between the alternative answers is in the wording, which is potentially confusing (Batanero et al., 2015).

Frequency tables have been used in research literature to analyse student and teacher understanding of probability and statistics for many years worldwide. Watson and Callingham (2015) described frequency tables as a link between probability and statistics, although Shaughnessy (2003) portrays the distinction as artificial. According to Batanero et al. (1996), frequency tables were first used in 1955 in a study by Piaget and Inhelder who found that children could understand these tables after approximately 15 years of age, although Batanero et al. (1996) reported that Smedslund (1963) found many adults could not interpret these tables. In their work with pre-university students, Batanero et al. (1996) described the frequency tables as fundamental to other concepts in statistics, probability and sciences and especially useful to uncover the thinking and reasoning of the participants. Reaburn (2013) used frequency table problems with her first-year university students and found no significant differences depending on their prior studies, which prompted the use of this problem in the current study. More recently these problem types were used in research with pre-service teachers (Batanero et al., 2015) and teachers (Watson & Callingham, 2014; Watson &

Nathan, 2010) to discover Pedagogical Content Knowledge (PCK) (Callingham & Watson, 2011; Shulman, 1986; Watson et al., 2008). PCK is the teaching knowledge required for particular content, for example relating to an understanding of which representations are effective in supporting student learning of a particular mathematical concept. Watson and Callingham (2014) used frequency table problems to create and trial a four-level statistics PCK framework, as described in the literature review.

Frequency table problems have been used in research for many years worldwide. A frequency table problem was included in a large longitudinal study in Australia involving 3000 students over grades 5–9 (Watson & Callingham, 2014), resulting in the construction of a statistical literacy scale. A frequency table was a research item in the large longitudinal study in the USA which produced the GAISE report *Guidelines for assessment and instruction in statistics education* (Franklin et al., 2007, p. 57). This report influenced the Australian curriculum review (ACARA, 2012) and described how to encourage statistical literacy, reasoning and thinking. Batanero et al. (2015) reported that using frequency tables was a less fearful method for students than the formula method of investigating conditional probability, and frequency tables with whole numbers were easier than using proportions (Pfannkuch et al., 2002). Being able to solve the problems by both formal and informal methods is desirable and represents a higher order of thinking using the SOLO taxonomy (Watson & Kelly, 2007).

Context is also very important, and Watson and Callingham (2015) found that students demonstrated different understanding of two ways tables depending on context. For example, they found in problems regarding smoking and lung disease, the students reported a stronger relationship between the factors, while with skin allergies and active lifestyle problems, they found less of a relationship, even when the numerical relationships were the same. Watson and Callingham (2015) suggested this was due to the students using their previous knowledge of smoking and lung disease rather than mathematics. Watson and Nathan (2010) also suggested teachers use contexts from the local media and figures which challenged the student's beliefs.

Suggestions regarding the use of frequency tables found in the research include:

1. Teachers should aim to understand the statistics, but also the common student errors and misconceptions. Students should be asked to explain their solutions, to inform teachers of their true understanding (Callingham & Watson, 2011).
2. Particular attention should be paid to the wording of problems (Batanero et al., 2015; Garfield & Chance, 2000).

3. Students understanding of proportional reasoning is a limiting factor, so the use of integer numbers rather than fractions or decimals is desirable initially (Shaughnessy, 2003).
4. Emphasis should be on statistical literacy and the development of statistical thinking (Franklin et al., 2007).
5. Real data should be used wherever possible (Franklin et al., 2007).
6. Conceptual understanding should be stressed rather than mere knowledge of procedures (Franklin et al., 2007).

These suggestions were considered when selecting the particular problems used for the student and teacher interviews. The frequency table problem was also in the curriculum statements from Year 8 to VCE Mathematical Methods, and both were trialled in the pilot study and deemed appropriate. This ends the description of the data collection methods with the data analysis methods utilised in the current study now presented.

3.4.3 Data Analysis

This section sets out the data analysis methods for the document analysis and the student and teacher interviews. The data sources are analysed separately in the results chapters, and then combined in the discussion chapter. The research questions, and the theorical frameworks were used as the basis for the analysis.

3.4.3.1 Curriculum—Analysis.

The senior secondary VCE Mathematical Methods curriculum (VCAA, 2015a) and the Foundation to 10 curricula from the Victorian Curriculum website (VCAA, n.d.-f) were considered for this study as the intended mathematics curriculum. The curriculum statements were classified according to the Two-Tiered Thinking Framework. Each statement was classified according to the verb in the statement. The list of verbs was taken from the various thinking frameworks described in the literature review, using the revised Bloom's taxonomy (Krathwohl & Anderson, 2002) as a starting point. Verbs including *define*, *solve,* and *calculate* were considered to represent skills associated with lower order thinking. Verbs such as *compare*, *design,* and *verify* were associated with higher order thinking. These statements were classified by myself and independently by another experienced teacher and researcher, with total agreement between us in relation to the classifications.

The implemented curriculum was viewed through the recommended textbook, which was written specifically for the VCE Mathematical Methods subject. The first probability chapter was selected for analysis to correspond with the two probability problems used in the interviews for the teachers and students. The probability review problems in the review chapters, were also analysed.

The textbook was the only document used as an indication of the implemented curriculum as students and teachers indicated this was the predominate resource used.

The textbook was arranged with explanations, example problems, and exercises for students to practice. The exercises and the review problems were analysed according to the order of thinking required, the context, and problem type. Classifying the textbook and examination probability problems could not be carried out using the key verbs, as most of the problems used the prompt: *'find the probability of….'.* As such, the thinking required was not evident from the statement or question. The potential responses to the problems needed to be analysed differently. Once the textbook and examination problems were solved by me, using all reasonable methods, the responses were classified as higher order thinking problems if they involved procedural complexity (four or more steps), required two or more concepts, and did not involve repetition of a similar example problem. This is consistent with the SOLO method of analysing tasks, and the TIMSS Video Study method of classifying problems (Hiebert, 2003; Vincent & Stacey, 2008). The order of thinking categories used were the Two-Tiered Thinking Framework, as defined in Section 3.3.2, and summarised in Table 3.1.

The student practice problems, as an indication of the implemented currculum, were also categorised according to the level of thinking expected and the context—abstract, games, personal or occupational context. The review chapters of the textbooks were also categorised into the problem type, whether a multi-choice problem, a short-answer problem intended to be attempted without a calculator, or an extended-answer, which involved several sub-problems. These categorises are summarised in Table 3.4.

Table 3.4

Categories for the Implemented and Attained Curriculum

Thinking	Context	Problem type
Higher order thinking	Abstract	Short-answer, technology-free
Lower order thinking	Games	Multi-choice
	Personal	Extended-answer
	Occupational	

The summative assessments for the senior secondary Mathematical Methods subject were also analysed according to the categories in Table 3.4, as an indication of the attained curriculum. Nine School Assessed Coursework (SAC) tasks and three years of examination papers were analysed. The SACs were described according to the thinking skills required, their style and context. The probability problems within the examinations were classified according to the Two-Tiered Thinking Framework, context, problem type and the proportion of probability problems. The aspects of the curriculum were described, analysed, and compared, to evaluate for internal and external consistency and alignment. The results are described in Chapter 4.

3.4.3.2 Interviews with Teacher and Students—Analysis

Teacher and student interviews were transcribed and checked several times with responses coded using NVivo 12 software, using a reflexive thematic analytic approach (Clarke & Braun, 2013; Teddlie & Tashakkori, 2009; Thomas, 2013). Deductively, reflexive thematic analysis starts with the research question(s), then codes are developed from the qualitative data and themes generated. In the current study, the themes were based around the activity system elements of rules, tools, tensions, division of labour and student and teacher agency for learning responsibility. Braun and Clarke (2020) argue that while the researcher might aim to be objective, bias cannot help but influence any analysis, thus it is important that potential bias is uncovered, discussed and used to advantage, with assumptions articulated. As such, using a theoretical framework to analyse the data "proclaims the bias" and improves validity (Braun & Clarke, 2020).

According to Braun and Clarke (2019), in a reflexive thematic analysis there are many questions to consider: the research questions, the questions asked in the interview and data collection process, and those that guided the coding and analysis of the data. These three groups of questions should not be the same, and simple summaries and superficial analysis results if the questions do correspond (Braun & Clarke, 2019). The difference between coding and finding themes is important in reflexive thematic analysis (Braun & Clarke, 2019). Codes are the small, practical ways to group the data and are based on the interview questions. The codes are modified as the data is read and reread, with themes created, beginning initially with patterns and interesting ideas from the data. In the current study, for example, the interview question might have been about the use of calculators, but the response of the participant demonstrated a procedural method of teaching as opposed to an inquiry style of learning. In this example, the code was *calculator*, but the theme was *rules*. Themes are common recurring patterns across a data set, clustered around a central concept, describing different facets of the concepts.

The teacher and student data were analysed separately, with themes identified and results provided in Chapter 5. The themes identified were then analysed together, compared, and contrasted to find common themes across the two data sets. Themes, as described by Clarke and Braun (2013), can be based on the data, theoretical frameworks and the assumptions of the researcher (Braun & Clarke, 2019). Braun and Clarke (2019) emphasise themes do not passively emerge from the data but are developed, constructed, or generated. They also warn against extending the qualitative data into quantitative data. For example, from the interviews with the students, five of the twenty students expressed the opinion that calculators were very helpful, but this should not be transposed to a generalisation of the population, or even of this sample, because other students might have agreed but not mentioned this in the interview. Interview data interpreted with reflexive thematic analysis might be described with numerical data, but this cannot be generalised.

The interview data were analysed through the activity system framework (Engeström, 2001), with references to the Two-Tiered Mathematical Thinking Framework and Curriculum Alignment. This supported the move from the pragmatic nodes established through NVivo 12 to deeper themes which explain the system. For example, the nodes of the interview data which are used in the results chapters of this research include, calculators, textbooks, and examinations. The themes mirror the theoretical framework more than the interview questions.

3.4.3.3 Combined Analysis.

Once the initial data analysis was completed, the final component was to compare and contrast the sets of data with reference to the research questions. The intended curriculum documents and textbooks were analysed regarding content, context, problem type, and discovering the amount and range of mathematical thinking, using the Two-Tiered Mathematical Thinking Framework. On the other hand, the interview data were purely qualitative and described the teachers' and students' perceptions of the themes. The themes of Mathematical Thinking and Curriculum Alignment were used within the overarching theoretical framework of Activity Theory, as indicated in Figure 3.3.3. Tensions, contradictions, and possibilities between the activity system elements were described and analysed, as these encouraged a change in the subject and system.

The combined analysis is presented in the discussion chapter, where the overarching research question is discussed and responded to. Tensions and possibilities are highlighted, and possible ways to improve the development of thinking skills in our senior secondary students are presented. The current study generated several recommendations relating to best practice to support curriculum developers, teachers, and students in the area of teaching and learning senior secondary mathematics, particularly in the probability topic.

3.4.4 Establishing Trustworthiness of Qualitative Data

Trustworthiness in interpretation is a potential issue with qualitative research, although there are methods that can be applied to help establish trustworthiness in relation to credibility and validity (Lincoln & Guba, 1986). Credibility addresses the consistency of the information and whether it is aligned with the participants' views (Nowell et al., 2017). The concept of authenticity, which Babbie (2011) suggests is interchangeable with credibility, refers to the ability of the research conclusions to make sense, or the extent to which the findings are consistent with the reality, all of which can be increased through the use of participant quotes, member checking and audit trails (Merriam, 2009). Validity involves measuring what you say you are measuring (Babbie, 2011). Braun and Clarke (2006) detail concerns with the interpretation of qualitative data in their work on reflexive thematic analysis which includes, oversimplification of interpretation, weak analysis and a mismatch between the data and the claims. These pitfalls can be reduced with the explication of the analysis method, and with detail around the use of the theorical framework.

Methodological issues can be minimised with clear descriptions of the data collection and analysis methods undertaken, or an "audit trail" (Guba, 1981, p. 83). An audit trail also supports transparency and coherence (Bryman, 2012). Reliability of case study research is important, so the protocol for the interviews and analysis methods need to be well documented (Yin, 2011) with an audit trail (Lincoln & Guba, 1986; Merriam & Tisdell, 2015). All interviews in the current study were voice recorded with permission, and the questions were forwarded to participants prior to the interview where possible, to provide more opportunity for the provision of considered and thoughtful responses. This can lead to participants providing biased responses, based on what they feel is most appropriate, however as these interpretations assume participants responded with their own perceptions, this is expected and is a component of the descriptions and explanation (Johnson & Christensen, 2014). As such, quotes from the participants' narratives are used in the descriptions and analysis in the current study to increase reliability (Yin, 2013). It is important that interpretation is free from undeclared bias (Babbie, 2011) with all decisions described and justified. This is where reflexivity plays a part, such that researchers use their own experiences to inform the research but in a clear and declared way (Nowell et al., 2017).

Interpretive validity can be increased if the researcher can understand the participant's viewpoints and inner world (Johnson & Christensen, 2014; Stake, 1995). Practical knowledge in the field of teaching mathematics is very important, and is supported by case study method (Flyvjerg, 2006; Thomas, 2011). As an experienced teacher in the system being studied, I had an advantage while interviewing and analysing this information due to familiarity with the language and some potential associated issues. However, I was also open to my own personal biases, and asked the participants to explain their thoughts in a detailed and descriptive way, rather than presume I understood them. As a previous teacher of secondary mathematics, I appreciated the situation and experiences of the teachers and students. The language was familiar, and I was able to establish trust with the participants. I enabled a rich and open dialogue during the interviews. Reflexivity was also ensured through reflecting on my own experiences to inform the research process (Braun & Clarke, 2019; Nowell et al., 2017).

The validity of qualitative research can be increased through strategies such as member checking (Creswell & Miller, 2000). As previously explained, the interview transcripts were returned to all teacher participants who were encouraged to check for accuracy, to ensure they reflected their thoughts without any misrepresentations. The participants also had an opportunity to add extra information, with several teacher participants taking up this offer. None of the student participants were interested in reviewing their transcripts, despite being offered this opportunity. Misinterpreting cause and effect can be a problem in interpreting interview data (Yin, 2011) but can be minimised by member checking, suggesting rival explanations, and pattern matching, which is where similar ideas are presented by several participants. Each of these strategies were utilised during the data analysis stage of the current study.

There can be a likelihood of bias towards verification when utilising case study methodology (Flyvbjerg, 2006). In the interpretation of the large quantity of data, almost any findings could be highlighted regardless of methodology (Merriam, 2014). Verification bias and data interpretation comes down to the integrity of the researcher. According to Tashakkori and Teddlie (2003), collecting data from a variety of sources ensures that pre-existing assumptions from the researchers are less likely. In the current study, interviews with teachers and students, and analysis of the written curriculum triangulated the information and created a deeper study (Creswell & Miller, 2000). To minimise interpretation bias in this project, a critical peer, who was a fellow independent researcher, critiqued the interpretation methods and cross analysed a sample of the data interpretation (Yin, 2011). Another way to reduce research bias in qualitative research is to look for counterexamples (Creswell & Miller, 2000) which was a feature of the analysis phase of the current study.

Peer debriefing was also carried out between myself and another experienced non-participating teacher to enhance the credibility of my findings. My interpretations, alternatives and counterexamples were discussed, and coding was checked. For example, the coding of the curriculum documents was independently coded and checked by the teacher to check for consistency. Additionally, some of the participant quotes were checked for other possible interpretations, for example, in relation to comments about bound reference books, which I had interpreted as being used by some teachers as a teaching and learning strategy, but the experienced peer suggested that these books are generally used as a revision tool only. The transcripts were then reread with these alternative interpretations in mind.

Case studies can become large studies with a multitude of data sources from a variety of research methods. This was a concern during the implementation of the current study, thus it was limited to a local geographic area and purposively sampled (Merriam, 2014). A small but varied selection of teachers and students assisted in containing the documentation and analysis required, while still enabling the discovery of the wide range of enablers and blockers to the implementation of a curriculum which encourages higher order thinking. After a range of participants were interviewed, the diversity of the participants was considered and it was decided to broaden the participant group through inclusion of a young, less experienced teacher in the far east of the Gippsland region, and an extremely high scoring VCE student.

A lack of generalisability is an acknowledged limitation of small qualitative case studies (Stake, 1995; Yin, 2011) as the participants are not randomly selected and are often few in number. In relation to the current study, the teacher and student participants were selected to gain a variety of views (Thomas, 2013). While this case study involved an investigation into one domain of mathematics in one geographic location, it was anticipated however that the knowledge gained would be valuable for the wider mathematics field and other geographic areas. The results may not be directly transferrable, but the identification of tensions and possibilities to the development of higher order thinking skills through the curriculum-teacher-student relationship is worthy of consideration in other areas. Stake (1995) described two types of case studies in relation to generalisability. First, an intrinsic case study, which is popular with education program evaluators, where the primary interest is to understand a specific case, while a secondary goal is to understand the more general situation (Johnson & Christensen, 2014; Punch & Oancea, 2014), similar to the aims of the current case study. Second, Stake (1995) describes instrumental case study as a means for investigating the bigger questions. Yin (2018) identified a third, collective case study, where multiple cases (Yin, 2018) are investigated and compared. The current project utilised a fourth type, a multiple embedded case study design, where a big picture question was examined from several embedded viewpoints in parallel and then in comparison (Yin, 2013). Case studies such as this project, do not attempt to be generalisable or repeatable (Yin, 2018), but a snapshot of a situation at a time and place. Issues and possibilities may however be relevant to other situations.

3.5 Ethical Considerations

Ethical considerations are fundamental to any form of research from a legal and moral viewpoint. Research needs to be centred on ethical design and implementation (Babbie, 2011). For the current study, ethics approval was gained from Federation University Australia Human Research Ethics Committee (HREC) (see Appendix F), and further permission to conduct research from the Victorian Department of Education and Training (DET) (see Appendix G). The project was also informed by the requirements set out in the National Statement on Ethical Conduct in Human Research (NHMRC, 2018). Ethical concerns with data collection of qualitative research relate to issues of access, consent, confidentiality, recognition and social responsibility (Babbie, 2011; Bryman, 2012; Lincoln & Guba, 1986). These concerns are addressed here.

The current study involved teachers working within government schools under the direction of the Victorian DET. However, as the identity of the schools was anonymous, the permission of principals was not required, but teacher participants were to be approached outside of the school environment, and interviews were to take place in public cafes or libraries. Teachers were invited via a casual conversation at regional professional development sessions. Appointments were made via email so the teachers could feel more comfortable with accepting or rejecting participation. Plain Language Information Statements were provided, and Consent Forms were read and completed by all participants prior to the interviews. The teacher interviews were conducted face to face although the option of phone interviews was available if more convenient for the teachers. Transcribed teacher interviews were emailed back to the participants for accuracy checking.

The student participants were post VCE students studying at a regional university in Gippsland in the Engineering, Science, Medical Science, Arts and Education faculties. None of the students were in classes that were currently being taught by myself, to avoid undue influence. Students were interviewed in the local university café. Participants were asked not to refer to their secondary schools, teachers, or peers by name and if they did, these details were not transcribed. Teachers and students were given pseudonyms. Any data that could have identified students, teachers or schools were not included. Participation was voluntary with the option to withdraw at any time until data collection was complete. The interviews took less than one hour, and no judgement was expressed by myself, within the interviews.

The intended curriculum material was coded by me and another independent researcher and recoded after 6 months, to demonstrate consistency. The title of the textbook was not used. Overall, every effort was made to ensure that the standards set out in the National Statement on Ethical Conduct in Human Research (NHMRC, 2018) and all the specific requirements associated with the ethics approval for the current study were met, to ensure that the experience for all participants, and the collection and analysis of the data provided, was as ethical as possible.

3.6 Chapter Summary

This chapter presented the methodological framework within a case study design that was used in the current study. The research design was underpinned by a constructivist epistemology with an interpretivist perspective and was conceptualised within the theoretical frameworks of Activity Theory, using TIMSS curriculum alignment and the Two-Tiered Thinking Framework.

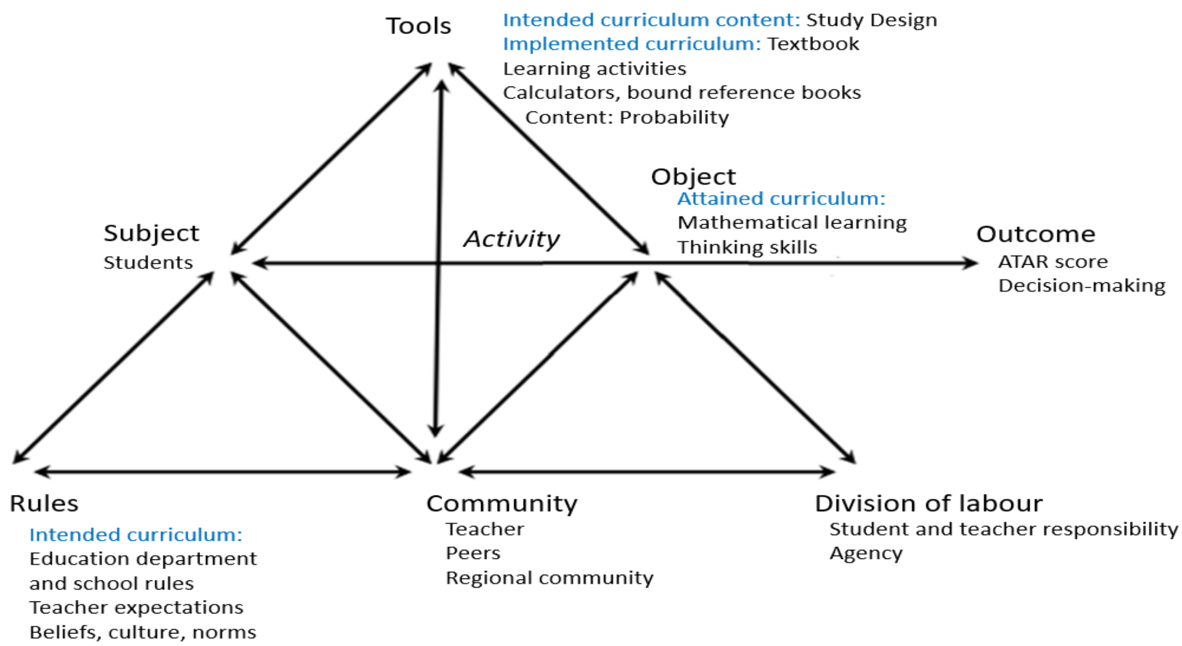
Also included in the chapter were the issues and solutions surrounding the theorical perspective underpinning the study, the conceptual frameworks used, and the interactions between them. This chapter also described the methods of collecting and analysing the data, both the document and interview data. Finally issues around trustworthiness and ethical considerations were discussed. The next two chapters present the findings from the data analysis. In Chapter 4, the document analysis details the three aspects of the curriculum, followed in Chapter 5 by the results of the analysis of data provided in the student and teacher interviews.

Chapter 4: Findings—Curriculum Documentation

This chapter presents the results of the document analysis of Victorian senior secondary mathematics curriculum associated with the study area of probability within the subject of Mathematical Methods. Three sections are presented. The first corresponds to the intended curriculum which analyses the Victorian Certificate of Education (VCE) Study Design (VCAA, 2015a), and the mathematics components within the Victorian Curriculum (VCAA, 2015b), with a focus on the probability topic. The second relates to the implemented curriculum and provides a description and analysis of relevant sections within the recommended mathematics textbook used for Units 3 and 4 of Mathematics Methods, as a tool for implementing the curriculum. The third presents an analysis of the summative assessments used in the senior secondary mathematics subjects, which provided a form of measurement for the attained curriculum. These three aspects of the curriculum correspond to the TIMSS Curriculum Model (Mullis & Martin, 2013), and are investigated and compared regarding the content, context and thinking skills targeted for development. As well as analysing curriculum alignment, the documentation has been analysed through the overarching lens of the Activity Theory (AT) framework (Engeström, 2001), as described in Chapter 3. To briefly recapitulate, Engeström's (2001) framework of activity systems proposes that student learning is influenced by many elements including tools, rules, community, and division of labour, as illustrated by Figure 4.1.

Figure 4.1

Influence of the Intended Curriculum using an Activity System



Note. Adapted from "Expansive Learning at Work: Toward an activity theoretical reconceptualization" by Engeström, Y. (2001). *Journal of Education and Work*, *14*(1), p. 135. Copyright 2001 by Taylor & Francis Ltd.

These three aspects of the curriculum (intended, implemented and attained) fit into the activity system, with the intended curriculum acting as tools with rules, the implemented curriculum acting as a tool (with rules and division of labour), and the attained curriculum acting as an object. The students are the subject of the activity, which involves transformation of student learning and knowledge towards the object, which is development and attainment of mathematical and thinking skills. Learning and change occur through the tensions and contradictions within and between elements of an activity system. Tensions are not always negative in this context. The results of the document analysis of the three aspects of the curriculum are now described and cross referenced with the elements within the activity system (Engeström, 2001).

4.1 Intended Curriculum

This section presents the intended mathematics curriculum for the VCE and how the Victorian curriculum prepares students for VCE mathematics. For the purposes of this study, the intended curriculum is considered to be a tool within the activity system in Figure 4.1, which supports (or hinders) the students' attainment of their object and outcomes. The intended curriculum has rules, which also influence the students' attainment of their object (development of mathematical and thinking skills) and outcomes (pass in VCE MM, the option of attending university, and hopefully improved decision making in the future).

The 2016–2022 Victorian Certificate of Education (VCE) Mathematics Study Design (VCAA, 2015a) is the official intended curriculum document for VCE mathematics. Associated support documents include: Advice to Teachers, Study Summary, VCE Mathematics School-assessment Coursework (SAC) resource 2000, and the VCE and VCAL Administration Handbook, all available on the VCAA website (VCAA, 2019d). Background information on the VCE and the senior secondary mathematics subjects was included in Chapter 1 Section 1.2.

4.1.1 Probability Content in Mathematics Intended Curriculum

The focus in this section will be on the mathematical content area of probability, and how it develops through the intended curriculum levels. The content areas of the curriculum fall into the tools element of an activity system, as the content is a vehicle for developing thinking skills, as well as important in its own right as a set of mathematical skills to support students as they move onto further study and the workforce. The results of the intended curriculum analysis regarding the content will be compared to the implemented curriculum as encompassed in the textbook, and the attained curriculum as indicated by the assessments.

As previously stated, the Victorian Curriculum (VCAA, 2015b) is the official curriculum for the Years Foundation to 10 in Victorian schools, with strands covering number and algebra, measurement and geometry, and statistics and probability. Within senior secondary mathematics, the topic of probability was predominantly covered within the senior secondary subject of Mathematical Methods (see Table 1.1), which lists the proportion of curriculum statements relating to the content area of probability from Year 7 to the final year of schooling.

Table 4.1

Proportion of Curriculum Statements Relating to Probability in each Year Level

Year	Proportion of content statements relating to probability	Percentage of content statements relating to probability %
7	2/34	6
8	3/29	10
9	3/26	12
10	2/28	7
MM12	9/50	18
MM34	17/42	40

Note. MM12 is generally studied in Year 11. MM34 is generally studied in Year 12. In MM12 and MM34 the curriculum area of study is called Probability and Statistics, but the content is predominantly Probability. The division between Probability and Statistics is undefined.

Table 4.1 illustrates that the probability content in Years 7–10 of secondary school is varied but less than in the senior secondary level, with an increase in the VCE Mathematical Methods units and particularly MM34. Although each point in the curriculum does not attract the same focus, the fact that in MM34, 17 of the 42 curriculum statements relate to the Probability and Statistics area of study indicates that they form a substantial component of the course. This is a large increase from the two curriculum statements relating to probability out of 28 in Year 10. This emphasis on the topic of probability in MM34 was further supported by the rule that at least one of the three SACs needed to cover the Probability and Statistics area of study.

Probability content developed slowly through Years 7 to 11, with a large increase in complexity in Year 12. This is the result of a range of new concepts being introduced including discrete and continuous probability distributions; functions which incorporate calculus integration; probability distribution including the binomial distribution and normal distribution; sampling and confidence intervals; and the relationships between all these components. CAS calculators are recommended to support the use of probability distributions, graphing and calculus. The increase in number of interconnecting concepts resulted in the probability content escalating in complexity in the final year of schooling, in MM34. As the number of probability concepts and their complexity increases in senior secondary subject of Mathematical Methods compared to the lower secondary years, this impacts on the development of thinking and reasoning within these concepts, as outlined in the next section.

4.1.2 Thinking Skills in the Mathematics Curriculum

Mathematical thinking and reasoning are promoted in the Victorian mathematics curriculum documentation through Years Foundation–10 and onto the senior secondary mathematics. Across Foundation–10, the curriculum states "The proficiencies of Understanding, Fluency, Problem Solving and Reasoning are fundamental to learning mathematics and working mathematically and are applied across all three strands Number and Algebra, Measurement and Geometry, and Statistics and Probability" (VCAA, 2015c, para. 1). Reasoning is defined as the ability to explain thinking (VCAA, 2015c). The focus on encouraging a variety of thinking skills continues into the senior secondary curriculum, with the requirements for application tasks, problem-solving and modelling as the school assessed coursework (VCAA, 2015a). This is further supported by the scope of the study of all VCE mathematics which states "Essential mathematical activities include: conjecturing, hypothesising and problem posing; estimating, calculating and computing; abstracting, proving, refuting and inferring; applying, investigating, modelling and problem-solving" (VCAA, 2015a, p. 6), which shows that use of a variety of thinking skills are expected.

The curriculum statements in the Foundation–10 Probability and Statistics strand and the Mathematical Methods Study Design, were classified according to the Two-Tiered Mathematical Thinking Framework as described in the Methodology Chapter, Section 3.3.2 (See Table 3.1). The proportion of outcome statements classified as lower and higher order thinking skills are displayed in Table 4.2, with examples.

Table 4.2

Percentage of Probability Curriculum Statements Involving Higher and Lower Order Thinking, with Examples

Education level	LOT %	Example LOT statement	HOT %	Example HOT statement
7-10	57	Year 10- Describe the results of two- and three-step chance experiments, both with and without replacements.	43	Year 10 - Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data
VCE MM	55	MM34- calculate sample proportions and confidence intervals for population proportions	45	MM34 - inferences from analysis and their use to draw valid conclusions related to a given context

Note. LOT is lower order thinking, while HOT is higher order thinking.

For probability and statistics in Years 7–10, 43% (n = 17 of 30) of statements were classified as involving higher order thinking skills, which is statistically similar to senior secondary VCE Mathematical Methods having 45% (n= 29 of 65) of the statements being classified as involving higher order thinking. Using a Chi-squared test for independence, the higher order thinking skills of the two groups of curriculum statements were found to be similar ($\chi2 = 0.069$, df = 1, $p= 0.793$) (Moore & McCabe, 2003). There was no evidence of a difference in the level of thinking between the probability sections of Years 7–10 and VCE MM. As such, the intended mathematics curriculum for Victoria at Years Foundation to 10 and VCE aims to support the development of a diverse range of thinking skills.

4.1.3 Summary of Intended Curriculum

The aim of this section was to analyse how the intended curriculum influences the development of mathematical and thinking skills, in its role as an influential tool within an activity system (Engeström, 2001). The rules of the intended curriculum (for example the use of calculators and assessment types) also influence the system and are described in the next sections of this chapter. The probability content of the various subjects was presented and found to form a small component of Years 7–10, and a large component of senior secondary Mathematical Methods, especially Units 3 and 4. The probability content increases in volume and complexity in the final year of Mathematical Methods. Primary, secondary, and senior secondary mathematics intended curriculum aimed to encourage the development and use of a range of thinking skills. The thinking requirements in the probability curriculum, utilising the measure of the Two-Tiered Mathematical

Thinking Framework, increases between the primary and secondary level, and then is consistent in senior secondary mathematics.

4.2 Implemented Curriculum

As previously discussed, mathematics textbooks are an indication of the implemented curriculum, and used as a tool to support student learning, and also to support teachers in their implementation of the curriculum (Remillard & Kim, 2017). This is illustrated by the activity system in Figure 4.1. This section analyses the textbook content, context and levels of thinking expected to solve the textbook problems. The method used for analysing the textbook is then carried over to the SACs and examinations in the next section on the Attained Curriculum.

4.2.1 Description of the Textbook

One textbook was referred to by the VCAA online support material and used by most teachers and students interviewed in the current study. The textbook was written specifically for the Victorian Mathematical Methods units. Some key features of the textbook, important to this study, include:

- Colour coded descriptions of the mathematical theory were provided, as well as examples, formula and many practice exercises. Many problems were referenced back to specific examples to scaffold students.
- Examples were explained with a mixture of written language and mathematical symbols, with and without calculators.
- Problems included a mixture of multiple-choice, short-answer and extended-answer both with and without technology requirements.
- Answers were provided to all problems, but no worked solutions were available.
- Between 2–4 extra online support activities were included with each chapter, involving a combination of videos, quizzes, animations, or skill sheets. These were introductory learning activities for basic concepts, e.g., animations demonstrating a tree diagram and a video explaining a conditional probability.

4.2.2 Textbook and Context

The context of the mathematical examples and exercises is important as this can help or hinder students' understanding of mathematical concepts (Pfannkuch, 2011). The problems from the selected sections of the textbook were classified into four categories namely, abstract, games, personal and occupational. To explain the categories, examples of probability problems classified into these four categories are listed in Table 4.3.

Table 4.3

Context Categories Definitions with Examples

Category	Definition	Example
Abstract	No attempt is made for any context	Given that for two events A and B, $\Pr(\mathrm{A}) = 0.6, \Pr(\mathrm{B}) = 0.3$ and $\Pr(\mathrm{A} \cap \mathrm{B}) = 0.1$, find: $\Pr(B\|A)$, $\Pr(A\|B)$.
Games	Questions were referring to coins, dice, marbles or spinners	Two-coins are tossed, and a die is rolled. Use a tree diagram to show all the possible outcomes.
Personal	A question which the VCE student might relate to personally	Suppose the probability that a student owns a smartphone is 0.7, the probability that they own a laptop is 0.6, and the probability they own both is 0.5. What is the probability that a student owns either a smartphone or a laptop or both?
Occupational	Context to do with businesses or government	A business consultant evaluates a proposed venture as follows. A company stands to make a profit of $10,000 with a probability 0.15, to make a profit of $5000 with probability 0.45, to break even with probability 0.25, and to lose $5000 with probability 0.15. Find the expected profit.

These categories are similar to those of PISA (De Bortoli & Macaskill, 2014; Ernst, 2018; Siswono et al., 2017). Contexts that students could relate to are seen as more meaningful and desirable (Hiebert, 2003). For the purposes of the current study, the textbook, SACs, and examination problems were all categorised in this manner.

In the current study, a few sections of the textbook were chosen as most suitable for analysis. The first chapter of the probability section was chosen as it related to the problems asked of both student and teacher participants in the interviews, using discrete probability. The probability problems within the review chapters were also chosen as they contained a range of areas of study and were the chapters which included formative assessment problems for students to use to gauge their readiness for the examinations. The textbook review chapters are now compared to the SACs and examinations. Table 4.4 below shows the number of problems in the selected sections of the VCE MM34 textbook, and their context.

Table 4.4

VCE MM34 Textbook Probability Exercise Categorised by Context

Chapter on discrete probability		No. of questions	Abstract	Games	Personal	Occupational
A	Sample spaces and probability	17	0	6	8	3
B	Conditional probability and independence	19	4	2	10	3
C	Discrete random variables	19	3	11	5	0
D	Expected value	18	6	7	3	2
Sum mary	Short-answer	11	6	2	1	2
	Multiple-choice	7	7	0	0	0
	Extended-answer	13	1	2	6	4
		104	**27**	**30**	**33**	**14**

	Review chapter	Probability questions	Abstract	Games	Personal	Occupational
A	Short-answer	8	2	1	5	0
B	Multiple-choice	7	5	0	2	0
C	Extended-answer	14	5	1	2	6
		29	**12**	**2**	**9**	**6**

Overall, the textbook included many probability exercises for students to attempt, extra review problems at the end of each chapter and at the end of the unit. It contained a variety of problems with an abstract, games and personal context. In the review chapter at the end of the textbook, approximately one third of the problems (29) involved the content of probability. This is slightly less than the 40% of curriculum content statements from the MM34 intended curriculum statements.

There were fewer problems that were classified as occupational. The textbook contained multiple-choice, short-answer, and extended-answer problems, to correspond with the design of the examinations. Problems were also explicitly classified as being answerable with or without the use of technology, which is consistent with the outcomes of the intended curriculum and the format of the examinations. The results of the textbook analysis will be compared to the context in the SACs and examinations later in this chapter.

4.2.3 Textbook and Thinking

This section involves the classification of the probability questions in the textbook according to the thinking skills expected in student responses to the learning activities. Within the activity system (see Figure 4.1), the textbooks are a tool towards the object of the development of thinking skills. The intended curriculum statements were classified as involving higher or lower order thinking according to the verbs used in the curriculum statements using the Two-Tiered Mathematical Thinking Framework. For example, a lower order thinking task might require students to list, describe or apply a mathematical concept. A higher order thinking task might ask students to compare, design or justify a mathematical concept. When analysing textbook problems and other learning and assessment tasks, other factors need to be taken into account. For example, prior knowledge is an important factor when classifying learning and assessment tasks. If an activity is a repeat of a previous example or activity, the activity might require the lower order thinking classification of remembering, regardless of the verb used in the problem or the degree of difficulty of the task. The number of steps in a task, and the number of decisions also affects the degree of difficulty of a task, hence the Two-Tiered Mathematical Thinking Framework was used to classify the textbook exercises, SACs and examination problems. The sections of the textbook used in the context Section 4.2.2 were used for the thinking analysis, that is the first probability chapter involving discrete probability and the review chapters. Table 4.5 shows the number of exercises classified by their levels of thinking.

Table 4.5

VCE MM34 Textbook Problems Categorised by Lower Order Thinking (LOT) and Higher Order Thinking (HOT)

Chapter on discrete probability		No. of exercises	LOT	HOT
A	Sample spaces and probability	17	17	0
B	Conditional probability and independence	19	18	1
C	Discrete random variables	19	17	2
D	Expected value (mean), variance and standard deviation	18	14	4
Chapter summary	Short-answer	11	7	4
	Multiple-choice	7	6	1
	Extended-answer	13	5	8
		104	**84**	**20**
Review chapter		PROB	LOT	HOT
A	Short-answer	8	7	1
B	Multiple-choice	7	5	2
C	Extended-answer	14	3	11
		29	**15**	**14**
	Combined chapters	133	99	34

As illustrated in Table 4.5, there were more exercises in the chapter summary and review section relating to probability which were classified as requiring higher order thinking, but also a higher proportion of extended-answer problems with multiple parts covering a variety of concepts. Some of the extended-answer problems combined probability concepts, for example, starting with a discrete probability and moving to a binomial distribution. They were classified as higher order thinking problems as they combined concepts, however, the problems were scaffolded with the use of keywords and references back to examples. Each of the individual parts could be considered repeats of examples but when combined, they become complex. The majority of exercises in the first probability chapter of the textbook could be solved using lower order thinking skills. The thinking skills required increased in the chapter summary, and then increased again for the review chapter, where appropriately, around one half of the exercises required higher order thinking skills.

4.2.4 Summary of Textbook Section

The textbook recommended for the teaching of senior secondary Mathematical Methods, which acts as an indication of the implemented curriculum, has been described and analysed, with a focus on the content, context, problem type and thinking skills involved. The textbook is a tool within the activity system, to support student learning. The teacher and student roles in the use of the textbook will be detailed in Chapter 5.

The context of the textbook exercises included a range of abstract, game related, personal, and occupational contexts, attempting to cater to a range of student interests, support student learning, and correspond to the examinations. There were very few probability problems of the problem-solving and modelling type, which were required in the SACs. The textbook contained a variety of problem types, multiple-choice, short-answer, and extended-answer problems, with and without technology, to correspond with the examinations. The textbook had little online support material, and none to support the more complex concepts. It covered all the content required by the official curriculum, except there were no problems that required the use of simulations as stipulated by the Study Design.

The textbook as a tool of the activity system (see Figure 4.1), was also described and compared to the types of thinking required to solve the mathematical problems, which is the object of the activity system. The Two-Tiered Mathematical Thinking Framework was used to classify the textbook problems. The review section of the textbook included more higher order thinking problems than the earlier chapters, especially extended-answer problems. As such, students who completed the chapter exercises would receive limited experience in a variety of thinking skills, whereas students who completed the review exercises as well, experienced a range of thinking skills. Within the review exercises, the higher order exercises tended to be at the end of the section. This supports that how the textbook is used by students and teachers has an influence on its effectiveness as a tool for developing higher order thinking skills.

The previous section described and analysed the intended curriculum, as described by the VCAA (2015a, 2015b), in its role of rules and tools of supporting the activity of the development of thinking skills. This section described and analysed the recommended textbook in its role as a tool to support student learning as an indication of the implemented curriculum. The next section examines the assessment as an indication of the attained curriculum. These sections will be compared in the Chapter 7.

4.3 Attained Curriculum

School assessed course work and external examinations are an indicator of the attained curriculum. The attained curriculum forms the object of the activity system, shown in Figure 4.1, which includes the development of both mathematical and thinking skills. This section describes and analyses the school assessed coursework (SACs) or internal assessment tasks and the end of year examinations of VCE Mathematical Methods.

4.3.1. School Assessed Coursework

SACs can be used as an indication of the attained curriculum. They can also be used as learning tasks and as formative and summative assessment. The SACs are tools for student learning and have implementation rules surrounding them, as part of the activity system.

One of the three SACs for Mathematical Methods Year 12 (MM34) was required to focus on the area of probability and was in the style of a problem-solving or modelling task (VCAA, 2015a). The probability task was required to be of 2–3 hours duration over the period of one week and completed in class to ensure it was the student’s own work. SACs rules aim to encourage students to demonstrate a wider range of skills than those demonstrated in examinations, as indicated by this extract:

> School-assessed Coursework enhances the validity of student assessment by providing the opportunity for a context to be explored mathematically in greater depth and breadth than is possible in an examination, with non-routine and open-ended elements and aspects engaged in more fully. (VCAA, 2017b, p. 1)

Prior to 2000 (VCAA, 2019d), students were encouraged to take the SACs home to complete. While this generally made for higher quality work, it also provided problems relating to authenticity. Authentication was addressed by a small test on the content of the assessment task, but with a different context. As the use and abuse of the internet increased, the rules around the tasks were modified so they were completed within class time under test conditions (VCAA, 2019b). This is relevant as Activity Theory proposes that the history of an activity needs to be considered as it influences the activity system (Engeström, 2001).

The probability SACs which are analysed here, comprise two samples from VCAA, five commercially available SACs and two SACs supplied by teacher participants, as described in Table 4.6.

Table 4.6

Context, Style and Marking Scheme of SAC Samples

SAC source	Context	Style	Marking
VCAA	Abstract graphing	Modelling, simulations with cards	Rubric
VCAA	Politician popularity	Graphing, simulations with technology	Rubric
Commercial	Basketball	Short-answer examination style Two independent sections with and without technology	Marks
Commercial	Football	Problem-solving	Rubric
Commercial	Solar activity	Modelling with open-ended section	Rubric
Commercial	Jellybeans & school uniforms	Extended-answer examination style	Marks
Commercial	School uniform	Extended-answer examination style	Marks
Teacher	Music store	Short-answer examination style Three independent sections	Marks
Teacher	Eleven different personal contexts	Short-answer examination style Three independent sections	Marks

Table 4.6 shows that all but one of the analysed SACs had contexts which utilised experiences generally relatable to senior secondary students, the exception being the VCAA SAC with an abstract context. The VCAA sample SACs (VCAA, n.d.-b, n.d.-c) were open-ended tasks which involved a practical simulation section. They were the only SACs with a simulation section. Five of the nine SACs were of the same style as the examinations, which is contra to the rules of the VCAA, which require open-ended problem-solving or modelling tasks (VCAA, 2015a). Three of the SACs were broken into two or three independent sections to be completed over several days, with one commercial SAC having independent sections with and without the requirement for use of technology.

Four of the analysed SACs had a marking scheme involving a rubric aligned with the three Outcomes: skills, applications and technology, as recommended by the Study Design. The other five SACS had marking schemes where each part of each problem was allocated a mark in the same style as the examinations, which was against the recommendations for SACs. This contradiction was attempted to be overcome by the use of a cross-referenced spreadsheet which broke the SAC problems into the three Outcomes, as provided by some of the commercially available SACs.

The details of how the marks of the SACs should be broken down into the Outcomes is in Table 4.7, which all show the level of thinking skills expected by the rules of the Study Design (VCAA, 2015a).

Table 4.*7*

Expected Level of Thinking for the Probability SAC

Outcome	Description	Mark	%	Level of thinking
Outcome 1	Skills	7	28	LOT
Outcome 2	Application	10	40	HOT
Outcome 3	Technology	8	32	LOT
	Total	25		

The probability SAC accounted for 25% of the total SAC marks. The marks were required to be allocated to the Outcomes as detailed in Table 4.7, with 40% of the marks for this SAC expected to be for Outcome 2, where students needed to apply, analyse and discuss applications in non-routine situations, which are tasks that align with higher order thinking skills. To demonstrate Outcome 1, students needed to define, explain, and apply mathematical content. Outcome 3 required students to select and use technology, the CAS calculators, to solve mathematical problems. Outcome 1 and 3 tended to involve lower order thinking skills. These requirements meant that at least 40% of the marks should have been allocated for higher order thinking skills.

Several of the SACs did not align with the requirements of the Study Design. The sample SACs on the VCAA website provided open-ended inquiry tasks with simulations, different from the examinations and requiring a range of thinking skills. Two of the commercial sample SACs were structured like extended-answer examination problems but had higher order thinking components with prompts to compare, comment and explain the mathematical concepts. Three of the commercial SACs and the two tasks created by the teacher interview participants were the most similar to the examinations. Teachers' comments regarding which SACs they used, along with their thoughts on the designing and implementing of SACs is detailed in Chapter 5, Section 5.2.2.5.

The school assessed coursework tasks were described in their role as rules and tools in the activity system, which aims to support the mathematical learning and thinking of senior secondary students. Tensions and contradictions with the implementation of these tasks, from the perceptions of the students and teachers, will be analysed in Chapter 5. Having analysed the nine SACS, the results will now be compared with the examinations and conclude with a discussion of the attainment of the curriculum by the whole student cohort, as indicated by the examination results. The probability components of the examinations will be analysed and compared regarding the context, level of thinking required by students, and the probability content.

4.3.2 Examinations and Context

The context of the mathematical problems within the examinations is important as this can help or hinder students' ability to respond. The same categories were used for the examination problems as for the textbook classification, namely, abstract, games, personal and occupational, as outlined in Table 4.8.

Table 4.8

VCE MM34 Examinations – Probability and Context 2016-2018.

A comparison of probability content compared to the whole exam and the context of the questions

Year Exam	Problem type	Total marks	Marks for probability section	Abstract	Games	Personal	Occupat -ional
2016							
Exam 1	Short-answer	40	12	6	0	0	6
Exam 2	Multi-choice	20	7	4	2	1	0
	Extended-answer	60	16	0	0	16	0
				10	**2**	**17**	**6**
2017							
Exam 1	Short-answer	40	11	5	0	6	0
Exam 2	Multi-choice	20	6	3	1	1	1
	Extended-answer	60	19	0	0	19	0
				8	**1**	**27**	**1**
2018							
Exam 1	Short-answer	40	6	2	4	0	0
Exam 2	Multi-choice	20	5	4	1	0	0
	Extended-answer	60	16	0	0	16	0
				6	**5**	**16**	**0**

Note. Examination 1 did not allow calculators or bound reference books.

The majority of short-answer and multiple-choice problems in the MM34 exams were of a purely abstract context or related to games, for example, dice, cards, spinners, or marbles in a jar. In 2016 and 2017, the first examination contained 12 or 11 marks of probability problems out of the 40 marks in total. In 2018 there were only six marks associated with probability content. No comments or explanations of this were provided in the Examiner's Report (VCAA, 2019a), which is the written report the panel of examiners publish each year with the results, selected solutions, problems faced, and hints for teachers and students for improving the learning for the next year. All the Examination 2 extended-answer problems, with their high proportion of marks, were of a personal context, which students would be expected to relate to. For example, battery life of computers, heart rate of exercising students, or amount of time spent on homework. Very few problems were of a context relating to occupations, for example, sales reports of car yards, or data about insurance, which would have had less meaning for students.

4.3.3 Examinations and Levels of Thinking

The level of thinking of the examination problems was categorised according to the Two-Tiered Mathematical Thinking Framework, in the same way as the intended curriculum and the textbook problems were previously categorised. If a problem was a repeat of an example from the textbook, it was considered to be a lower order thinking problem, as it would be expected that students had previous experience at these and possibly remember the processes. This does assume the textbook was used by the teachers and students, which will be investigated in the next chapter. extended-answer problems were difficult to classify. They were multi-part problems, which often involved several aspects of different probability concepts, but some sections of the problems were independent of other sections, so they could be considered to be a group of lower order thinking problems loosely based around a theme or context.

The examinations for the three year period of 2016-2018 were analysed, with the problems involving probability classified as involving lower or higher order thinking using the Two-Tiered Mathematical Thinking Framework and are presented in Table 4.9.

Table 4.9

VCE MM34 Examination – Level of Thinking 2016-2018

Year Exam	Problem type	Total marks	Marks for probability section	LOT %	HOT %
2016					
Exam 1	Short-answer	40	12	50	50
Exam 2	Multi-choice	20	7	57	43
	Extended-answer	60	16	69	31
			35	**60**	**40**
2017					
Exam 1	Short-answer	40	11	27	73
Exam 2	Multi-choice	20	6	67	33
	Extended-answer	60	19	47	53
			36	**44**	**56**
2018					
Exam 1	Short-answer	40	6	100	0
Exam 2	Multi-choice	20	5	40	60
	Extended-answer	60	16	38	63
			27	**52**	**48**

Note. LOT is lower order thinking, while HOT is higher order thinking.

As illustrated in Table 4.9, over the three years 2016–2018, the expected level of thinking varied between the sections of the examinations. In the area of study of probability, the level of thinking was lower in 2016, the first year of this Study Design. The proportion of higher order thinking problems varied over the sections of the examinations and the years, however, was consistently about 40% (at 40, 56, 48%).

4.3.4 Examinations and Probability Content

The Mathematical Methods examinations combined problems from the function and graphs, algebra, calculus and probability and statistics areas of study. Table 4.10 shows the proportion of probability content in the examinations, as a proportion of the marks allocated. This was calculated using the Examiner's Report for each of the three years (2016–2018), which includes the mean score of the individual problems (VCAA, 2019a). The mean score of the probability problems is compared to the mean score overall, as an indication of the degree of difficulty that students found with those problems. This is also illustrated in Table 4.10.

Table 4.10

VCE MM34 Examinations– Student Results 2016–2018

Student Results for Probability Area of Study Compared to the Whole Examination

Year Exam	Problem type	Proportion of probability content %	Mean probability result %	Mean overall result %
2016				
Exam 1	Short-answer	30	36	50
Exam 2	Multi-choice	35	43	62
	Extended-answer	27	51	46
	Mean	29	45	50
2017				
Exam 1	Short-answer	28	54	50
Exam 2	Multi-choice	30	55	59
	Extended-answer	32	50	51
	Mean	30	52	52
2018				
Exam 1	Short-answer	15	68	54
Exam 2	Multi-choice	25	49	54
	Extended-answer	27	44	51
	Mean	23	53	52

Note. The Proportion of probability content is calculated by the proportion of marks allocated. Adapted from the examiner's reports (https://www.vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx).

Table 4.10 shows that the probability component of the examinations varied over the three years, with less marks allocated for probability in 2018 than in 2016 and 2017. The students' mean scores on the probability component were generally lower than the mean score for each section of the examination, an indication that students found these problems more difficult. In 2016, the first year of the Study Design, students received results of 45% for the probability problems, and 51% average for the examinations overall. This evened out the following year with 52% for both the probability component and the examinations overall. Over the three years the marks allocated to the probability problems decreased, and the students' grades for these sections increased slightly, although this is not a clear pattern.

4.3.5 Summary of Attained Section

Nine School Assessed Coursework tasks (SACs) and three years of end of year examinations for VCE senior secondary mathematics subjects were used as an indication of the expected attained curriculum, the object of the activity system. These two types of summative assessments tasks were analysed for content, context, problem type and expected level of thinking. The SACs provided by the curriculum authority (VCAA), and those available commercially and created by the teacher participants, varied greatly in style and in all the criteria investigated. The SACs did utilise contexts that students would relate to. Few of the SACs matched the criteria recommended by the intended curriculum, as outlined in the Study Design and support material. Few of the SACs were different to the examinations in style of problems and were neither open-ended nor modelling tasks. The way the SACs were implemented by the teachers will be investigated in Chapter 5.

The examinations included a variety of problem types and supports, for example multiple-choice, short-answer and extended-answer problems, to be completed with and without CAS calculators and bound reference books, to assess students in a range of formats. More than half of the probability examination problems involved a context which a student might relate to, for example, time on homework, heart rate while exercising and battery life of computers. Abstract problems with pronumerals and functions accounted for a quarter of the examination problems, with the small number of remaining contexts centred around dice, cards, or occupations. The higher order thinking required to solve the probability problems in the examinations over the three years of the Study Design did vary in the probability component of the examinations but was always higher than the proportion of higher order thinking problems in the probability sections of the textbook. The textbook review sections came closest to the corresponding proportion of higher order thinking problems in the examinations. Probability is a large area of study within the Mathematical Methods course, accounting for a quarter of the examination problems in all sections of the examinations. From the state-wide results in the examinations, the probability problems were found to be slightly more difficult for students than the examination as a whole.

4.4 Summary of the Curriculum Document Analysis

This chapter analysed the components of the curriculum, in senior secondary mathematics, focusing on the probability section of Mathematical Methods. Starting with the intended curriculum of VCE senior secondary mathematics as defined by the Study Design (VCAA, 2015a), the implemented curriculum was represented by the textbook, with the attained curriculum represented by the SACs and examinations. The intended curriculum was a tool, with rules, in the activity system, the implemented curriculum also acted as a tool, while the attained curriculum became a measure of the object. Findings regarding the VCE MM34 probability curriculum document analysis included:

- The Victorian Curriculum (F–10 and VCE) supported the development of a variety of thinking skills by its stated aims and objectives, and the curriculum statements.
- The probability area of study increased in amount and complexity in the VCE senior secondary subject of Mathematical Methods compared with earlier mathematics subjects.
- Probability content forms a large part of the subject, as indicated by 40% of the curriculum statements, and can be used to support the development of higher order thinking skills in students.
- The recommended textbook for VCE MM34 provided an appropriate combination of examples and problem types to support the implementation of the curriculum, and preparation of examinations, but not SACs.
- The content chapters from the recommended textbook included predominately lower order thinking problems, while the review chapters included more higher order thinking problems in a style related to the examination problems.
- A variety of SACs were analysed and showed a wide range of styles, some inconsistent with the recommendations of the Study Design.
- The textbook, SACs and examinations generally utilised contexts students could relate to.
- The three years of examinations analysed (2016–2018) required a range of thinking skills.

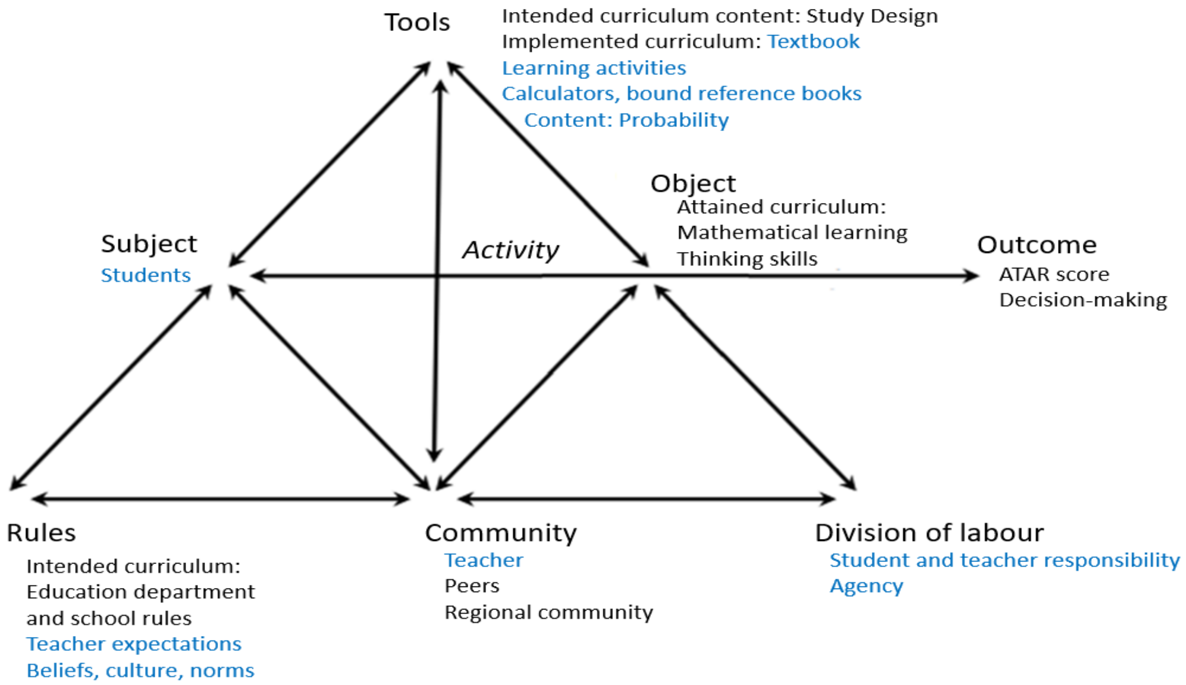
The tensions and contradictions surrounding the implementation of these curriculum tools and rules, as perceived by the participating students and teachers, will be discussed and analysed in Chapter 5. The alignment of the intended, implemented and attained curriculum, and the tensions and contradictions within and between these elements and with the student and teacher’s views, will be the focus of the Discussion and Conclusion, Chapters 6 and 7.

Chapter 5: Findings—Student and Teacher Interviews

This chapter reports on the data obtained from the semi-structured interviews conducted with 20 post-VCE student participants and 14 teachers, analysed through the lens of an activity system (Engeström, 2001) as part of Activity Theory. The students and teachers reported on their experiences of the implementation of the VCE Mathematical Methods curriculum, and their experiences of learning and teaching this subject. Questions were asked about the use of tools and rules of the curriculum, including the Study Design, textbook, learning activities, calculators, and bound reference books. Students and teachers also considered the division of labour, roles, and responsibilities they played in the development of mathematical and thinking skills. These questions involved all areas of the activity system model, as illustrated by Figure 5.1.

Figure 5.1

Activity System Framework for the Student and Teacher Interviews



Note. Adapted from “Expansive Learning at Work: Toward an activity theoretical reconceptualization” by Engeström, Y. (2001). *Journal of Education and Work*, *14*:1, p. 135. Copyright 2001 by Taylor & Francis Ltd.

The student, then teacher responses are reported, and then details of specific probability problems are investigated. The data is presented mainly in the form of tables with representative quotes from participants included where appropriate to further highlight particular findings. These are italicised and indented on the left margin to differentiate them from quotes from literature sources. Pseudonyms in brackets follow participant quotes.

5.1 Student Interviews

This section reports on data collected from the 20 post-VCE students, who had all previously completed at least the first two units of Mathematical Methods (MM) and were currently attending a regional university in Gippsland Victoria, Australia, studying a variety of programs including Engineering, Science, Medical Science, Arts and Education. The student participants were invited from tutorial groups of mathematics and statistical subjects within the university. The details relating to how the students were chosen and the practical details of the interviews are included in the Methodology Section 3.4.3.2 of Chapter 3. This small sample is not intended to be representative of the student body, however students with a range of experience, home locales within Gippsland, and university programs were included. Table 5.1 provides details of the student participants using pseudonyms, and includes nominated gender, VCE mathematics classes previously completed, and current university program being studied. The first part of the interview with the student participants focused on background questions to confirm they had completed VCE mathematics in schools within the Gippsland region and to ascertain their current enrolment. These introductory questions were intended to help the students feel comfortable.

Table 5.1

Student Participants, Mathematical Background and University Program (N= 20)

Pseudonym	Units 3 & 4 Maths studied	Program at University
Cathy	FM	Science/Biology
Hazel	FM	Science/Medical
Quin	FM	Science/Medical
Pete	FM	Science/Vet
Vinnie	FM MM	Arts
Olive	FM MM	Education/Mathematics
Rob	FM MM	Education/Mathematics
Foster	FM MM	Engineering/Medical
Nick	FM MM	Health PE with Math minor
Tracey	FM MM	Medical Science
Lenny	FM MM SM	Engineering
Dan	FM MM SM	Engineering
James	MM	Engineering
Sue	MM	Science
Gabby	MM	Science/Medical
Ivy	MM	Science/Medical
Ken	MM SM	Engineering
Max	MM SM	Engineering
Eddy	MM SM	Engineering
Wes	MM SM	Science/Mathematics

Note. MM = Mathematical Methods, FM = Further Mathematics, SM = Specialist Mathematics.
All participants had completed Units 1 and 2 Mathematical Methods.

The 20 student participants were studying a range of university courses at the regional university at the time of the research. All participating students had completed Mathematical Methods Units 1 and 2 (MM12, generally studied in Year 11), with 16 of the 20 students also completing Mathematical Methods Units 3 and 4 (MM34, generally studied in Year 12). Twelve of the student participants had studied more than one senior secondary mathematics course, with two participants completing three senior secondary mathematics Unit 3 and 4 courses.

5.1.1 Student Descriptions of their Mathematics Classes

Students were asked to describe a typical senior secondary mathematics class: *Please tell me about your VCE mathematics classes, what was a typical class like?* (see Appendix D). This open-ended question invited the student participants to influence the direction of the interview, so the information they provided was a true indication of their priorities and views. This information helped me analyse the degree to which the learning was perceived as being student or teacher led, as a component of the community and division of labour element of the activity system (illustrated in Figure 5.1).

The focus of the learning activities of the student participants varied based on their personal experiences, between being perceived as student or teacher focused. Table 5.2 shows the spread of focus between the students' reported perceptions including examples of representative or typical comments provided.

Table 5.2

Student Perceptions of the Focus of their VCE Mathematics Classes

Focus of the class	No. of students (N = 20)	Typical comments
Teacher and textbook	7	The teacher would explain a few ideas and examples, and then we would work through the textbook on our own
Teacher, textbook and student	7	The teacher would write up on the board some examples; we would then do textbook problems, and get help from our friends and the teacher, both in and out of class
Teacher and student	2	The teacher made assignments for us to do at your own pace with the help of my friends
Textbook and student	4	We did self-study, individually with the textbook and with our friends

Table 5.2 shows that four of the twenty students reported they worked predominantly alone, with the textbook providing a focus for their learning. Two students described their classes as very teacher centred, with all the learning activities managed by the teacher. Seven students reported their classes had components of all three foci over the course of the year.

The agency or responsibility for the learning, as divided between the students and teachers, is another aspect to consider. It was difficult to gain information about the students' perceptions of responsibility or division of labour from a single interview, however, some comments did indicate how the responsibility for learning was perceived by the students. Quin explained how his teacher directed the learning experiences by constructing notes for the class, to supplement the textbook:

> *In our class the teacher printed off a set of notes for each topic and those notes would specifically cover a certain area in the textbook which we were allowed to staple all together at the end of the year to take into our exam. At first the teacher would construct like a lecture. At first, we would fill in the notes. The class would actively listen and fill in. The teacher would demonstrate how to complete that task. Demonstrate in the notes. Then he would go on to do questions either in the textbook or when or printed worksheet, made by the teacher (Quin, student).*

Ivy also described her mathematics class as very teacher centred, with the teacher controlling the learning by demonstrating then assigning the learning tasks, predominately from the textbook:

> *The teacher would stand up the front and do examples and explain what we're going to do and probably do a practice question and then we'd have a list of questions to do, and we sit and do them by ourselves (Ivy, student).*

In contrast, Vinnie reported that the textbook was the focus of learning and the teacher had a lesser bearing on his learning:

> *We have four or three classes a week. For the first one, which is the shorter one, it tends to be the one where the teacher talks the most. To introduce the work for the week, and then after that he kind of names the chapter we have to do, and then we do that chapter work (Vinnie, student).*

In a similar situation however, Rob showed his initiative by working ahead while the teacher explained the mathematical ideas, which was indicative of taking charge of his own learning:

> *My maths teacher would talk for about the majority of the lesson, maybe 30 to 40 minutes. She would ask you to try and get the exercises done during the rest of the class or, do it for homework. That was the general structure, although most of the students who were good at math [including me] were kind of doing it as she was talking just. Like less homework that way (Rob, student).*

Nick explained how he and his friends also took control of their own learning in and out of class:

> *During study periods, we were always round in the library… So we had 4 of my best mates. We'll be in Methods and we always do stuff together at home or in study periods (Nick, student).*

Wes described the teamwork and peer teaching which occurred in his class:

> *My class size in math methods was quite large, which meant that the teacher couldn't always get around to each student immediately ... it was also good because it meant he encouraged us to talk to the students around us and get them to try to help us which then furthered their learning … the level of cooperation in the classes was excellent (Wes, student).*

Wes also explained how flexible his senior mathematics teacher was, unlike his teachers from the lower year levels who checked homework and gave detentions for missing work. Wes explained that his peers did not do much homework or study, but they worked well in class. He completed all the relevant chapter work from the textbook in class. Hazel described how she also took control of her learning. As she had been unwell for most of her VCE year, she sought support from a tutor:

> *Because I was really sick, I only went like 1 day a week. But that specific teacher was—like both Year 11 and 12, and students were mainly working at their own pace, and when you needed help they would go through it, and the questions [from the textbook] that everybody struggled with they would go through together (Hazel, student).*

Three students (Hazel, Sue and Tracey) explained that they had employed mathematics tutors outside school to assist them. This could have been an indication of their own desire to improve their learning, or it could have been encouraged by their parents.

The learning activities reported by student participants consisted predominately of textbook activities. Interviews with students highlighted the substantive influence that the textbook was perceived as having on learning, which will be further discussed in the next chapter. The students' agency towards their own learning can be glimpsed from the interviews, as an indication of the division of labour between the students and their teachers. A full range of attitudes and expectations were evident, even in this small sample, between students and teachers' responsibility in teaching and learning. From the students' perspective, students and teachers were both in control of the learning experience to varying degrees. Textbooks were also seen to be both a supporting and controlling factor, and more than just a learning tool. The teachers' reflections on this question will be outlined in Section 5.2.2.1.

5.1.2 Tools to Support Student Learning

The next focus of the interviews with the students related to the tools of the activity system (see Figure 5.1), which support learning of mathematics: the learning activities, calculators, bound reference books (summary book), school assessed coursework (SAC), and examinations. The questions were:

Which teaching and learning activities did you do, which did you find most helpful?

Which resources, textbook, calculator, and exam preparation material did you use?

Did you do practice exams throughout the year?

How did you make and use your summary book? (see Appendix D)

These open-ended questions aimed to enable the students to steer the direction of the conversation and prioritise their own responses.

5.1.2.1 Learning Activities—Simulations

Learning Activities are one of the tools used to support learning as described in the activity system. The formal written curriculum explicitly expects mathematical simulations to be used as learning experiences (VCAA, 2015a). The curriculum content statements include: “Statistical inference, including definition and distribution of sample proportions, simulations and confidence intervals” (VCAA, 2015a, p. 74). However, few of the students mentioned experiencing mathematical experiments or simulations in their senior secondary mathematics classes. Of the 20 student participants, five detailed how they or their teachers used dice, coins or smarties for probability games or simulations in senior mathematics and another four discussed using dice or coins in middle school. Ken and Dan for example described an activity with smarties, while Wes described an activity arranging classmates as an introduction to permutations in probability in Mathematical Methods:

In Methods, I remember our first lesson from probability. When the teacher asked for four volunteers. We all went up to the front of the room and we sat and chairs and then we were asked how many different ways we could arrange ourselves (Wes, student).

Rob discussed playing a coin flipping game, but the teacher was the only one with the coins. Eddy, and Ivy explained how some teachers would bring things to show the senior mathematics students, usually on the projector screen. Vinnie’s response was one of four that outlined how physical manipulatives were used to support learning in lower levels of mathematics classes, but not senior mathematics classes:

Grade 6 is probably just flipping coins and saying 50/50. In Year 7 we got the weird dice out, the eight-sided ones. We did do a lot of dice rolling. Once we hit year 11 it was all in the mind (Vinnie, student).

Across the interviews, there appeared to be little physical involvement attached to the learning activities of the student participants. According to Olive:

> *So they were into drilling in formulas like you know no maluables [equipment], all in your head that kind of stuff (Olive, student).*

In summary, all but two of the twenty student participants described the learning activities as very textbook based. The textbooks are based on the formal intended curriculum, the Study Design (VCAA, 2015a). The Study Design included the expectation of the use of mathematical simulations, however very few participating students reported any involvement in simulations. These interviews identified a potential gap in the expected flow in the curriculum implementation from the intended curriculum, to curriculum implemented by the teachers via the textbook, to the attained curriculum of the students as described in Chapter 3, Section 3.3.3. Another tool used by the students was the calculator, which is described in the next section.

5.1.2.2 Calculators

Calculators are tools in the activity system. The rules for using calculators are defined in the formal curriculum documents (VCAA, 2015a). The student participants explained that they were able to use CAS calculators and scientific calculators in their SACs and in one of the two examinations at the end of the year. Table 5.3 summarises student perceptions of CAS calculators.

Table 5.3

Student Perceptions of the CAS Calculators

Student perceptions of CAS calculators	No. of students (N = 20)	Typical comment
Calculators are very helpful	5	The calculator was great
Calculators used to check answers	2	I just used it to check my answers
Took too long to learn or hesitant to rely on them	5	The calculator was hard to learn
Dependant on the teacher's skill	3	It depends on the teacher's competency with it
Calculators are not helpful	5	The batteries went flat You are not learning by typing in a calculator

The calculators were thought of as helpful by only 7 of the 20 student participants, with a further ten reporting calculators as less helpful, to varying degrees. The remaining three students reported that the value was dependent on teacher's skills in the use of the calculators. Dan explained that although the CAS calculator was difficult to use initially, with practice they were supportive of learning:

> *Going from year 10 to year 11, going from the scientific to the CAS calculator, I kind of had a tantrum as it was so out of the ordinary. Now I love it, it's so easy (Dan, student).*

Several students mentioned that in one of the two examination they were not allowed to use the calculator. As Foster explained:

> *A lot of students rely on their calculators way too much for specific types of questions and then you'll go into the non-calculator side of the exam and you'll get one of those questions and you'll be stuffed because you're always using a calculator for it and you don't actually know how to do it by hand (Foster, student).*

Similarly, some university courses do not allow the use of calculators, which at least three student participants students found frustrating, as exemplified by Max's comment:

> *Compared to Uni probably a bit of a waste of time having the graphics [CAS calculator]. I find that at university we never use them (Max, student).*

James and Lenny both commented on how a focus on learning to use the calculator might actually limit mathematical skill development times, with some students becoming too dependent on them, as highlighted by James:

> *They [the calculator] were really good at what you could. They could solve everything you required but it was more teaching you how to use the calculator than actually learning the basics behind it, … it was restricting your own skills (James, student).*

Most comments relating to the usefulness of calculators as a tool to support learning related to solving, checking, and working quickly with their mathematics. However, several comments identified limitations including the initial difficulties of learning how to use the calculators; the potential limited skills of the teachers; the need to complete one examination without a calculator; and the lack of calculators in some university programs. Several students also responded that they did not think they were learning the mathematics if they used a calculator. Students perceived calculators as both a support and hindrance to the development of mathematical and thinking skills.

5.1.2.3 Bound Reference Book

The *bound reference book* (VCAA, n.d.-d), or summary book as the students and teachers referred to them, is a book that students are permitted to use while completing their SACs, and one of the two examinations. It could be a published book, or a student made book created as the learning occurs or made near the end of the year as part of examination preparation. The summary books can be used as a tool for learning (see Figure 5.1), and this is quite individual, as student participants explained in interviews. For instance, some students reported creating their summary book throughout the year, while others developed them during exam preparation. There was acknowledgement by a number of participants of following their teacher's direction in creating the book, while others described having the initiative and self-motivation to make it themselves. A small number of students reported not finding the summary books helpful, but that is possibly a reflection of feeling more confident in their knowledge. The key perceptions of the student participants about the summary book are presented in Table 5.4.

Table 5.4

Student Perceptions on the Bound Reference Book

The usefulness of the bound reference book		No. of students (N = 20)	Typical comment
Very helpful	Purposely created	8	I made summary notes most weeks, with exams and annotations. Especially before SACs
	Used class notes	4	I made my class notes clear enough so they could be used as a summary book
	Used textbook	1	I just took my textbook
Not helpful	Formula sheet was enough	3	Making a summary book was annoying; I did not need it.
	Too disorganised	1	My summary book was disorganised, paper and examples everywhere. It was no help
No comment		3	

The way in which the summary book was created and used provided insights into the students' attitude to learning as demonstrated through the following examples. Foster transferred the notes he took in class straight into his summary book then annotated the examples suggesting he perceived maths as a set of steps, like a recipe:

> *The summary book would be where I would usually take most of my notes in class. We would be doing examples on the board, I'd probably do them in my summary book and I'd go back through and … down to step A, step B, step C, here's your answer is usually how I roll, break it down into steps (Foster, student).*

Tracey used her summary book as part of her self-directed learning and self-assessment:

> *Before every SAC as revision, the teacher always tells us to update our summary books … I*

choose the questions after I've done the questions from the textbook and I always choose the ones that I had trouble with (Tracey, student).

Eddy and Nick found their summary books useful in their senior mathematics classes, and also for their university classes, to refresh their memory of the mathematical concepts:

I still have the (summary) book at home, and I still refer to it for uni maths (Eddy, student).

I have a summary book for Further and Methods and I still use today [at university] (Nick, student).

In contrast to the students who described the summary book as helpful in some way, Max and Sue were not as positive:

Summary books were a nuisance (Max, student).

Sue used the textbook as her summary book:

You just take the textbook [rather than a personalised summary book] into the exam (Sue, student).

The students' descriptions of the use of the summary books also alluded to their sense of agency in the learning process. For example, Pete used his experiences in Year 11 to improve his study strategies in Year 12, and his use of the summary books explains this:

In Year 11 I did it [made the summary book] all at the end, as my revision. In Year 12, I did it for each topic as I went along, and so by the end, I had it already done, which would make it easy to go over the practice exams (Pete, student).

In contrast, Vinnie's description of how he made his summary book hinted at his dependence on the teacher to guide his learning:

We hadn't been given any strict instructions on how to use it [the summary book] or when to use it or when we should be updating it (Vinnie, student).

The students' descriptions of their use of the bound reference book (summary book) indicated their attitude to study and the division of labour between the student and teachers in the learning process. Most student participants (thirteen out of the twenty) found the bound reference book helpful in their mathematical studies and used the task of making it as a learning tool. Another tool used by the students to focus their learning was the SACs, which are described in the next section.

5.1.2.4 School Assessed Coursework

School Assessed Coursework (SACs) are assessment tasks (tools within the AT system, see figure 5.1) to support the learning and assessing of senior students in Victorian schools. The results of the SACs account for 34% of the overall grades of students in Mathematical Methods Units 3 and 4 (MM34). The SAC results are moderated against the examination results. Teachers create their own SACs, or use commercially created ones, which are then adapted to comply with the rules of the VCAA. The SACs represent an indication of the attained curriculum, and of the development of mathematical and thinking skills, which is the object of the activity system.

Student participants highlighted that familiar contexts were often used in SACs, in an effort to relate to practical mathematical skills:

> *There was a rollercoaster. There was a train, there was one of about fishing, there was just there were always like a weird question about like ice cream and just things they weren't straightforward. They were always trying to refer to the real world. But there is no way you have to refer to all of trains going this fast through a tunnel like this (Olive, student).*

Students spoke of the SACs attempting to relate to real-life, but also to the examinations:

> *SACs were like, I guess mini exams, like some multiple choice at the start, a few short answer and then maybe one or two longer ones that stay on like one topic - things like building a rollercoaster or football players or something like that (Vinnie, student).*

The style of the SACs were meant to be problem-solving, modelling tasks or applications tasks, as described by the rules of VCAA (2015a) as described in Section 1.2.2. Five students described the style of the SACs as extended-answer response, similar to the second examination, as Pete clearly volunteered:

> *Sort of like a condensed exam for the topic (Pete, student).*

Whereas Olive explained the SACs were like examinations, but also involved thinking:

> *They were difficult, they were like part two of the exam where they're all odd. Like it's not straightforward questions. So it's very much that you have to think about how to do it without being told (Olive, student).*

Although SACs are meant to cater for the needs of students, with teachers able to adapt the tasks to suit their own classes, Wes explained that he felt some students were disadvantaged:

There was a huge emphasis on worded problems. We had a lot of, in both classes, there was a large portion of international students for whom English wasn't the first language. And in the exams, there was lots of worded problems (Wes, student).

In summary, eight of the twenty student participants described the SACs as mini examinations. The students could see that their teachers were attempting to relate the mathematics to real life, but the wording of the problems was seen as a potential concern. The students' descriptions of the SACs demonstrate the dilemma faced by teachers between supporting student preparation for the examinations and conducting problem-solving and modelling assessment tasks as required by the rules of the governing body, VCAA. The issues associated with examinations are different again, as highlighted in the next section.

5.1.2.5 Practice Examinations

The end of year examinations are the main assessments used in senior secondary mathematics. The rules for the examinations are detailed in Section 1.2.2. In general, students complete previous and practice examinations to prepare for the end of year examinations, which could be used as formative assessment, to identify student strengths and weaknesses, or as practice tasks to familiarise students with the exam format. When asked about their personal experience of practice examinations, a variety of responses were provided by student participants. These ranged from practice examinations considered an important component of the learning over the whole year from week one, to being perceived as a last-minute activity at the end of the year. Nick and James supplied two contrasting responses with Nick explaining how he used examination problems supplied by the teacher from week one:

I did probably 15 Methods practice exams. Yep. And about six Further Mathematics. Over the year. Even in the first week you'd be doing questions out of practice exams. It slowly built up into a fair few practice exams. So, when it came to crunch time, I did about 10 in the last month (Nick, student).

In contrast, James completed just one practice examination towards the end of the year:

Yes, so our school did a practice exam about – I think it was about a month prior and that was just like an older exam that they give us (James, student).

Ivy used the practice examinations as formative assessment tasks to identify the areas of the content she needed to work on:

I did a few [practice exams], I did maybe four, I think, and then I wrote down all the areas that I didn't get right and then I asked my teacher for extra resources for that, and then I was

> *just working on those up until the exams. The teacher had a whole lot of practice exams, the school put a lot of money into purchasing exams, practice exams so we had a lot more practice instead of just having the textbook (Ivy, student).*

As demonstrated from the interviews, all the participating students reported using practice examinations and understood the focus of the class to be the final examinations. The ways in which the practice examinations are used could be an indicator of the student's sense of agency or influence in their own learning. Student participants reported using practice examinations in various ways, to identify their strengths and weaknesses in the mathematical content, and to prepare for the high stakes examinations by practicing the specific style of problems.

5.1.2.6 Summary of Tools—from the Student Perspective

The tools influencing student learning of mathematics included the learning activities, textbooks, calculators, bound reference books, SACs, and practice examinations. Descriptions of these tools also involved discussion on the division of labour between the teachers and students in the learning process. The rules, beliefs and norms of the learning environment also emerged as an influencing factor in how these tools were used to develop mathematical and thinking skills. Engeström's (2001) framework of activity systems proposes that all the elements; rules, tools, community and division of labour, cause tensions and contradictions for achieving the object, in this case the development of mathematical and thinking skills, or the outcome, which is passing the VCE mathematics subject, and even to gaining a high enough score to enter university. These tensions and contradictions are both positive and negative, within an element and between elements.

Tensions developed between the students and teachers as they shared the division of labour or responsibility for learning. The tools of learning—textbooks, SACs, examinations, bound reference books and calculators, were all used to support student learning, while the bound reference books and calculators were judged to be counterproductive by some students. This was due to the extra time needed to learn how to use the calculators or produce the bound reference books, and the perceived lack of mathematical rigour if calculators were used. The interview discussions around the use of the various tools or learning materials helped demonstrate the division of labour, as the responsibility of assigning, completing and assessing the learning tasks moved between the students and teachers. Contradictions and tensions uncovered in the student interviews included:

- Formal curriculum rules stating the need for simulations contrasted with the lack of student experiences in simulations.
- Students being encouraged to study together and provide peer support yet also competing against each other in the VCE.

- Calculators and bound reference books being seen as either a hurdle or support by students, although others feeling they were a learning task, or unnecessary additional task.
- Differences in relation to the perceived value of completing practice exams, with simple completions seen as enough by some students, but others seeing a need for mastery of them.

These findings from the student interviews in relation to the tools of learning, will be compared to those of the teachers, and linked to the rules and tools within the curriculum and literature in Chapter 6. The next section delves into one content area of study within mathematics, examining the student responses to problems on the content area of probability and two mathematical problems.

5.1.3 Student Perceptions of Probability

The next section of the student interviews focused specifically on the area of study of probability. The mathematical topic of probability is a large part of the Mathematical Methods subject and is a tool in the development of thinking skills. The student participants indicated mixed opinions regarding their study of probability, with eleven of the twenty reporting positive views as they found it an interesting topic which related to real life or added variety to the subject of mathematics. Eight of the students disliked the topic, as they found it confusing or different, and for some, they perceived the topic had been rushed or was too big a jump in difficulty in their final year at school. Just one student could not remember learning probability.

Five students explained they found probability interesting. For example, Gabby liked the topic for its real-life applications:

> *I love it. It's interesting. I thrived on it. It's in the real world. So all probability needs to have a real-world focus. It can't be just theory. We did the whole smarties and the chances of getting a red one or whatever. And I think that schools teaching should be a lot more real world examples in the classroom. I mean that's one of the reasons I love doing probability and statistics because it is very real world (Gabby, student).*

Ken liked how it added variety to the mathematics subject:

> *I found it OK, a lot of the maths courses was calculus, so it was good to do something else a little bit different (Ken, student).*

Foster liked probability, and after he started his university course in Engineering, he commented he would have liked to have more knowledge in this area of study:

I personally quite like probability. But probably a little light on. There's not that much probability in Year 12 Methods … There's not enough of that in Methods really to be able to slot easily into a first-year uni stats course, for engineering or science (Foster, student).

In contrast, Sue, Rob and Wes all disliked the topic of probability as it was different, and did not seem to fit readily into the Mathematics Methods subject:

It's different, all basically just graphs and stuff with different types of graphs until you get to probability and then it's completely different and it was weird doing probability. I don't think I was very good at it to be honest (Sue, student).

Rob reported some students who did well in the other parts of the subject found probability difficult:

I found I was actually good in Methods but whereas the students I found a lot more intelligent and could pick things up at a quicker pace really struggled with it [probability]. I don't know if it was due to their way of thinking or whether that was due to the teaching (Rob, student).

Wes found probability different to the rest of the subject, and did not like it:

There are set rules, but tricky and then very often situational. I just couldn't get my head around them. So that's why I did not like probability (Wes, student.)

Rob, James and Wes indicated the topic was difficult. Rob thought he had not had much experience with the topic in the past:

Yet it was the first time I've done anything like it. Did I actually do much probability in Years 7-10? I don't remember it (Rob, student).

James agreed, and added the topic was rushed at the end of the subject:

But one of the issues was we spent a lot of time on getting calculus and functions, so we didn't get any time to do probability; that was even in Year 12 we only spent maybe a few weeks on it. I think all of us struggled with probability. I don't think we even covered it in Year 10 like my probability was almost zero coming into Year 11 and I did Advanced Maths all the way through. I don't really remember doing much on it, because again it was only a few weeks that we did spend on it and I think I even just skipped the [probability] questions on the exam because I just didn't have the knowledge (James, student).

Ivy was especially detailed in her reasoning as to why she disliked probability. She stated the calculus component of the subject was agreeable, but the probability content was difficult, disjointed and less predictable. The symbols and language were also complicated:

> *Yeah, it [probability] was my least favourite part. I really enjoyed calculus and I'm into physics and all of that but yeah, I found probability really challenging and … like it should have been in the Further course – it seemed a bit disjointed … I've had to get used to the terminology and the symbols for everything, it's a bit different. Probably because it wasn't quite set in stone. It wasn't a, here's the question, here's how you figure it out, that's the answer - it was sort of a bit more wishy washy and it had a lot of reading through text trying to find how to … what you're looking for … (Ivy, student).*

Dan and Vinnie suggested probability in Year 12 was much more difficult than in Year 11, as Dan explained:

> *In Year 11 probability was wasn't too drastic, but in Year 12 it got more extreme really quickly. It was a lot of hard work (Dan, student).*

Vinnie agreed, but could also see how the probability linked in with the calculus part of the subject:

> *I liked it in Year 11 when it was kind of its own thing. But it kind of stopped being its own thing in Year 12, it really just became another version of calculus. But you get the sense that it had an essence of practicality in Year 11 because we were dealing with physical problems and stuff, and then this year it's going to go back to your more methods abstract less practical side, it starts to become it just feels less like probably more like clever like calculus. Sneaky calculus (Vinnie, student).*

Overall, there were many different perceptions of probability among the student participants, with some reporting the topic was easy or difficult, relevant or irrelevant and added variety or confusion. Probability was an area of study which was not routine. Some points which consistently emerged during interviews were that the topic did significantly increase in difficulty in the final year, and understanding was dependant on the wording and context. Five student participants appeared to like the topic of probability as it related to real life, while seven students disliked it as it was different and less structured than the algebra and calculus part of the senior secondary mathematics subjects, or more complex without appropriate preparation in the earlier years of mathematics. Five students could not remember much about the topic of probability and felt the school did not focus on it. The remaining three students thought it was ok, and a bit of a change to the graphs and calculus sections of the subject. The topic was seen both as an extension of the functions and integral to calculus but also an unrelated topic.

5.1.4 Summary of Findings from Student Interviews

Twenty university students who had previously attended a variety of Gippsland secondary schools reflected on their experience studying the topic of probability within the subject of Mathematical Methods, as they began a range of university programs. The student participants were open and keen to share their ideas during interviews.

From the interview data, it was apparent that the *division of labour*, or the responsibility for learning, fell between the students and teachers to varying degrees. The learning activities the students participated in, involved textbooks, SACs, examinations, bound reference books and calculators, while simulations and experiments were used irregularly, if at all. Textbooks were used extensively in the learning of senior secondary mathematics. The students also relied heavily on their teachers and peers. The students' use of the tools of learning, practice exams, bound reference books, SACs and calculators helped identify their sense of personal agency.

The formal assessments were SACs and examinations. All student participants had attempted practice exams to varying degrees, with some using them as motivation, learning tools, formative assessment, or hurdles which the teachers prescribed. Most of the student participants found the bound reference books and practice exams helpful in their learning and assessments. The usefulness of the CAS calculators was more divisive, with half of the student participants negatively commenting about the value of calculators, due to difficulty of use, or perceived disruption to learning the mathematical concepts.

5.2 Teacher Interviews

This section presents the findings from the analysis of data collected from the interviews with the senior secondary mathematics teachers. The interviews with 14 senior Mathematical Methods teachers investigated perceptions of their role in supporting students in their learning. Within the activity system framework (Engeström, 2001) used in the current study, the teachers form part of the community which influences student learning activities (see Figure 5.1). The balance between the agency of the students and teachers is a component of the division of labour. The way in which the teachers use and recommend the tools (for example; learning tasks, calculators, textbooks and SACs), and implement the rules (formal curriculum rules, school rules, unwritten cultural rules) affects the students, and their development of mathematical and higher order thinking skills. Table 5.5 introduces the fourteen teacher participants via pseudonyms and provides details relating to gender, teaching experience generally and specifically relating to Mathematical Methods.

Table 5.5

Teacher Participant Details (N= 14)

Name	Teaching experience in years	Mathematical Methods experience in years	Major at uni
Kerri	11	5	Mathematics / Statistics
Elaine	11	11	Mathematics / Statistics
Matt	26	12	Mathematics / Statistics
John	20	20	Mathematics
Beth	20	8	Mathematics /French
Claire	30	15	Mathematics /Health
George	6	6	Mathematics /Chemistry
Libby	5	4	Mathematics /Geography (PhD)
Ian	6	6	Mathematics /History
Nathan	1	1	Engineering then Mathematics
Danielle	16	3	Health
Fred	30	10	Physics
Arthur	30	2	Biology
Harry	20	7	Biology

Table 5.5 also outlines university majors of participants, demonstrating that nine of the fourteen teachers had Mathematics as one of their majors at university, and ten teachers had Mathematics teaching methods in their Education studies. Two teachers had qualified in other fields, (Health Science and Engineering) and then retrained as Mathematics teachers. Three teachers were qualified Biology or Physics teachers, but also taught senior secondary Mathematics. The teachers had a range of experience, from 1 to 20 years in teaching Mathematical Methods, and between 1 to 30 years in teaching in secondary schools.

5.2.1 Teacher Descriptions of their Mathematics Classes

The first part of the interview with teachers involved background questions to confirm they taught VCE Mathematical Methods in the Gippsland region, and for how long. This was followed by questions related to use of textbooks and calculators and then participants were asked to describe their typical senior secondary mathematics classes. The question about their classes aimed to help determine the degree to which they perceived teaching and learning as being more student or teacher focused, as a component of the division of labour element of the activity system. This question was also associated with understanding the associated tools and rules employed in the secondary mathematics classes. Responses were varied, with two teacher participants focusing their discussion on the textbook, four explaining their classes were predominately teacher focused, and six reporting implementing a combination of textbook, teacher, and student-focused teaching strategies. Two teacher participants did not elaborate on the style of their classes. This section presents commentary from the teacher interviews, with examples, to demonstrate the various approaches to teaching with a focus on teacher and student agency.

Four of the fourteen participating teachers described themselves as the main influence of the classes, through assigning tasks, monitoring, and assessing students constantly. George supported his students by creating notes for each lesson, which the students followed and filled in the blanks. This was intended to give students more time to complete mathematical problems themselves:

> *I've been giving them pre-prepared notes. But there's bits in it missing … a couple of examples but they all have blanks to fill in based on what I've actually put on the board … So in terms of going through those notes if I've had time, I've created that little note packet for them (George, teacher).*

Other teachers led lessons through the use of checklists, videos, demonstrations and mind maps. Matt, for instance, used rigorously monitored checklists:

> *I make a checklist of the questions the kids should do, for each chapter which I give them at the beginning of the topic. I check their books, just over their shoulder, about once a week. I feel that they should get a pat on the back for the work they do. Some will feel discouraged if the teacher didn't notice if they were doing it or not. They need someone to nag or encourage them if they get behind (Matt, teacher).*

All the teachers reported they influenced their students, but these four teachers reported they perceived themselves as the major influence.

Several teacher participants described their lessons as traditional: involving an introduction at the whiteboard of the theory and examples of the mathematical concept of the lesson, leading to textbook problems for students to complete, sometimes in cycles within a lesson. Harry, Matt, and

Ian provided similar descriptions of their classes as quite structured, as elucidated in Harry's comment:

A typical class would be, problem from the previous exercise for 20-30 minutes, then go through the next topic on the board, and try and keep that to 20-25 minutes, so we could get 20-25 min to go through some of those questions in class. The rest would be set for homework. In the middle section, that would be on the whiteboard, so time to do a question from the textbook (Harry, teacher).

Most teacher participants reported that a mixture of teacher activities was appropriate. The style of teaching depended on the time of the year, the needs of the class and the priorities of the individual students. For example, Beth explained that at the beginning of the year, staff wanted to ensure students were in the correct courses, so classes tended to be very structured. This was particularly the case when students were being introduced to a subject that they were intending to complete the following year:

In Headstart[4], it is very teacher centred, I've had to give the students all this information, but normally, they should just be working in class, especially at the beginning when it's revision. You don't want them just listening all class (Beth, teacher).

Teacher participants also spoke of the need to adjust their expectations due to the array of outside commitments that many students had including work, sport and family commitments. As such, there was a focus on trying to include as much as possible in class time because this was the only time that some students actually focused on the subject. Harry and Ian commented on the adjustments they made once they got to know their students:

I would keep going to the new work, did the new work as quickly as I could in a sense, so I could spend more time with them as they worked in class, I had to realise that there was half the students who weren't going to do work except in the time I gave them in class (Harry, teacher).

One day per fortnight, one class was primarily homework, so that everyone would have a chance to revise while I was there for advice and to ask me questions on the difficult concepts. I compacted the teaching a little bit to free up that one day a fortnight (Ian, teacher).

[4] Headstart is a two-week period where the classes for the next year run, to prepare the students and teachers for the year ahead. This occurs in several of the Gippsland schools.

Nathan agreed, and as a recent graduate, noticed that there was more to being a mathematics teacher than presenting mathematical concepts and setting problems:

> *It was surprising to me that as a teacher I expected to go through the content mainly but then I realised that for year 11, I need to be more of a psychologist and counsellor for them. I need to help them with planning and emotional planning of the workload because Methods has a lot of new concepts. The students thought that they were good at maths, and then they realised they didn't understand. Most of them have this fixed mindset (Nathan, teacher).*

There was evidence in the interviews of teacher participants knowing their students well enough to cater for individual needs. Discussion suggested that they knew which students worked better alone or in groups, and which ones needed extra help, as evidenced in the following comments:

> *The matching cards and concept maps and stuff are covered in groups. And I encourage the kids to have a study partner. Someone to talk about the maths with. The best kids, the one who get scores of 80 and 90[5], all have study partners (Matt, teacher).*

> *I encourage the students to "take action" rather than "get help", as get help is difficult for many students especially the males. I give them options including after school help. The way I discuss it is very supportive, not punitive (Danielle, teacher).*

Nathan was the least experienced senior mathematics teacher, with the interview taking place at the end of his first year of teaching. His description of his class was detailed and involved many learning activities per class. These learning activities involved a mixture of teacher and student-centred activities, individual and group activities. Nathan's class had just four students at his regional school, and he explained that many of the classes ran as group activities:

> *We talk about the topic for maybe 2 or 3 minutes, sometimes I have a hook, like a video or something interesting that is talked about. Sometimes I take cards in a class, for probability I took a deck of cards or dice … then I asked them questions. We make it like scattered, like I put one question at a time and ask them to solve one question each on the board. I had one question for each student on the board. So they solve it simultaneously. Sometimes I asked them to check each other's solutions and then they returned to their desk and they work on the worksheet or the book … They work on questions, I walk around and see if I need to help*

---

[5] The university ranking score or Australian Tertiary Admission Rank (ATAR), up to 100.

them through and then the last 4-5 minutes we wrap up it. If it is the end of the topic we will have a mind map activity (Nathan, teacher).

Some teacher participants assumed that by senior secondary school, the students would be self-motivated. They anticipated the students would drive the learning, with the teacher, textbook and videos as support. Elaine explained that she was keen for students to also have time to support their peers:

I try to allow lots of time in class for the students to work. I try to keep explanations to a minimum. Normally a bit of a spiel at the beginning, but basically just trying to get through the basic concepts that are crucial. Then give kids lots of time to have a go, talk with each other, ask for help. The kids tend to be very much spread in their skills (Elaine, teacher).

Videos and computer activities used for homework to save time in class and to cater for student differences, were only mentioned by two of the teacher participants. George described the use of videos as a form of the trend for “flipped classrooms” (Muir & Geiger, 2016), which saved time in class by introducing ideas prior to classes:

I know one teacher is trialling a … flipped classroom, and in fact as part of that project I interviewed every student in that class to see what they thought of it. They did like it. I think their main concern was they liked it because it was an easy topic and it saved time and they were able to do harder problems in class. But they weren't a fan of it when the course got harder and they preferred teacher instruction in particular classes, I suppose there are limitations. The problem is you can't interact with the video (George, teacher).

Libby, on the other hand, used videos to review and support students to self-assess their learning:

The kids use the online stuff so they do the review questions online and they do the quiz online and we've also got some videos. A program that we've bought is a video package, so they can use those to practice as well as a video practice questions for most VCE topics. … it's really good, I really love it. So they have examples for every question (Libby, teacher).

A couple of teacher participants used metaphors to describe their teaching, which indicated they had thought about their strategies and linked them to other parts of their lives. Ian compared learning mathematics to learning music and reflected on the need to work with others:

I saw from my days in an orchestra that when you had a question about some music you start by asking peers. And then you'd have your section and in between you come up with it and you'd ask the conductor. So that's how I set up the class. You have people paired off, and *if there was a difficulty during the study session [they discuss this with their partner] and then ask me [if they still need help] (Ian, teacher).*

Whereas Danielle likened it to cooking and highlighted the need for constant reviewing of previous skills, in mathematics and other areas of learning:

For the first 10 minutes of their double class each week I do a quick revision of something from an old topic. I talk about when you are a cook, you might make a casserole and put it on the back burner while you go on to make the dessert. Unless you stop and stir the casserole every now and again the casserole will be ruined. I encourage the students to work on their summary books every week. Add in the new work every week (Danielle, teacher).

Matt, a teacher with 12 years' experience, provided a pertinent example of the changing balance between student and teacher responsibility, based on the needs of students. In his interview, he discussed participating in a professional development program which involved teachers observing each other. This influenced his view of a teacher focus compared to a student focus in the classroom, and enabled him to see students as well as teachers needed to be active in lessons:

We had this 'learning walk' thing a few years back, where principals from several schools wandered around and watched us teach, the whole school. They said that the teachers were doing heaps of work, but the kids weren't. We were making summary sheets, exam revision, concepts maps, and the kids weren't, so now I try and get the kids to participate in that (Matt, teacher).

Teacher participants discussed adapting their teaching to the needs of their students and used a range of strategies to support the development of mathematical and thinking skills. Teachers demonstrated an awareness of the importance of not dominating class time to allow students time to practice their skills, and work with their peers. The teachers appeared to judge the success of a strategy, for example, group work, videos or a computer program, based on whether students liked the initiative, and how enthusiastically they participated. The balance between teacher talk and students practising individually or in small groups was a common point raised in all the interviews. The perceptions relating to the degree of influence of teachers on the agency of students varied greatly. Additionally, this can be complicated by the fact that sometimes when teachers perceive they are supporting students in their learning, they might in fact reduce students' control over their own learning, for example when teachers provide summary notes, they may be taking away learning opportunities.

5.2.2 Tools to Support Teaching

The next focus of the interviews with the teachers, related to the tools of the activity system (see Figure 5.1) that support learning of mathematics—the learning activities, calculators, bound reference books, SACs, and examinations. The questions were:

> *Which resources, textbook, calculator and exam preparation material did you use when teaching VCE mathematics?*
>
> *Which teaching and learning activities do you use with your VCE classes, which did you think the students found most helpful?* (See Appendix E)

These questions were intended to help uncover how the tools of the activity system supported the development of mathematical and thinking skills, around probability in particular. The tools used to support student learning included: textbooks, simulations, SACs, examinations, bound reference books and calculators.

5.2.2.1 Textbooks.

Textbooks were used extensively for teaching senior secondary mathematics by the participating teachers, although in a variety of ways. Some teachers, such as Danielle, stated that they would ask students to complete every problem from the whole textbook:

> *I recommend the students do everything in the textbook, or at least most of it. I give out tick sheets and weekly schedules that had the SACs and micro-tasks dates. I don't collect their books as the answers will be all the same (Danielle, teacher).*

Other teachers would select particular textbook problems for students to complete and make checklists, as highlighted in Fred's comment:

> *I look at the textbook questions and pick those they should not bother with, otherwise I say do the whole lot. But if you find a section too easy, just move on to the next question and pace yourself, according to your own needs, decide for yourself which questions need to be done, which ones you can push past and you know your own abilities are and what easy and what's more challenging. I don't like to tell students exactly which ones they should do (Fred, teacher).*

All teacher participants used textbooks extensively. The examination was also used as an interpretation of the intended curriculum. George explained how he used the textbook and the examinations together to guide his teaching just like a written curriculum:

Well, I've been setting them pretty lightly in terms of what's in the textbook because I mean there is just so much repetition in there … I just try to graduate in terms of difficulty so a couple of easy ones, couple of medium ones, a couple of hard ones but then I always keep an eye out for the ones that look very exam like, like this appeared in the exam, every year. I will set a lot of those ones, so they get really good practice at them (George, teacher).

Teacher participants indicated that they used the textbooks extensively. While some teachers complemented the use of the textbooks with other resources, others used the textbooks as a pseudo-curriculum.

5.2.2.2 Learning Activities—Simulations.

The formal written curriculum explicitly expects mathematical simulations to be used and as such they are tools and learning experiences (VCAA, 2015a). The recent inclusion of simulations as a component of the formal written curriculum is an example of one of the rules of the curriculum. There was mixed opinion on use of experiments and simulations to teach Mathematical Methods classes amongst the teacher participants. Of the fourteen teachers interviewed, three reported they would not use simulations or experiments in the teaching of probability at all. Six teachers would use them in senior MM, three reported they would use simulations and experiments in younger years, and two teachers did not comment.

Three teachers reported they would not use experiments or simulations at all, in any year level of mathematics classes. Both Arthur and Danielle explained it was unnecessary to conduct experiments in mathematics, to throw coins, dice or pick marbles out of a jar:

I never did that [experiments]. I never did physical rolling of a die or whatever, just theoretical, straight out of the book, because if anything else it would be time-consuming to do the experiment and it is pretty obvious before you start. And it's messy and noisy and you lose coins (Arthur, teacher).

I would not ask the students to throw a coin, not younger aged students either … I would not do a simulation (Danielle, teacher).

Three teachers incorporated experiments with games, beads, dice, coins or lollies into their Year 7, 8 and 9 classes, but did not use them in senior mathematics classes. George explained the physical rolling of the dice would help the younger students overcome their misconceptions:

For Year 7s and 8s I normally would get them rolling die a hundred times to look at the distribution … I've had some weak classes before. And they would tell me like I'm all the *chances of rolling a six is much lower than rolling any of the other numbers (George, teacher).*

Harry and Libby stated they would not include experiments in the senior years as they knew students had previously engaged with them in Years 7–9:

No, you haven't really got the time [in Year 11 and 12], and there is so much data available, I would do dice and coins in Year 7–9, … , I could refer back to it (Harry, teacher).

Libby did acknowledge that sometimes activities helped if students were struggling:

In 7 to 9, we'll get the jar of marbles out, especially for the lower kids but definitely not in Year 11 and 12 but if they're struggling, we do. We go back to it. Yeah, it's very hard because when it comes to probability it's very abstract (Libby, teacher).

Seven of the 14 interviewed teachers reported they would use experiments with the senior Year 11 and 12 students to explain the complex ideas of sampling and expected values. Fred described how sampling beads helps demonstrate the difficult concept of the difference between the sample and population mean:

Yes, I did actually do a bead counting exercise where I take out a sample of beads, I have a particular portion of the population and so there is a population in a bucket and I randomly physically select a sample and look at what the proportions are within that sample. So they can relate to the population proportion as well as sample proportion. How they are different theoretically, and what will happen with the sample, the most likely outcome. So those discussions I do have (Fred, teacher).

Several of the teachers including Claire explained that hands-on activities were important, even for the senior students:

If you just talk about it, they don't remember, you actually have to do it, so I used to get them counting smarties, chucking coins, and make a horrible noise and people clambering around on the floor, but they remember the hands on stuff … . Definitely in Year 11. Because I never assumed anything they had learnt in Year 10 (Claire, teacher).

Nathan and Ian were the only teachers to suggest a combination of actual physical experiments and simulations would be appropriate to support student understanding in senior secondary classes:

I go with a combination of coins and simulations with a small number of physical things. I realise that would work, like if they're all die or if they pass the coin they connect with *physical items better but if they tend to do it a hundred times, they realise that they just get bored … . They then developed some kind of hypothesis (Nathan, teacher).*

I've done both. I've had them use the hands on, and I had them download an app (Ian, teacher).

At least half of the teacher participants appeared to regard the use of simulations or experiments as important tools in probability lessons to support student learning and engagement. However, a number of teachers did not follow the rules of the curriculum by including simulations in Mathematical Methods classes, which might be due to the requirement for use of simulations being a recent addition to the curriculum. The use of textbooks and simulations by the participating teachers has been described, and their use as learning activities for students. The next sections explore the use of calculators, examinations, and practice examinations as tools with rules within the learning activities.

5.2.2.3 Calculators.

In senior secondary mathematics in Victoria, the use of CAS and scientific calculators is encouraged for the longer of the two examinations. The full details of the rules on the use of calculators are explained in Section 1.2.2. Calculators have been used to check answers, solve problems and teach concepts, so would be considered tools of learning.

Teacher participants had mixed attitudes toward the use of calculators as summarised in Table 5.6. All but two of the fourteen teachers interviewed had a positive attitude towards calculator use. Five teachers reported the main advantage of calculators was to save time, with five also claiming they used calculators predominantly to teach mathematical concepts. Two teachers reported calculators saved the students cognitive load and improved learning while two teachers also stated that calculators were unnecessary and added another complication to the subject.

The teacher participants expressed a variety of opinions in relation to the benefits and limitations of CAS calculators in senior mathematics classes. For example, three teachers pointed out the time needed for students to be actually taught how to use calculators in order to make them effective aids, as described by Claire:

There's not much point just plugging stuff into the calculator unless you know what the heck you are doing, garbage in—garbage out (Claire, teacher).

Teachers also used the calculators to teach mathematical concepts. To show how the mathematics fitted together, between algebra, graphs and solutions. Fred connected graphing to algebra:

I have a huge focus on the calculator, I bring up the calculator emulator on the smart board, I love making the connects between the solving by algebra and what it means to the graph (Fred, teacher).

Matt focused on the value of calculators in building up the difficulty to explain general concepts:

The bit [of the PD] I found most interesting was how to teach the concepts on the calculator. Build it up. Use the calculator to show the easy questions then the harder questions, then the general case. I was blown away by that (Matt, teacher).

Nathan then highlighted how calculators can assist with working on mathematical concepts rather than the mechanics of solving a mathematical problem:

The CAS calculators allow them to think in concept level of thinking, makes their cognitive load less, and actually focus on the actual problem rather than the technicality of solutions (Nathan, teacher).

An issue raised by a number of interviewed teachers was that students needed to learn when to use the calculator, and when to do the calculations by hand. Three teachers explicitly mentioned the issue of students needing to know when to use calculators, as explained by John:

It's about the maths and the technology is just a tool that you can use to enhance the maths. And it's not always the most appropriate method. So by hand or by thinking (John, teacher).

There were some negative attitudes and comments regarding calculator use, with Ian suggesting that calculators added work for students and added unnecessary options:

So there are challenges using calculators … I mean why are we giving them options when they don't need to do it that way? (Ian, teacher).

Some universities in Victoria do not allow calculator use in mathematics related courses (Weiss & Tobin, 2016), which led a number of teachers, such as John, to question whether this disadvantaged students:

Well I just think the universities need to catch up … . And it's not that they become de-skilled, because if you ask clever questions you can actually make sure that you ask questions where you really use the technology to enable you to take the drudgery out of it … . And I gather in some universities students still get a normal distribution table, I said are we in some third world country, you don't mean Australia, to me that's just crazy (John, teacher).

Overall, the CAS calculators were seen as an important and integral component of the mathematics course by twelve of the fourteen participating teachers, the other two said they were an unnecessary complication. The calculators were a tool which supported students by helping them save time, cognitive load, and helped them visualise and learn mathematics. Limitations were also acknowledged including adding unnecessary complications to the subject especially as one examination does not allow use of calculators as is the case with some universities. This last factor is an element of the rules of the subject impacting the use of calculators as a tool, causing tension.

5.2.2.4 Bound Reference Books.

One of the tools of learning that students create is a bound reference book, as an element of the activity system (see Figure 5.1). A bound reference book is a single book of unlimited pages which can be taken into some of the final year SACs and one of the two examinations for Mathematical Methods. In the curriculum documents, this book is referred to as a bound reference book (VCAA, 2015a), but during interviews, both teachers and students called it a summary book. It could be created by students, teachers or commercially made. The only conditions are that it be of A4 size or less, have no loose pages or foldouts. This curriculum rule and how it is implemented can affect the development of mathematical and thinking skills, the object, as evidenced in the discussion within the interviews.

It was apparent from the data provided by the teacher participants that some liked to play a greater role in controlling the development of this learning resource. A number of teachers outlined how they directed and supported students in making this book while others expected students to create the book themselves. Some students make the book as they study during classes, while others leave the creation of the book until just before the SACs and examinations. Claire explained how the creation and review of the summary book can be used as a learning task:

> *How useful they (summary book) are when they reach the exams is a debatable point, but doesn't matter, because the actual exercise of putting the exams book together means they are doing really good revision. I think the kids who do it well, and are constantly updating and refining it, it's a really valuable reference, which they probably will not refer to in the exam, but the act of putting it together has been incredibly valuable (Claire, teacher).*

Matt and Harry provided examples of how they actively supported students in constructing their summary books:

I give the kids a formula sheet on coloured card at the beginning of the year, which I get them to add to, the log laws and trig rules, that is just as helpful as a summary book for most kids (Matt, teacher).

Our school had a policy of having a summary book. We allow to them take their summary books into SACs. At my son's school, they are given notes where the kids fill in the blanks, and these are all bound in a book and become the summary book … . Some kids say they can't be bothered writing it, and say to me, you can't stop me taking my textbook into the exam (Harry, teacher).

Ian used the creation of the summary book as a bridge between Years 10, 11 and 12:

I said let's start the formula book routine in Year 10. Start giving them guidance on how to structure it, allow them to use it in all tests, so they were accustomed to using it in assessment items, come Year 12 they are conformable with it. So that for me was a big accomplishment in setting up the three years, end of the school as an integrated VCE structure (Ian, teacher).

Elaine claimed the summary book made the mathematics subject too easy for students, and they might not really understand the mathematics:

I can get a kid through that can do very little work, I can give them a crash course on a subject have a good summary book and a calculator and they can get through. Would they be able to get a 70? No but you can get them through and there's something wrong if that's possible, because I'm not convinced the kids actually know stuff (Elaine, teacher).

Whereas Matt added that the old rules of just two A4 pages encouraged students to do more revision:

I don't know about summary books, I think the old one page of A4 notes was better. The kids would have to summarise down, again and again, that's what I did at school, a summary of a summary. A whole book makes the kids lazy and they think they don't have to remember anything. Some kids just photocopy stuff. You don't learn by photocopying (Matt, teacher).

Although limitations were acknowledged by teacher participants, comments also highlighted that the making and use of the bound reference books can be a positive learning experience for students or a tool for learning. The rules around the creation and implementation of the bound reference books were also perceived as impacting on student learning. Other tools of learning which are controlled by the rules of the curriculum are the SACs, which will be discussed in the next section.

5.2.2.5 Teacher Perceptions of School Assessed Coursework.

The teacher participants had mixed responses to the question on the use of SACs with regard to the restriction of style to application tasks, problem-solving and modelling, the use of realistic contexts, and similarities to examination problems. Two teachers provided probability SACs they had created for analysis, which were analysed in Chapter 4, Section 4.3.1. Other teachers were hesitant to share their resources or used commercial SACs.

Four of the teachers appreciated the opportunities the SACs provided to deal with real-life mathematics, and encouragement of problem-solving, with John stating:

> *I like the changes to the SACs … they are fantastic … because it is putting the focus back on problem-solving and modelling, not just on routine computations (John, teacher).*

Claire perceived the SACs as an effective way to focus the study of mathematics, by including relevant mathematics in context with problem-solving. She reported:

> *I always say to the kids, you learn stuff in maths. What you have been doing all the way up in your school, you have been learning all the way up new tools to put in your mathematical tool chest. But now you are in senior years, you are going to do practical things, and you will have to open up the lid of your tool chest, and decide which tools do you take out for the problem. Problem-solving in SACs is what real-life problem-solving is like. If you are an engineer or a scientist, you have got to decide what Tools you will need. You have acquired most of the tools you will need by Year 11. You have got the tool chests there, what skills do you need (Claire, teacher).*

Ian used SACs not just as an assessment tool but also to introduce university mathematics and to teach a concept, through structured tasks that developed into new mathematical ideas, espousing:

> *So I guess the whole reason I got onto the idea of playing around with SACs structure was being introduced to assessment as learning. And how would you use your assessment tasks as a learning tool as opposed to me as somewhat of a summative assessment experience which once again came through a long-standing interest in formative and summative assessment (Ian, teacher).*

Fred liked the opportunity to include real-life mathematics problems in extended SACs and was amused when the real-life problems became comical in the commercial SACs, stating:

> *I like the real-life applications, the realistic stuff, I try and have as much as possible, not abstract. I looked through other suggested material, and one was about a wedding, Wendy's wedding, and at one point she looks up at the leadlight and says, that looks like an expediential graph, I thought what a ridiculous thing, I'm not doing that. The silly girl shouldn't be focusing on the leadlight she should be looking at her husband (Fred, teacher).*

Five of the teachers reported concerns with one or more aspects of the SACs including expected skills in writing SACs, lack of time to create or conduct SACs, and student cheating. In relation to writing SACs, a number of concerns were raised by teacher participants. For instance, despite Kerri's five years of teaching Mathematical Methods, she experienced anxiety over creating SACs, which was intensified when her school failed the audit for the SACs. She appreciated the motivation to use SACs, which was to increase the range of assessment types, but believed she did not have the skills to appropriately create the tasks:

> *So I suppose this is where my, I suppose I'm going to call it anxiety and my issue is with perhaps setting SACs like this and look I get the whole idea that you can't just have exam questions and we need to give the students an opportunity to show what they understand not through these little discrete quantitative questions that we ask in an exam format (Kerri, teacher).*

Ian, Arthur and Matt also found writing SACs within the rules of VCAA challenging, as evidenced in their comments:

> *The questions within SACs gradually get more difficult, but they still seem to be very closed and similar to examination 2. It's much harder to write a SAC to the course description. It needs to have that open-endedness. Even teachers that are buying the commercial tasks, those tasks are made up of closed questions (Ian, teacher).*

> *The students also had to do some internally assessed work, SACs, which I was never happy about. I never felt confident that I knew what the right standard was (Arthur, teacher).*

> *I used to use old exam 2 questions as a starting point and expand on them. Now change this … now change that... . I went to an MAV making session, in [regional town] and they had this guy from this private school in the city. He showed us his SAC which was way too hard for my kids, they would have trouble with question one, way outside the course (Matt, teacher).*

There were also concerns raised in relation to the time required to create and mark the SACs and perform moderation across groups, as outlined in the following comments:

> *We teachers are so busy, and we have no time to make SACs, one teacher said he spent 50 hours making a SAC. I can't do that. We made our SACs from … commercial SACs, and adjusted them, but failed the audit (Kerri, teacher).*

> *I can't help but feel that cross marking this kind of open-ended question would be an absolute nightmare especially when there are more than two of you running parallel classes (Kerri, teacher).*

Matt described how he wrote his SACs, and how they changed to be more exam-like over the years:

> *And then we started printing SACs in booklets, with spaces, so they looked even more like exams. The graphing SACs were easy to make open-ended, find 3 graphs which go through these points and have a gradient of something or other, the probability was trickier (Matt, teacher).*

Libby explained how she created her SACs, with both open-ended and structured problems. Her main concern was with the time needed to teach to the style of the SACs:

> *So I started from the commercial SAC once and then I modified them to make what I wanted to test and I modified them to teach our students because we had an ESL student so I modified the language to make it a bit simpler. And then I tried to make it as open-ended as possible. Yeah, it's quite difficult to make them open-ended (Libby, teacher).*

Libby also clarified that the skills required for the open-ended style of the SACs, was not taught in her school, which made it difficult to teach and assess in senior classes:

> *So the issue is, how do you teach them that skill when you have got so much content, that open-ended problem-solving skill? When do I have time to teach that? I only see them three times a week, it's quite impossible. So I try and find a nice balance between the two. I try and have it so some are open-ended and some are really structured and that gets me the separation that I need (Libby, teacher).*

One of the problems with SACs is the potential for plagiarism. Concern was expressed by a number of teacher participants about students helping each other and getting help from paid tutors or the internet to complete the tasks. This was a particular concern if the SAC was conducted over several days, as required in the VCE SACs rules (VCAA, 2015a). Schools and teachers tried to avoid this problem by dividing the SACs into independent parts. The following comments highlighted some of the concerns relating to plagiarism:

Security was a major problem [with SACs]. There was no guarantee that there wasn't cheating of one kind or another. The exam was much more objective and purer (Arthur, teacher).

To start with we had one big question, and the kids worked on them for several periods. We collected their book and calculators between classes. But then everyone got worried about cheating, so we had to break the task into bits (Matt, teacher).

My experience of SACs that weren't using discrete pieces was that there was collaboration, which is perhaps a polite way to say it. There are even some attempts to photograph pages secretly and take them home to work on (Ian, teacher).

Problems with SACs occurred if there was more than one group of students, or if students were absent. Kerri and Matt discussed how in order to minimise any time related advantage, parallel classes completed the SACs over a short time period:

So we normally run a session after school that everybody begins the SAC in like a two hour session and then we give them two in class lessons in that same week to get the SAC done within a timeframe of a week (Kerri, teacher).

If they miss a SAC class they can do Wednesday afternoon, or afterschool. If they fail, they do a redemption, to demonstrate the outcomes. Mainly the skills (Matt, teacher).

While adapting the SAC conditions to discourage plagiarism and adapt to the needs of the schools, the school-based assessment became more like examinations, which diminishes the aim of the SACs, as an alternative assessment type.

There were both internal and external tensions reported by teacher participants around the writing and use of SACs. Teachers had the flexibility to make the SACs to cater for their students, and also used these tasks as a form of preparation for the end of year examinations in both content and style of problems. The teacher participants demonstrated that they understood the reasoning behind the rules of the SACs, to encourage a variety of assessment styles and to support students in their learning of applications of mathematics. However, there were also concerns raised regarding the writing and implementation of the SACs, both from a teaching of content and context view, and a practical authentication view. Almost all the teacher participants reported that they needed support in writing SACs. In past, SACs had been provided by the Education Department, so teachers writing their own SACs is a comparatively new idea.

5.2.2.6 Examinations.

Examinations are the focus of the year for many senior secondary mathematics teachers and students. The examinations are the high stakes summative assessment which can be interpreted as tools with rules around the activity of mathematical learning within the activity system (see Figure 5.1). Teacher participants, like the student participants, were very aware of the importance of the end of year examinations. Danielle explained how her students looked at past examinations in the first weeks of classes:

> *In the first week of class, we will look at the VCAA website and look at the past exams. We will look at the two types of exams and see how the examiners write their reports (Danielle, teacher).*

Elaine and Ian both explicitly planned their use of the examinations in their classes:

> *My focus for this year is going to be to do a lot more exam 2 style stuff because we tend to have to focus on getting the skills through, … we tend to do a lot of exam 1 style stuff, because that's going through the content anyway, but not enough exam 2 style for my liking (Elaine, teacher).*

> *We had piles of practice exams. We had six or seven that we got every year … I think there were six that I bought every year (Ian, teacher).*

Claire perceived examinations as predictable, recommending students complete as many practice examinations as possible:

> *Yes, if the kids haven't done a heap of exams, they will get nasty surprises in them. Partly because of the language and the style of the questions. My mantra was always if you have done enough practice exams, there is only so many questions they can ask, if you have done enough practice exams, you will come to a question, oh, that's an old friend … . Those who listened and those who did the practice exams, really benefited from that I think (Claire, teacher).*

Teachers also used examiner's reports for past examinations as George explained:

> *The examiner's report … they're definitely learning from them. They're a great resource to do well in the exams (George, teacher).*

John, who completed his teacher education in Canada, compared the examinations to those in his home country. He pointed out that, for example, examination 2 includes analysis problems which benefited students who were problem-solvers*:*

The analysis questions in exam 2, you don't do that by doing repetition. They [students] have to consider different solutions, methods … . So I think that is one of the problems with Australia compared to some other countries. Students have had very little practice at true problem-solving (John, teacher).

Beth valued the impact of her own teaching and her school's reputation on students' scores:

… I got the highest [in the school] in maths again, not that great, but two 41s. … The good thing is our school average was better than the state average, by 3%. We were better in every category than the state average (Beth, teacher).

The examinations were at times, used as inspiration for the SACs, as explained by Fred:

Sometimes if there are more examples, I will use exams questions as a worksheet in class. Otherwise I will use some as inspiration for SACs (Fred, teacher).

These examples demonstrate that the examinations are used as an intended curriculum, further confirmed by Matt's comment:

I try and have real exam questions as early as possible, just a few questions scattering through the weeks (Matt, teacher)

The practice and past examinations become a tool of learning for students, and a way to enforce the rules of the system. The curriculum included the mathematical content and the rules of use regarding calculators and bound reference books. The teachers and students used textbooks and examinations to plan their teaching and learning, and very few other resources. As such, the textbooks and past examinations supplement the intended curriculum (Study Design) and become the Implemented curriculum, as delivered by the teachers. Some teacher participants used examinations as learning tasks for students and reported that to do well in the examinations, students needed to complete many practice examinations, so as to avoid surprises. Other teachers perceived the examinations, especially Examination 2, contained enough original material of a problem-solving style to encourage students to develop problem-solving skills, which in turn would encourage the development of thinking skills. Practice examinations were also used as formative assessment to support teachers and students in their planning and learning.

5.2.2.7 Summary of Tools—from the Teacher Perspective.

The interviews with teachers revealed the range of teaching methods and priorities to support the students in their development of mathematical and thinking skills. The interviews focused on the use of the curriculum rules and tools, but also on the student and peer community, and the student and teacher division of labour, as modelled by an activity system (see Figure 5.1). The data revealed that there was a reliance on traditional approaches to teaching mathematics whereby the teacher explained the content and ideas and then the students completed practice problems from the textbooks, with only two teachers reporting that they supplemented the textbooks with notes and activities they designed themselves. There did appear to be an awareness of the need to balance between time for teacher talk and student practice, and an attempt to cater for individual cohorts once they got to know the students. Just three of the fourteen teachers explained specific strategies for enabling group work, while others informally encouraged it.

Videos or computer programs were suggested as homework by just two of the fourteen participating teachers. Simulations of probability were reported to have been carried out by half of the teachers, although often as a demonstration, and physical experiments by three of the teachers. Calculators were seen as both a hurdle to master and a scaffold to support students, as a time saver and learning tool. The bound reference book was intended as a scaffold for students, which was appreciated to varying degrees. Some teachers incorporated the construction of the bound reference book into their teaching program, others left it up to students.

The creation of SACs was considered a challenging task, which is why a number of teachers reported using commercially available tasks and adapting them. Tensions existed around creating and adapting SACs to conform with the rules of the official curriculum, to be application tasks, problem-solving or modelling tasks, with a context the students would relate to, while supporting students as they prepared for examinations, which included those required aspects. Past examinations were used as learning tasks and formative assessment, occasionally from the start of the year, but enthusiastically at the end of the year.

Examples of contradictions and tensions within the teaching community as they supported students in their learning that emerged from the data included:

- The teacher's desire to explicitly teach, explain, discuss and write the mathematics, and the conflicting desire to let students practice the mathematics themselves within the limited class time.
- Calculators and bound reference books were seen as a combination of time savers, supports, teaching aids and another burden to be appropriately mastered.
- Achieving a balance between appropriate assessment tasks with authentic contexts, which met the rules of the official curriculum but also avoided plagiarism.
- The balance between the use of SACs as learning activities and assessment tools and using real contexts while preparing students for examinations.

- The need for application tasks, problem-solving or modelling tasks as SACs, but the lack of student experience with these types of tasks.
- The textbooks and previous examinations acted as both the implemented and pseudo-intended curriculum.
- The tensions between the aims of the SACs to cater for individual cohorts of students, relate to the real world, and prepare students for the examinations.

Responses to the teacher interview questions in relation to the tools of learning, will be compared to those of the student participants and linked to the rules and tools within the curriculum and literature in Chapter 6. The next section now delves into the content area of probability examining the teacher participant responses to solving two mathematical problems.

5.2.3 Teacher Perceptions of Probability

Probability was described as interesting and as contributing to increased mathematical thinking, by five of the teachers. For example, Arthur described calculus as mechanical, while probability involved original thinking:

> *I thought probability was more interesting, calculus gets a bit mechanical after a while … but there is more original thinking involved in probability, less recipes (Arthur, teacher).*

Claire taught probability with hands-on activities and videos which she thought improved student interest and learning:

> *Probability can be awfully dry if you just talk about it, the more I taught it the more interesting I found it. The more practical stuff we could do, the more it would stick (Claire, teacher).*

George stated that he liked the topic due to the possible learning activities, while Nathan agreed that his students enjoyed probability lessons due to the activities:

> *I've liked the sampling part of the course that they've actually put in because I was able to … well it kind of lent itself to some different activities I could do. So for example I bought the whole class in groups of four, a packet of M and Ms [coloured chocolates] … and I got them to write down how many blue ones are in each packet and then we'll basically be sampling the number of blue ones in each packet. And to calculate proportion means and all those sorts of things with it in terms of what each one had (George, teacher).*

> *The class enjoyed probability a lot more. So my feedback was that the marks were higher on this topic and their class participation was better. Yeah they were more engaged maybe*

because I could take more props into class compared to say graphs and algebra, this case with almost every other session we had something going on in class (Nathan, teacher).

Elaine explained that probability was very relevant to students' future careers and life, although she also described the confusion that often exists between theoretical and experimental probability:

Probability was a bit of a passion so in terms of content as I say I can see the relevance of it. You've got to get the idea through to them, of the experimental versus the theoretical, … they tend to always confuse that aspect. (Elaine, teacher)

The teacher participants explained how probability fitted into the subject of Mathematical Methods. Arthur described how probability was left until the end of the year, and often covered in a rush. George reported he felt the topic of probability did not fit comfortably into the subject, as the rest of the course was about algebra, graphs and transforming graphs:

It's I suppose it is a bit left to centre in terms of what's in the course because everything seems to build on each other (George, teacher).

Fred explained that the probability topic was an application of integral calculus and flowed from the calculus component of the subject:

With Year 12 methods it does overlap, you have got that probability distribution which is all about area under the graph, nice connections to calculus, but even before you get into the probability distributions, looking at the discrete data, the discrete distributions, and normal distributions and things like that can be nice applications so many things that have normal distributions in nature (Fred, teacher).

The Mathematical Methods subject is about functions and graphs, and probability. Probability has components of transforming functions, in particular the normal and binomial distributions, which some teachers pointed out. Several teachers reported that their students tended not to like probability which John suggested was because it was less routine, while Matt suggested it was counter-intuitive:

Probability was their least favourite. The favourite was algebra and calculus. You just learn a routine and you just crank the handle and out comes the answer. But with probability you have to understand what's going on (John, teacher).

I like probability, some of my best calculus students don't get probability. It's counter intuitive. The wording of the questions too. And some kids don't play cards and board games anymore. They don't have a feel for it (Matt, teacher).

Three teachers reported the language and symbols of the topic of probability were difficult for students to understand:

> *Probability can be deadly for the kids to have to deal with, and I see a lot of frustration there as well. Like all the other worded problems, and on the binomial seems to throw them a bit (George, teacher).*
>
> *I don't mind probability, not all teachers like it. Kids hate it for the same reason I think, translating the English into maths, is the problem. Probability is more difficult, some of it is ambiguous (Harry, teacher).*
>
> *You've got p hats and you've got ps, in specialist lower case p hats an upper case p hats and lower case xs and upper case xs, unless it's taken down one step at a time (John, teacher).*

Ian pondered whether the fractions within the probabilities were what caused the students to dislike the topic of probability:

> *It would be really interesting if I could go back and find out which students had difficulty working with fractions in Year 6 … . And find out five years later those same students don't like working with probability. Because of fractions they shut down (Ian, teacher).*

In general, the area of study of probability within the senior secondary mathematics units of Mathematical Methods was seen as important by the teacher participants, with some describing the topic as interesting due to the learning activities which could be included. Not all could see the links to the other parts of the subject and acknowledged that some students did not like the topic due to the lack of predictability, language, symbols and involvement of fractions. Overall, probability was described as important, relevant, a bit different to the rest of the subject, or a suitable avenue for encouraging students to think a bit differently in their mathematics.

5.2.4 Summary of Findings from Teacher Interviews

Fourteen teachers from a variety of Gippsland government schools from a range of geographical areas and teaching experience reflected on teaching their VCE mathematics subjects, with a focus on the subject of Mathematical Methods, and specifically the probability area of study. The participating teacher were open and keen to share their ideas and concerns.

From the interview data, the teachers explained that they supported and guided students in the activity of the development of mathematical and thinking skills, using the tools of textbooks, learning activities, calculators, bound reference books, SACs and previous examinations. Calculators and bound reference books were seen as positive and helpful tools to support students during their learning and during assessments. The teacher participants relied heavily on the textbook and previous examinations to support their implementation of the official curriculum, with few teachers using other resources for their own teaching or for student learning. SACs were used as formative assessments tasks, with tensions between encouraging the need for modelling and problem-solving, and the desire to use SACs as examination preparation. Tensions also surrounded the creation and implementation of the SACs, due to the workload of teachers, and the issues of timing and authentication of student work. The teachers encouraged the division of labour, or the responsibility of learning to fall between the students and teachers to varying degrees. All of these influences were explained and linked using the activity system, as shown in Figure 5.1.

5.3 The Probability Problems

The final part of the interviews with the student and teacher participants involved the *Two-coins problem* and the *Frequency table problem* See Chapter 3 Section 3.4.2.3 for the background and justification for inclusion of these problems. The possible solutions, use of these problems in the literature, and the misconceptions surrounding these problems were also discussed. In this section, both the student and teacher responses to these problems will be reported and compared. The responses will also reveal the participants' wider attitudes to learning and teaching of probability, mathematics and thinking skills. Students should have had previous experience with these problem types as similar problems were in the intended curriculum from Years 7–12.

5.3.1 The Student Perspective on the Two-coins Problem

The first mathematical question, asked of the student participants, was the probability of gaining two heads on the toss of two coins, with the context of a game within a family. They were then asked if the game was played a thousand times, how many times would two heads occur so that Mum could win, and finally how accurate would their answer be, and how would they approach the problem. The format of the problems is in the Methodology Chapter 3, as described in Figure 3.3. Of the 20 students interviewed, 16 answered the question correctly, with either 25%, 1/4, or 0.25. Four students answered incorrectly, with responses of 1/3, 2 out of 6, 2 sixths, 0.5 x 0.5=1, and 25 or 50. The two responses of 1/3 or its equivalent are consistent with the common misconception for this question of *equiprobability bias* (Garfield, 2003), which is falsely justified if the students incorrectly thought that a head and a tail with the two coins was the same as a tail and a head.

Possible ways to explain this solution included a list of the sample space or tree diagrams. Most of the participating students reported that they thought about the list of possible outcomes. Of the twenty student participants, eight agreed the answer could have been found with a tree diagram but added that these questions were straight forward and did not need a tree diagram, as tree diagrams grew too big too quickly to be used in other questions. Just one student, Ken, thought a tree diagram would be his first method of solving this question:

> *There are a couple ways you could do it—you could either draw like a tree diagram, display each outcome (Ken, student).*

Most students visualised a list, as Ivy explained:

> *I would write down all the possible outcomes for the head-head, tail-tail, head-tail and tail-head and then ... that's ¼, 25% (Ivy, student).*

The students were then asked if the game was played 1000 times, how many times would you expect mum to win and how many times would you expect two heads from tossing two-coins? All students knew to multiply the probability from the first question by 1000, and all but one could do this successfully. Most students could then explain the answer would be the mathematically expected value, but the real experimental value would be in a range around that figure. There were varieties of ways put forward to explain this range or confidence level, as evidenced in the following comments:

> *Would it be exactly 250? If it was distributed perfectly and both coins were perfect it should, but due to different errors and different—well when you say I'm looking at random well you put a random error into your calculations well no it wouldn't be. But the larger your sample size the closer you'll get to that 250 mark (James, student).*
>
> *So 250 times mum would win, but then you need to look at. Obviously, you'd have a confidence interval (Gabby, student).*
>
> *Oh no, nothings ever exact with probability, around 250 … . Between 220 and 280 (Nick, student).*

A couple of the students were incorrect and confused, either with their answer or with their explanation, such as Foster:

> *1/3 of a thousand, which is—whatever that is—150? No that's … . Whatever 1/3 of a thousand is (Foster, student).*

Pete's response was unexpected. He correctly answered the chance of getting two heads from tossing two-coins was 25% but with uncertainty in his voice. He then went on to predict between 250 to 500 double heads out of 1000 instead of the correct answer of a confidence interval evenly spread around 250. Pete had th*e bilateral not unilateral* understanding which teacher participant

Nathan had explained. Pete then continued to respond with an expected value of 300, flipping back to the *equiprobability bias* (Garfield, 2003):

> *If I was going to give an estimation based on what I already knew, I say at least between 250 to 500 times. Out of the three people, maybe 300, a third of a chance of winning (Pete, student).*

Ivy answered the probability questions correctly, but her explanation demonstrated her belief in subjective chance, luck or throwing ability:

> *No, because its luck, someone could throw it badly or you could have one person throw it and another person throw it and their throwing abilities could be very different to each other, yeah and variables could change (Ivy, student).*

Three students related the problem to a context which interested them. Cathy related the questions to the study of genetics, within her specialist area of study, Biology. James related it to errors in measurement in engineering (as he is studying Engineering), while Rob remembered his teacher using the historic gambling game Two-up, for the two-coins problem. Their comments included:

> *So, I'm very interested in genetics so I am doing [Punnett] squares and stuff like that so they show me also that they're the same but different (Cathy, student).*
>
> *That's more due to engineering in general; we actually do a full unit about measurement and then we look at the degree of error in our calculations and where we expect them to be (James, student).*
>
> *We covered that [two-coins game,] I think at school—I think that it was because our teacher was of an age, and she explained about Two-up (Rob, student).*

The probability question about the chance of getting two heads from tossing two coins should have been straightforward for the students as the probability content of this type is in the Year 7 to the senior secondary curriculum, and these students had all successfully completed Year 12. Nonetheless, the part of the questions about variation was confusing to some students, which may not have been covered so well in the subject material, perhaps due to a lack of coverage in the lower levels and being a recent addition to the subject (VCAA, 2015a).

Overall, student participants did not have a problem with fractions, decimals or percentages with their probability problems. Only two of the twenty students exhibited the *equiprobability bias*, or lack of understanding of independence, as described by Garfield (2003). While eight participants identified a tree diagram as a possible method of solving the two-coins problem, only one used this method, with most making a list of the event space to solve this problem. Most students could calculate the expected value of double heads from 1000 two-coin throws and could explain with estimates the confidence interval. Two students were confused by the questions, and responded with incorrect solutions, demonstrating a misunderstanding with the probability concepts, which is surprising as they should have been considering these questions for five or more years, according to the intended curriculum. Finally, three students related the two-coins problem to other areas of their studies, which is a sign of higher order thinking according to SOLO.

5.3.2 The Teacher Perspective on the Two-coins Problem

The two-coins problem has been discussed from the point of view of the student participants. The responses of the teacher participants are now reported and analysed. All fourteen teachers correctly identified the probability of two heads from the toss of two coins as one quarter, with one teacher correctly answering in the ratio format of 1 in 4, and one as 25%. They were then asked about the solution methods they would recommend for their students. Seven of the fourteen participating teachers thought students would use tree diagrams to answer the first question about the probability of Mum winning, the other teachers did not suggest which methods students would use:

> *A tree diagram, definitely. Once you know that more than one thing's going on it's a tree diagram or common sense because that one is quite straightforward (Libby, teacher).*

> *I would expect the students to do a tree diagram. I like tree diagrams (Danielle, teacher).*

> *Well I tend to get them to draw a tree, I'm a big fan of trees so then there won't be any misconception, so that would be my first thing to them, any probability problem, draw a tree, a very physical picture of what it looks like (Elaine, teacher).*

> *Tree diagrams are really useful for methods because even with trying to get your head around really complex questions, even if the tree is too big to draw on the page, if you start drawing the tree it gives you a sense of what it is that is happening. And how the questions might be tackled (Fred, teacher).*

George was the only teacher to actually draw a tree diagram during the interview.

As the interview questions had been provided to teacher participants prior to the interviews, Nathan had asked his three VCE students the two-coins problem. He explained that in the class environment, under the guise of a revision question, all his students had used tree diagrams and answered the questions correctly:

The students came up with 25 per cent and 250 times. Exactly. All three of them. All used a tree diagram. One of them used a table as well. One student said Mum would win at least 250 times so he realised that's not an exact value but he didn't realise that it's a bilateral not unilateral difference though (Nathan, teacher).

Six of the teacher participants could suggest possible problems or misconceptions that students might have in relation to the question. Danielle and Arthur expected all students to find these questions straightforward, while Harry suggested that students would be able to answer this instinctively, but then get confused after they started to learn the formula:

I would ask straight first, see what answers we get, generally they were pretty good, if you asked them this sort of thing cold. After a little bit of teaching, the probability of an outcome divided by the total number of outcomes, and then they will start answering this question a third, a third, a third. They are more likely to answer a third, a third, a third, after they have learnt the rule, than before. Once you have learnt the rule you would go onto a tree diagram, and then they will get back to a half, quarter, quarter (Harry, teacher).

Both George and Matt predicted that students would have the misconception that a head-tail combination was the same as a tail-head combination and see just 3 possible outcomes:

I think the misconception that they could say is that they would see heads tails and tails heads and they might actually count that as one outcome. And then they just have to see three things in their samples space as opposed to four (George, teacher).

Some would say a third, as there are three possibilities. I would make a list of the possibilities, maybe a tree diagram, but more likely a list. Some kids might make a tree diagram with each branch as throw. That's the sort of silly thing they would do. Or not realise a head-tail is different to a tail-head (Matt, teacher).

Overall, half of the teacher participants expected students to use a tree diagram to solve this problem and expected students to find this problem straight forward.

The next part of the question required calculating the expected number of two-heads after 1000 throws of the pair of coins. The teachers all answered 250 correctly, with most also adding the answer would be appropriate, with correct explanations of why there would be an expected variation. Ian extended his response on how he would explain this to his students:

For a student that just doesn't like the idea of expected value, I find that that building from scratch. If you throw it four times, you'd expect one of them. How do we expand that one out of four to scale up to one thousand? I had success with that strategy (Ian, teacher).

When the teachers were asked about the accuracy of the expected value, some teachers expressed surprise and interest in this approach, although others thought it was not important as it was not required in the curriculum, although it had recently been added (VCAA, 2015a). This is an indication of the bigger issue that teachers may not be aware of changes to the Study Design.

Libby described an issue with the students who also studied psychology as they learned about surveying and sampling, which includes subjective results, which can add confusion in experimental and theoretical probability:

Well, it depends. We could calculate the expected number, but we could also do it theoretically in you know we teach some experimental and theoretical probability. And one thing we do is these experiments and you find out that the more you do it the closer you get to the expected value. So it's particularly tricky the kids that come from psychology where they start talking about beliefs and causation whereas in maths we don't do any of that (Libby, teacher).

Arthur expressed concern that teaching probability might lead to gambling, so was keen not to make it too enjoyable:

One thing that might be worth thinking about is ethics, I think it would be general agreement you shouldn't do things that they enjoy so much that it encourages them to gamble. They might start gambling using real money, that would be a question in my mind, is it going to encourage them to gamble? (Arthur, teacher).

The misconception about luck being a factor when learning about probability was not mentioned while discussing these questions. George did mention that when working with dice students sometimes thought that rolling the number 6 was less likely than rolling a lower number:

Relating it back to the dice one. I've had some weak classes before. And they would tell me the chances of rolling a six is much lower than rolling any of the other numbers. There would be some sort of perceived notion that rolling higher numbers is much more difficult (George, teacher).

Overall, the teachers reported that students would get the first part of these questions correct and agreed the two-coin problem should be straight forward for students. Half of the teacher participants thought students would use tree diagrams to solve the two-coins problem, but this was not evident in student participant responses. Some teachers suggested strategies for the teaching of the expected values and confidence intervals, while three did not see the relationship between the variation in expected values as relating to the new part of the Mathematics subject. This could have been as the interviews took place earlier in the year, before the teachers had taught the probability section, which tended to be taught last. While discussing the two-coin problem, some general points around the teaching and learning of probability were made. There was one comment from a teacher participant around the possible problem of teaching probability which might lead to gambling, and student views on luck and subjectivity which might be associated with studying psychology.

5.3.3 Overview of Responses to the Two-coins Problem

The student and teacher participants' responses to the two-coins problem indicated a lack of alignment. While teachers expected students to use tree diagrams for the question, students thought a tree diagram was unnecessary and would complicate the solution. Few teachers could explain any misconceptions students might have, yet a number of students did have problems with the question. One interesting observation is that the three students (whole rural class) who were asked to solve the two-coins problem in class by teacher participant Nathan, all used a tree diagram, yet none of the student participants used a tree diagram when interviewed for this research project. Most of the students responded to the problem correctly and explained their solutions and those who could not answer correctly displayed the misconceptions expected from the literature. As anticipated, the students found explaining the variation more challenging than calculating the expected value. A more detailed comparison of the responses made by student and teacher participants, and links to the literature around these questions and associated misconceptions is covered in the Discussion Chapter, Section 6.1.6.

5.3.4 The Student Perspective on the Frequency Table Problem

The second mathematical question asked during interviews involved a frequency table, as described in Figure 3.5 in the Methodology Chapter, Section 3.4.2.3. The problem had a frequency table comparing the gender and employment status of people in a small town. The student participants were asked to answer the multiple-choice problem, but also to explain how they would approach the questions, and how this problem could be confused. The first multiple-choice response was the correct answer to; *What percentage of females were unemployed?* The difference between the alternative answers is in the wording. The second response was the answer to; *What is the percentage of unemployed females from the population*? While the third response was the correct answer to; *What percentage of unemployed people were female*?

Twelve of the twenty students responded correctly with choice A, eight students responded incorrectly, with choice B chosen by six and choice C by two students. All but two students identified the issue with the problem was the wording. The two students who answered incorrectly with C, were the only students who thought the wording of the question was not an issue. This suggests that students are not always aware of what they do not know, and that confidence is not always related to competence. Although this is a question of a type these students would have been exposed to since Year 8, most students re-read and pondered this question before responding, as Cathy explained:

> *I had to read it a couple of times … . If it was worded differently as well, what percentage of maybe unemployed were females or, it depends on the wording? I remember we were told often before the exams to read the question carefully (Cathy, student).*

The students explained their thinking, for both the correct and incorrect responses. For example, Gabby responded to the question correctly, while Tracey was confident but incorrect:

> *I mean we've got to learn what's in the question and what isn't in the question. That's so important, to be able to analyse that question and say well, they haven't said the total population. That's not in there, must only relate to the females (Gabby, student).*
>
> *It would be B. Because if it's females that are unemployed then it would be ninety-eight over the total people (Tracey, student).*

Pete linked the question to his general knowledge and logic, stating:

> *I did it by a process of elimination, essentially, I think 7 percent is too low, 50 percent is definitely high …so by default I got A (Pete, student).*

All of the students were happy to explain their answers. Most students thought the wording of the question would confuse some students. Vinnie explained how the wording was confusing:

> *It's just the word order… just flipping unemployed and females will change the question to make that one [response C] right. Because then what percentage of unemployed were female. … I can see someone getting a bit confused about it especially if they read it quickly (Vinnie, student).*

From this information on the probability questions in the student interviews, some of the tensions and contradictions were evident within the activity of learning probability. The frequency table problem was similar to problems the students would have seen for many years, but eight of the twenty students responded incorrectly. The wording of the problem could have been the main issue, even if the students did not realise it.

5.3.5 The Teacher Perspective on the Frequency Table Problem

The frequency table problem was also asked of the teacher participants during interviews, and they were also asked to explain their thinking behind their answer. They were asked, *Which answer is correct and why? What might be confusing about this question? Would you like to reword this question?* All of the teacher participants correctly answered this question, except one, (Elaine, who had mathematics and statistics majors at university), who suggested rewording the question to match the answer she gave, which involved a percentage of the total group rather than just females. All but one teacher (Harry) thought the question would be confusing to students due to the wording. A number of suggestions for rewording the question were provided, including:

> *Using the information in the table above, which is the correct calculation for finding the percentage of females unemployed? (Danielle, teacher)*

> *From the population what percentage of females were unemployed? (Elaine, teacher)*

> *What is the percentage of unemployment among females? (Nathan, teacher)*

Some teachers would have liked to reword the questions but were not confident in an alternative. Fred and Libby's tentative attempts at rewording the questions were:

> *What percentage of females were unemployed out of the total number of females? (Fred, teacher).*

> *Considering the female population. What percentage of the females were unemployed? Oh, I don't know how I could put it (Libby, teacher).*

Nathan rationalised the correct answer by suggesting the question would be asking about the percentage of unemployed females so it could be compared to the percentage of unemployed males at a later stage. Nathan asked the frequency table problem of another teacher at his school and his four Year 11 Mathematical Methods students, all five of whom answered incorrectly:

> *My students, picked B. So they got the base as the total population. And then one of them got C. She actually got the rates of females being unemployed. Then when I was talking to one of my colleagues, she teaches maths as well. She said no, based on this wording she would use the overall total. I thought it was controversial but then I realised I wanted to rewrite the question as; What is the percentage of unemployment among females? (Nathan, teacher).*

Nathan justified his students and fellow teacher's responses by comparing them to textbook questions which he suggested normally ask for the percentage out of the whole total number of participants. Elaine suggested the confusion with the wording as a general issue in probability:

> *So, the question would be—are you just referring to females or are you talking about the percentage of females within the population. So, it's either going to be [answer] a) or b) depending on the focus of the problem and that's always the thing with probability (Elaine, teacher).*

Fred also elaborated on the frequency table problem, explaining that multiple-choice questions can be used to draw out these misconceptions, so he questioned the definition of unemployed in this context:

> *Multiple-choice questions are good when they actually highlight a misconception and allow you to deal with the misconception and give feedback on what they have done incorrectly. They don't necessarily take that on board the whole time, because often with multiple-choice questions they will get the wrong answer but they will not think about why. The idea of reading the question and understanding what is relevant, because it has more information than what the questions really require to solve the problem (Fred, teacher).*

Ian proposed that the confusion could be due to the way in which questions are presented in the textbooks. If the textbooks use a particular style of question, students tend to follow that pattern in answering questions:

> *And in my experience I've wondered to what extent it is that a student has done a certain number of homework questions, where there were more questions which use the row rather than the column, and then under exam stress they use the column or the row, because that's what they had done more of. And so I'm thinking maybe they resorted to an instinctive way, I have no idea how I would go about untangling that (Ian, teacher).*

Ian expanded on how he purposely set up situations like this, with 'trick' wording of questions, so he could discuss with students as to how to identify and avoid these issues. Matt suggested that some people have problems with percentages, and thought this was a big problem as probability and percentages were so important in daily life, especially in terms of interpreting the news and current affairs:

> *The wording will throw some kids. Even adults. Percentages, some people have a real hang-up about percentages … I don't know how you could word it better, but that sort of thing is*

> *so important because that sort of thing is in the news, and the politicians use these badly too. Like the crime statistics of refugees (Matt, teacher).*

In mathematics teaching, reasonableness and authenticity are often contextual. For example, given the low socioeconomic status associated with the Gippsland region, an unemployment rate of 58% might not seem unreasonable but to students who live in high SES areas, it may be difficult to relate to. Another complicating factor is if the questions are examined critically, how is unemployment defined? For example, would a person who is employed for one hour a week be classified as unemployed?

5.3.6 Overview of Responses to the Frequency Table Problem

The frequency table question was an appropriate question to present in the interviews, as it is the type of question students would commonly have seen during mathematics classes at secondary school, but also of a type which demonstrates common misconceptions. It is a realistic question on an issue which might be of interest to students. Through discussing this problem, a number of complexities associated with teaching and learning probability were highlighted.

Eight of the twenty students answered the frequency table problem incorrectly. This could be due to the wording of the question, or students following precedents set by past problems, or relating the problem to their general knowledge. The teachers correctly answered the frequency table problem except one teacher who reworded the question to match her answer.

The teachers were aware of the potential misconceptions and complications around the frequency table problem, as were the students. They could explain the potential problems with the wording, format, and possible errors with the frequency table problem, more so than the two-coin problem.

The wording of the frequency table problem was identified as an issue by all the teachers except one. The participating teachers identified other issues. For example, the way in which textbooks present similar problems might influence student responses to assessment problems. The style of the problem, being multiple-choice, was also mentioned as common misconceptions being included in alternative answers can also influence student responses.

5.4 Summary of the Analysis of the Student and Teacher Interviews

This section reported on the findings from the data provided during interviews with students and teachers. Elements described included the usefulness of the various tools to support student learning, the community around the students, and the division of labour or agency. The interview data provided insights into the activity of student learning, and highlighted supports and tensions in the activity system (see Figure 5.1). Main findings from the interview analysis included:

- Students and teachers demonstrated heavy reliance on textbooks as a tool in the implementation of the curriculum.
- The tools of calculators and bound reference books supported the development of mathematical and thinking skills, however there were tensions due to the extra time required to learn how to effectively utilise these tools. Students were less positive than teachers about the value of calculators and bound reference books.
- The rules regarding the use of calculators and bound reference books, which also limited their use for some assessments, created tensions for students.
- The use of the various tools (textbooks, learning activities, calculators, bound reference books and previous examination papers) provided insight into the division of labour between students and teachers, which was varied.
- Tensions existed around the creation and implementation of the SACs, with conflicting aims of preparing students for examinations and the objective of diversifying assessment types to encourage a wider range of thinking skills.
- Participants had mixed emotions toward the topic of probability, finding it both related to real life, and irrelevant.
- Simulations of probability events were not utilised as much as expected by the intended curriculum.
- The two probability problems discussed in the interviews provided insight into some of the tensions in the teaching and learning of mathematics. Just a few students still had misconceptions in these concepts even though they had been exposed to these problems for years, according to the intended curriculum. Most teachers seemed to be aware of issues students would have in the understanding of these concepts.

Chapter 6 will discuss and compare results from all data sources, namely the interviews and document analysis. Discussion of the analysis of the elements will be conducted with regard to the research questions and the literature, using the framework of the related activity system (Engeström, 2001).

Chapter 6: Discussion

The previous two findings chapters provided details of the description and analysis of the data collected associated with the current study, which examines the development of thinking skills within the probability section of senior secondary mathematics curriculum in Victoria. The data originated from document analysis of the Study Design, textbook, assessment tasks and support material, and interviews conducted with students and teachers.

This chapter addresses the research questions, which are repeated below, by analysing these multiple data sources with regard to the development of thinking skills by students, using the Two-Tiered Mathematical Thinking Framework, the TIMSS Curriculum Model (Mullis & Martin, 2015) and the overarching conceptual framework of Engeström's second generation Activity Theory (2001).

The elements of the activity system (subject, tools, rules, community, division of labour, object, and outcomes) can be used to focus the discussion of complex system issues and fits well with education due to the interconnectedness of elements that affect learning and teaching. Figure 3.1 from Chapter 3 illustrates the activity system. Research using Activity Theory looks for tensions and contradictions within and between the elements, and over time, with the view that these tensions and contradictions can be opportunities for learning, development, or expansion. Tensions and contradictions are not the same as problems or conflicts, as they may be perceived or latent (Engeström, 2001; Goodchild & Jaworski, 2005).

Research question

How does the study of probability impact on the development of thinking skills?

Sub-questions

1. *In what ways are thinking skills developed through the senior secondary Victorian mathematics probability curriculum?*
2. *What factors impact on the development of thinking skills through the teaching and learning of the senior secondary Victorian mathematics probability curriculum?*

Following a discussion of each sub-question in this chapter, the overarching question will then be discussed in the concluding Chapter 7.

6.1 Discussion: Curriculum Documentation

Sub-question 1: In what ways are thinking skills developed through the senior secondary Victorian mathematics probability curriculum?

The first research sub-question asked about the ways in which the curriculum supports (or hinders) the learning of mathematics and development of student thinking, using a case study of the Victorian senior secondary mathematics curriculum in relation to the probability area of study. For the purposes of discussion, this sub-question has been broken into five sections relating to:

1. How all three aspects of the curriculum support the development of thinking skills.
2. How the rules and tools of VCE mathematics curriculum support the development of thinking skills.
3. How the context of the curriculum supports the development of thinking skills.
4. How the topic of probability curriculum supports the development of thinking skills.
5. How the VCE mathematics curriculum supports a range of students to develop thinking skills.

6.1.1 Thinking Skills within the Three Aspects of the Curriculum

This section will discuss the three aspects of the curriculum exploring how these aspects support the development of both mathematical skills and a variety of thinking skills. The three aspects of the curriculum considered in this study, as defined by TIMSS (Mullis & Martin, 2013) are,

- Intended curriculum: Study Design (VCAA, 2015a)
- Implemented curriculum: the textbook
- Attained curriculum: SACs and the examinations

Each of these three aspects have been investigated individually in the literature review and results chapters. In this discussion chapter, they will be compared and contrasted regarding their potential for the development of thinking skills to determine whether there is a mismatch or tension between them which means the intended curriculum is less likely to be attained. Using an activity system, the intended curriculum is a tool which is impacted by rules, to support the desired object of development of mathematical and thinking skills. The textbook, as implemented curriculum, is also a tool, where the object of the system is the development of mathematical and thinking skills. Activity Theory looks for tensions and contradictions within and between the elements.

In the intended curriculum for VCE mathematics, a variety of thinking skills for mathematics were encouraged by the requirement (rule) that students must demonstrate the three VCE Outcomes of concept understanding, applications, and use of technology (VCAA, 2015a). This is unusual, as traditionally senior secondary mathematics students are considered successful through passing examinations, using purely quantitative methods (Li & Lappan, 2014). This creates tension as a pass in the mathematics subject is gained by demonstrating the three VCE Mathematics Outcomes, while the grade, study score and university entrance ATAR score is gained by a combination of SAC and examination results. This requires students to meet two criteria to complete the subject.

The expected level of thinking skills within each of the three aspects of the curriculum was classified using the Two-Tiered Mathematical Thinking Framework, with the results listed in Table 6.1, which synthesises several tables of results from Chapter 4.

Table 6.1

Proportion of Lower and Higher Order Thinking in VCE MM34 Probability Section

Aspect of curriculum	Tool	LOT %	HOT %
Intended	Curriculum statements	57	43
Implemented	Combined probability textbook problems	74	26
Attained	SACs	60	40
	Examinations	52	48

Note. LOT is lower order thinking, while HOT is higher order thinking.

Overall, the probability curriculum aspects analysed here, contained 40%, or more, higher order thinking statements or problems in all aspects of the curriculum, except the parts of the textbook where students first learned and practised the concepts. The intended curriculum in senior secondary mathematics has about 43% of the curriculum content statements from the VCE MM34 Study Design as higher order thinking according to the Two-Tiered Mathematical Thinking Framework. Textbook problems include 26% higher order thinking problems overall. The review chapters had a larger proportion of higher order thinking problems at 52%. The review chapters consisted of problems in the style of the three types of examination problems: the technology-free short-answer problems of the first examination and the multiple-choice and extended-answer problems of the second examination. Many of the higher order thinking problems were in the extended response sections of the review chapter. The marking scheme for the SACs ensured that 40% of the SACs involved higher order thinking. The examinations varied from 40 to 56% of problems with an expectation of higher order thinking. This is an appropriate amount of higher order thinking considering the amount of new mathematical content, and when compared to the related literature.

While comparisons with other studies were difficult to make due to the lack of a uniform way of comparing the thinking skills, complexity or cognitive demand of curriculum or textbooks, it seemed that the senior secondary mathematics curriculum described above did contain more higher order thinking problems than the curriculum described in the literature. Studies examining thinking skills in the intended curriculum of senior secondary mathematics were rare, and none involved only probability. Several studies investigated the lower years of schooling. For example, Porter (2002) examined Year 7 mathematics curriculum whereby the USA state curriculum was compared to the USA National Curriculum, and it was found that none contained more than 10% higher-level thinking problems. Atweh, Miller, et al. (2012) reported that the four required proficiencies were unevenly represented, with the Year 8 curriculum statements in the Australian Mathematics Curriculum including 53% understanding, 56% fluency, 12% problem-solving and only 7% reasoning, where reasoning was defined as the ability to describe thinking. Most studies which classify curriculum according to thinking skills, focus on the implemented and attained curriculum, via textbooks or assessments, rather than the curriculum statements, whereas this study compared all three aspects of the curriculum, the intended, implemented and attained.

Textbooks can act as implemented curriculum. In the current study the probability chapters of the textbook examined, contained just 26% higher order thinking problems. Using the TIMSS 1999 study as a benchmark, Year 12 textbooks in Australia were analysed (Lokan & Greenwood, 2001), with approximately 40% of problems involving the higher order thinking skills of investigating, problem-solving and reasoning. The corresponding figures for these higher order thinking skills in Year 12 Japanese, American, and Swedish textbooks were approximately only 20%, while Israel and Norway included approximately 70% higher order thinking skills. More recently, Wijaya and Kaur (2018) analysed three Year 10 Indonesian textbooks, finding only 29% of problems were of higher cognitive demand, with problem-solving or connections between concepts.

Assessment tasks, SACs, and examinations are an indication of the attained curriculum in the current study. Of those investigated, 40–58% of the probability examination problems required higher order thinking. This compared favourably to examples from the literature. Mathematics examinations in Pakistan contained 18% (Grade 9) and 42% (Grade 10) higher order application and analysis problems using Bloom’s Taxonomy (Saeed & Naseem, 2014). The level of thinking in UK school leaving mathematics assessments, were classified using a variation of Bloom’s taxonomy and the three level MATH framework (Smith et al., 1996) by Darlington (2015). In the UK, some universities use A-level school examinations as university entrance criteria, while some universities use independent specific university entrance examinations. A-level examinations contained 90% of marks allocated to the lower Group A skills, with just 3% of the marks allocated to the higher-level Group C skills. The independent university entrance examinations contained between 13 and 68% higher-level Group C skills, which compared to 52% higher-level Group C skills for undergraduate mathematics courses. In comparison, the Australian VCE Mathematical Methods probability sections of the examinations that were looked at in the current study contained more higher-order thinking problems than the UK A-levels, but less than some of the independent university entrance examinations.

The alignment of the three aspects of the curriculum demonstrates a variety of thinking types for students to potentially develop. The curriculum is one of the tools of the activity system, which also contains rules of implementation. The mathematical and thinking skills are the object of the activity system. The amount of higher order thinking in the VCE mathematics probability area of study compares favourably to the findings of the small number of studies found in the literature. The probability curriculum does not appear to be aligned in terms of higher order thinking. The intended curriculum indicated by the VCE Mathematics Study Design and attained curriculum as denoted by the examinations included 40% or more of the probability problems involving higher order thinking. The textbook was another tool of the system, was more varied with just 26% of problems involving higher order thinking. How the textbooks are used by teachers and students (the community and subject of the system) has a substantive impact on the development of mathematical and thinking skills (Remillard et al., 2014), which will be investigated later in this chapter in response to the next research sub question.

6.1.2 Curriculum Rules and Tools Supporting Thinking Skill Development

The tools and rules within the curriculum can be used to support students to develop thinking skills. The VCE Study Design for Mathematics is a tool which describes the use of tools and the rules around their use. Tools include calculators, bound reference books, SACs and examinations, while an example of a rules is the calculators and bound reference books could be used in one examination but not the other.

With the use of calculators and bound reference books as supporting tools for completing SACs and examinations, the pressure to memorise procedures is decreased. This means the emphasis can be on understanding, applying and interpreting the mathematics to non-routine situations (Block, 2012; Settlage & Wollscheid, 2019). The use of CAS calculators as a tool can streamline procedures with the mathematical problems, as calculators can solve and simplify problems automatically. Technology has been found to increase the level of thinking in tasks associated with the learning of probability (Batanero et al., 2016; Gürbüz et al., 2018). Technology use in senior secondary mathematics has also been associated with more variety in, and complexity of, learning tasks (Goos, 2012). VCE Mathematical Methods requires students to master the use of technology, but also to select the functionalities, develop mathematical ideas, problem solve, model and investigate while using technology (VCAA, 2015a). These are all indicators of higher order thinking as described by Stillman (2017), Watson and Fitzallen (2016), and the Two-Tiered Mathematical Thinking Framework.

The mathematics examinations are designed to provide opportunities for demonstration of different types of thinking through a variety of problem types, including short-answer problems without support material, multiple-choice response problems and extended-answer problems supported by use of a CAS calculator and a bound reference book. The VCE Mathematical Methods SACs aim to provide an alternative way for students to demonstrate their skills, by way of application tasks, problem-solving and modelling tasks. This variety encourages diversity of thinking skills, but also creates tension, as teachers and students attempt to adapt to the variety of tasks.

These rules around the use of the tools of the bound reference book and calculators were perceived in the current study, as supporting student development of mathematical and thinking skills. However, this could lead to tensions as both tools can be used well, or not, and time is required to learn how to use the tools appropriately, for example the syntax of calculators (Pierce & Bardini, 2015). Another tension is that the bound reference books and calculators can be used for most of the assessments, however one examination precludes students from having access to these supporting tools, requiring memorisation of the mathematical content and processes.

As evidenced in the current study, and supported in the literature, tensions within and between the elements of the activity system, especially the tools and rules of the Study Design, calculators and bound reference book, can support but also discourage the development of thinking skills by students. It is possible that the focus could be on the use of the tools such as the textbooks and calculators, rather than on the mathematical learning.

6.1.3 Curriculum Context Supporting Thinking Skill Development

This section describes how the curriculum supports thinking by using a mixture of context, especially a personal context that students can relate to, thus supporting their development of mathematical and thinking skills (Gravemeijer et al., 2017; OECD, 2019; Stanley, 2008). Starting with the intended curriculum of the VCE Mathematics Study Design, it recommended both familiar and unfamiliar contexts, practical and theoretical, routine, and non-routine problems. In the implemented curriculum, the textbook contained a variety of questions to cater for a diversity of students, across the range of problem types. Problems with an abstract, game or personal context were evenly utilised in the probability chapter in the textbook, while occupational contexts were less common. In the review chapter, the game context was less popular. Overall, problems with an abstract or personal context appear to be most popular in mathematics textbooks. A summary of the context of the textbook and examination problems is in Table 6.2. A breakdown of these results was described in Chapter 4, in Tables 4.4 and 4.8.

Table 6.2

Proportion of Probability Problems Classified by Context

	Abstract	Games	Personal	Occupational
Textbook	34	18	31	17
Examinations (2016-2018)	24	9	60	7

In the attained curriculum as demonstrated in the assessments analysed in the current study, the context of the SACS was not able to be classified, as they were different for all teachers and schools. For the commercially available SACs, some were related to the one context (for example, voting polls, abstract functions, football, solar flares) while the SACs created by the teachers tended to include a mixture of personal contexts. Over the three years of examinations (2016–2018) analysed in the current study, the context most underlying the problems (60% on average) was a personal context, especially the extended questions in Examination 2. For example: the battery life of computers, heart rate of exercising students, or the amount of time spent on homework. Multiple-choice questions in Examination 2 and short-answer questions in Examination 1 were more likely to be abstract. Examinations had a higher proportion of problems with a personal context, due to the high number of extended-answer problems with a personal context. The intention of providing a variety of question types, to cater for a wider variety of students and ensure all the content areas were included, was evident in the analysis.

If the context of a problem is familiar, students can focus on the mathematics, which encourages higher order mathematical thinking (Mullis & Martin, 2013), equally, students' value realistic scenarios (Barkatsas & Seah, 2015). Context can also hinder students if they make assumptions, for example in an Australian study, the students assumed a smoking–cancer relationship using frequency tables, but not in other contexts with similar numerical results (Watson & Callingham, 2014).

In summary, the tools within the activity system, the textbook, SACs and three years of examinations, were analysed in the current study all used a mixture of abstract, games and personal contexts for the probability problems. The balance between these contexts was inconsistent, which could cause tensions for the students. The use of a personal context for the complex extended response section of the examination was intended to scaffold the subjects, the students, in understanding the content of the situation, so they could concentrate on the mathematics of the problem, which may be one way of enabling higher order thinking, which was the object of the activity system.

6.1.4 Probability Content Supports Thinking Skill Development

Within the senior secondary mathematics curriculum, the content area of probability potentially supports student development of thinking skills, however several tensions exist within the topic of probability in VCE MM34. For instance, the study of probability involves more than remembering formula and procedures. It requires a new way of interpreting, thinking and reasoning (Fischbein & Schnarch, 1997) as the classical and frequentist interpretations of probability clash with the subjective and counter-intuitive views. Thinking and reasoning with probability skills is important in many aspects of one's lives, including medicine, science, interpreting news reports, finance and sports (for example; Garfield, 2002; Kahneman, 2012; Watson & English, 2015b)

Probability is inter-related with other mathematical areas of study. Algebra, function, graphs and calculus are all necessary for the probability content in senior secondary mathematics, and as such probability is an application of the other components of the subject (VCAA, 2015a). Combining content areas increases the level of thinking according to the SOLO framework (Biggs & Collis, 1991), and making connections is an indicator of higher order thinking in the *Performance type categories for mathematics* as demonstrated in Chapter 3, Figure 3.1 (Ziebell et al., 2017). An example of this interrelationship is demonstrated in the probability problem (4e) on the 2018 Examination 2, shown in Figure 6.1.

Figure 6.1

A Probability Problem Relating to other Branches of Mathematics

VCE MM34 2018 Examination 2 Question 4e.

The time taken by a randomly selected student to reach the top of the hill has the probability density function M with the rule

$$M(t) = \begin{cases} \dfrac{3}{50}\left(\dfrac{t}{50}\right)^2 e^{-\left(\frac{t}{50}\right)^3} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

where t is given in minutes.

e. Find the expected time, in minutes, for a randomly selected student from Mathsland Secondary College to reach the top of the hill. Give your answer correct to one decimal place. 2 marks

---

This problem required integrating probability concepts, graphing and calculus to solve. The problem links many of the concepts of the whole Year 12 Mathematical Methods subject together, including strategic use of the CAS calculator. The examination problem in Figure 6.1 also demonstrates a context which students would relate to, the time taken to reach the top of a hill, however it seems unlikely this situation would arise in reality.

While the topic of probability can provide an effective vehicle for supporting students as they develop thinking skills, an associated tension is the increased amount and complexity of probability content in the final year of secondary schooling in comparison to previous years. Probability content in the secondary mathematics intended curriculum begins with between 6–12% of the curriculum statements but in MM12 the proportion increases to 18% and increases again to 40% in MM34. In 2016 the probability area of study increased within MM34, with the reintroduction of variation, inference, sample proportions and confidence intervals, which also created a tension as teachers adapted to this change.

Another tension in the progression of probability through the year levels, is the large increase in complexity of the probability area of study in Mathematical Methods from Year 11 to Year 12, as described in the results Chapter 4, Section 4.1.1. Year 12 probability within Mathematical Methods includes the new concept of distributions, but also links to algebra, functions, calculus and requires the use of many new functions on the CAS calculators.

In summary, probability can be an important content area to support students in their future life and it also provides a vehicle for the development of thinking skills. The probability area of study relates to many other branches of mathematics which increases the order of thinking required. The historical influences on the activity system are evident as the changes to the probability content in the VCE curriculum. Tensions which may hinder this include the large increase in probability content and complexity in the final year of senior secondary schooling resulting from the 2016 reaccreditation. Probability is a difficult area of study, as indicated by the examination marks received for the topic, as outlined in Chapter 4, Section 4.3.4. Probability also moved from being part of the popular General Mathematics subject in past years, to inclusion in the higher-level Mathematical Methods subject, which means less students are learning probability (Ernst, 2018).

6.1.5 VCE Mathematical Methods Curriculum Supports a Range of Students

The aims and objectives associated with the VCE are multipurpose, with a focus on catering for a diverse range of students. Completion of the VCE opens up a broad range of pathways to further study or training at university or TAFE and to employment (VCAA, n.d.-e). As such, the VCE intended curriculum (Study Design) attempts to provide learning opportunities for a range of students to develop a variety of thinking skills. An example includes the option of passing the senior secondary subjects by demonstrating the required outcomes, even without attempting or passing the associated school assessed coursework (SACs) and examinations. This caters for students who are easily stressed and do not perform well under the time constraints of examinations (VCAA, 2019b). However, students who do not sit the SACs and examinations do not gain an ATAR score for university entry, hence cutting off some career options.

Another way in which VCE Mathematics supports diversity is through enabling teachers to choose topics to cater for their students in several of the mathematics subjects. For example, in the Year 12 subject of Further Mathematics, the compulsory areas of study include ‘Data Analysis’, ‘Recursion and financial modelling’, but teachers can choose two of the remaining areas of study from a choice of topics including ‘Matrices’, ‘Networks and decision mathematics’, ‘Geometry and measurement’, ‘Graphs and relations. The subject Mathematical Methods does not have any content options. Teachers also cater for their students and community by creating their own SACs as internal summative assessments. These SACs and their role in the curriculum are discussed earlier in Chapter 1, but the fact the teachers can adapt the tasks to their students and community supports the argument that the VCE intended curriculum attempts to cater for a range of students. This will be discussed in the second half of this chapter. SACs have changed in style over the years, with an increase in application, problem-solving and modelling tasks in the study period of 2016. This creates a tension over time as teachers need to adapt, which will be further unpacked in the next research sub question.

The outcome of the activity system includes the grades which contribute to the study score and university entrance score (ATAR). These scores are standardised and scaled according to a normal distribution bell curve as described by the Victorian Tertiary Admissions Centre (VTAC, 2018). The average of 49.75 % in 2016 (VCAA, 2017a), or the overall examination score, corresponds to a grade of C+. This means a student could correctly respond to the lower order thinking questions only, and gain a score of C+, which would correspond to a study score of around 30 out of a possible of 50 for each subject (VTAC, 2018). A study score of 30 in Mathematical Method is higher than the prerequisite study score of 25 for most Victorian university programs (VTAC, 2020). As such, this presents a tension within the VCE system, in that the flexibility which supports a range of students to complete VCE senior secondary mathematics, can make it possible for a student to complete most of the textbook chapter work, pass the SACs and examinations and pass the subject, even get average scores in the summative assessments, get a score high enough to get into university, and not demonstrate higher order mathematical and thinking skills. The supportive tools of the activity system could enable students to complete their studies, without developing the thinking skills which are an object of the system.

6.1.6 Summary of the Discussion Concerning the Curriculum Documentation

This section summarises the key points in response to the first sub-question, *In what ways are thinking skills developed through the senior secondary Victorian mathematics probability curriculum?* There are several factors within the probability area of study in the Year 12 VCE Mathematical Methods subject which influence the development of thinking skills in students:

- All three aspects of the curriculum, intended, implemented and attained, contained 40% or more statements or mathematical problems relating to higher order thinking skills, except the practice and learning chapters of the textbook. This compared favourably to the few relevant studies in the literature.
- Using an activity system, the intended and implemented aspects of the curriculum were tools to support students' development of higher order thinking skills, while the attained curriculum was the object.
- Tools which were encouraged and even assumed to be used by the curriculum documents included calculators and a bound reference book. The rules around the use of these tools formed part of the intended curriculum. These tools can support the development of thinking skills; however, they can also cause tensions as their implementation requires time and new skills to use the tools effectively, and some of the assessments must be completed without them.
- The curriculum used a variety of contexts for the mathematical problems, including a focus on problems students would be expected to relate to personally. This supports the development of thinking skills as students can apply, interpret, and evaluate the mathematics and solutions rather than just memorise and apply formula and processes.
- The content area of probability was a tool that can be used to support students to develop thinking skills, particularly higher order thinking skills. It links many other topics of mathematics together, can be applied to real life, and involves interpreting results, all of which increase the level of thinking required.
- Tensions which may hinder students in developing thinking skills include the sharp increase in quantity and complexity of probability content in the Year 12 subject.
- The assessment methods and the ways in which students can gain a pass in the VCE Mathematics subjects are flexible and attempt to cater for a range of students, from those aiming to go to university and study courses which require a background in probability, to those who wish to leave formal education. This flexibility creates tension within the activity

system as students can pass the subject, and even gain a score high enough to gain entry into university courses, without demonstrating many of the higher order thinking skills.

It is also important to examine how student and teachers' implementation of the curriculum influences student outcomes as outlined in the next section.

6.2 Discussion—Student and Teacher Interviews

Sub-question 2. What factors impact on the development of thinking skills through the teaching and learning of the senior secondary Victorian mathematics probability curriculum?

Research sub-question 2 broadens the earlier investigation of alignment between the aspects of the curriculum and their influence on the learning of mathematics and the development of thinking skills. As demonstrated in the current study, students and teachers influence through their agency in the class learning activities and pedagogy, within the constraints of the rules defined by the intended curriculum (VCAA, 2015a) and the cultures and norms of the communities of the local schools. Utilising the Activity Thinking framework through the creation of an activity system, curriculum was investigated with regard to community and division of labour, tools and rules, the content area of probability, and contradictions and tensions within and between these elements, and over time. This section focuses on the community of students and teachers in the current study, and their influence on the activity sytem that underpinned this research.

This second research sub-question has been broken into three sections for discussion:

1. How the community and division of labour between the students and teachers supports the development of thinking skills.
2. How implementation of the rules and tools by the students and teachers supports the development of thinking skills.
3. How implementation of the topic of probability curriculum supports the development of thinking skills, from the point of view of the students and teachers.

6.2.1 Community and Division of Labour in Relation to Thinking

The *community* which influences students in their learning of mathematic skills and the development of thinking skills is predominantly made up of teachers and peers. Other influences might also include families, tutors, universities, sports clubs, and part-time employment. This section discusses how the student and teacher interview participants described the division of labour in teaching and learning of the intended curriculum, and how this might influence the development of thinking skills. The interview responses are compared, contrasted, and analysed around the theme of division of labour and agency with the aim of evaluating whether the curriculum is aligned and consistent between aspects, and how this might impact on thinking skill development. Teacher and student agency and sense of responsibility within the curriculum has also been found to support deeper learning for students (Bandura, 2006; Reeve, 2009) and as such was seen as an important part of the teacher responses that occurred in interviews.

Student and teacher participants both reported that teachers exerted considerable influence on the division of labour within the mathematics classes. Generally, teachers established the structure of the classes, the norms, habits, and protocols, but students also influenced the division of labour through demonstrating agency. Considering the results of the analysis of the interviews, 16 of the 20 students described their teachers as having a considerable impact on their use of class time and the learning activities. For example, one student described their teacher as being totally in control:

> *In our class the teacher printed off a set of notes for each topic and those notes would specifically cover a certain area in the textbook which we were allowed to staple all together at the end of the year to take into our exam. At first the teacher would construct like a lecture. At first we would fill in the notes. The class would actively listen and fill in (Quin, student).*

However, when teachers dominated the class time and decision making, students could still enact their agency and self-motivation. For example, Rob described his teacher as dominating class time demonstrating a 'learning is knowing' attitude (Bobis et al., 2019). However, Rob went on to say:

> *…That was the general structure, although most of the students who were good at math [including me] were kind of doing it as she was talking. Like less homework that way (Rob, student).*

This comment demonstrated a degree of proactivity and self-motivation, with Rob explaining that his teacher allowed students to work in this way. This is consistent with Liu et al. (2019), who explain that when students demonstrate engagement, teacher's motivation increases, and they are more likely to support student autonomy.

George (teacher) reported being very proactive in his classes. He explained how he developed prepared notes for students to fill in and add to. Other teachers, such as Matt and Claire, also reported making checklists for students to keep track of their work and monitoring them with praise and encouragement. Wes (student) commented that senior secondary teachers were flexible and supportive with their setting of tasks, without the threat of punishments for unfinished work from the lower year levels, which gave him a sense of autonomy, similar to that reported in research

(Jang et al., 2016). Wes (student) explained that his reaction to this autonomy was to complete all tasks with a high level of motivation and cognitive engagement.

Several of the student participants reported receiving little individual support from their teachers, but this was seen in two ways, either as teacher neglect or enabling student agency. Some students were self-motivated and proactive, explicitly demonstrating agency (Bandura, 2006). For instance, Hazel's (student) class had a small number of students and was a combined class with Year 11 and 12 students (Units 1, 2 and 3, 4). The students worked individually which suited Hazel as she was unwell for most of the year. Wes and Vinnie (students) perceived the lack of teacher input due to a large class (20 students) was a positive situation as it encouraged students to support each other. Wes explained this as:

> *My class size in math methods was quite large, which meant that the teacher couldn't always get around to each student immediately ... it was also good because it meant he encouraged us to talk to the students around us and get them to try to help us which then furthered their learning … the level of cooperation in the classes was excellent (Wes, student).*

Max and Lenny (students) also described how they partnered up with a peer to complete all classwork and homework. Thus, the students developed their own ways for overcoming a lack of individual support from the teacher, thus becoming more agentic in the process.

Most teacher participants perceived an appropriate balance between time in class to explain the ideas and example problems and time for students to practice solving problems. Some student participants perceived their teachers as dominating the class and would have liked more time for their own practice. This suggests that greater use of the 'flipped classroom' could better address student needs. Flipped classrooms use videos of teachers or experts explaining concepts, or resources to be completed at home (Clark, 2015; Muir & Geiger, 2016), to give students more time in class. When asked about this in interviews, the student participants reported not having thought to look up online support (even though they do for other areas of their lives) and most of the participating teachers did not direct them to any. Libby (teacher) was the only participant who regularly assigned video tutorials from a school purchased online video program. George (teacher) described a colleague from his school who trialled 'flipped classrooms' where George was the critical friend but even though the students reported to have found the video homework helpful, it was only because it was an easy revision topic, so the trial was discontinued.

Teacher participants reported adjusting their control of the class time depending on student need. For instance, Beth (teacher) explained that she adjusted her style of teaching and management according to the content and time of year:

> *In Headstart [transition classes at the end of school year in preparation for the next year], it is very teacher centred, I've had to give the students all this information, but normally, they should just be working in class, especially at the beginning when it's revision. You don't want them just listening all class (Beth, teacher).*

When considering the division of labour (Engeström, 2001), handing students all the responsibility for their learning was also perceived as problematic by some of the student participants. For example, Vinnie (student) described his mathematics classes with teacher-talk for one part of one lesson per week, and the rest of the time was individual work from the textbook. While some students did support each other in very informal ways, which had potential for developing agency, Vinnie did not appear to be ready for it, requiring more direction from his teacher. This was evident in his comments presented in the previous chapter, for example his explanation for not making a summary book over the year as his teacher had not asked him to.

The student participants reported that teachers rarely, if ever, suggested group work, demonstrations, investigation or open-ended tasks, which were all suggested within the literature as helpful for creating student agency and self-motivation (Bobis et al., 2019). Most participating teachers reported they did not actively encourage students to work together or to discuss or support each other, counter to findings by Hattie et al. (2017) who found cooperation and discussion between students had a positive effect on depth of learning. However, most teacher participants approved students assisting each other, which is consistent with Reeve (2009) who described how teachers would like their students to be self-sufficient learners but find it difficult to hand over the control. Only one older teacher, Arthur, was firm in his belief that students should not discuss their work with each other and believed thinking was not possible while students were talking, adopting the attitude that students should have little influence in their own learning experiences in class.

Professional development supported Matt (teacher) to empower his students. This was also noted in the literature (Bobis et al., 2019; Calvert, 2016; Philpott & Oates, 2017). Matt described how the leaders of the school completed an observational learning walk which involved observing a wide range of classes for a short period of time. According to Matt, the school leaders found the teachers were doing more preparation and work than the students, which was reported back at a professional development session. As a result, Matt adjusted his classes, so students had opportunities to demonstrate agency and develop deeper thinking around the content area and study skills. As Matt explained, he encouraged his students to begin making concept maps, summaries and linking the mathematical ideas together, without his input:

> *We had this 'learning walk' thing a few years back … they said that the teachers were doing heaps of work, but the kids weren't, so now I try and get the kids to do that. They have to decide what goes in their summary book, they have to make the concept maps. I think it's better. I think the kids need to take responsibility. But I still tell them what to do. I tell them 'this would be a good one for the summary book'. And set aside time in class to make a concept map of all the different say, types of trig [onometry] questions (Matt, teacher).*

The literature as described in the literature review, emphasised the positive effect of professional development on teacher agency, leading to improved student agency (Bobis et al., 2019; Philpott & Oates, 2017). For example, Bobis et al. (2019) describe how ongoing professional development programs can alter classes from teacher centred to student centred, while Philpott and Oates (2017) report that school-based *Learning Rounds* can do the same. Most of the teachers interviewed (eleven out of fourteen) had participated in professional development relating to teaching senior secondary mathematics in the previous three years, and two had delivered professional development sessions. The professional development as described by the teachers, included information about the use of calculators, writing and implementation of the SACs, and information about the examinations. The professional development for the use of calculators included "*tips and tricks*" (John, teacher) for their use, but also how the calculators can be used for teaching concepts. This was reported by John (teacher) who was involved in running some of the sessions, and teachers Matt and Nathan who attended the sessions. Interestingly, most of the reported PD appeared to focus on assessment and associated resources rather than pedagogy.

While professional development would appear to be essential in supporting teachers and consequentially improving student learning, within the current activity system, it was not possible to establish a direct relationship. If a third-generation activity system was utilised (Engeström, 2001), a second triangle with teachers as the subject could be created, so the influence of the tool of the professional development on teachers could then be followed to the influence on student learning. As such, the current activity system with the students as the subject cannot fully consider the full effects of professional development on teachers and students.

The conventions involved in activity systems also consider habits, norms, and culture. For example, some students demonstrated a minimalist culture of engagement with their learning. Nathan, a newly graduated teacher, was especially surprised by the relaxed attitude of students to mathematics and reported his students did very little homework, which was also reported by other participating teachers, for example Harry who said:

> *I had to realise that there were half the students who weren't going to do work except in the time I gave them in class (Harry, teacher).*

Senior school students generally have other commitments including sport, family and part-time work, which at times makes school a low priority. However, most students in the current study indicated that they did prioritise their studies, both in and out of class time. Sue would email her teacher, while James worked with peers:

> *Usually it's like if you have any trouble with the homework and stuff you just email her outside of class sometimes (Sue, student).*

> *We would self-study and we would work together between the three of us and if we came across a problem, we would ask each other (James, student).*

It was apparent in the student interviews that student's habits and priorities influenced their learning, which using an activity system, is a component of the community and division of labour elements. There was some evidence that tensions within and between the priorities of teachers and students influenced student learning, and the attainment of mathematical and thinking skills.

In summary, the balance between the student and teacher's division of labour in the activity system was varied, which is consistent with the literature. Some students were proactive in their learning, demonstrating agency, which the teachers generally encouraged. Similarly, some teachers encouraged student agency, which was occasionally misinterpreted as a lack of guidance. Teachers need to make their learning intentions clear to students to avoid this miscommunication. Tools such as group-work, investigations, and open-ended tasks, which have been recorded in the literature as enablers of student agency, were rarely initiated. Another supportive tool of professional development did support a small number of teachers to encourage students to take more responsibility for their learning. Teachers supported students in their learning and their development of thinking skills, but both groups also depended on the tools of the system, the intended curriculum, textbooks, calculators, and support material as tools to support the objective of the development of thinking skills, which is discussed in the next section.

6.2.2 Implementation of the Tools and Rules Supports the Development of Thinking Skills

The tools and rules within the curriculum, specifically the probability curriculum within Year 12 VCE Mathematical Methods, can be used to support the development of mathematical and thinking skills, as discussed in the first part of this chapter. Within the current study, the use of these tools was both a support and a tension for the students, and teachers, in their learning of the mathematical content, and the development of thinking skills. The use of the tools; learning activities, calculators, bound reference books, assessments, and the content area of probability will be discussed, and the rules surrounding their use, from the student and teacher's point of view.

6.2.2.1 Learning Activities

Textbooks, examinations, and the other elements of the intended curriculum are discussed in research sub-question, but the focus here is the teachers and students' views and insights regarding how they transform these learning tools into learning activities to support thinking skill development.

A key theme to emerge, was that participating teachers and students reported that they used the textbook extensively for the learning activities, so the textbook could be seen as part of the pseudo-curriculum. The recommended textbook for senior secondary mathematics had many features to support student learning of mathematics, and learning of thinking and reasoning skills, as described earlier in this chapter. The textbook included detailed examples, practice questions, review questions, and answers. Teacher participants reported that they generally explained the mathematical concepts with rules and examples, then assign some or all of the textbook problems, or encouraged students to select their own. Some teacher participants outlined how they supported (or controlled) students with tick sheets and book checks, but most relied on students to keep track of their own learning. It would be possible for students to just complete the early sections of each problem set and feel they had mastered the concepts or processes when this was really just following the patterns of the examples. However, in general teacher participants reported satisfaction with the textbooks, with Danielle appreciating the explanations and Fred commenting on the range of questions:

> *I use textbooks and VCAA website as my main teaching tools. The textbook is very good. The explanations use exactly the language I would use and they explain things just as I would (Danielle, teacher).*

> *The new textbook has lots of good questions, more than enough, which is good, good to have a good range of questions. That's the good point about this textbook (Fred, teacher).*

Students also found the classes focused around the textbooks, as evidenced by Gabby's comment:

> *It was a good textbook. I remember I think the structure of the class is very much textbook orientated (Gabby, student).*

The teacher and student participants all relied on the textbooks to varying degrees, which seemed appropriate as the textbooks had been written for the VCE Mathematical Methods subject, covered the mathematical material and contained problems mimicking the style of the examinations. The textbook did not include problems in the expected style of the SACs, as the SACs were expected to include simulations, problem-solving or modelling. One quarter of the interviewed students commented that they completed few simulations in their Year 12 classes, although half the participating teachers reported they did include simulations, although the students may have been observers rather than participants. The final review chapters of the textbook were a combined selection, to give students practice at deciding on a process and method. These review sections provided important preparation for the examinations and contained a high proportion of the higher order thinking problems. Problems from previous examinations (or practice examinations) also supplemented the textbook problems, which are discussed later in the chapter.

Online textbooks, and online apps, videos and widgets were rarely used by the teachers or students in the current study, even though some were referred to in the textbooks. Rob (student) did mention the online (PDF) version of the textbook and how he was original in using this resource, which seemed to indicate a lack of initiative by the other students:

> *I don't think even most students know that there is even a PDF version with a book to be honest. I don't know if it's because they hadn't read the first page or because the teacher hadn't told them (Rob, student)*

In summary, the textbook appeared to be the dominant source of guidance and learning activities for teachers and students in the current study and were interpreted and used as the pseudo intended curriculum, as a tool within the activity system. The textbook was found to be appropriate as the basis for the learning activities, as long as students and teachers accessed all the problems especially the review chapters, which contained the majority of the higher order thinking problems, showing the method of utilisation of the tools to be an important factor. There was a gap in the appropriateness and value of the textbook, as there were no SAC type problems included. Few other tools or resources were used other than previous examinations. By establishing this dominance of textbook use, the analysis of the learning and teaching activities became an analysis of the textbook activities, which was carried out in Chapter 4 Section 4.2, when the textbook was investigated. The next tool to support the objective of student mathematical learning and the development of thinking skills, was the CAS calculator.

6.2.2.2 Calculators

Student and teacher participants had mixed reactions to the usefulness of CAS calculators. Calculators can support students by completing the routine components of the mathematics quickly, leaving students time and freeing cognitive load for the complex setting up and interpreting of the mathematical problems, as explained by several teachers, including Matt:

> *The calculators give the kids confidence, the calculator can do the busy work, and the kids can concentrate on what the answers mean (Matt, teacher).*

Nathan (teacher) and Dan (student) also expressed positive support for calculator use. Nathan, in his first year of teaching, said calculators supported students' conceptual understanding of a problem, rather than the technical complications of using complex formula:

> *I really like it the CAS calculator for probability, we use the calculator a lot … makes their cognitive load less, and actually focus on the actual problem rather than the technicality of solutions (Nathan, teacher).*

Dan initially found the calculator difficult to adapt to, but now appreciated it:

> *Going from Year 10 to Year 11, going from the scientific to the CAS calculator, I kind of had a tantrum as it was so out of the ordinary. Now I love it, it's so easy (Dan, student).*

Teacher participants appeared to be generally comfortable with the use of calculators. For example, teachers Matt and Fred explicitly explained how they used the calculator and smart board to demonstrate mathematical concepts to students, which is consistent with Goos and Bennison (2008) finding that more than half of the teachers in their study agreed that technology supported students to understand concepts.

Several of the teacher participants explained their use of technology to support the teaching of particular mathematical concepts, for example demonstrating the effect of changing variables on a graph on binomial distributions or using the graph of functions on the calculators to understand the calculus concepts. One of the teachers explained:

> *So, with the calculator, the graphing function or the solving functions, or the fact it can do all the differentiation and integration, itself, and relating that to what it means to the graph (Fred, teacher).*

Calculators can create tensions as both student and teacher participants reported that calculators took time to learn, for example calculators have their own syntax or language. Some student participants reported that learning how to use calculators was not helpful as there was a non-calculator examination, and the effort required to learn the calculator outweighed their advantages. This is consistent with research by Pierce and Bardini (2015) who stated that the additional syntax of the calculators complicated the mathematical learning. Student participants explained that calculators provided a scaffold by carrying out routine procedures and checking solutions but were rarely used to learn or demonstrate concepts. Claire (teacher) and Foster (student) expressed concern at the over reliance of students on calculators, which Claire described in terms of, "*garbage in, garbage out*". Danielle (teacher) highlighted the importance of students understanding the concepts first, and then using the calculator as a time saver. Max (student) perceived calculators were a waste of time, while James (student) described them as restricting his skills:

> *That's what I found very challenging coming into first semester uni was—well I knew how to do it on a calculator, but I didn't know how to do it by hand (James, student).*

Calculators could not be used all the time due to the rules of VCAA, and now the student participants were at university, some found they could not use them in their engineering mathematic courses.

Calculators were however, the main technology tool used by the student participants in their senior mathematics studies. Although students often had access to the electronic version of the textbook, few used it and videos to support mathematical learning were generally not used, nor were spreadsheets or Mathematica dynamic software programs. Just one teacher, Libby, reported that she used an online video program:

> *A program that we've bought is a video package, so they can use those to practice as well as a video practice questions for every topic every VCE topic … they do the quiz online and we've also got some videos … it is really good and I can see what they've watched (Libby, teacher).*

It was surprising other technological resources were not used more, as many are free and technology was often used in other areas of students' lives (Oviedo-Trespalacios et al., 2019).

The use of the tool of technology, in particular CAS calculators, can increase higher order thinking (Pierce & Bardini, 2015), which was the object of the activity system. The use of calculators as a tool in mathematics education can help meet the needs of a range of students and lead to increased engagement and effective use of class time, which all aid in ensuring objects of the activity such as mathematical learning and thinking skill development are reached. This potential for the use of technology to improve learning, particularly in relation to probability, suggests an opportunity for further research. The interviews highlighted a tension between the teacher and student participants around their views on calculators, with teachers generally more positive and students more divided, with half of the students expressing some negative comments about calculators, and their ability to encourage deeper thinking. The increased functionality of the calculators as the technology developed over time, has strengthened the tension in the use of these tools, with teachers (as the supportive community) and students (as the subjects) not always keeping up with the potential of these tools.

6.2.2.3 Bound Reference Book

Victorian secondary mathematics (VCE MM34) students have the unique support of the use of a bound reference book in some of their assessments, including one of the two examinations (VCAA, 2015a). This book, which is generally called a *summary book* by students and teachers, can be created by students or commercially made. Linking to an activity system (Engeström, 2001), the summary book is a tool used to scaffold students' mathematical learning and the development of thinking skills, relating to the rules of the activity system. The participating students and teachers had mixed attitudes towards the summary book, which is consistent with the research around the use of 'cheat sheets' and open book examinations (Larwin et al., 2013).

The history of the support material for Victorian Year 12 mathematics started with a list of formula in the examination, and then in years 2000–2006 students were permitted to take one A4 page of notes into examinations. In 2006 the rules changed to allow a single book to be used in the examination, corresponding with the introduction of CAS calculators which could contain many pages of notes within the calculator storage (Ernst, 2018). This is an example of how the sociocultural-historical setting of the school system influenced the development of the rules of the curriculum.

More than half, or thirteen out of the twenty student participants reported finding the summary books helpful, either for building confidence or better understanding as a result of the process of creating the books. Few students actually used the bound reference books often in the examinations, which is consistent with Burns (2014) study on cheat-sheets, although they did use them in the SACS, which supports the idea that SACs are learning tasks. Some students worked on developing the summary books regularly throughout the year, while others made them as a review task at the end. A number of students mentioned waiting for instructions from teachers before making their summary books while others just went ahead and developed them, which is an indication of their agency and *division of labour* in their relationship.

The student and teacher participants described the summary book as an intended support for students, to scaffold and guide them in their study techniques. Claire (teacher) explained that creating the summary book was a helpful study technique:

The actual exercise of putting the exams [summary] book together means they are doing really good revision (Claire, teacher).

Similarly, Tracey (student) described how she used the creation of her summary book as a learning experience, using the textbook practice questions she had struggled with, but then mastered the content:

Before every SAC as revision, the teacher always tells us to update our summary books … I choose the questions after I've done the questions from the textbook and I always choose the ones that I had trouble with (Tracey, student).

Eddie and Nick (students) reported that they both continue to use their Year 12 summary books in their university studies, as a scaffold to help remember the mathematical content, which is an indication of how helpful these students found the summary books. This description of the summary book as a support, is also reflected in the research on assessment support material such as cheat sheets or open book examinations, which aimed to reduce the anxiety of students, and focus study skills (Block, 2012; Erbe, 2007; Gharib et al., 2012).

Teacher participants were diverse in their scaffolding of the use of the summary books. Some teachers explicitly taught students to create and use the books. For example, Ian (mathematics coordinator and Mathematical Methods teacher) implemented a school policy on the creation and use of summary books in Years 10–12. Other teachers left it totally up to the students. This is consistent with data from the interviews with the student participants. Some students purposefully created their summary books throughout the year enabling them to focus on practice examinations earlier in their revision time while others created their summary books as a component of their end of year revision strategies. Some students demonstrated agency by being in control of the creation and use of the summary books, while others relied on their teachers to control its use. Burns (2014) described how students require support in how to create and use assessment support material (in that case the cheat sheets and open book examinations), with high achieving students relying less on the assessment support material.

Summary books can be used to decrease rote memorisation and increase higher order thinking (Agarwal et al., 2008; Settlage & Wollscheid, 2019). For example, George (teacher) described it is a scaffold like the formula sheets given in the examinations:

[A summary book] was useful for formulas that aren't on the formulas sheet (George, teacher).

In contrast, Elaine (teacher) suggested some students could pass the assessments with a good summary book, rather than fully understanding the mathematics:

> *With a good summary book and a calculator and they can get through … I'm not convinced the kids actually know stuff (Elaine, teacher).*

Teachers and students both mentioned that summary books could make students complacent in their studies. For example, Sue (student) used her textbook as a summary book, saying "*You just take the textbook into the exam".* Matt (teacher) explained how some students would photocopy pages of the textbooks or peers notes for their summary books, which he described as ineffective:

> *A whole [summary] book makes the kids lazy and they think they don't have to remember anything. Some kids just photocopy stuff. You don't learn by photocopying (Matt, teacher).*

Disadvantages associated with assessment support tools like the bound reference book, include less preparation time for the assessments (although creating the bound reference book may be assessment preparation), loss of time in assessments due to searching for information, and little impact on improvement of grades or increasing long term retention of material (Gharib et al., 2012).

The description of the use of the tools of calculators and bound reference books to support the objective of development of mathematical and thinking skills highlighted many tensions and possibilities within the activity system. As the rules of use of the tools developed over time, with the calculators improving in functionality and the bound reference books replacing the double A4 summary pages, the teachers and students use of these scaffolds did not appear to be fully realised. It was apparent that teachers and students required explicit instruction on how to make full use of these tools, as the best use of the CAS calculator and larger bound reference book cannot be assumed. The usefulness of calculators and bound reference books depend on many factors including school policies (school rules), the importance placed on them by teachers, the agency and roles of teachers and students. One limiting factor on the use of resources, is the rule that they can be used in all learning activities, most SACs, but only one of the two examinations (VCAA, 2015a). The participating student and teacher views on the tools of assessment will now be discussed.

6.2.2.4 Assessment

The VCE Mathematical Methods assessments have the potential to support the development of thinking skills in the students. The assessment discussed here includes the SACs (school assessed coursework) and practice examinations tools. Neither the student nor teacher participants had access to the students' final examinations to comment on. SACs are tools for the activity system of student learning, to support and formally assess students' mathematical learning.

The rules related to the SACs as contained within the Study Design, state that the application tasks, problem-solving or modelling tasks need to be completed over a certain period of time. For example, “One of the modelling or problem-solving tasks is to be related to the **Probability and Statistics** area of study. The modelling or problem-solving tasks are to be of 2–3 hours duration over a period of 1 week” (VCAA, 2015a, p. 79, original emphasis). This requirement can cause tensions with school timetables. As one teacher participant pointed out, their school had a mixture of single 50-minute classes, and double 100-minute classes, neither of which accommodated the full completion of a SAC. Furthermore, if there were two groups of students, then students could be completing the SACs under different conditions. This causes tensions for the teachers and schools as they also want to avoid cheating, plagiarism, and make the conditions of the SACs as consistent as possible, which was highlighted by a number of teachers, as evidenced in Danielle’s comment:

> *If the SAC goes for more than one class, the calculators and summary books of the students are kept until the next class to try and eliminate cheating (Danielle, teacher).*

Teacher participants reported tensions, as they aimed to assess student knowledge of the mathematical concepts but were given limited guidance on how to create this type of assessment task. Sample SACs were provided on the VCAA website, but teachers were asked to create their own to avoid plagiarism. Audits are carried out on the schools, but even if schools do not pass the audits, they are not provided with support on how to improve, as highlighted by Kerri:

> *We teachers are so busy, and we have no time to make SACs, one bloke said he spent 50 hours making a SAC. I can't do that. We made our SACS from [publisher] and other commercial SACs, and adjusted them, but failed the audit (Kerri, teacher).*

As explained by a number of teacher participants, it was not unusual for teachers and schools to buy commercially produced SACs, but not all passed the VCAA audits, as they were not open-ended. The authors of the commercially made SACs clearly stated that they were only intended to be a starting point for teachers to modify to suit the needs of their own students, therefore teachers could adapt the tasks to make them more open-ended and suitable. It was also raised that the open-ended problem-solving and modelling tasks were not always familiar to teachers and therefore were difficult to design, as Libby stated:

> *We're not used to that open-ended problem-solving (Libby, teacher).*

Kerri described this as anxiety with setting SACs:

> *So I suppose this is where my, I suppose I'm going to call it anxiety and my issue is with perhaps setting SACs like this and look I get the whole idea that you can't just have exam*

questions and we need to give the students an opportunity to show what they understand not through these little discrete quantitative questions that we ask in an exam format. But I just have a little bit of angst about (Kerri, teacher).

Teachers also need to consider grading and moderation between classes and even moderations between schools if classes are small. The aim of the SACs is to have assessment types that differ from the examinations, so students have a chance to demonstrate their knowledge in different ways, but teachers are also under pressure to prepare students for the examinations.

Teacher participants also expressed concern that students would give each other hints on the SACs to either advantage or confuse other students. Participating teachers explained how they introduced school rules, where the student completed SACs under a different timetable or after school. School rules around access to calculators and summary books were also introduced for fairness. The SACs were also broken into discrete parts, with one part completed each day, which was a contradiction of the guidelines of the longer open-ended assessment tasks, as explained by Matt:

> *To start with we had one big question, and the kids worked on them for about 6 periods. We collected their book and calculators between classes. But then everyone got worried about cheating, so we had to break the task into bits. So annoying. And then we started printing them in booklets, with spaces, so they looked even more like exams (Matt, teacher).*

It is not uncommon to find informal formative assessments also occurring during senior mathematics classes (Ernst, 2014) including quizzes, matching cards and micro-tasks, and this was also highlighted by some student and teacher participants:

> *We had little micro tasks, it’s just like a little like a little quiz … maybe halfway through the chapter and then we just write down the answers the questions and just to see how we're going and what we maybe need to go back over for the SAC (Tracey, student).*

> *So now we do micro tasks most weeks. 10-15-minute tasks which are done about one a week on particular textbook exercises. A micro task is one page of questions, just a few key questions (Danielle, teacher).*

These tasks were used to inform teachers and students of the student’s specific strengths and weaknesses (Wiliam, 2006), enabling teachers to provide feedback and re-teach students individually or as a class on misunderstood concepts.

Student and teacher participants reported using practice examinations in various ways to learn the mathematical material and prepare for the high-stake examinations. Some teachers used examinations as learning tasks for students and reported that to do well in the examinations they needed to complete many practice examinations, in order to avoid surprises. Other teachers perceived that the examinations, especially Examination 2, contained enough original material of a problem-solving style, to encourage students to develop problem-solving skills, which in turn would encourage higher order thinking. Practice examinations could also be used as formative assessment to support teachers and students in their planning and learning. Wes (student) explained how the previous examinations could be used well, and misused:

> *I was able to answer almost every single question in the [practice] exams. Sometimes I get it wrong, but I was able to answer them after some revision, so many of the other students ended up just skipping over, up to a quarter of the [practice] exam because they didn't know how to do it. And then moving on to the next exam, but that doesn't help at all (Wes, student).*

Wes further explained how he worked though all parts of the past examinations, doing extra revision and study to complete them. Meanwhile, some peers just completed parts of the past examinations which Wes felt was not helpful in preparing for the final examinations. Ivy used the practice examinations as formative assessment, to identify areas which needed revision:

> *I did a few [practice exams], I did maybe four, I think, and then I wrote down all the areas that I didn't get right and then I asked my teacher for extra resources for that (Ivy, student).*

The past examinations were used by some teacher and student participants as pseudo intended curriculum. They could be used to predict the final examinations, to avoid needing to do higher order thinking during the examination. Claire, a teacher, explained this:

> *My mantra was always if you have done enough practice exams, there is only so many questions they can ask, if you have done enough practice exams, you will come to a question, oh, that's an old friend … . Those who listened and those who did the practice exams, really benefited from that I think (Claire, teacher).*

The combinations of the tools of SACs and examinations as summative assessments will potentially encourage a range of thinking skills, an object of the activity system. However, teacher participants mentioned the need for more support in creating, implementing and assessing the SACs. Assessments including SACs, informal quizzes and practice examinations can be used as learning tools, as formative assessment, to practice mathematical and thinking skills, but also to reduce the need for higher order thinking in the examinations. The development of the SACs over time, from tasks created by the Board of Education, then teacher created, are an example of how the sociocultural-historical changes can create tensions with the activity system, especially as the teachers did not feel they had support in developing these high stakes assessments. The content area of probability can also be considered a tool to support the object of the development of mathematical and thinking skills, which will be discussed next.

6.2.3 Probability Topic and Content

The content areas of study within the intended curriculum as described by the Study Design are also tools to support mathematical learning and the development of thinking skills. Probability is an important content area as it combines many other content areas of mathematics, and can involve real life problems, modelling, problem-solving and simulations. The two probability focus problems from Chapter 3, Section 3.4.2.3 will be used to demonstrate the potential for probability to improve thinking skills.

Students and teachers expressed a variety of opinions about the topic of probability. Student participants appeared to either like the topic as it related to real life or dislike it as it was different and less structured than the algebra and calculus part of the senior secondary mathematics subjects. For example, Gabby loved the topic, while Ivy found it less structured:

> *I love it. It's interesting. I thrived on it. It's in the real world (Gabby, student).*
>
> *Probability was my least favourite part ... it seemed a bit disjointed … it's a bit different. Probably because it wasn't quite set in stone. It wasn't a, here's the question, here's how you figure it out, that's the answer—it was sort of a bit more wishy washy (Ivy, student).*

The topic was seen both as an extension of the functions and integral to calculus but also an unrelated topic. Probability within VCE Mathematical Methods was reported by the participants to have been substantially more difficult in the final year, which is a tension consistent with the curriculum analysis in the first section of this chapter. Another tension was around the wording and context of the problems. These factors could encourage the development of higher order thinking (Fischbein & Schnarch, 1997). Teachers described the topic of probability as important, relevant, different to the rest of the course, and a way of encouraging students to think more divergently in their mathematics. Not all could see the links to the other parts of the subject and acknowledged that some students did not like the topic due to the lack of predictability, the language and symbols.

VCE Mathematical Methods probability content is completed towards the end of the year, as it is an application of graphs and calculus content and can be used to relate and consolidate other content areas. In this way probability can be seen as an ideal content area to develop and demonstrate thinking skills. For example, transformations of graphs was a concept throughout the subject, which is also linked to the normal distribution curve, which is described in the MM34 Study Design curriculum statement:

> standard normal distribution, N(0,1), and transformed normal distributions, $N(\mu,\sigma^2)$, as examples of a probability distribution for a continuous random variable. (VCAA, 2015a, p. 74)

Questions around the normal distribution function could be completed by memorising the question types and plugging into the calculator, however the wording of this curriculum outcome intends links will be made to the transformation of the function or graph. Linking different areas with mathematics like this, is an example of the higher order thinking skill—*relational thinking,* as described by the SOLO framework (Biggs & Collis, 1991).

Modelling and simulations are particularly important in probability, and explicitly mentioned in the Study Design and literature (for example, López-Martín et al., 2016). However, while teacher participants mentioned including modelling or simulation, dice, cards, smarties, this appeared to be rare. Several of the teachers including Claire (teacher) explained that hands-on activities were important, even for senior students:

> *If you just talk about it, they don't remember, you actually have to do it, so I used to get them counting smarties, chucking coins, and make a horrible noise and people clambering around on the floor, but they remember the hands on stuff. … Definitely in Year 11. Because I never assumed anything they had learnt in Year 10 (Claire, teacher).*

Wes (student) described a learning activity arranging classmates as an introduction to permutations in probability in Mathematical Methods:

> *In Methods, I remember our first lesson from probability. When the teacher asked for four volunteers. We all went up to the front of the room and we sat in chairs and then we were asked how many different ways we could arrange ourselves (Wes, student).*

In contrast, two teacher participants claimed that experiments and simulations were a total waste of time, while most thought they were more useful in the lower year levels, as reported by Harry:

> *No, you haven't really got the time [in Year 11 and 12], and there is so much data available, I would do dice and coins in Year 7–9, …, I could refer back to it (Harry, teacher).*

Lack of time in class was a reported tension within each class and over the school year. Lack of time to teach the wide range of content areas and to prepare for the variety of types of SAC and examination problems, hence some learning activities were left out, or conducted as teacher demonstrations.

The focus probability problems from the student and teacher interviews were the two-coin problem, and the frequency table problem. These two problems were appropriate for the interviews as the mathematical content was part of mathematics subjects from Year 8 onwards. The participants could all respond to the problems, although some of the responses highlighted errors and misconceptions. The context of these problems were ones which the students would be able to relate to. The problems also started conversations around mathematics education in general, but also highlighted a range of associated tensions.

The student participants found the two-coin problem straightforward, except two of the twenty students who demonstrated the *Equiprobability bias* (Garfield, 2003). The students generally found the sample space with a list, while the teachers thought they would use a tree diagram, which is an example of a tension between the student and teacher responses. Students related the two-coin problem to other areas of their studies (errors in measurement, games, genetics) without prompting, which demonstrated higher order thinking according to SOLO (Biggs & Collis, 1991). The last part of the two-coins problem involved the new part of the course with variation, which the students estimated with realistic solutions, although some of the teachers did not relate this problem to the recently added (2016) part of the subject. The wording of the frequency table problem was the main issue, as expected. All participants except two students and one teacher thought the wording would be confusing for some students. This multiple-choice problem was an appropriate format for this problem, and would support preparation for examinations and formative assessments, according to the teachers. Teachers demonstrated pedagogical content knowledge (PCK) with suggestions on how to teach this content, and suggestions on how to avoid and overcome the common misconceptions.

Probability content was found to be an appropriate topic to relate to real life and is a tool which can be used to support the objective of the development of mathematical and thinking skills. Interview questions around the content within the topic of probability also identified some possible tensions within the activity system under investigation in this study, including the use of practical verses abstract models for teaching, language issues, and the balance between providing examples and encouraging original thinking.

6.2.4 Summary of the Discussion Concerning the Student and Teacher Interviews

This section summarises the key points in response to the second sub-question, namely, *What factors impact on the development of thinking skills through the teaching and learning of the senior secondary Victorian mathematics probability curriculum*? Within the activity system, the students were the *subjects*, and the peers and teachers as the *community* influenced the students' development of higher order thinking skills through their implementation of the curriculum. The findings which respond to the research question, and some of the *tensions* identified by the student and teacher participants include:

- The division of labour between the students and teachers was diverse, creating inconsistent tensions. Generally, teachers maintained control, and allowed their students to exhibit some agency when they demonstrated responsibility. Some students demonstrated agency by forming study groups, working ahead, creating bound reference books, or trialling extra practice examinations, without their teacher's encouragement, but this was not common.
- The teachers as part of the community with the activity system, and student participants as the subjects, were consistent with each other in their dependence on the textbook as a tool to support learning and teaching.
- Participants expressed mixed responses in regard to the use of the tools of calculators and bound reference books. There was acknowledgement of the value of these tools in reducing reliance on memorisation and increasing higher order thinking in relation to the mathematical problems attempted, but the time and skills required to learn how to effectively use these tools was seen as a limitation, particularly by student participants.
- The use of practice exams as tools to support the development of thinking skills compared to relying on memorisation was also a contested issue. All student participants had utilised practice exams to varying degrees, for the purposes of motivation, as learning tools or formative assessment. They were also perceived by some as hurdles which the teachers prescribed.
- The mathematical problems in the tools of textbooks, SACs and examinations were perceived by the participants as attempting to relate to contexts that students would relate to and understand. This could support the object of the developing thinking skills.
- Teachers followed the rules of the VCE Mathematical Methods Study Design and relied heavily on the recommended textbook as tools. However, they identified tensions within the rules relating to the creation and implementation of the SACs, with the requirement of problem-solving or modelling tasks over a longer period of time contradicting the issues of avoiding plagiarism and preparing students for the examinations.

- The teacher and student participants had mixed reactions to the topic of probability as a tool within the activity system, with some seeing it as real life mathematics which linked other mathematical topics together, and others perceiving it as unconnected and an illogical topic, for which they were not prepared by their previous school mathematical experiences.
- Several students related what they covered in the topic of probability in VCE Mathematical Methods to other subjects they had studied in VCE and at university, which is an example of higher order thinking as defined in this study by the Two-Tiered Mathematical Thinking Framework.

The next and concluding chapter responds to the overarching research question, through combining the investigation of the document analysis of the curriculum with the analysis of the student and teacher interviews.

Chapter 7: Conclusion

This chapter concludes the study, beginning with an overview and summary of the findings in response to the overarching research question that underpinned the study. The significance and limitations are outlined, and a set of recommendations proposed.

7.1 Overview of the Study

This thesis was motivated by the changes to the Victorian senior secondary mathematics curriculum that were introduced in 2016. These changes included an increased focus on probability and a difference in the style of the school assessed coursework (SAC). Another influencing factor was the TIMSS and PISA international studies which highlighted a concerning decrease in Australian students' mathematics results, especially in terms of demonstration of deep thinking, which warranted investigation. The aim of this study was to explore the extent to which probability curriculum is perceived as supporting the development of thinking skills in mathematics students at the senior secondary level. Although the case study involved regional students and teachers, regionality did not emerge as a theme, having no specifically contextual influence on the tensions or possibilities that were presented in the interview data.

This study combined an investigation of curriculum materials and student and teacher perceptions. It aimed to discover factors that supported and caused tensions in developing mathematical and thinking skills, using probability to bound the case study. The TIMSS Curriculum Model describes three aspects of the curriculum (Mullis & Martin, 2013) and the mathematical thinking skills were classified by the Two-Tiered Mathematical Thinking Framework (see Chapter 3, Section 3.3.2). The overarching theoretical framework used an activity system (Engeström, 2001) as part of Activity Theory, where many interpersonal and material elements are used to describe the influences on the development of mathematical and thinking skills.

The curriculum was analysed regarding the expected levels of thinking for the intended, implemented and attained curriculum, represented by the VCE Mathematics Study Design, recommended textbook, and assessments. Current university students who had previously completed Mathematical Methods (n = 20) and Mathematical Methods teachers (n = 14) were interviewed to discover their views on the teaching and learning that they experienced in their Mathematical Methods classes, their perceived role within the classes and the tools they found helpful. Two probability problems were also discussed which developed the conversations and uncovered tensions.

The discussion of the conclusions, recommendations and limitations associated with the current study are organised in the following way:

7.2 Response to the Research Question

The main research question underpinning this study was:

How does the study of probability impact on the development of thinking skills?

This research question was broken into two sub-questions in the previous Discussion chapter, with the curriculum documents and interviews discussed separately. These two data sources will now be combined, and the main research question responded to in relation to the elements of the activity system, namely the tools, rules, community and division of labour, within the context of this case study, which focuses on the topic of probability. The main findings in relation to the various elements within the activity system that was studied include:

Outcome - higher order mathematical thinking

- The requirement for higher order thinking, which was an anticipated outcome of the activity system, was evident in all aspects of the senior mathematics documented curriculum related to the topic of probability.

Tools

- The recommended textbook included material to support the development of mathematical and thinking skills, however, it was possible that the method of use limited the value of textbooks for students.
- Teachers viewed support tools such as calculators and bound reference books as beneficial to the development of mathematical and thinking skills however students were less convinced of their value, particularly in relation to calculators.

Rules

- The rules regarding the SACs supported the development of higher order thinking, however, there were tensions identified by teachers that hindered the implementation of the SACs.

Division of labour

- Agency is important for both teachers and students in building the necessary thinking skills required to effectively learn probability concepts within the senior mathematics curriculum.

Community

- While the relationship between teachers and students was described and perceived as important, especially in relation to the development of student agency, the aspect of the regional community did not emerge as important within the study.

Probability and Context

- Probability content provides an appropriate context in which to build higher order thinking skills in senior secondary students.
- The context of the mathematical problems presented in the probability content is important in providing an appropriate milieu for the development of thinking skills

These points will now be described in more detail.

7.2.1 Higher Order Thinking

Higher order thinking is evident in all aspects of the related mathematics curriculum.

The VCE MM34 curriculum, contains 40% or more statements or mathematical problems relating to higher order thinking skills, in the intended curriculum as denoted by the mathematics Study Design, and the attained curriculum as measured by the assessments. Forty percent higher order thinking content is appropriate as it compares favourably to content proportions reported in the associated literature. Forty percent higher order thinking is also appropriate given the large amount of new content in VCE MM34 probability curriculum. The implemented curriculum was not found to contain forty percent higher order thinking, with lower order thinking in the learning chapter (19%), and additional higher order thinking in the review chapter (52%). The use of the curriculum materials by the teachers and students will greatly influence the thinking skills developed. This does not mean students necessarily complete all the higher order thinking tasks, but it does mean the possibilities are there.

This is appropriate considering the large amount of new content in the senior secondary probability curriculum that emerged from the 2016 revisions. However, while the expectation is that students attempt or complete all the higher order thinking tasks, this does not mean the mathematics curriculum is aligned in this regard, only that the possibilities are there.

7.2.2 Tools

The recommended textbook included material to support the development of mathematical and thinking skills, however, the method of use could limit the value of textbooks.

Both student and teacher participants reported the textbook as the main instructional resource used in senior mathematics classes. In the current study, the recommended textbook was used as an indication of the implemented curriculum. The textbook included a variety of explanations, examples, and problems of a variety of contexts and types, to support students in their development of mathematical and thinking skills. However, most of the higher order thinking problems were in the review sections at the end of the textbook, which was problematic if this section was not looked at by students. The textbook contained a variety of problem types, corresponding to the examination problems, however, they did not include problems in the expected style of the SACs, which should include simulations, problem solvers or modelling. The way in which the textbook is used by teachers can thus greatly influence the nature and depth of thinking skills developed by students. Teacher and student participants reported teachers playing a main role in setting all or selected problems from the textbooks, but there was little evidence of teachers formally checking student work. It is therefore probable that some students did not complete all the problems, and particularly the review problems at the end of the textbook, resulting in missing out on engaging with the section with the most potential to provide practice at higher order thinking.

While teachers viewed support tools such as calculators and bound reference books as beneficial to the development of mathematical and thinking skills, students were less convinced of their value, particularly in relation to calculators.

The VCE Mathematics Study Design assumed students used a CAS calculator, and a bound reference book was allowed to be used in the SACs and second examination. However, student participants expressed mixed responses to the value of using these resources. While a third of the student participants in the current study responded positively to the value of calculators for checking answers, solving complex equations, and sketching graphs, the rest were less favourable. This was due to the time taken to learn how to use the calculators, the requirement for a non-calculator examination, perceived lack of skills of the teachers and the inability to use calculators at university. A number of student participants reported that they did not feel they were learning when using a calculator. Bound reference books were viewed more positively, as a learning and revision tool. A couple of the more able student participants felt they were not needed, while a small number found them very helpful, and were even using their VCE bound reference books to support their subsequent university studies.

The teacher participants held more positive views towards both the CAS calculators and bound reference books than the students. Just two of the fourteen teachers felt the calculators were unnecessary due to the increased workload involved to learn them, while the other teachers reported they saved time and cognitive load. Only a few teachers reported that they used calculators to teach mathematical concepts. Teacher participants recommended the bound reference books to students in different ways, with some actively directing how they were to be used and incorporating this as a learning activity. Others reported that they gave hints for their use, while some teachers left the preparation and use entirely up to the students. The use of CAS calculators and bound reference books were expected to be supports for students and teachers, as described in the literature, to enable students to focus less on memorisation and procedures and more on higher order thinking. However, the student participants did not find them as helpful as expected.

7.2.3 Rules

The rules regarding the SACs aimed to support the development of higher order thinking, however, teachers identified tensions that hindered the implementation of the SACs.

The VCE Mathematical Methods Study Design aimed for the SACs to add a variety of assessment styles and include modelling and problem solvers. The rules relating to the SACs required inclusion of mathematical problems which necessitated the use of higher order thinking. Teacher participants reported that they followed the rules of the Study Design to the best of their ability, however, they identified several tensions in the creation and implementation of the SACs. During the interviews the teacher participants expressed a degree of reluctance to share their personally created SACs, as they did not want their SACs to be analysed or they wanted to keep them confidential.

While adapting the SAC conditions to discourage plagiarism and adapt to the needs of the schools, the school-based assessment became more like examinations, which diminishes the aims of the SACs, as an alternative assessment type. This was an ongoing tension that the teacher participants did not know how to solve. Textbooks were not seen as particularly helpful for SAC preparation as they did not contain problems of the style expected in the SACs. These tensions meant teachers did not always follow the rules in setting SACs. Commercially available SACs did not always follow the VCE Mathematical Methods rules either, and some teachers explained they could not design and implement the required SACs due to lack of skill, time, or confidence.

7.2.4 Division of Labour

Regarding the division of labour within the school community, the agency demonstrated by both teachers and students can support student learning in relation to thinking skills.

The mathematics curriculum, represented by the Study Design and textbook, set up the learning system, but ultimately, the teachers and students had the greatest influence on the implementation of the curriculum. The findings from the interviews demonstrated that teachers set up the dynamics of the mathematics classes. Teachers directed the classes, set the routines for whole class explanations, and even directed students' individual study from the textbooks in what they called a 'traditional' way. Some student participants demonstrated agency in their studies by working ahead or forming study groups in and out of class. In general, teacher participants accepted this student initiative. This mixture of control or division of labour was flexible and depended on the content and student needs.

Teachers reported that they felt they adapted to cater for the needs of their students. However, the findings demonstrated that the teacher participants rarely used alternative teaching methods, for example, open-ended tasks, student developed concept maps or used technology other than calculators. This lack of alternative pedagogy was also reported by the student participants in relation to their VCE Mathematical Methods teachers. The few teacher participants who did try alternative teaching methods reported that they found them to be effective, leading to more student discussion of alternative solutions, collaboration, and arguably to the development of higher order thinking. Professional development attended by the teachers tended to be around the use of CAS calculators or assessments, with no mention of support to develop more creative approaches to teaching senior mathematics.

7.2.5 Community

Supportive relationships between students and teachers were seen as important in helping students develop mathematical and thinking skills, but the support within a regional community was not acknowledged as important by participants.

Students and teachers formed part of the community component of the activity system, but regional community was also included due to the research context which was a regional case study. All research participants had attended or taught at regional secondary schools and all student participants were attending a regional university, so the expectation was that the regional community may emerge as a theme, particularly in terms of presenting certain tensions in relation to achievement of the outcome. For instance, regionality may have been deemed as enhancing skill development due to the possibilities of closer relationships with teachers in smaller, more intimate class settings. Alternatively, it may have produced tensions and limitations due to a lack of opportunities often associated with regional education as evident in the discussion of the teaching and learning of mathematics in Gippsland in Chapter 1, Section 1.3.

The lack of discussion of regionality within the current study emerged for a number of reasons, most of which were structural ones and for this reason, it has been included as a limitation of the study design, the details of which are provided in Section 7.4. Additionality the need for this to be a focus of future research is highlighted in Section 7.5.2.

7.2.6 Probability and Context

The study of the topic of probability potentially supports the learning of higher order thinking in senior secondary students.

Probability is a mathematical topic which relates to real-life. Probability lends itself to problems of many types including closed multiple-choice, short-answer problems and extended-answer problems. According to the Study Design, probability provides a means for encouraging application, modelling and problem-solving, all of which contrast with lower order thinking of memorisation of skills and procedures. The probability problems in the textbook and assessments used for VCE mathematics, which formed part of the current study, included a range of lower and higher order thinking skills. When problems link different concepts together, this is also an indication of higher order thinking. Student participants described the tension around the topic of probability as an area that combined other mathematics topics, for example, algebra, functions, calculus, but also as an unrelated topic that seemed to have little structure.

A tension identified through the analysis of the curriculum documents, was the large increase in the amount and complexity of probability in the final year of school, in Mathematical Methods. Together with the finding that only about 30% of students enrol in VCE Mathematical Methods Units 3 and 4, and are thus exposed to this important probability content, it is worth considering whether some of this content could be covered be in earlier year levels.

The context of the mathematical problems can support the development of thinking skills.

The mathematical problems in all aspects of the curriculum, the textbooks, SACs, and examinations, were perceived by study participants as representing a variety of contexts, some of which were practical and relatable while others were quite abstract. Research supports that if students understand the context of mathematical problems, they can focus on the mathematical concepts (Vincent & Stacey, 2008). The curriculum documents studied included a range of problems with contexts that students could relate to, and the participants (both students and teachers) highlighted the context of problems as a supporting factor in developing their mathematical understanding.

The wording of some of the probability problems was also described as confusing for students by several student and teacher participants. This was a tension identified in both textbooks and assessments, and for one of the specific probability problems discussed in the interviews. The context of some problems could be seen as contrived, as in the examples in Figure 6.1 in Chapter 6, Section 6.1.4. This examination problem involved the time taken for students to climb a hill, but as a complex combined polynomial/exponential function. Another example of a contrived context was described in one of the teacher interviews, involving a commercial SAC which suggested leadlight in a church window was an exponential function on a bride's wedding day, a situation which is very unlikely to be of interest to 18-year-old students. While all aspects of the curriculum used a variety of contexts to develop a range of thinking skills, some 'real-life' contexts were contrived or largely irrelevant for the age group.

7.2.7 Overall Conclusion

The probability topic within the Victorian senior secondary mathematics subject of Mathematical Methods potentially supports the development of higher order thinking. Utilising an Activity Theory framework to describe the relationships within a particular activity system, the subjects (students) were supported in the object (development of thinking skills), by the tools, (Study Design, textbooks, calculators, and bound reference books) and rules (Study Design, school and class norms). The community (teachers, students, and peers) worked together, while dividing their labour, to support the object, and the ultimate outcome (ATAR score to have the option of attending university and potentially improved decision-making skills). Addressing the tensions and contradictions within and between the elements can lead to improvements in the object and outcomes of the activity system. Some of the tensions and contradictions include: potential missed opportunities in the use of the textbooks, calculators, bound reference books, alternative learning activities including technologies, lack of support for teachers and students in preparation for the SACs, and the large increase in amount and complexity of probability.

7.3 Significance of this Study

This study contributes to the field of mathematics education by identifying the possibilities and tensions for the development of thinking skills in senior secondary mathematics, particularly in relation to the topic of probability. Some significant insights emerged from the study, regarding the implementation of SACs, use of textbooks, pedagogy of teachers, and missed opportunities of students in utilising the resources. The study also adds to knowledge on the teaching and learning of probability and use of the theoretical frameworks to investigate curriculum and thinking skills.

7.3.1 Contributions to Knowledge

Greater understandings on several levels associated with the teaching of probability at the senior secondary level have been generated through the current study. These have ramifications at the departmental level, school level, and then at the class and student level.

At the departmental level, SACs which makes up a third of the high stakes' summative assessments in Victorian senior secondary mathematics subjects, are an under-researched area for the final year of schooling. The requirement for teachers to design and implement the SACS adds unreasonable stress on teachers, who do not appear to have the time or necessary skills to create appropriate tasks, while even the commercially available tasks do not always pass the audit process. This study found that the requirement for teachers to create their own SACs was generally not effectively implemented, with reports of test type tasks being produced, which would not meet the VCAA guidelines. This is consistent with the findings from a study that is now twenty years old (Barnes et al., 2000), demonstrating that little support appears to have been provided to overcome this issue.

Textbooks appear to significantly influence the learning activities of senior secondary mathematics classes, as demonstrated by the participants in this study. This will not be a surprise for teachers and students, but is not explicitly mentioned in the literature, as widespread studies on this topic have not been conducted since TIMSS 1999 (Valverde et al., 2002). The textbook in this study had a range of contexts, content, and levels of thinking, but much of the higher order thinking problems were contained in the review sections, so teachers and students need to be aware of this when using textbooks. The VCE Mathematical Methods recommended textbook did not prepare students for SACs, which is an area the publishers may need to address.

At the school level, there was evidence within this study that senior secondary mathematics classes in Gippsland schools continue to be very traditional, heavily focused on a teacher-centred, textbook reliant method to teach students. While it appears some teachers in primary schools adapt to more creative teaching methods, using group work, formative assessment, open-ended tasks and a wide range of technology (Attard et al., 2020; Hunter et al., 2020; Jones, 2006), most of the teachers and students in this study did not report experience of or with these strategies.

Students missed possible opportunities by underutilising the available resources. The use of CAS calculators and bound reference books for some assessments aimed to support students in developing their thinking skills, by taking the pressure off memorisation of skills and procedures.

Some students did not find these tools as helpful as the teachers anticipated. This was due to the time and energy required to learn how to use these skills, the teacher's lack of expertise or interest, and one of the final examinations precluding the use of calculators or bound reference books.

Probability continues to be an important and relevant topic for students, which combines and applies many of the other topics within mathematics. Students notice and appreciate when they can relate to the context of mathematical problems. While the textbooks and assessments reported on in the current study aimed to cover a range of contexts, some were quite contrived, although still helpful for students to relate to the context of the problems, and hence focus on the higher order thinking of the application, interpretation and evaluation of a problem and its solution.

7.3.2 Contributions to the Use of Theoretical Frameworks

This study contributes to mathematics education research by its use of a range of theoretical frameworks. The three frameworks used in this study were the Two-Tiered Mathematical Thinking Framework (see Chapter 3, Section 3.3.2), TIMSS Curriculum Model (Mullis & Martin, 2013) and the overarching theoretical framework of Activity Theory as interpreted by Engeström's (2001) framework of activity systems. (Engeström, 2001) This study contributes to the understandings of the value of each of these frameworks in mathematics education research.

While Bloom's taxonomy is the most popular thinking framework used in schools, it has limitations in terms of its use in Mathematics, as outlined in the review of the related literature. Bloom's taxonomy and the SOLO thinking framework are both used in local Gippsland schools: to support teachers' planning processes, as well as being incorporated into learning intentions, designing assessments, and as a language to support the discussion of thinking skills in classes. Many other thinking frameworks have been used in mathematics education, all with limitations. Thus, while no single thinking framework appears to be ideal for every situation, the combined *Performance type categories for mathematics* (Ziebell et al., 2017) has many worthwhile features and should be considered for future mathematics research. The current study used the combined *Performance type categories for mathematics,* however it was adapted to include aspects relating to SOLO (Biggs & Collis, 1991), procedural complexity, and the repetition factor (Vincent & Stacey, 2008). This combined thinking framework was also simplified into two levels as demonstrated by the *Two-Tiered Mathematical Thinking Framework* (Chapter 3 Section 3.3.2), which was particularly appropriate in this context to classify both the curriculum statements and probability problems which formed part of the document analysis component of the research. This framework is a straightforward, practical way for teachers to classify and evaluate their implemented curriculum.

The current study suggests a method of classifying curriculum using the TIMSS Curriculum Model (Mullis & Martin, 2013) and the three aspects of the intended, implemented and attained curriculum. In this study, these aspects were represented by the Study Design curriculum statements, textbooks, and assessments. A review of the literature demonstrated a lack of knowledge around curriculum alignment across all three aspects of the curriculum. Curriculum alignment of one or two of the aspects has been encouraged in Victorian schools by the HITs curriculum support documents (DET, 2020a). It has also been examined in previous research, especially with the introduction of the National Curriculum in the USA (for example, Porter et al., 2011; Remillard & Heck, 2014). However, little research has been conducted across the three levels, as in the current study, which examined curriculum alignment across all three levels, supplementing the data with the perspectives of students and teachers via interviews.

Engeström's (2001) framework of activity systems has been used in a range of research in the Education discipline, especially in research involving the implementation of new technologies. However, it has not been used to analyse the influence of curriculum on learning. In any activity system such as education, curriculum is a tool, to support learning, but it also creates tensions in its implementation. Activity Theory provided an appropriate theoretical lens for this study, as it was able to examine different components of an activity system to determine how they interacted with each other and their ultimate influence on the intended outcome, which in this instance was the development of mathematical and thinking skills. Within the activity system, the tools and rules (for example the textbook, calculators, and the rules surrounding their use) could be studied, but also the community and division of labour, so the influence on students, teachers and peers could also be included. The use of an activity system to investigate the influence of curriculum on student learning, provided an effective analytical lens and a framework for structuring the discussion.

7.4 Limitations of this Study

This section addresses the limitations of the investigation, some of which are inherent in the research design, and others related to circumstances that arose during the implementation of the study. The main limitation is the lack of generalisability of this case study.

The students and teachers in this study came from the Victorian regional district of Gippsland, as described in Chapter 1. This is a diverse region, but it is likely that metropolitan or other regional students and teachers may have had different experiences. Regionality did not emerge as a theme, mainly because no specific interview questions related to teaching or learning in the regional area of Gippsland. The students and teacher participants had little experience outside the Gippsland area, so may have not felt the need to be specific about aspects of learning or teaching in a regional context. As the research design did not include recruiting participants outside the Gippsland region, there was no comparative aspect and therefore it was not possible to make any conclusions about the described experiences as being related to regionality.

The sample size of 20 students and 14 teachers from one geographic region means that the results cannot be generalised beyond the particular case, although an attempt was made to recruit a diverse range of eligible students and teachers and the groups differed in terms of their backgrounds and experiences. The identified tensions and possibilities that emerged from the current study is nonetheless anticipated to have broader applicability as indicators of general issues with mathematics curriculum implementation and higher order thinking development. Another possible limiting factor related to the sample, was the lack of comparative analysis of students, teachers, or schools. To keep within the ethical requirements of the study, schools were not identified, and participants were discouraged from mentioning names of students, teachers, or schools. While this could be considered a limitation, the aim was not to compare or critique teachers or schools, but to discover possibilities and tensions.

The focus of the current study was the official intended Victorian curriculum, which, although like the Australian Curriculum, has different assessment methods, so this study cannot be used as representative of Australian or any other curriculum. The Victorian curriculum is unique, with its use of SACs, CAS calculators and bound reference books, which makes it an interesting case study (Yin, 2013).

The data were collected from students and teachers as they reflected on their previous year studying or teaching mathematics. Interviews were conducted rather than class observations. The teachers were continuing to take the VCE Mathematical Methods classes; however, the students had completed their VCE and had moved onto university. This was because school principals were reluctant to allow any interruptions to VCE students during the stressful final year. The interviews were also based on perceptions of participants in just one moment in time. These factors were limitations; however, they were also a positive as students had had time to reflect on their complete experience of year 12 studies and consider the implications as they continued in their university studies.

The recruitment method for student and teacher participants also presented a limitation. Recent students of VCE Mathematical Methods were recruited from one campus of a regional university in their first year of studies, from mathematics or statistics classes where VCE Mathematical Methods students were likely to be enrolled. Mathematical Methods students in other courses or who were not attending this university were not represented. Teachers were recruited from external professional development programs, in particular, sessions aiming to support VCE Mathematics teachers with SACs, preparing students for examinations and incorporating the CAS calculators into their classes. Participating teachers then recommended the study to their peers. These recruitment methods may bias the results.

To prevent researcher bias, another experienced researcher assisted me in checking the classifications of a sample of the document analysis and regarding the interpretation of participant quotes. Views of both students and teachers were compared to the documentation of the curriculum, to uncover a range of tensions and possibilities surrounding the use of the curriculum. Other issues around researcher bias in interpretation were addressed in Chapter 3, Section 3.5. Therefore, while the results might not be directly generalisable to other similar settings, theories and recommendations developed from the research could still provide useful guidance for other similar settings and regarding further research.

The findings of this study would not be surprising to secondary teachers in the area in which it was conducted, and certainly not to me, the mathematics teacher turned researcher. It was reassuring to have the information confirmed, that while the development of higher order thinking was a design feature of the curriculum, it was possible for students to miss this opportunity, especially with the limitations of the some of the tools including the 'unbalanced' textbook. This research project came about as an opportunity for a secondary teacher turned researcher to investigate this under researched area of mathematics education, providing a glimpse of the reality of senior secondary teaching in a well-meaning but overcrowded curriculum.

7.5 Recommendations

The literature review and analysis of the curriculum documents identified many possibilities for students to develop mathematical and higher order thinking skills. This study has identified a range of missed learning and teaching opportunities that many teachers appear not to be effectively utilising. This is likely due to the time and energy required to change teaching practices, and the lack of knowledge of the possibilities.

7.5.1 Recommendations to Improve the Thinking Component within Senior Secondary Mathematics

Recommendations that emerged from this study to improve the development of thinking skills in the senior secondary students, include recommendations regarding the topic of probability within the intended curriculum, textbook design, teacher support, student support and use of thinking frameworks in classes.

7.5.1.1 Probability within the Intended Curriculum

The current study demonstrated the large increase in the amount and complexity of probability content in the final year of secondary school resulting in a number of challenges for students. This could be overcome with a gradual increase in probability content in Years F–10. Watson and English (2015a) suggest students are ready for probability earlier than the curriculum recommends, especially with the assistance of technology simulations like Tinkerplots (Konold & Miller, 2011). Jones and Tarr (2007) report some middle years teachers leave probability to the end of the year, and sometimes even miss probability. This will amplify the problem of increased content and complexity in the final schooling years. Also, just 30% of VCE students enrol in Mathematical Methods. If the probability content was increased in middle school, this would result in more students learning this important topic.

7.5.1.2. Textbooks

Recommendations for textbooks for senior mathematics include ensuring there is a variety of problem types including SAC type problems and improving the guidance for using the books and the accompanying resources. Textbooks need to include problems of the style of the SACs, with problem solvers, modelling, application tasks and simulations. Textbooks need to include higher order thinking tasks in the earlier chapters, rather than leaving them to the review sections, or make it very clear to teachers and students that the earlier chapters are not enough to prepare for the examinations. Although valued as a resource, the textbooks were not used to full advantage by the teachers or students in the current study, with students reporting that they did not use the associated online activities. Similarly, other online activities such as videos and quizzes which were popular in the literature, were not used by most of the participating students and teachers, although this may have improved since the interviews were conducted for this study, particularly in light of the focus on remote learning and teaching that occurred as a result of the COVID-19 pandemic. However, if textbook publishers cannot support these changes, the Department of Education and Training may need to supply these additional supports.

7.5.1.3. Teacher Support

This case study found that teachers require support in the implementation of the curriculum for several reasons:

- Senior secondary mathematics teachers in the Gippsland region appear to be hesitant to trial innovative teaching strategies, with teacher participants reporting reliance on lectures and textbooks for teaching. It is important that they are provided with more guidance in how to support students to fully utilise the textbooks and bound reference books.

- If VCE senior secondary subjects change in content, teachers and students need support and guidance in terms of implementing such changes. This is particularly important for teachers who are teaching outside their field of expertise, something that more commonly occurs in regional areas (Lyons et al., 2006).
- There are many uses for technology in mathematics teaching and learning, including probability, which teachers appear not to be aware of, or have little time to trial (Goos & Bennison, 2008). For example, calculators can be used for teaching concepts, as well as answer checking. Videos can be used to explain concepts, to support missed or rushed classes, and enable class time to be used in a wider variety of ways. In the current study, even the online resources provided with the textbook were not fully utilised.
- Teachers need support in creating and implementing SACs, especially if the style of assessments change (Barnes et al., 2000).

Support for teachers might be in the form of a teacher edition of the textbook (Vincent & Stacey, 2008), professional development, subject support networks, lesson study or publications. Research into supporting regional teachers through professional development over a longer period of time (Goos et al., 2011), combined with setting up and resourcing learning communities (Brodie, 2020), could help. There are still issues in scaling up this support. The Department of Education and Training publishes general pedagogy support, for example, the *High Impact Teaching Strategies (HITS)* (DET, 2020a), but senior secondary mathematics teachers require guidance in implementing these in their particular context.

7.5.1.4 Students

Recommendations for students focus on encouraging students to take full advantage of the resources available to them. Powerful calculators and bound reference books can be used to reduce the need for memorisation, and to reduce anxiety. The textbook and practice examinations provide the full range of problems of a variety of levels of thinking, if all the resources are used, especially the review sections of the textbooks. The practice examinations are only useful if used as formative assessment tools and used to identify strengths and weakness which are then acted on. Senior secondary students should be encouraged to demonstrate agency, be proactive in their learning, form study groups, use the freely available online technology resources, and work with their teachers by asking questions and providing feedback to improve their mathematical skills and range of thinking skills.

7.5.1.5 Thinking Frameworks

An increase in higher order thinking needs to be emphasised in senior secondary schools. Explicit discussions about the variety of thinking skills and why they are important need to be acknowledged. As schools focus on *learning intentions* and *success criteria* as recommended by *High Impact Teaching Strategies (HITS)* (DET, 2020a), it is appropriate to incorporate higher order thinking skills as well. Blooms taxonomy (Krathwohl & Anderson, 2002) is one important thinking framework, but others, especially SOLO (Biggs, 1989) need to be considered as well.

7.5.2 Future Research Directions

Some interesting avenues for future research include:

Thinking Skills. Further investigation into how to increase a range of thinking skills into probability and mathematics in schools is needed. This could include creative and critical thinking skills as well as higher and lower order thinking skills.

Pedagogy. Improving senior secondary teaching pedagogy, possibly through action research (for example, Ernst, 2014)

Technology. Using technology to improve the development of thinking skills in senior secondary mathematics. I found many studies conducted in lower school year levels, and at university, but little research covering the important final years of school.

Agency. As senior secondary students are aged 16–18 years and preparing to move into higher education or the workforce, it is important that they are given opportunities to be proactive in their learning. However, this was not demonstrated by many of the student participants in the current study. More research into how to improve senior secondary student's demonstration of agency in their studies is needed.

Regionality. Mathematics and in fact secondary education in general, has been found to be experienced in different ways depending on locale (Plunkett & Dyson, 2011). So regional and metropolitan students and teachers would be expected to highlight different tensions within an activity system such as the one used in the current research. It is therefore important that future research focus more specifically on this aspect in the development of the study design and that comparative studies are conducted to ensure that differences are identified, as emergent recommendations may need to be more nuanced with regard to locale than those in the current study.

7.6 Concluding Remarks

A range of thinking skills were evident in the curriculum under investigation in the current study, namely the Victorian senior secondary mathematics probability curriculum. The Study Design, textbook and assessment represent the intended, implemented and attained curriculum, all of which contained a variety of thinking skills as categorised by the Two-Tiered Thinking Framework. This study demonstrated how activity systems (Engeström, 2001) can be used to describe the combined influence of the tools and rules of the intended curriculum, with the divisions of labour within the community.

In summary, the influential recommended textbook included material to support the development of mathematical and thinking skills, however the method of use could limit the textbooks' value. While teachers viewed support tools such as calculators and bound reference books as beneficial to developing mathematical and thinking skills, students were less convinced of their value, particularly concerning calculators. The rules within the curriculum, for example the rules around the SACs, aimed to support the development of higher order thinking, however teachers identified tensions that hindered the implementation. Regarding the division of labour within the school community, the agency demonstrated by teachers and students, can support student learning. The context of the mathematical problems can support the development of thinking skills, and finally the study of the topic of probability potentially supports the learning of higher order thinking in senior secondary students.

The current study contributes to knowledge by providing a voice to a sample of senior secondary students and teachers, to enable the sharing of their perceptions of the possibilities and tensions related to their personal experience. The classes they experienced tended to be traditional, lecture and textbook based, with surprisingly little technology used except the compulsory CAS calculators. The Mathematics Study Design encouraged problem-solving, modelling and application tasks, however the implementation of the textbooks and school-based assessments minimised these opportunities. Recommendations include an increased focus on probability at the lower levels of schooling, increased support for teachers particularly in this regional area, modifications to the textbooks to increase the variety of problem types, and explicit use of thinking skills in schools. Some areas of future research were also recommended.

7.7 Postscript

This study commenced in 2015 and was submitted for examination in 2021, with the data collection occurring between 2016–2018. The writing up of this thesis was mainly completed in 2020, which turned out to be a very unique year, the COVID-19 year of lockdowns and associated stress. Schools went online, with some students still attending and some staff working from home. It will be interesting to see the changes to VCE senior secondary mathematics teaching after this transformative time. Have teachers and students adjusted their practices? Have teachers used technology more effectively, and will these changes continue once students and teachers are back in the classrooms? In 2020, all VCE subjects were trimmed to attempt to cater for the changes resulting from the Victorian government's response to the COVID-19 pandemic. VCE Mathematical Methods was trimmed through reducing the content and removing one of the SACs. The content area trimmed was probability, with about half of the probability content removed, just for the year 2020. The SAC removed was the probability SAC. This could be seen as an indication of the perceived lower value of this topic, but it is more likely that this content is taught last in the year and relies on the previous content (especially integral calculus), and so is the most logical to trim. It will be interesting to see how the 2020 VCE students perform at university and in their future. Overall, the pandemic has created a number of opportunities for further research into the topic of mathematics and probability in particular.

The VCE Mathematics Study Design was accredited for the period of 2016–2021 but this was extended to 2022. The disruptions caused by COVID-19 meant the normal reaccreditation process had to be delayed. The 2016 Study Design was informed by educators and international research, and it seems the next reaccreditation process will do the same. I look forward to seeing what the next iteration of the mathematics subjects will be. Rumours are that coding and algorithmics may be included in some mathematics subjects, and the discontinuation of the alternative senior secondary certificate, Victorian Certificate of Applied Learning (VCAL) will also influence the content and delivery of the mathematics subjects. All these factors may change the position of probability, possibly back to the popular General/Further Mathematics as it was in 1978 (Ernst, 2018; Fitzpatrick, 1974), which would make it more available as an area of study for all students.

References

Abbott, R. C. (2016). Embracing digital technologies in classroom practice: The impact of teacher identity. *Australian Educational Computing, 31*(2). http://journal.acce.edu.au/index.php/AEC/article/view/93

ACARA. (2010). *Australian curriculum: Mathematics*. https://www.australiancurriculum.edu.au/f-10-curriculum/mathematics/

ACARA. (2012). The Australian Curriculum. Senior Secondary Mathematics - Information sheet. Retrieved 23/02/2019, from http://www.acara.edu.au/verve/_resources/Senior_Secondary_Mathematics.pdf

ACARA. (2018). *National assessment program—Literacy and Numeracy achievement in Reading, Writing, Language Conventions and Numeracy: National report for 2018*. https://nap.edu.au/docs/default-source/resources/2018-naplan-national-report.pdf?sfvrsn=2

Agarwal, P. K., Karpicke, J. D., Kang, S. H. K., Roediger III, H. L., & McDermott, K. B. (2008). Examining the testing effect with open‐ and closed‐ book tests. *Applied Cognitive Psychology, 22*(7), 861-876. https://doi.org/10.1002/acp.1391

Agarwal, P. K., & Roediger, H. L. (2011, 2011/11/01). Expectancy of an open-book test decreases performance on a delayed closed-book test. *Memory, 19*(8), 836-852. https://doi.org/10.1080/09658211.2011.613840

Aizikovitsh-Udi, E., & Amit, M. (2011). Developing the skills of critical and creative thinking by probability teaching. *Procedia-Social and Behavioral Sciences, 15*, 1087-1091. https://doi.org/10.1016/j.sbspro.2011.03.243

Aizikovitsh-Udi, E., Clarke, D., & Kuntze, S. (2014). Hybrid tasks: Promoting statistical thinking and critical thinking through the same mathematical activities. *PNA. Revista de Investigación en Didáctica de la Matemática, 8*(3), 115-126.

Aline, F., Zeng, S., & Yu, Y. M. (2013, May 17-19, 2012). Using Bloom's Taxonomy in a peer-led workshop in probability and statistics. Conference Proceedings of the Peer-Led Team Learning International Society, New York.

Allen, K. C. (2011). Mathematics as thinking. A response to "Democracy and School Math". *Democracy and Education, 19*(2), 10. https://democracyeducationjournal.org/cgi/viewcontent.cgi?referer=https://scholar.google.com.au/&httpsredir=1&article=1036&context=home

Anderson, J., White, P., & Wong, M. (2012). Mathematics curriculum in the schooling years. In *Research in Mathematics Education in Australasia 2008–2011* (pp. 219-244). Sense Publishers.

Ang, L. H., & Shahrill, M. (2014). Identifying students' specific misconceptions in learning probability. *International Journal of Probability and Statistics, 3*(2), 23-29. https://doi.org/10.5923/j.ijps.20140302.01

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. International Academy of Education. http://www.ibe.unesco.org

Apino, E., & Retnawati, H. (2016). Creative problem solving to improve students' higher order thinking skills in mathematics instructions. Proceedings of 3rd International Conference on Research, Implementation and Education of Mathematics and Science, Yogyakarta.

Appelbaum, P., & Stathopoulou, C. (2016). Mathematics education as a matter of curriculum. *Encyclopedia of educational philosophy and theory*, 1-6. https://doi.org/10.1007/978-981-287-532-7_515-1

Arnold, P., & Pfannkuch, M. (2018). *Critiquing investigative questions.* Looking back, looking forward. Proceedings of the Tenth International Conference on Teaching Statistics (ICOTS10, July, 2018), Kyoto, Japan.

Assor, A. (2012). Allowing choice and nurturing an inner compass: Educational practices supporting students' need for autonomy. In *Handbook of research on student engagement* (pp. 421-439). Springer. https://doi.org/10.1007/978-1-4614-2018-7_20

Attard, C., Calder, N., Holmes, K., Larkin, K., & Trenholm, S. (2020). Teaching and learning mathematics with digital technologies. *Research in Mathematics Education in Australasia 2016–2019*, 319-347. https://doi.org/10.1007/978-981-15-4269-5

Attard, C., & Holmes, K. (2019). *Technology-enabled mathematics education: Optimising student engagement*. Routledge.

Atweh, B., & Goos, M. (2011). The Australian mathematics curriculum: A move forward or back to the future? *Australian Journal of Education, 55*(3), 214-228. https://doi.org/10.1177/000494411105500304

Atweh, B., Goos, M., Jorgenson, R., & Siemon, D. (2012). *Engaging the Australian curriculum mathematics: perspectives from the field*. Mathematics Education Research Group of Australasia Incorporated.

Atweh, B., Miller, D., & Thornton, S. (2012). The Australian curriculum: Mathematics-world class for déjà vu. In *Engaging the Australian curriculum mathematics: perspectives from the field* (pp. 1-18). Mathematics Education Research Group of Australasia.

Avis, J. (2009). Transformation or transformism: Engeström's version of activity theory? *Educational Review*, *61*(2), 151-165. https://doi.org/10.1080/00131910902844754

Azizan, U. H., & Ibrahim, F. (2012). Identifying pupil's cognitive level in fractions using Bloom's Taxonomy. *International Journal of Business and Social Science, 3*(9), 254-256.

Babbie, E. R. (2011). *The basics of social research* (Fifth ed.). Wadsworth Cengage Lerarning.

Bakker, A., Derry, J., & Konold, C. (2006). *Using technology to support diagrammatic reasoning about center and variation* Proceedings of ICOTS-7. International Association for Statistical Education, Salvador de Bahia, Brasil.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education, 59*(5), 389-407. http://kprcontentlibrary.kprdsb.ca:8080/docushare/dsweb/Get/Document-9589/content_knowledge_for_teaching_what_makes_it_special.pdf

Ball, L. (2014). *Use of computer algebra systems (CAS) and written solutions in a CAS allowed Year 12 mathematics subject: Teachers' beliefs and students' practices.* [Doctoral dissertation, The University of Melbourne]. http://hdl.handle.net/11343/42231

Bandura, A. (2006). Toward a psychology of human agency. *Perspectives on psychological science, 1*(2), 164-180.

Barab, S., Schatz, S., & Scheckler, R. (2004). Using activity theory to conceptualize online community and using online community to conceptualize activity theory. *Mind, Culture, and Activity, 11*(1), 25-47. https://doi.org/10.1207/s15327884mca1101_3

Barkatsas, A. T., & Seah, W. T. (2015). Learners' preferred mathematical task types: The values perspective. In *Diversity in mathematics education* (pp. 63-79). Springer. https://doi.org/10.1007/978-3-319-05978-5_4

Barkatsas, T., Gialamas, V., & Orellana, C. (2016). Secondary students' attitudes toward learning mathematics with computer algebra systems (cas): A structural equation model. In *Global Learning in the 21st Century* (pp. 121-137). Brill Sense.

Barnes, M., Clarke, D., & Stephens, M. (2000). Assessment: the engine of systemic curricular reform? *Journal of Curriculum Studies, 32*(5), 623-650. https://doi.org/10.1080/00220270050116923

Batanero, C. (2014). Probability teaching and learning. In *Encyclopedia of Mathematics education* (pp. 491-496). Springer. https://doi.org/10.1007/978-94-007-4978-8

Batanero, C., & Borovcnik, M. (2016). *Statistics and probability in high school*. Springer.

Batanero, C., Burrill, G., & Reading, C. (2011). *Teaching statistics in school mathematics- Challenges for teaching and teacher education*. Springer.

Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). *Research on teaching and learning probability*. Springer. https://doi.org/10.1007/978-3-319-31625-3_1

Batanero, C., Contreras, J. M., Díaz, C., & Sánchez, E. (2015). Prospective teachers' semiotic conflicts in computing probabilities from a two-way table. *Mathematics Education, 10*(1), 3-16. https://doi.org/10.12973/mathedu.2015.101a

Batanero, C., Contreras, J. M., Fernandes, J. A., & Ojeda, M. M. (2010). *Paradoxical games as a didactic tool to train teachers in probability.* Data and context in statistics education: Towards an evidence-based society. Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia. .

Batanero, C., Estepa, A., Godino, J. D., & Green, D. R. (1996). Intuitive strategies and preconceptions about association in contingency tables. *Journal for Research in Mathematics Education, 27*(2), 151-169.

Batanero, C., Henry, M., & Parzysz, B. (2005). The nature of chance and probability. In *Exploring probability in school* (pp. 15-37). Springer.

Biehler, R. (1994). Probabilistic thinking, statistical reasoning, and the search for causes—Do we need a probabilistic revolution after we have taught data analysis? *Research papers from ICOTS, 4*, 20-37.

Biesta, G., Priestley, M., & Robinson, S. (2015). The role of beliefs in teacher agency. *Teachers and Teaching, 21*(6), 624-640. https://doi.org/10.1080/13540602.2015.1044325

Biggs, J. (1989). Towards a model of school-based curriculum development and assessment using the SOLO Taxonomy. *Australian Journal of Education, 33*(2), 151-163.

Biggs, J. (2003). Aligning teaching for constructing learning. *Higher Education Academy*(1), 1-4. https://www.researchgate.net/profile/John_Biggs3/publication/255583992_Aligning_Teaching_for_Constructing_Learning/links/5406ffe70cf2bba34c1e8153.pdf

Biggs, J. (2012). What the student does: teaching for enhanced learning. *Higher Education Research & Development, 31*(1), 39-55. https://doi.org/10.1080/07294360.2012.642839

Biggs, J., & Collis, K. (1982). *Evaluating the quality of learning: The SOLO taxonomy (Structure of the Observed Learning Outcome)*. Academic Press.

Biggs, J., & Collis, K. (1991). Multimodal learning and the quality of intelligent behaviour. In *Intelligence: Reconceptualization and measurement* (pp. 57-76). Lawrence Erlbaum.

Biggs, J., & Tang, C. (2011). *Teaching for quality learning at university: What the student does* (Fourth ed.). McGraw-Hill Education (UK).

Block, R. M. (2012). A discussion of the effect of open-book and closed-book exams on student achievement in an introductory statistics course. *Primus, 22*(3), 228-238. https://doi.org/10.1080/10511970.2011.565402

Bloom, B. S. (1956). Taxonomy of educational objectives. Vol. 1: Cognitive domain. *New York: McKay*, 20-24.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29*(1), 41-62.

Boaler, J. (2019). Developing mathematical mindsets: The need to interact with numbers flexibly and conceptually. *American Educator, 42*(4), 28. https://files.eric.ed.gov/fulltext/EJ1200568.pdf

Bobis, J., Khosronejad, M., Way, J., & Anderson, J. (2019). “Sage on the stage” or “Meddler in the middle”: Shifting mathematics teachers’ identities to support student engagement. *Journal of Mathematics Teacher Education*. https://doi.org/10.1007/s10857-019-09444-1

Boggiano, A. K., Flink, C., Shields, A., Seelbach, A., & Barrett, M. (1993). Use of techniques promoting students' self-determination: Effects on students' analytic problem-solving skills. *Motivation and Emotion, 17*(4), 319-336.

Boulton‐Lewis, G. M. (1995). The SOLO taxonomy as a means of shaping and assessing learning in higher education. *Higher Education Research & Development, 14*(2), 143-154. https://doi.org/10.1080/0729436950140201

Bowman, D. (2020). *The bushfire royal commission has made a clarion call for change. Now we need politics to follow*. https://theconversation.com/the-bushfire-royal-commission-has-made-a-clarion-call-for-change-now-we-need-politics-to-follow-149158

Braham, H. M., & Ben-Zvi, D. (2019). Design for reasoning with uncertainty. In G. Burrill & D. Ben-Zvi (Eds.), *Topics and trends in current statistics education research: International perspectives* (pp. 97-121). Springer International Publishing. https://doi.org/10.1007/978-3-030-03472-6_5

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-101. https://doi.org/10.1191/1478088706qp063oa

Braun, V., & Clarke, V. (2019). Reflecting on reflexive thematic analysis. *Qualitative Research in Sport, Exercise and Health, 11*(4), 589-597. https://doi.org/10.1080/2159676X.2019.1628806

Braun, V., & Clarke, V. (2020). One size fits all? What counts as quality practice in (reflexive) thematic analysis? *Qualitative Research in Psychology*, 1-25. https://doi.org/10.1080/14780887.2020.1769238

Bray, A., & Tangney, B. (2016). Enhancing student engagement through the affordances of mobile technology: a 21st century learning perspective on realistic mathematics education. *Mathematics Education Research Journal, 28*(1), 173-197. https://doi.org/10.1007/s13394-015-0158-7

Brehmer, D., Ryve, A., & Van Steenbrugge, H. (2016). Problem solving in Swedish mathematics textbooks for upper secondary school. *Scandinavian Journal of Educational Research, 60*(6), 577-593. https://doi.org/10.1080/00313831.2015.1066427

Brodie, K. (2020). Professional learning communities in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics education* (pp. 693-696). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_130

Brown, J. P. (2015). Visualisation tactics for solving real world tasks. In G. A. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice* (pp. 431-442). Springer. https://doi.org/10.1007/978-3-319-18272-8

Brown, M., Brown, P., & Bibby, T. (2008). “I would rather die”: Reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education, 10*(1), 3-18. https://doi.org/10.1080/14794800801915814

Bryman, A. (2012). *Social research methods* (4th ed.). Oxford University Press. (Oxford)

Burns, K. C. (2014). Security blanket or crutch? Crib card usage depends on students’ abilities. *Teaching of Psychology, 41*(1), 66-68. https://doi.org/10.1177/0098628313514181

Burton, L. (2002). Methodology and methods in mathematics education research: Where is "The why"? In S. Goodchild & L. English (Eds.), *Researching mathematics classrooms: A critical examination of methodology* (pp. 1-10). Greenwood Publishing Group.

Cain, A., Grundy, J., & Woodward, C. J. (2018). Focusing on learning through constructive alignment with task-oriented portfolio assessment. *European Journal of Engineering Education, 43*(4), 569-584. https://doi.org/10.1080/03043797.2017.1299693

Callingham, R., & Watson, J. (2011). Measuring levels of statistical pedagogical content knowledge. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics-Challenges for teaching and teacher education* (pp. 283-293). Springer. https://doi.org/10.1007/978-94-007-1131-0

Callingham, R., Watson, J., & Oates, G. (2021). Learning progressions and the Australian curriculum mathematics: The case of statistics and probability. *Australian Journal of Education, 0*(0), 1-14. https://doi.org/10.1177/00049441211036521

Calvert, L. (2016). The power of teacher agency. *The Learning Professional, 37*(2), 51-56.

Campbell, K. (2006). *The SOLO Taxonomy and probability: Development and validation of a set of superitems*. [Doctorial dissertation, State University of New York at Albany]. https://www.proquest.com/docview/304946944?pq-origsite=gscholar&fromopenview=true

Cargile, L. A., & Harkness, S. S. (2015). Flip or flop: Are math teachers using Khan Academy as envisioned by Sal Khan? *TechTrends, 59*(6), 21-28.

Chan, M., O'Connor, T., & Peat, O. C. (2016). *Using Khan Academy in community college developmental math courses: Results and lessons learned from developmental math demonstration project*. www.nebhe.org

Chang, C. C., & Silalahi, S. M. (2017). A review and content analysis of mathematics textbooks in educational research. In *Problems of education in the 21st Century* (Vol. 75, pp. 235-251).

Chernoff, E. (2019). The Canadian math wars. *Canadian Journal of Science, Mathematics Technology Education, 19*(1), 73-76. https://doi.org/10.1007/s42330-018-0037-9

Chernoff, E. J., & Sriraman, B. (2014). Commentary on probabilistic thinking: Presenting plural perspectives. In *Probabilistic thinking* (pp. 721-727). Springer.

Chick, H. (2010). A Lakatosian encounter with probability. *Australian Mathematics Teacher, 66*(4), 32-40.

Chong, S. Y., Herng Han, S., Azura Abdullah, N., Siew Fang Chong, M., Widjaja, W., & Shahrill, M. (2017). Utilizing lesson study in improving Year 12 students' learning and performance in mathematics. *Mathematics Education Trends and Research, 2017*(1), 24-31. https://doi.org/10.5899/2017/metr-00095

Clark, K. R. (2015). The effects of the flipped model of instruction on student engagement and performance in the secondary mathematics classroom. *Journal of Educators Online, 12*(1), 91-115.

Clarke, D., Stephens, M., & Wallbridge, M. (1993). The instructional impact of changes in assessment. *Contexts in mathematics education*, 177-182.

Clarke, V., & Braun, V. (2013). Teaching thematic analysis: Overcoming challenges and developing strategies for effective learning. *The Psychologist, 26*(2), 120-123.

Clarkson, P. (1993). Gender, Ethnicity and Textbooks. *Australian Mathematics Teacher, 49*(2), 14-16.

Clements, D. (2007). Curriculum research: Toward a framework for "research-based curricula". *Journal for Research in Mathematics Education*, 35-70. https://www.jstor.org/stable/30034927

Collins, R. (2014). Skills for the 21st century: Teaching higher-order thinking. *Curriculum & Leadership Journal, 12*(14). http://www.curriculum.edu.au/leader/teaching_higher_order_thinking,37431.html?issueI

Conant, D. L. (2013). *The impact of an instructional intervention designed to support development of stochastic understanding of probability distribution.* [Doctoral dissertation, University of Maryland]. https://www.proquest.com/docview/1461769809?pq-origsite=gscholar&fromopenview=true

Connor, S. (2012). *Make maths compulsory for all students until the age of 18, leading scientists urge*. Retrieved 14/12/2017 from http://www.independent.co.uk/news/education/education-news/make-maths-compulsory-for-all-students-until-the-age-of-18-leading-scientists-urge-7966272.html

Coupland, M. (2004). *Learning with new tools* [Doctoral dissertation, The University of Wollongong]. Wollongong. https://ro.uow.edu.au/cgi/viewcontent.cgi?referer=https://scholar.google.com.au/&httpsredir=1&article=1198&context=theses

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative* (4th ed.). Pearson.

Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage publications.

Creswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory into practice, 39*(3), 124-130. https://doi.org/10.1207/s15430421tip3903_2

Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. Sage.

Daniels, H. (2016). *Vygotsky and Pedagogy: Classic Edition*. Routledge.

Darlington, E. (2013). *The use of Bloom's taxonomy in advanced mathematics questions* Proceedings of the British Society for Research into Learning Mathematics Bristol, UK.

Darlington, E. (2015). What benefits could extension papers and admissions tests have for university mathematics applicants? *Teaching Mathematics and Its Applications: International Journal of the IMA, 34*(4), 179-193. https://doi.org/10.1093/teamat/hrv003

David, M. M., & Lopes, M. (2002). Students-teacher interactions and the development of students' mathematical thinking. In S. G. L. English (Ed.), *Researching mathematics classrooms: A critical examination of methodology* (pp. 11-38). Praeger.

Davis, P. (2007). How cultural models about reading mediate classroom (pedagogic) practice. *International Journal of Educational Research, 46*(1-2), 31-42.

De Bortoli, L., & Macaskill, G. (2014). *Thinking it through: Australian students' skills in creative problem solving*. ACER.

de Moirve, A. (1756). *The doctrine of chances: A method of calculating the probabilities of events in play* (The third edition ed.). Chelsea Publishing Company. https://archive.org/details/doctrineofchance00moiv/page/n6

Dede, C. (2010). Comparing frameworks for 21st century skills. In J. Bellanca (Ed.), *21st century skills: Rethinking how students learn* (Vol. 20, pp. 51-76). Solution Tree Press.

Delmas, R., Garfield, J., Ooms, A., & Chance, B. (2007). Assessing students' conceptual understanding after a first course in statistics. *Statistics Education Research Journal, 6*(2), 28-58.

DelMas, R. C., Garfield, J., & Chance, B. (1999). A model of classroom research in action: Developing simulation activities to improve students' statistical reasoning. *Journal of Statistics Education, 7*(3).

DET. (2020a). *High impact teaching strategies*. V. S. Government. https://www.education.vic.gov.au/Documents/school/teachers/support/high-impact-teaching-strategies.pdf

DET. (2020b). *The pedagogical model*. https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/practice/pedagogicalmodel.pdf

Drijvers, P., Kodde-Buitenhuis, H., & Doorman, M. (2019). Assessing mathematical thinking as part of curriculum reform in the Netherlands. *Educational Studies in Mathematics, 102*(3), 435-456. https://doi.org/10.1007/s10649-019-09905-7

Duron, R., Limbach, B., & Waugh, W. (2006). Critical thinking framework for any discipline. *International Journal of Teaching and Learning in Higher Education, 17*(2), 160-166.

Easton, G. (2010). Critical realism in case study research. *Industrial marketing management, 39*(1), 118-128. https://doi.org/10.1016/j.indmarman.2008.06.00 4

Echazarra, A., & Schwabe, M. (2019). *Programme for International Student Assessment (PISA) Results from PISA 2018: Australia*. https://www.oecd.org/pisa/publications/PISA2018_CN_AUS.pdf

Edwards, G. (2012). Snippet: The Monty hall problem. *Vinculum, 49*(2), 20.

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work, 14*(1), 133-156. https://doi.org/10.1080/13639080020028747

Engeström, Y. (2015). *Learning by expanding* (Second edition ed.). Cambridge University Press.

Ennis, R. H. (1985). A logical basis for measuring critical thinking skills. *Educational Leadership, 43*(2), 44-48.

Erbe, B. (2007). Reducing test anxiety while increasing learning: The cheat sheet. *College Teaching, 55*(3), 96-98. https://doi.org/10.3200/CTCH.55.3.96-98

Ernest, P. (2015). The social outcomes of learning mathematics: Standard, unintended or visionary? *International Journal of Education in Mathematics, Science and Technology, 3*(3), 187-192.

Ernest, P. (2018a). The ethics of mathematics: Is mathematics harmful? In *The philosophy of mathematics education today* (pp. 187-216). Springer. https://doi.org/10.1007/978-3-319-77760-3

Ernest, P. (2018b). The philosophy of mathematics education: An overview. In *The Philosophy of Mathematics Education Today* (pp. 13-35). Springer. https://doi.org/https://doi.org/10.1007/978-3-319-77760-3

Ernst, H. (2014). Formative assessment in senior mathematics. *Vinculum, 51*(2), 6-7.

Ernst, H. (2018). *Senior secondary probability curriculum: What has changed?* Making waves, opening spaces (Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia), Auckland.

Ernst, H., & Morton, A. (2020). Probability across the curriculum. *Australian Mathematics Education Journal, 2*(2), 4.

Ernst, M. D. (2011). Active learning? not with my syllabus! *Teaching Statistics, 34*(1), 21-24.

Even, R., & Kvatinsky, T. (2010). What mathematics do teachers with contrasting teaching approaches address in probability lessons? *Educational studies in mathematics, 74*(3), 207-222. https://doi.org/10.1007/s10649-010-9234-9

Fan, L., Trouche, L., Qi, C., Rezat, S., & Visnovska, J. (2018). *Research on mathematics textbooks and teachers' resources: Advances and issues*. Springer. https://doi.org/10.1007/978-3-319-73253-4

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? *Educational Studies in Mathematics, 15*(1), 1-24.

Fischbein, E., & Schnarch, D. (1997). The evolution with age of probabilistic, intuitively based misconceptions. *Journal for Research in Mathematics Education*, 96-105. http://www.jstor.org/stable/749665

Fitzallen, N. E. (2012). *Reasoning about covariation with TinkerPlots* [Doctorial dissertation, University of Tasmania]. https://eprints.utas.edu.au/14717/

Fitzpatrick, J. B. (1974). *New general mathematics* (Third ed.). The Jacaranda Press.

Fitzsimons, G. E. (2005). Technology mediated post-compulsory mathematics: An activity theory approach. *International Journal of Mathematical Education in Science and Technology, 36*(7), 769-777. https://doi.org/10.1080/00207390500271123

Flyvbjerg, B. (2006). Five misunderstandings about case-study research. *Qualitative Inquiry, 12*(2), 219-245. https://doi.org/10.1177/1077800405284363

Forgasz, H., & Ledfer, G. (2016). Numeracy and Australian teachers. Opening up mathematics education research (Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia), Adelaide MERGA.

Forgasz, H., & Tan, H. (2010). Does CAS use disadvantage girls in VCE mathematics? *Australian Senior Mathematics Journal, 24*(1), 25-36.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report*. American Statistical Association.

Fray, L., Gore, J., Harris, J., & North, B. (2019). Key influences on aspirations for higher education of Australian school students in regional and remote locations: a scoping review of empirical research, 1991–2016. *The Australian Educational Researcher*. https://doi.org/10.1007/s13384-019-00332-4

Frejd, P. (2013). Modes of modelling assessment—A literature review. *Educational Studies in Mathematics, 84*(3), 413-438. https://doi.org/10.1007/s10649-013-9491-5

Fuaad, N. F. A., Nopiah, Z. M., Tawil, N. M., Othman, H., Asshaari, I., Osman, M. H., & Ismail, N. A. (2014). Blooms' separation of the final exam of engineering mathematics ii: Item reliability using rasch measurement model. Proceedings of the 3rd International Conference on Mathematical Sciences, In *AIP Conference Proceedings* (Vol. 1602, No. 1, pp. 508-513). American Institute of Physics. https://aip.scitation.org/doi/abs/10.1063/1.4882533

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM, 38*(2), 143-162. https://link.springer.com/content/pdf/10.1007/BF02655886.pdf

Garfield, J. (2002). The challenge of developing statistical reasoning. *Journal of Statistics Education, 10*(3), 58-69. https://doi.org/10.1080/10691898.2002.11910676
Garfield, J. (2003). Assessing statistical reasoning. *Statistics Education Research Journal, 2*(1), 22-38.
Garfield, J., & Chance, B. (2000). Assessment in statistics education: Issues and challenges. *Mathematical Thinking and Learning, 2*(1-2), 99-125.
Garfield, J., delMas, R., & Zieffler, A. (2010). Assessing important learning outcomes in introductory tertiary statistics courses. In P. Bidgood, N. Hunt, & F. Jolliffe (Eds.), *Assessment methods in statistical education: An international perspective* (pp. 75-86). Wiley & Sons Ltd.
Gharib, A., Phillips, W., & Mathew, N. (2012). Cheat sheet or open-book? A comparison of the effects of exam types on performance, retention, and anxiety. *Psychology Research,, 2*(8), 469-478. https://files.eric.ed.gov/fulltext/ED537423.pdf
Gobby, B. (2017). What is curriculum? In B. Gobby & R. Walker (Eds.), *Powers of curriculum: Sociological perspectives on education* (pp. 5-34). Oxford University.
Goldberg, A. E., & Allen, K. R. (2015). Communicating qualitative research: Some practical guideposts for scholars. *Journal of Marriage and Family, 77*(1), 3-22.
Goodchild, S., & Jaworski, B. (2005). *Using contradictions in a teaching and learning development project* Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Melbourne.
Goos, M. (2012). Digital technologies in the Australian curriculum: Mathematics–a lost opportunity. In B. Atweh, M. Goos, R. Jorgensen, & D. Siemon (Eds.), *Engaging the Australian National Curriculum: Mathematics – Perspectives from the field* (pp. 135-152). MERGA.
Goos, M., & Bennison, A. (2008). Surveying the technology landscape: Teachers' use of technology in secondary mathematics classrooms. *Mathematics Education Research Journal, 20*(3), 102-130.
Goos, M., Dole, S., & Geiger, V. (2011, 2011/06/24). Improving numeracy education in rural schools: a professional development approach. *Mathematics Education Research Journal, 23*(2), 129. https://doi.org/10.1007/s13394-011-0008-1
Goos, M., Stillman, G., & Vale, C. (2007). *Teaching secondary school mathematics: Research and practice for the 21st century*. Allen & Unwin.
Gough, J. (2012). How to review a secondary mathematics textbook. *Vinculum, 49*(3), 12.
GPE. (2020). *21st century skills: What potential role for the global partnership for education?* (Ramya Vivekanandan, Ed.). Global Parnership for Education. https://www.globalpartnership.org/content/21st-century-skills-what-potential-role-global-partnership-education
Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education, 15*(1), 105-123. https://doi.org/10.1007/s10763-017-9814-6
Gray, D. E. (2013). *Doing research in the real world* (Third ed.). Sage.
Green, D. R. (1982). *Probability concepts in school pupils aged 11–16 years* [Doctoral dissertation, Loughborough University]. https://repository.lboro.ac.uk/articles/thesis/Probability_concepts_in_school_pupils_aged_11_16_years/9375947/1
Groth, R. E., & Bergner, J. A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning, 8*(1), 37-63.
Guan, T. E. (2011). Think of a number: Adapting the birthday problem for the classroom. *Australian Mathematics Teacher, 67*(4), 22-25.
Guba, E. G. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Ectj, 29*(2), 75.
Gürbüz, R. (2010). The effect of activity-based instruction on conceptual development of seventh grade students in probability. *International Journal of Mathematical Education in Science and Technology, 41*(6), 743-767.

Gürbüz, R., Yüksel, D., & Doğan, M. F. (2018). The role of computer-assisted instruction in the teaching of probability. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 33*(3), 705-722. https://doi.org/10.16986/HUJE.2018036943

Hadar, L. L. (2017). Opportunities to learn: Mathematics textbooks and students' achievements. *Studies in Educational Evaluation, 55*, 153-166. https://doi.org/10.1016/j.stueduc.2017.10.002

Hadar, L. L., & Ruby, T. L. (2019). Cognitive opportunities in textbooks: the cases of grade four and eight textbooks in Israel. *Mathematical Thinking and Learning, 21*(1), 54-77. https://doi.org/10.1080/10986065.2019.1564968

Halsey, J. (2018). *Independent review into regional, rural and remote education*. Literature Review. Commonwealth of Australia. https://www.nintione.com.au/resources/rao/independent-review-into-regional-rural-and-remote-education-literature-review

Handal, B., Watson, K., Petocz, P., & Maher, M. (2013). Retaining mathematics and science teachers in rural and remote schools. *Australian International Journal of Rural Education, 23*(3), 13.

Hasan, H., & Kazlauskas, A. (2014). Activity Theory: who is doing what, why and how. In H. Hasan (Ed.), *Being practical with theory: A window into business research*. THEORI Research Group. http://eurekaconnection.files.wordpress.com/2014/02/p-09-14-activity-theory-theori-ebook-2014.pdf

Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. Routledge.

Hattie, J., Biggs, J., & Purdie, N. (1996). Effects of learning skills interventions on student learning: A meta-analysis. *Review ofReducational Research, 66*(2), 99-136.

Hattie, J., Fisher, D., Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2017). *Visible learning for mathematics, grades K-12: What works best to optimize student learning*. Corwin Press.

Hiebert, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*. ACER. http://research.acer.edu.au/timss_video/5

Hogan, J. (2012). Mathematics and numeracy: Has anything changed?: Are we any clearer?: Are we on track? *Australian Mathematics Teacher, 68*(4), 4-7.

Hollingsworth, H., McCrae, B., & Lokan, J. (2003). *Teaching mathematics in Australia: Results from the TIMSS 1999 video study*. Australian Council for Educational Research.

Holmes, V.-L. (2012). Depth of teachers' knowledge: Frameworks for teachers' knowledge of mathematics. *Journal of STEM Education: Innovations and Research, 13*(1), 55-70.

Hook, P., Gravett, C., Howard, M., & John, E. (2014). *SOLO taxonomy in mathematics: Strategies for thinking like a mathematician*. Essential Resources.

Hudson, S., & Hudson, P. (2019). "Please help me find teachers for my rural and remote school". *Rural Education, 29*(3), 1-15.

Hunter, J., Miller, J., Choy, B. H., & Hunter, R. (2020). Innovative and Powerful Pedagogical Practices in Mathematics Education. In *Research in Mathematics Education in Australasia 2016–2019* (pp. 293-318). Springer. https://doi.org/10.1007/978-981-15-4269-5

Husbands, C. T. (1976). Ideological bias in the marking of examinations: a method of testing for its presence and its implications. *Research in Education, 15*(1), 17-38. https://doi.org/10.1177/003452377601500102

Iannone, P., & Jones, I. (2017). Special issue on summative assessment. *Research in Mathematics Education 19*. https://doi.org/10.1080/14794802.2017.1334578

Iraola-Real, I., & Gonzales-Macavilca, M. (2020). Anxiety and poor Performance Facing Exams and Studying Mathematics: Predictive Research on Applicants to the Universidad Nacional de Ingenieria of Lima IEEE World Conference on Engineering Education (EDUNINE) Peru.

Ireland, S., & Watson, J. M. (2009). Building a connection between experimental and theorectical aspects of probability. *International Electronic Journal of Mathematics Education, 4*(3), 339-370. https://www.iejme.com/download/building-a-connection-between-experimental-and-theoretical-aspects-of-probability.pdf

Jacobs, B., Wright, S., & Reynolds, N. (2017). Reevaluating the concrete–explanatory animation creation as a digital catalyst for cross-modal cognition. *Mind, Culture, and Activity, 24*(4), 297-310. https://doi.org/10.1080/10749039.2017.1294181

Jaiswal, P. (2019). Using constructive alignment to foster teaching learning processes. *English Language Teaching, 12*(6), 10-23. https://doi.org/10.5539/elt.v12n6p10

Jang, H., Reeve, J., & Halusic, M. (2016). A new autonomy-supportive way of teaching that increases conceptual learning: Teaching in students' preferred ways. *The Journal of Experimental Education, 84*(4), 686-701. https://doi.org/10.1080/00220973.2015.1083522

Johnson, R. B., & Christensen, L. (2014). *Educational research: Quantitative, qualitative, and mixed approaches* (Fifth ed.). Sage.

Jolliffe, F. (2005). Assessing probabilistic thinking and reasoning. In D. G. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 325-344). Springer. https://www.semanticscholar.org/paper/Exploring-Probability-in-School%3A-Challenges-for-and-Jones/a8418cc0b1794e067f0686a385ee3c35882d4874

Jonassen, D. H., & Rohrer-Murphy, L. (1999). Activity theory as a framework for designing constructivist learning environments. *Educational Technology Research and Development, 47*(1), 61-79.

Jones, D., & Tarr, J. (2007). An examination of the levels of cogitive demand required by probability tasks in middle grades mathematics textbooks. *Statistics Education Research Journal, 6*(2).

Jones, G. A. (2006). *Exploring probability in school: Challenges for teaching and learning* (Vol. 40). Springer. https://www.semanticscholar.org/paper/Exploring-Probability-in-School%3A-Challenges-for-and-Jones/a8418cc0b1794e067f0686a385ee3c35882d4874

Jones, G. A., Langrall, C. W., & Mooney, E. S. (2007). Research in probability. *Second handbook of research on mathematics teaching and learning*, 909-956.

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational studies in mathematics, 32*(2), 101-125.

Jorgensen, R., & Larkin, K. (2017). Analysing the relationships between students and mathematics: a tale of two paradigms. *Mathematics Education Research Journal, 29*(1), 113-130. https://doi.org/10.1007/s13394-016-0183-1

Jun, L. (2000). *Chinese students' understanding of probability* [Unpublished doctorial dissertation, Nanyang Technological University]. Singapore. https://iase-web.org/documents/dissertations/00.Li.Dissertation.pdf

Jürges, H., Schneider, K., Senkbeil, M., & Carstensen, C. H. (2012). Assessment drives learning: The effect of central exit exams on curricular knowledge and mathematical literacy. *Economics of Education Review, 31*(1), 56-65. https://doi.org/10.1016/j.econedurev.2011.08.007

Kahneman, D. (2012). *Thinking, fast and slow*. Penguin.

Kaptelinin, V., & Nardi, B. A. (2006). *Acting with technology: Activity theory and interaction design*. MIT press.

Kelly, D. (2018). *What do we know about Khan Academy? A review of the literature and justification for further study*. https://doi.org/10.13140/RG.2.1.2462.5044

Khan, S. (2019). *Khan Academy*. https://www.khanacademy.org/

Klymchuk, S., & Kachapova, F. (2012). Paradoxes and counterexamples in teaching and learning of probability at university. *International Journal of Mathematical Education in Science & Technology, 43*(6), 803-811. https://doi.org/10.1080/0020739X.2011.633631

Koenig, G. (2009). Realistic evaluation and case studies: stretching the potential. *Evaluation, 15*(1), 9-30. https://doi.org/10.1177/1356389008097869

Konold, C., Madden, S., Pollatsek, A., Pfannkuch, M., Wild, C., Ziedins, I., Finzer, W., Horton, N. J., & Kazak, S. (2011). Conceptual challenges in coordinating theoretical and data-centered estimates of probability. *Mathematical Thinking and Learning, 13*(1-2), 68-86. https://doi.org/10.1080/10986065.2011.538299

Konold, C., & Miller, C. (2011). *TinkerPlots: Dynamic data exploration.* In (Version 2.0) [computer software]. Key Curriculum Press. http://www.srri.umass.edu/tinkerplots/

Koparan, T. (2021). The impact of a game and simulation-based probability learning environment on the achievement and attitudes of prospective teachers. *International Journal of Mathematical Education in Science and Technology*, 1-19. https://doi.org/10.1080/0020739X.2020.1868592

Krathwohl, D., & Anderson, W. (2002). A revision of Bloom's taxonomy: An overview theory into practice. *The Ohio State University*.

Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. *Theory into Practice, 41*(4), 212-218. https://doi.org/10.1207/s15430421tip4104_2

Kuntze, S., Aizikovitsh-Udi, E., & Clarke, D. (2017, November 01). Hybrid task design: Connecting learning opportunities related to critical thinking and statistical thinking. *ZDM, 49*(6), 923-935. https://doi.org/10.1007/s11858-017-0874-4

Kyei, L., Apam, B., & Nokoe, K. S. (2011). Some gender differences in performance in senior high mathematics examinations in mixed high schools. *American Journal of Social and Management Sciences, 2*(4), 348-355. https://doi.org/10.5251/ajsms.2011.2.4.348.355

Larkin, H., & Richardson, B. (2013). Creating high challenge/high support academic environments through constructive alignment: student outcomes. *Teaching in Higher Education, 18*(2), 192-204. https://doi.org/10.1080/13562517.2012.696541

Larkin, K. (2011). You use! I use! We use! Questioning the orthodoxy of one-to-one computing in primary schools. *Journal of Research on Technology in Education, 44*(2), 101-120. https://doi.org/10.1080/15391523.2011.10782581

Larwin, K. H., Gorman, J., & Larwin, D. A. (2013). Assessing the impact of testing aids on post-secondary student performance: A meta-analytic investigation. *Educational Psychology Review, 25*(3), 429-443. https://doi.org/10.1007/s10648-013-9227-1

Leigh-Lancaster, D. (2010). *The case of technology in senior secondary mathematics: Curriculum and assessment congruence?* ACER 2010 - Teaching Mathematics? Make it count., Melbourne, Australia. https://research.acer.edu.au/cgi/viewcontent.cgi?article=1074&context=research_conference

Lemos, M., Pereira-Querol, M. A., & Almeida, I. M. d. (2013). The historical-cultural Activity Theory and its contributions to education, health and communication: Interview with Yrjö Engeström. *Interface-Comunicação, Saúde, Educação, 17*(46), 715-727. https://www.scielo.br/pdf/icse/v17n46/18.pdf

Leont'ev, A. N. (1981). The problem of activity in psychology (J. Wertsch, Trans.). In J. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp. 37-71). Sharpe.

Lepik, M., Grevholm, B., & Viholainen, A. (2015). Using textbooks in the mathematics classroom–the teachers' view. *Nordic Studies in Mathematics Education, 20*(3-4), 129-156. http://ncm.gu.se/wp-content/uploads/2020/06/20_34_129156_lepik.pdf

Li, Y., & Lappan, G. (2014). *Mathematics curriculum in school education*. Springer. https://doi.org/10.1007/978-94-007-7560-2

Lidster, S., Watson, J., Collis, K., & Pereira-Mendoza, L. (1996). *The relationship of the concept of fair to the construction of probabilistic understanding,* Technology in Mathematics Education, Melbourne.

Lincoln, Y. S., & Guba, E. G. (1986). But is it rigorous? Trustworthiness and authenticity in naturalistic evaluation. *New directions for program evaluation, 30*, 73-84.

Liu, W. C., Wang, C. K. J., Reeve, J., Kee, Y. H., & Chian, L. K. (2019). What determines teachers' use of motivational strategies in the classrooms? A self-determination theory perspective. *Journal of Education*, 1-11. https://doi.org/10.1177/0022057419881171

Lloyd, M., & Cronin, R. (2002). *A community of teachers: Using Activity Theory to investigate the implementation of ICTE in a remote indigenous school*. Institute of Education Sciences. https://files.eric.ed.gov/fulltext/ED473469.pdf

Lokan, J., & Greenwood, L. (2001). *Maths & science on the line: Australian year 12 students' preformance in the third international mathematics and science study*. ACER.

Longhofer, J., Floersch, J., & Hartmann, E. (2017). A case for the case study: How and why they matter. *Clinical Social Work Journal, 45*, 189-200. https://doi.org/10.1007/s10615-017-0631-8

López-Martín, M. M., Batanero, C., Contreras, J. M., & Ortiz, J. J. (2016). *Characterizing the probability problems proposed in the enterence to university tests in Andalucia.* 13th International Congress on Mathematical Education, Hamburg.

Lowe, I. (2009). What is a textbook? *Vinculum, 46*(1), 17.

Lowe, I., & Money, R. (2017). Gambling and mathematics education. *Vinculum, 54*(1), 5-9.

Lucas, B. (2019). *The impact of critical and creative thinking on achievement in Literacy and Numeracy*. Victorian State Governement.

Lyons, T., Cooksey, R., Panizzon, D., Parnell, A., & Pegg, J. (2006). *Science, ICT and mathematics education in rural and regional Australia: The SiMERR national survey*. https://simerr.une.edu.au/national-survey/

Madhuri, G. V., Kantamreddi, V., & Prakash Goteti, L. N. S. (2012). Promoting higher order thinking skills using inquiry-based learning. *European Journal of Engineering Education, 37*(2), 117-123. https://doi.org/10.1080/03043797.2012.661701

Maher, N. (2019). *Perspectives on pedagogical content knowledge in the senior secondary mathematics classroom* [Doctorial dissortation, University of Tasmania]. http://dx.doi.org/10.25959/100.00031741

Maher, N., Chick, H., & Muir, T. (2016). "I believe the most helpful thing was him skipping over the proof": Examining PCK in a senior secondary mathematics lesson. 39th annual conference of the Mathematics Education Research Group of Australasia, Adelaide. https://files.eric.ed.gov/fulltext/ED572316.pdf

Marsh, C., & Stafford, K. (1984). *Curriculum: Australian practices and issues*. McGraw-Hill.

Martin, D., & Peim, N. (2009). Critical perspectives on activity theory*, Educational Revie, 61(2),* 131-138. https://doi.org/10.1080/00131910902844689

Masters, G. (2019). *Focus on basics leaves schoolkids short in essential deep thinking*. ACER. https://rd.acer.org/article/focus-on-basics-leaves-schoolkids-short-in-essential-deep-thinking

Masters, G. N. (2020). *The role of the curriculum in creating the future*. ACER. https://www.teachermagazine.com.au/columnists/geoff-masters/the-role-of-the-curriculum-in-creating-the-future?utm_source=CM&utm_medium=Bulletin&utm_content=30June

McCoy, L. P., Buckner, S., & Munley, J. (2007). Probability games from diverse cultures. *Mathematics Teaching in the Middle School, 12*(7), 394-402.

McCulloch, A. W., Hollebrands, K., Lee, H., Harrison, T., & Mutlu, A. (2018). Factors that influence secondary mathematics teachers' integration of technology in mathematics lessons. *Computers & Education, 123*, 26-40. https://doi.org/10.1016/j.compedu.2018.04.008

Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Education. Revised and Expanded from "Case Study Research in Education"*. ERIC.

Merriam, S. B. (2009). *Qualitative research : A guide to design and implementation*. Wiley.

Merriam, S. B., & Grenier, R. S. (2019). *Qualitative research in practice: Examples for discussion and analysis*. John Wiley & Sons.

Merriam, S. B., & Tisdell, E. J. (2015). *Qualitative research: A guide to design and implementation*. John Wiley & Sons.

Meyers, N. M., & Nulty, D. D. (2009). How to use (five) curriculum design principles to align authentic learning environments, assessment, students' approaches to thinking and learning outcomes. *Assessment & Evaluation in Higher Education, 34*(5), 565-577. https://doi.org/10.1080/02602930802226502

Mikk, J. (2000). *Textbook: Research and Writing. Baltische Studien zur Erziehungs und Sozialwissenschaft, Band 3 (Baltic Studies for Education and Social Sciences, Volume 3)*. ERIC.

Mills, P. (2016). Minecraft and mathematics. *Vinculum, 53*(1), 15.

Mitchell Institite. (2015). *Senior school years: school completion uneven across Australia*. http://www.mitchellinstitute.org.au/fact-sheets/senior-school-years-school-completion-uneven-across-australia/

Money, R. (2015). The biggest loser. *Vinculum, 52*(1), 5-7.

Mooney, E. S., Langrall, C. W., & Hertel, J. T. (2014). A practitional perspective on probabilistic thinking models and frameworks. In E. J. Chernoff & B. Sriraman (Eds.), *Probabilistic thinking* (pp. 495-507). Springer. https://doi.org/10.1007/978-94-007-7155-0_27

Moore, D. S., & McCabe, G. P. (2003). *Introduction to the practice of statistics.* W.H. Freemann and Company.

Moseley, D., Baumfield, V., Elliott, J., Higgins, S., Newton, D. P., Miller, J., & Gregson, M. (2005). *Frameworks for thinking: A handbook for teaching and learning*. Cambridge University Press.

Mousley, J. (2008). *The National "Numeracy" Review*. Mathematics in Australia. Retrieved 26 August 2008 from https://austmaths.wordpress.com/2008/08/26/the-national-%E2%80%9Cnumeracy%E2%80%9D-review/

Muir, T. (2018). *It's more than the videos: Examining the factors that impact upon students' uptake of the flipped classroom approach in a senior secondary mathematics classroom* Making waves, opening spaces. Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia, Auckland. https://eric.ed.gov/?id=ED592454

Muir, T., & Geiger, V. (2016). The affordances of using a flipped classroom approach in the teaching of mathematics: a case study of a grade 10 mathematics class. *Mathematics Education Research Journal, 28*(1), 149-171. https://doi.org/10.1007/s13394-015-0165-8

Mullis, I. V., & Martin, M. O. (2013). TIMSS 2015 assessment frameworks. *Retrieved from Boston College, TIMSS & PIRLS International Study Center website:*. http://timssandpirls.bc.edu/timss2015/frameworks.html

Mullis, I. V., & Martin, M. O. (2015). *TIMSS advanced 2015 assessment frameworks*. TIMSS & PIRLS International Study Center. https://timss.bc.edu/timss2015-advanced/frameworks.html#:~:text=The%20TIMSS%20Advanced%202015%20Assessment,2015%E2%80%94Advanced%20Mathematics%20and%20Physics

Mullis, I. V., Martin, M. O., Ruddock, G. J., O'Sullivan, C. Y., & Preuschoff, C. (2009). *TIMSS 2011 Assessment frameworks*. ERIC.

Murphy, P. E. (2017). Student approaches to learning, conceptions of mathematics, and successful outcomes in learning mathematics. In *Success in higher education* (pp. 75-93). Springer.

Murphy, S. (2019). School location and socioeconomic status and patterns of participation and achievement in senior secondary mathematics. *Mathematics Education Research Journal, 31*(3), 219-235. https://doi.org/10.1007/s13394-018-0251-9

Nardi, B. A. (1996a). *Context and consciousness: Activity theory and human-computer interaction*. MIT Press.

Nardi, B. A. (Ed.). (1996b). *Studying context: A comparison of activity theory, situated action models, and distributed cognition* (Vol. 69102). http://sonify.psych.gatech.edu/~ben/references/nardi_studying_context_a_comparison_of_activity_theory_situated_action_models_and_distributed_cognition.pdf

Näsström, G. (2009). Interpretation of standards with Bloom's revised taxonomy: A comparison of teachers and assessment experts. *International Journal of Research & Method in Education, 32*(1), 39-51. https://doi.org/10.1080/17437270902749262

Näsström, G., & Henriksson, W. (2008). Alignment of standards and assessment: A theoretical and empirical study of methods for alignment. *Electronic Journal of Research in Educational Psychology, 16*(6), 667-690. http://repositorio.ual.es/bitstream/handle/10835/565/Art_16_216_eng.pdf?sequence=1

Nicholas, K., & Fletcher, J. (2017). What is happening in the use of ICT mathematics to support young adolescent learners? A New Zealand experience. *Educational Review, 69*(4), 474-489. https://doi.org/10.1080/00131911.2016.1237476

Nisbet, S., & Williams, A. (2009). Improving students' attitudes to chance with games and activities. *Australian Mathematics Teacher, 65*(3), 25-37.

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods, 16*(1), 1-13. https://doi.org/10.1177/1609406917733847

O'Toole, J., & Beckett, D. (2013). *Educational research: Creative thinking and doing* (Second Edition ed.). Oxford.

O'Keeffe, L., & O'Donoghue, J. (2015). A role for language analysis in mathematics textbook analysis. *International Journal of Science and Mathematics Education, 13*(3), 605-630. https://doi.org/10.1007/s10763-013-9463-3

OECD. (2003). The PISA 2003 assessment framework. Mathematics, reading. In *Science and problem solving knowledge and skills.* OECD.

OECD. (2019). *PISA 2018 assessment and analytical framework*. https://doi.org/10.1787/b25efab8-en

Onwuegbuzie, A. J., & Leech, N. L. (2005). On becoming a pragmatic researcher: The importance of combining quantitative and qualitative research methodologies. *International journal of Social Research Methodology, 8*(5), 375-387.

Orlando, J., & Attard, C. (2016). Digital natives come of age: The reality of today's early career teachers using mobile devices to teach mathematics. *Mathematics Education Research Journal, 28*(1), 107-121. https://doi.org/10.1007/s13394-015-0159-6

Oviedo-Trespalacios, O., Nandavar, S., Newton, J. D. A., Demant, D., & Phillips, J. G. (2019). Problematic use of mobile phones in Australia… is it getting worse? *Frontiers in Psychiatry, 10*, 105. https://doi.org/10.3389/fpsyt.2019.00105

Özgün-Koca, S. A. (2010). Prospective teachers' views on the use of calculators with computer algebra system in algebra instruction. *Journal of Mathematics Teacher Education, 13*(1), 49-71. https://doi.org/10.1007/s10857-009-9126-z

Pape, S. J., & Prosser, S. K. (2018). Barriers to technology implementation in community college mathematics classrooms. *Journal of Computing in Higher Education, 30*(3), 620-636. https://doi.org/10.1007/s12528-018-9195-z

Pease, R., Vuke, M., June Maker, C., & Muammar, O. M. (2020). A practical guide for implementing the STEM assessment results in classrooms: Using strength-based reports and real engagement in active problem solving. *Journal of Advanced Academics*, 367-406. https://doi.org/10.1177/1932202X20911643

Pegg, J. (2014). Structure of the Observed Learning Outcome (SOLO) Model. In *Encyclopedia of Mathematics education* (pp. 570-572). Springer. https://doi.org/10.1007/978-94-007-4978-8

Pegg, J., & Tall, D. (2005). The fundamental cycle of concept construction underlying various theoretical frameworks. *ZDM, 37*(6), 468-475.

Peña-López, I. (2012). *PISA 2012 assessment and analytical framework. Mathematics, reading, science, problem solving and financial literacy*. OECD. https://doi.org/10.1787/19963777

Peters, S. A., & Stokes-Levine, A. (2019). Secondary teachers' learning: Measures of variation. In *Topics and trends in current statistics education research* (pp. 245-264). Springer.

Pfannkuch, M. (2011). The role of context in developing informal statistical inferential reasoning: A classroom study. *Mathematical Thinking and Learning, 13*(1-2), 27-46. https://doi.org/10.1080/10986065.2011.538302

Pfannkuch, M., Budgett, S., Fewster, R., Fitch, M., Pattenwise, S., Wild, C., & Ziedins, I. (2016). Probability modeling and thinking: What can we learn from practice? *Statistics Education Research Journal, 15*(2), 11-37. https://doi.org/10.52041/serj.v15i2.238

Pfannkuch, M., Seber, G. A., & Wild, C. J. (2002). Probability with less pain. *Teaching Statistics, 24*(1), 24-30.

Pfannkuch, M., & Ziedins, I. (2014). A modelling perspective on probability. In *Probabilistic Thinking* (pp. 101-116). Springer. https://doi.org/10.1007/978-94-007-7155-0

Philpott, C., & Oates, C. (2017). Teacher agency and professional learning communities; what can Learning Rounds in Scotland teach us? *Professional Development in Education, 43*(3), 318-333. https://doi.org/10.1080/19415257.2016.1180316

Piaget, J., & Inhelder, B. (1975). *The origin of the idea of chance in children. (Trans L. Leake, P. Burrell & HD Fishbein)*. WW Norton.

Pierce, R. (2002). *An exploration of algebraic insight and effective use of computer algebra systems* [Doctorial dissertation, University of Melbourne]. Melbourne https://minerva-access.unimelb.edu.au/bitstream/handle/11343/39022/66478_00001526_01_Robyn_Pierce.pdf?isAllowed=y&sequence=1

Pierce, R., & Bardini, C. (2015). Computer algebra systems: Permitted but are they used? *Australian Senior Mathematics Journal, 29*(1), 32-42.

Pierce, R., & Stacey, K. (2001). Reflections on the changing pedagogical use of computer algebra systems: assistance for doing or learning mathematics? *Journal of Computers in Mathematics and Science Teaching, 20*(2), 143-161.

Pierce, R., & Stacey, K. (2013). Teaching with new technology: four 'early majority'teachers. *Journal of Mathematics Teacher Education, 16*(5), 323-347. https://doi.org/10.1007/s10857-012-9227-y

Plunkett, M., & Dyson, M. (2011). Becoming a teacher and staying one: Examining the complex ecologies associated with educating and retaining new teachers in rural Australia? *Australian Journal of Teacher Education, 36*(1), 31-47. https://doi.org/10.14221/ajte.2011v36n1.3

Polster, B. M., Ross, M., & Treeby, D. (2012). Book review: Essential Mathematical Methods 3&4 CAS. *Vinculum, 49*, 8-12.

Porter, A. (2002). Measuring the content of instruction: Uses in research and practice. *Educational Researcher, 31*(7), 3-14.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new US intended curriculum. *Educational Researcher, 40*(3), 103-116. https://doi.org/10.3102/0013189X11405038

Porter, A., Schmidt, W. H., Floden, R. E., & Freeman, D. J. (1978). Practical significance in program evaluation. *American Educational Research Journal, 15*(4), 529-539.

Porter, A., Smithson, J., Blank, R., & Zeidner, T. (2007). Alignment as a teacher variable. *Applied Measurement in Education, 20*(1), 27-51. https://doi.org/10.1080/08957340709336729

Pratt, D., Griffiths, G., Jennings, D., & Schmoller, S. (2019). A MOOC for adult learners of mathematics and statistics: Tensions and compromises in design. In *Topics and trends in current statistics education research* (pp. 351-371). Springer.

Prensky, M. (2001). Digital natives, digital immigrants part 1. *On the horizon, 9*(5), 1-6.

Priestley, M., Biesta, G., Philippou, S., & Robinson, S. (2015). The teacher and the curriculum: Exploring teacher agency. In *The SAGE handbook of curriculum, pedagogy and assessment* (pp. 187-201).

Priestley, M., & Philippou, S. (2018). Curriculum making as social practice: complex webs of enactment. https://doi.org/10.1080/09585176.2018.1451096

Prodromou, T. (2016). Secondary school students' reasoning about conditional probability, samples, and sampling procedures. *Statistics Education Research Journal, 15*(2), 106-125. http://iase-web.org/documents/SERJ/SERJ15(2)_Prodromou.pdf

Punch, K. F., & Oancea, A. (2014). *Introduction to research methods in education*. Sage.

Rafiepour-Gatabi, A., Stacey, K., & Gooya, Z. (2012). Investigating grade nine textbook problems for characteristics related to mathematical literacy. *Mathematics Education Research Journal, 24*(4), 403-421. https://doi.org/10.1007/s13394-012-0052-5

Razak, N. A., Jalil, H. A., Krauss, S. E., & Ahmad, N. A. (2018). Successful implementation of information and communication technology integration in Malaysian public schools: An activity systems analysis approach. *Studies in Educational Evaluation, 58*, 17-29. https://doi.org/10.1016/j.stueduc.2018.05.003

Reaburn, R. (2011). *Students' understanding of statistical inference: Implications for teaching* [Doctorial dissertation, University of Tasmania]. https://eprints.utas.edu.au/12499/1/Whole.pdf

Reaburn, R. (2013). Students' understanding of conditional probability on entering university. Mathematics Education Research Group of Australasia, Melbourne. https://files.eric.ed.gov/fulltext/ED573012.pdf

Reeve, J. (2009). Why teachers adopt a controlling motivating style toward students and how they can become more autonomy supportive. *Educational Psychologist, 44*(3), 159-175. https://doi.org/10.1080/00461520903028990

Remillard, J., & Kim, O.-K. (2017). Knowledge of curriculum embedded mathematics: Exploring a critical domain of teaching. *Educational Studies in Mathematics*(96), 65-81. https://doi.org/10.1007/s10649-017-9757-4

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246. https://doi.org/10.3102/00346543075002211

Remillard, J. T., Harris, B., & Agodini, R. (2014). The influence of curriculum material design on opportunities for student learning. *ZDM, 46*(5), 735-749. https://doi.org/10.1007/s11858-014-0585-z

Remillard, J. T., & Heck, D. J. (2014). Conceptualizing the curriculum enactment process in mathematics education. *ZDM, 46*(5), 705-718. https://doi.org/10.1007/s11858-014-0600-4

Rezat, S. (2009). The utilization of mathematics textbooks as instruments for learning. Proceedings of CERME6, Lyon France. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1060.4578&rep=rep1&type=pdf

Rezat, S. (2013). The textbook-in-use: students' utilization schemes of mathematics textbooks related to self-regulated practicing. *ZDM, 45*(5), 659-670. https://doi.org/10.1007/s11858-013-0529-z

Romeo, G., & Walker, I. (2002). Activity theory to investigate the implementation of ICTE. In *Networking the learner* (pp. 389-400). Springer.

Ross, E. J. (2017). *An investigation of teachers' curriculum interpretation and implementation in a Queensland school* [Doctorial dissertation, Queensland University of Technology]. https://eprints.qut.edu.au/107049/1/Emily_Ross_Thesis.pdf

Roth, W.-M. (2004). Introduction: Activity theory and education: An introduction. *Mind, Culture, and Activity, 11*(1), 1-8. https://doi.org/10.1207/s15327884mca1101_4

Roth, W.-M. (2007). The ethico-moral nature of identity: Prolegomena to the development of third-generation cultural-historical activity theory. *International Journal of Educational Research, 46*(1-2), 83-93. https://doi.org/10.3102/0034654306298273

Roth, W.-M. (2014). Activity theory in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics education* (Vol. 1, pp. 11-15). Springer. https://doi.org/10.1007/978-94-007-4978-8

Roth, W.-M., & Lee, Y. J. (2004). Interpreting unfamiliar graphs: A generative, activity theoretic model. *Educational Studies in Mathematics, 57*(2), 265-290. https://doi.org/10.1023/B:EDUC.0000049276.37088.e4

Rotherham, A. J., & Willingham, D. T. (2010). "21st-century" skills. *American Educator, 17*(1), 17-20.

Saeed, M., & Naseem, A. (2014). Analysis of secondary grade mathematics question papers of boards of intermediate and secondary education in punjab. *Pakistan Journal of Education, 31*(1). https://doi.org/10.30971/pje.v31i1.78

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H., Wiley, D. E., Cogan, L. S., & Wolfe, R. G. (2001). *Why schools matter: A cross-national comparison of curriculum and learning.* The Jossey-Bass Education Series. https://eric.ed.gov/?id=ED460093

Schoenfeld, A. H. (2004). The math wars. *Educational policy, 18*(1), 253-286. https://doi.org/10.1177/0895904803260042

Schoenfeld, A. H. (2016). Research in mathematics education. *Review of Research in Education, 40*(1), 497-528. https://doi.org/10.3102/0091732x16658650

Schukajlow, S., Kaiser, G., & Stillman, G. (2018). Empirical research on teaching and learning of mathematical modelling: A survey on the current state-of-the-art. *ZDM, 50*(1-2), 5-18. https://doi.org/10.1007/s11858-018-0933-5

Schwarz, B. B. (2020). Psychological approaches in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics education* (pp. 505-511). Springer. https://doi.org/10.1007/978-94-007-4978-8_167

Seah, W. T., Andersson, A., Bishop, A., & Clarkson, P. (2016). What would the mathematics curriculum look like if values were the focus? *For the Learning of Mathematics, 36*(1), 14-20. https://www.jstor.org/stable/44382694

Seitz, P. (2017). Curriculum alignment among the intended, enacted, and assessed curricula for Grade 9 mathematics. *Journal of the Canadian Association for Curriculum Studies, 15*(1), 72-94. https://jcacs.journals.yorku.ca/index.php/jcacs/article/view/40286/36255

Settlage, D. M., & Wollscheid, J. R. (2019). An Analysis of the Effect of Student Prepared Notecards on Exam Performance. *College Teaching, 67*(1), 15-22. https://doi.org/10.1080/87567555.2018.1514485

Severance, S., Penuel, W. R., Sumner, T., & Leary, H. (2016). Organizing for teacher agency in curricular co-design. *Journal of the Learning Sciences, 25*(4), 531-564. https://doi.org/10.1080/10508406.2016.1207541

Sharma, S. (2015). Teaching probability: a socio - constructivist perspective. *Teaching Statistics, 37*(3), 78-84. https://onlinelibrary.wiley.com/doi/abs/10.1111/test.12075

Shaughnessy, J. M. (2003). Research on students' understandings of probability. *A research companion to principles and standards for school mathematics*, 216-226.

Sheehan, G. R., & Mosse, J. (2013). Working with science teachers to transform the opportunity landscape for regional and rural youth: A qualitative evaluation of the science in schools program. *Australian Journal of Teacher Education, 38*(1), n1. https://doi.org/10.14221/ajte.2013v38n1.3

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 4-14.

Simons, H. (2009). *Case study research in practice*. SAGE publications.

Siswono, T. Y. E., Kohar, A. W., Rosyidi, A. H., & Hartono, S. (2017, 2018). Searching for authentic context in designing pisa-like mathematics problem: From indoor to outdoor field experience. *Journal of Physics: Conference Series, 953*, 012197. https://doi.org/10.1088/1742-6596/953/1/012197

Smedslund, J. (1963). The concept of correlation in adults. *Scandinavian Journal of Psychology, 4*(1), 165-173.

Smith, G., Wood, L., Coupland, M., Stephenson, B., Crawford, K., & Ball, G. (1996). Constructing mathematical examinations to assess a range of knowledge and skills. *International Journal of Mathematical Education in Science and Technology, 27*(1), 65-77. https://doi.org/10.1080/0020739960270109

Son, J.-W., & Kim, O.-K. (2015). Teachers' selection and enactment of mathematical problems from textbooks. *Mathematics Education Research Journal, 27*(4), 491-518. https://doi.org/10.1007/s13394-015-0148-9

Son, J., & Diletti, J. (2017). What can we learn from textbook analysis? In *What matters? Research trends in international comparative studies in mathematics education* (pp. 3-32). Springer. https://doi.org/10.1007/978-3-319-51187-0

Stake, R. E. (1995). *The art of case study research*. Sage.

Stanley, G. (2008). *National numeracy review report* [Report]. Council of Australian Governments. https://apo.org.au/node/4016

State Government of Victoria. (2014). *Gippsland regional growth plan; Background report*. https://www.planning.vic.gov.au/__data/assets/pdf_file/0018/106551/Gippsland-Regional-Growth-Plan-Background-Report-May-2014.pdf

Stephens, M. (2009). *Numeracy in practice: teaching, learning and using mathematics*. Education Policy and Research Division, Department of Education and Early Childhood Development.

Stephens, M. (2014). The Australian Curriculum: Mathematics—How did it come about? What challenges does it present for teachers and for the teaching of mathematics? In *Mathematics curriculum in school education* (pp. 157-176). Springer.

Stillman, G. (2017). Enabling anticipation through visualisation in mathematising real-world problems in a flipped classroom. In *Mathematical modelling and applications* (pp. 163-173). Springer. https://doi.org/10.1007/978-3-319-62968-1

Stillman, G., & Brown, J. (2011). *'Get down and get dirty in the mathematics': Technology and mathematical modelling in senior secondary,* Mathematics: Traditions and [new] practices: Proceedings of the 2011 AAMT-MERGA conference, Adelaide.

Stillman, G., Galbraith, P., Brown, J., & Edwards, I. (2007). *A framework for success in implementing mathematical modelling in the secondary classroom* Mathematics: Essential research, essential practice: Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart

Su, H. F. H., Ricci, F. A., & Mnatsakanian, M. (2016). Mathematical teaching strategies: Pathways to critical thinking and metacognition. *International Journal of Research in Education and Science, 2*(1), 190-200. https://files.eric.ed.gov/fulltext/EJ1105157.pdf

Sullivan, P., Mousley, J., & Zevenbergen, R. (2005). Increasing access to mathematical thinking. *The Australian Mathematical Society Gazette, 32*(2), 105-109.

Sutherland, L. (2004). Differentiating mathematics instruction and Bloom's Taxonomy. *Cross Section, 15*(3), 16-18.

Sweeney, T. (2010). Transforming pedagogy through interactive whiteboards: Using activity theory to understand tensions in practice. *Australian Educational Computing, 24*(3), 28-34.

Tan, C.-K., & Tan, C.-P. (2014). Effects of the Handheld Technology Instructional Approach on Performances of Students of Different Achievement Levels. *Computers & Education, 82*, 306-314. https://doi.org/10.1016/j.compedu.2014.11.011

Taylor, W., & Stacey, K. (2014). Gardner's two children problems and variations: Puzzles with conditional probability and sample spaces. *Australian Mathematics Teacher, 70*(2), 13-19.

Teddlie, C., & Tashakkori, A. (2003). Major issues and controveries inthe use of mixed methods in the social and behvioral sciences. In *Handbook of mixed methods in social & behavioral research* (pp. 3-50).

Teddlie, C., & Tashakkori, A. (2009). *Foundations of mixed methods research: Integrating quantitative and qualitative approaches in the social and behavioral sciences*. Sage.

The Good Schools Guide. (2019). *Every primary and secondary school in Australia*. The Good Schools Guide,. https://www.goodschools.com.au/compare-schools

Thomas, G. (2013). *Case Study Methods in Education*. Sage. https://doi.org/10.4135/9781446286425

Thomas, G., & Myers, K. (2015). *The anatomy of the case study*. Sage.

Thompson, T. (2008). Mathematics teachers’ interpretation of higher-order thinking in Bloom’s Taxonomy. *International Electronic Journal of Mathematics Education, 3*(2), 96-109.

Thomson, S., De Bortoli, L., & Underwood, C. (2016). *PISA 2015: A first look at Australia's results*. ACER. https://research.acer.edu.au/cgi/viewcontent.cgi?referer=https://www.google.com.au/&httpsredir=1&article=1021&context=ozpisa

Thomson, S., De Bortoli, L., Underwood, C., & Schmid, M. (2019). *PISA 2018: Reporting Australia's results* (Vol. Volume 1 Student Performance). ACER. https://research.acer.edu.au/cgi/viewcontent.cgi?article=1035&context=ozpisa

Thomson, S., Wernert, N., O'Grady, E., & Rodrigues, S. (2017). TIMSS 2015: Reporting Australia's results. *Melbourne: Australian Council for Educational Research (ACER),* https://research.acer.edu.au/timss_2015/2/

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science, 211*(4481), 453-458.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Springer Science & Business Media.

VCAA. (2010). *VCE Mathematical Methods (CAS) Assessment Handbook 2006-2015*. https://doi.org/ISBN 978-1-921702-34-1

VCAA. (2015a). *Victorian Certificate of Education: Mathematics Study Design 2016-2022*. https://www.vcaa.vic.edu.au/Documents/vce/mathematics/2016MathematicsSD.pdf

VCAA. (2015b). *Victorian curriculum: Foundation-10*. https://victoriancurriculum.vcaa.vic.edu.au/

VCAA. (2015c). *Victorian curriculum: Mathematics - Learning in mathematics*. http://victoriancurriculum.vcaa.vic.edu.au/mathematics/introduction/learning-in-mathematics

VCAA. (2017a). *2016 Grade distributions for graded assessments VCE*. Retrieved 16/01/2018 from https://www.vcaa.vic.edu.au/Documents/statistics/2016/section3/vce_mathematical_methods_ga16.pdf

VCAA. (2017b). *Mathematical methods 2016–2020: School-assessed coursework report*. https://www.vcaa.vic.edu.au/Documents/vce/mathematics/Maths_Methods_SAC_report.docx

VCAA. (2019a). *2018 VCE Mathematical Methods 1 examination report*. https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/mm1_examrep18.pdf

VCAA. (2019b). *VCE and VCAL Administrative handbook 2020* https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx

VCAA. (2019c). *VCE Mathematical Methods Unit completion outcomes 2018*. Retrieved 21/03/19 from https://www.vcaa.vic.edu.au/Documents/statistics/2018/section2/vce_mathematical_methods_18.pdf

VCAA. (2019d). *VCE Mathematical Methods: Support material*. VCAA. Retrieved 21/03/19 from https://www.vcaa.vic.edu.au/Pages/vce/studies/mathematics/cas/casindex.aspx

VCAA. (n.d.-a). *Examination specifications, past examinations and examination reports*. VCAA. https://www.vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx

VCAA. (n.d.-b). *Sample modelling or problem-solving task 2 – close to normal*. Retrieved 12/04/2019 from https://www.vcaa.vic.edu.au/Documents/vce/adviceforteachers/mathsmethods/Samp_Mod_Prob_unit_4_close.doc

VCAA. (n.d.-c). *Sample modelling or problem-solving task 3 - proportions of popularity*. Retrieved 12/04/2019 from https://www.vcaa.vic.edu.au/Documents/vce/adviceforteachers/mathsmethods/Samp_Mod_Prob_unit_4_proportions.doc

VCAA. (n.d.-d). *VCE assessment: Authorised materils and equipment*. Victorian State Government. https://www.vcaa.vic.edu.au/assessment/vce-assessment/materials/Pages/index.aspx#01

VCAA. (n.d.-e). *VCE curriculum*. https://www.vcaa.vic.edu.au/curriculum/vce/Pages/Index.aspx

VCAA. (n.d.-f). *Victorian curriculum: Foundation-10*. http://victoriancurriculum.vcaa.vic.edu.au/mathematics/curriculum/f-10#level=9

Vincent, J., & Stacey, K. (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS video study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal, 20*(1), 82-107 https://link.springer.com/article/10.1007/BF03217470

VTAC. (2018). *ABC of scaling 2018*. Victorian tertiary admissions centre. http://www.vtac.edu.au/files/pdf/publications/abcofscaling.pdf

VTAC. (2020). *VTAC Careers Hub*. VTAC. https://www.vtac.edu.au/

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. (A. R. Luria, M. Lopez-Morillas & M. Cole [with J. V. Wertsch], Trans.) Harvard University Press. (Original work [ca. 1930-1934]).

Wallen, M., & Tormey, R. (2019). Developing teacher agency through dialogue. *Teaching and Teacher Education, 82*, 129-139. https://doi.org/10.1016/j.tate.2019.03.014

Wang, L., Liu, Q., Du, X., & Liu, J. (2017). Chinese mathematics curriculum reform in the 21st century: A review. *Eurasia Journal of Mathematics, Science and Technology Education, 13*(8), 5311-5326.

Watson, J., & Callingham, R. (2014). Two-way tables: Issues at the heart of statistics and probability for students and teachers. *Mathematical Thinking and Learning: An International Journal, 16*(4), 254-284. https://doi.org/10.1080/10986065.2014.953019

Watson, J., & Callingham, R. (2015). Lung disease, indigestion, and two-way tables. *Investigations in Mathematics Learning, 8*(2), 1-16. https://doi.org/10.1080/24727466.2015.11790348

Watson, J., Callingham, R. A., & Donne, J. M. (2008). *Establishing PCK for teaching statistics* 31st Annual Conference of the Mathematics Education Research Group of Australasia, Brisbane.

Watson, J., & Chance, B. (2012). Building intuitions about statistical inference based on resampling. *Australian Senior Mathematics Journal, 26*(1), 6.

Watson, J., Collis, K. F., & Moritz, J. B. (1997). The development of chance measurement. *Mathematics Education Research Journal, 9*(1), 60-82.

Watson, J., & English, L. (2015a). Expectation and variation with a virtual die. *Australian Mathematics Teacher, 71*(3), 3-9.

Watson, J., & English, L. (2015b). Introducing the practice of statistics: are we environmentally friendly? *Mathematics Education Research Journal, 27*(4), 585-613. https://doi.org/10.1007/s13394-015-0153-z

Watson, J., & Fitzallen, N. (2016). Statistical software and mathematics education-Affordances for learning. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 563-594). Routledge. https://www.routledge.com/products/9780415832038

Watson, J., & Kelly, B. (2007). The development of conditional probability reasoning. *International Journal of Mathematical Education in Science and Technology, 38*(2), 213-235. https://doi.org/10.1080/00207390601002880

Watson, J., Kelly, B., & Izard, J. (2006). A longitudinal study of student understanding of chance and data. *Mathematics Education Research Journal, 18*(2), 40-55. https://doi.org/10.1007/BF03217435

Watson, J., & Moritz, J. B. (2003). Fairness of dice: A longitudinal study of students' beliefs and strategies for making judgments. *Journal for Research in Mathematics Education*, 270-304.

Watson, J., & Nathan, E. (2010). Assessing the interpretation of two-way tables as part of statistical literacy. Reading, C., Data and context in statistics education: Towards an evidence-based society. Proceedings of Eight International Conference on Teaching Statistics,

Way, J., Bobis, J., Lamb, J., & Higgins, J. (2016). Researching curriculum, policy and leadership in mathematics education. In *Research in mathematics education in Australasia 2012-2015* (pp. 49-71). Springer.

Webb, D. C. (2014). Bloom's taxonomy in mathematics education. *Encyclopedia of Mathematics education*, 63-68. https://doi.org/10.1007/978-94-007-4978-8

Webb, N. L. (1997). *Criteria for alignment of expectations and assessments in Mathematics and Science education. Research Monograph No. 6*. https://eric.ed.gov/?id=ED414305

Webb, N. L. (2007). Issues related to judging the alignment of curriculum standards and assessments. *Applied Measurement in Education, 20*(1), 7-25.

Weiss, V., & Tobin, P. (2016). Use of calculators with computer algebra systems in test assessment in engineering mathematics. *ANZIAM Journal, 57*, 51-65. http://journal.austms.org.au/ojs/index.php/ANZIAMJ/article/view/10445

White, P., Sullivan, P., Warren, E., & Quinlan, C. (2016). To investigate or not to investigate? The use of content specific open-ended tasks. *Australian Mathematics Teacher, 72*(3), 61-65.

Wijaya, T. O., & Kaur, B. (2018). *A study of grade 10 Indonesian mathematics textbooks* ICMI-East Asia Regional Conference on Mathematics Education, Taipei, Taiwan.

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223-248.

Wiliam, D. (2006). *Mathematics inside the black box: Assessment for learning in the mathematics classroom.* Granada Learning.

Williams, J., Davis, P., & Black, L. (2007). Sociocultural and Cultural–Historical Activity Theory perspectives on subjectivities and learning in schools and other educational contexts. *International Journal of Educational Research, 46*(1-2), 1-7. https://doi.org/10.1016/j.ijer.2007.07.001

Wilson, E. (2013). *School-based research. A guide for education students.* (J. Clark, Ed. 2nd ed.). Sage.

Wilson, J., Lawman, J., Murphy, R., & Nelson, M. (2011). A comprehensive probability project for the upper division one-semester probability course using yahtzee. *Journal of Statistics Education, 19*(1), n1. https://doi.org/10.1080/10691898.2011.11889602

Woo, E. (2019). *Wootube*. https://misterwootube.com/

Wroughton, J., & Nolan, J. (2012). Pinochle poker: An activity for counting and probability. *Journal of Statistics Education, 20*(2), n2. https://doi.org/10.1080/10691898.2012.11889642

Yamagata-Lynch, L. C., & Haudenschild, M. T. (2009). Using activity systems analysis to identify inner contradictions in teacher professional development. *Teaching and Teacher Education, 25*(3), 507-517. https://doi.org/10.1016/j.tate.2008.09.014

Yamagata-Lynch, L. C., & Smaldino, S. (2007). Using activity theory to evaluate and improve K-12 school and university partnerships. *Evaluation and program planning, 30*(4), 364-380. https://doi.org/10.1016/j.evalprogplan.2007.08.003

Yin, R. K. (2013). Validity and generalization in future case study evaluations. *Evaluation*, *19*(3), 321-332. https://doi.org/10.1177/1356389013497081

Yin, R. K. (2018). *Case study research and applications: Design and methods*. Sage publications.

Zaslavsky, O., & Sullivan, P. (2011). *Constructing knowledge for teaching secondary mathematics: Tasks to enhance prospective and practicing teacher learning* (Vol. 6). Springer Science & Business Media.

Zelkowski, J. J. T. (2011). The TI-Nspire CAS: A happy-medium mobile device for grades 8–16 mathematics classrooms. *TechTrends, 55*(3), 40-47. https://www.learntechlib.org/p/50564/

Zengin, Y. (2017). Investigating the use of the Khan Academy and mathematics software with a flipped classroom approach in mathematics teaching. *Journal of Educational Technology & Society, 20*(2), 89-100. https://www.jstor.org/stable/10.2307/90002166

Ziebell, N. (2014). *The alignment of valued performance types in assessment practices and curriculum in year 5 mathematics and science classrooms* [Doctorial dissertation, The University of Melbourne]. https://minerva-access.unimelb.edu.au/handle/11343/44112

Ziebell, N., Ong, A., & Clarke, D. (2017). Aligning curriculum, instruction and assessment. In T. Bentley & G. C. Savage (Eds.), *Educating Australia: Challenges for the decade ahead* (pp. 257-275). Melbourne University Press.

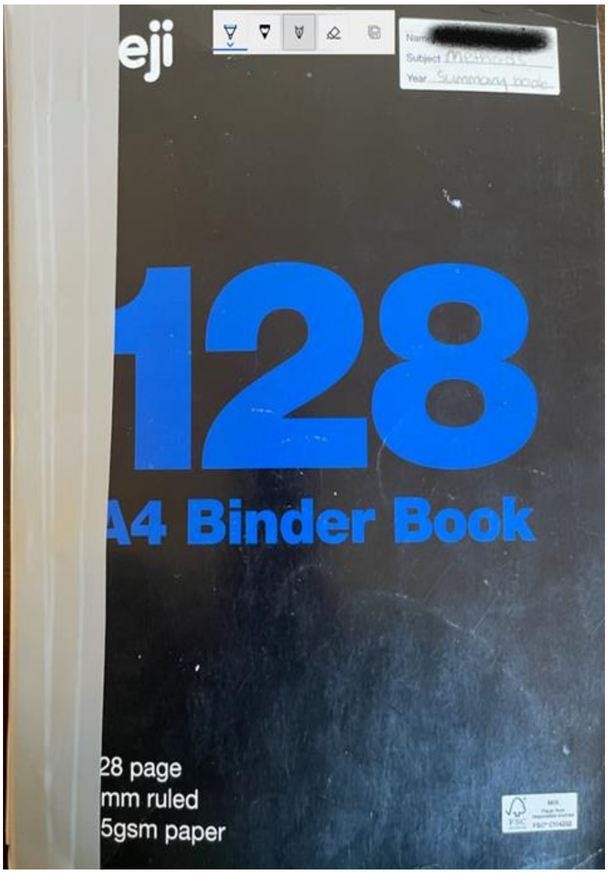
Zimmermann, G. (2002). *Students' reasoning about probability simulations during instruction* [Doctorial Dissertation, Illinois State University]. https://www.proquest.com/docview/275672645?pq-origsite=gscholar&fromopenview=true

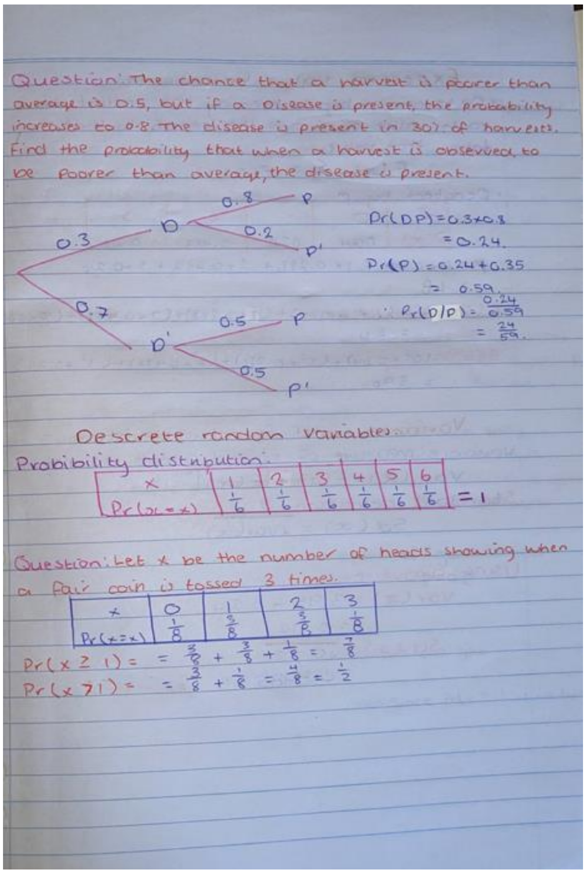
Zimmermann, G., & Jones, G. (2002). Probability simulation: What meaning does it have for high school students? *Canadian Journal of Math, Science & Technology Education, 2*(2), 221-236.

Appendices

Appendix A

Bound Reference Book Sample Page





Appendix B

Plain Language Information Statements for Students

Plain Language Information Statement

Federation UNIVERSITY • AUSTRALIA

SCHOOL OF EDUCATION, FACULTY OF EDUCATION AND ARTS

PROJECT TITLE:	Thinking in senior secondary mathematics: a case for probability
PRINCIPAL RESEARCHER:	Assoc Prof Margaret Plunkett
OTHER RESEARCHERS:	Ms Heather Ernst, Dr Peter Sellings

To the student participant,

As part of a study being carried out through the School of Education and Federation University, Heather Ernst (current candidate of a Doctor of Philosophy) will be conducting a research project titled,

"*Thinking in senior secondary mathematics: a case for probability*".

As someone who has recently completed senior secondary mathematics, you are invited to be part of this research project which will explore the implementation of the Australian Curriculum in VCE mathematics, especially in the area of probability. The project involves analysing the learning and assessment tasks recommended by the VCAA, MAV and commercially available resources with respect to the thinking skills involved in the tasks.

Your participation would involve completion of a short online or paper survey, which should not take longer than 20 minutes to complete. The survey would be accessed via a link provided to participants or through a paper copy handed out at the end of one of your classes, and would ask about the resources used in your mathematics classes, your views on the curriculum and about any support you may have received while completing VCE mathematics. A range of sample questions would also be used to investigate thinking processes used by students in mathematics.

You would also be invited to participate in an individual interview of no longer than 30 minutes, which could take place either face to face at FedUni or by phone. This interview, which would be audio-taped, would seek follow-up information relating to your VCE mathematics experience. Interviews would be conducted at a mutually agreed time and place.

Participation is voluntary and refusal to participate requires no explanation, you are also free to discontinue participation at any time up until data is processed, without prejudice. All collected data will be confidential and will be coded to ensure that there is no identifying data in any publication arising from the research. Data will be stored in a locked filing cabinet or password protected file, will only be accessibly by the researchers and will be destroyed after five years. Despite the sample size of the participants being small (potentially making it easier to identify participants), every effort will be made to ensure that information will not be identifiable. All the collected data will be confidential and no identifying information will be used in any publication or presentation arising from the research project.

Please note that any data collection procedure (e.g., interview) will be terminated should any participant show signs of personal distress. Should you experience any distress or personal concerns from your participation in this study please seek support from FedUni Counselling Services by phoning (03) 5327 9470, or emailing counselling@federation.edu.au, or for 24 hour Telephone Crisis Support contacting Lifeline on 131114.

Upon completion of the research, results will be used to produce a thesis and journal articles which will be available for you to view if requested.

If you would like to participate in this project, please indicate that you have read and understood this information by signing the accompanying consent form.

If you have any questions, or you would like further information regarding the project titled please contact the Principal researcher, Associate Professor Margaret Plunkett, Faculty of Education & Arts, Federation University Australia, Gippsland campus. **Email:** **margaret.plunkett@federation.edu.au** **Ph: (03) 5122 6980**
Should you (i.e. the participant) have any concerns about the ethical conduct of this research project, please contact the Federation University Ethics Officers, Research Services, Federation University Australia, P O Box 663 Mt Helen Vic 3353 or Northways Rd, Churchill Vic 3842. Telephone: (03) 5327 9765, (03) 5122 6446 Email: research.ethics@federation.edu.au CRICOS Provider Number 00103D

Appendix C

Plain Language Information Statements for Teachers

Plain Language Information Statement

Federation UNIVERSITY•AUSTRALIA

SCHOOL OF EDUCATION, FACULTY OF EDUCATION AND ARTS

PROJECT TITLE:	Thinking in senior secondary mathematics: a case for probability
PRINCIPAL RESEARCHER:	Assoc Prof Margaret Plunkett
OTHER RESEARCHERS:	Ms Heather Ernst, Dr Peter Sellings

To the teacher participant,

As part of a study being carried out through the School of Education and Federation University, Heather Ernst (current candidate of a Doctor of Philosophy) will be carrying out a research project titled,

"*Thinking in senior secondary mathematics: a case for probability*".

As a senior secondary mathematics teacher, you are invited be part of this research project which will explore the implementation of the Australian Curriculum in VCE mathematics, especially in the area of probability. The project involves analysing the learning and assessment tasks recommended by the VCAA, MAV and commercially available resources with respect to the thinking skills involved in the tasks. Teachers will be asked about the resources they use, their views on the curriculum and the support they receive in relation to teaching mathematics. Discussion of a range of sample questions will be used to determine perceptions about the thinking processes used by students in mathematics. You will be invited to share learning and assessment tasks used in VCE mathematics especially in the area of probability.

Your involvement in the project would require you to participate in an audiotaped face-to-face or phone interview conducted at a mutually agreed time and place, which would take approximately 30-45 minutes.

Participation is voluntary and refusal to participate requires no explanation, you are also free to discontinue participation at any time up until data is processed, without prejudice. All collected data will be confidential and will be coded to ensure that there is no identifying data in any publication arising from the research. Data will be stored in a locked filing cabinet or password protected file, will only be accessibly by the researchers and will be destroyed after five years. Despite the sample size of the participants being small (potentially making it easier to identify participants), every effort will be made to ensure that information will not be identifiable. All the collected data will be confidential and no identifying information will be used in any publication or presentation arising from the research project.

Please note that any data collection procedure (e.g., interview) will be terminated should any participant show signs of personal distress. Should you experience any distress or personal concerns from your participation in this study please seek support from any of the services listed at the end of this statement.

Upon completion of the research, results will be used to produce a thesis and journal articles which will be available for you to view if requested.

If you would like to participate in this project, please indicate that you have read and understood this information by signing the accompanying consent form.

COUNSELLING/SUPPORT SERVICES

The Employee Assistance Program (EAP)
A short term, strictly confidential counselling service for employees of the Victorian Department of Education and Training (support provided by OPTUM). The EAP is available 24/7 for up to four sessions for the Department's employees to discuss any personal or work related issues.
Phone: 1300 361 008

Australian Education Union
Morwell office, supporting the Gippsland region with any union or employment-related queries
Phone: (03) 5134 8844
Toll-free: 1800 013 979

Beyond Blue Support Service

A national initiative to raise awareness of anxiety and depression, providing resources for recovery, management and resilience.

Phone: 1300 22 4636

Lifeline

Phone: 13 11 14

If you have any questions, or you would like further information regarding the project titled please contact the Principal researcher, Associate Professor Margaret Plunkett, Faculty of Education & Arts, Federation University Australia, Gippsland campus. **Email: margaret.plunkett@federation.edu.au Ph: (03) 5122 6980**
Should you (i.e. the participant) have any concerns about the ethical conduct of this research project, please contact the Federation University Ethics Officers, Research Services, Federation University Australia, P O Box 663 Mt Helen Vic 3353 or Northways Rd, Churchill Vic 3842. Telephone: (03) 5327 9765, (03) 5122 6446 Email: research.ethics@federation.edu.au CRICOS Provider Number 00103D

Appendix D

Student Semi-structured Interview Questions

Thank you for agreeing to participate in this survey. Please read the Plain Language Information Statement and complete the Consent Form.

I would like to ask you to avoid using the names of schools, teachers or students if possible. You may miss a question or finish this interview whenever you wish. With your permission I will audio-tape this interview and email you a copy of the transcribed interview back to you for you to check and verify that you agree that it represents your views.

Semi-structured interview questions.

Which VCE mathematics did you do and when?

Please tell me about your VCE mathematics classes, what was a typical class like? Homework, calculators, group work/individual, whiteboard, computers, teacher talk v student talk…

Which teaching and learning activities did you do, which did you find most helpful?

Which resources, textbook, calculator and exam preparation material did you use? Did you do practice exams throughout the year? How did you make and use your bound reference (summary book)?

Thinking about probability:

What did you think about the topic of probability, compared to the other topics?

What did you remember about learning about topic of probability? Did you use any experiments or simulations?

Where there any questions within the course probability topic that you found particularly interesting? Confusing?

Are you studying Mathematics at University? Explain why or why not.

Probability questions.

I will now ask some probability questions, I am not so much interested in the answer, but how you will approach these questions,

Question 1 - The two-coin question.

Two fair coins are thrown

Mum wins if there are two heads

Dad wins if there are two tails

The kid wins if there is a head and a tail

e) What is the probability of mum winning?

f) If this game was played 1000 times, how many times would you expect mum to win?

g) Exactly?

h) Would you use a tree diagram, coins, or simulations?

Question 2 - Frequency table.

Smalltown	Male	Female	Total
Employed	580	645	1225
Unemployed	72	98	170
Total	652	743	1395

Using the information in the table above, which is the correct calculation for finding the percentage of females unemployed?

a) $\frac{98}{743} \times 100 \approx 13\%$ b) $\frac{98}{1395} \times 100 \approx 7\%$ c) $\frac{98}{170} \times 100 \approx 58\%$

Which answer is correct and why? What might be confusing about this question.

Is there anything else you would like to add?

For example; Do you have any opinions about calculators or summary books?....

Appendix E
Teacher Semi-structured Interview Questions

Thank you for agreeing to participate in this survey. Please read the plain language information statement and complete the consent form. These questions were emailed out prior to the interview.

Please do not mention schools, teachers or students by name. You may miss a question or finish this interview whenever you wish. With your permission I will audio-tape this interview and email it back to you for verification.

Semi-structured interview questions.

Background questions: What was your major at university?

How long have you been teaching? How long have you been teaching senior secondary mathematics? What subjects do you teach?

Please tell me about your teaching experience in schools and at VCE senior secondary mathematics, especially Mathematical Methods?

What professional development to do with VCE mathematics have you been to in the last 3 year?

Please tell me about your VCE mathematics classes, what is a typical class like? Homework, calculators, group work/individual, whiteboard, computers, teacher talk v student talk…

Which teaching and learning activities do you use with your VCE classes, which did you think the students found most helpful?

Which resources, textbook, calculator and exam preparation material did you use when teaching VCE mathematic?

Thinking about probability:

What did you think about teaching the topic of probability?

How long do you send on probability in the year in Year 11 or 12 Mathematical Methods??

Were there any question types you found particularly interesting? Confusing?

Could you please supply a copy of your yearly timeline for teaching VCE mathematics?

Could you please supply a copy of any Year 11 and/or 12 SACs relating to the topic of probability?

These materials may be emailed to the researcher at a later date.

Probability questions

I will now ask some probability questions, I am not so much interested in the answer, but how you will approach these questions,

Question 1- The two-coin question.

Two fair coins are thrown

Mum wins if there are two heads

Dad wins if there are two tails

The kid wins if there is a head and a tail

a) What is the probability of mum winning?

b) If this game was played 1000 times, how many times would you expect mum to win?

c) Exactly?

d) When you were teaching this, what would you expect the students to say?

e) Would you use a tree diagram, coins, or simulations?

Question 2- Frequency table.

Probability questions taken from Pearson's textbook.

Smalltown	Male	Female	Total
Employed	580	645	1225
Unemployed	72	98	170
Total	652	743	1395

Using the information in the table above, which is the correct calculation for finding the percentage of females unemployed?

a) $\frac{98}{743} \times 100 \approx 13\%$ b) $\frac{98}{1395} \times 100 \approx 7\%$ c) $\frac{98}{170} \times 100 \approx 58\%$

- Which answer is correct and why?
- What might be confusing about this question?
- Would you like to reword this question?

Is there anything else you would like to add?

For example; Do you have any opinions about calculators or summary books?....

Appendix F

Final Project Report Human Ethics Committee

Annual/Final Project Report
Human Research Ethics Committee

Federation UNIVERSITY•AUSTRALIA

Please indicate the type of report	☐ Annual Report ☒ Final Report
Project No:	A16-150
Project Name:	Thinking in senior secondary mathematics: A case study of probability
Principal Researcher:	Assoc Prof Margaret Plunkett
Other Researchers:	Dr Peter Sellings Heather Ernst
Date of Original Approval:	26/10/2016
School / Section:	School of Education
Phone:	0351226980
Email:	Margaret.plunkett@federation.edu.au

Please note: For HDR candidates, this Ethics annual report is a separate requirement, in addition to your HDR Candidature annual report, which is submitted mid-year to research.degrees@federation.edu.au.

1) Please indicate the current status of the project:				
1a) Yet to start **1b) Continuing** **1c) Data collection completed** **1d) Abandoned / Withdrawn:**				☐ ☐ ☒ ☐
1e) If the approval was subject to certain conditions, have these conditions been met? (If not, please give details in the comments box below)			☒ Yes	☐ No
Comments:				
1f) Data Analysis	☐ Not yet commenced	☐ Proceeding	☒ Complete	☐ None

1g) Have ethical problems been encountered in any of the following areas: **Study Design** **Recruitment of Subjects** **Finance** **Facilities, Equipment** **(If yes, please give details in the comments box below)**	☐ Yes ☐ Yes ☐ Yes ☐ Yes	☒ No ☒ No ☒ No ☒ No
Comments:		

2a) Have amendments been made to the originally approved project?	
☒ No	☐ Yes
2b) If yes, was HREC approval granted for these changes?	
☐ Yes	**Provide detail:** ☐ Yes **Application for Amendment to an Existing Project** ☐ Yes **Change of Personnel** ☐ Yes **Extension Request**
☐ No	**If you have made changes, but not had HREC approval, provide detail as to why this has not yet occurred:**
2c) Do you need to submit any amendments now?	
☒ No	☐ Yes **Application for Amendment to an Existing Project** ☐ Yes **Change of Personnel** ☐ Yes **Extension Request** *** NB: If 'Yes', download & submit the appropriate request to the HREC for approval:** **Please note: Extensions will not be granted retrospectively. Apply well prior to the project end date, to ensure continuity of HRE approval.**

3a) Please indicate where you are storing the data collected during the course of this project: (Australian code for the Responsible conduct of Research Ch 2.2.2, 2.5 – 2.7)
Data is currently being stored on the password protected computer of the student researcher, who is a member of the School of Education.
3b) Final Reports: Advise when & how stored data will be destroyed (Australian code for the Responsible conduct of Research Ch 2.1.1)

Data will be kept for a minimum of 5 years and then disposed of through the Federation University Secure data disposal system.

4) Have there been any events that might have had an adverse effect on the research participants OR unforeseen events that might affect continued ethical acceptability of the project?	
☒ No	☐ Yes * **NB: If 'yes', please provide details in the comments box below:**
Comments:	

5a) Please provide a short summary of results of the project so far (no attachments please):
All data in this qualitative case study has been collected and analysed. The data was collected via interviews with mathematics teachers and university students who had previously completed the Y12 subject Mathematical Methods. Activity Theory was utilised as the theoretical framework for the study. Thus far the findings have indicated that the topic of probability is not given enough attention prior to the senior mathematics subjects. The sharp increase in the volume of content and complexity lead to the content being covered superficially. Content was aligned between the intended, implemented and attained curriculum, however the expected depth of thinking was not generally attained. Textbooks acted as influential pseudo curriculum, which was deemed appropriate. Scaffolds to support learning, calculators, summary books, and practice examinations, were not fully utilised by the teachers or students.
5b) Final Reports: Provide details about how the aims of the project, as stated in the application for approval, were achieved (or not achieved). (Australian code for the Responsible conduct of Research 4.4.1)
The key research aim of this project was to explore the Victorian senior secondary mathematics curriculum (Mathematical Method in VCE; Victorian Certificate of Education) in the area of probability. The project aimed to investigate the implementation of this curriculum from the perspectives of the formal written curriculum, the teachers and the students. This investigation has achieved the stated aims through providing an informed description of current practice in Gippsland. It has highlighted the perceived enablers and blockers to the use of higher order thinking in the mathematics curriculum and generated valuable recommendations for teachers to support the teaching and learning of mathematics in VCE, particularly within the probability strand. While the investigation involved a case study conducted in the Gippsland region, it is anticipated that the findings and recommendations will be more widely applicable. Teacher participants have been provided with an avenue to discuss their practices and to highlight concerns relating to the teaching of probability. The benefits to participating students, as post-VCE and beginning their university studies has been an enriched understanding of the value of higher order

thinking in mathematics and a greater awareness of strategies for measurement and enhancement of thinking.

6) Publications: Provide details of research dissemination outcomes for the previous year resulting from this project: eg: Community seminars; Conference attendance; Government reports and/or research publications

Ernst, H. (2018). Senior Secondary Probability Curriculum: What has changed? In Hunter, J., Perger, P., & Darragh, L. (Eds.). *Making waves, opening spaces* (Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia) pp.290-297. Auckland: MERGA. https://merga.net.au/Public/Publications/Annual_Conference_Proceedings/2018-MERGA-CP.aspx

Ernst, H. & Morton, A. (2019 in print). Connecting Probability. In Hall, J. & Tan, H. (eds). *Making + connection: Mathematics numeracy* (Proceedings of the 56th annual conference of Mathematics Association of Victoria, MAV19) Melbourne: MAV.

7) The HREC welcomes any feedback on:

- **Difficulties experienced with carrying out the research project; or**
- **Appropriate suggestions which might lead to improvements in ethical clearance and monitoring of research.**

8) Signatures

Principal Researcher:	Print name: A/P Margaret Plunkett	**Date:**	17/10/19
Other/Student Researchers:	Print name: Heather Ernst	**Date:**	17/10/19
	Print name: Dr Peter Sellings	**Date:**	17/10/19

Submit to the Ethics Officer, Mt Helen campus, by the due date:
research.ethics@federation.edu.au

Appendix G

Department of Education & Training Permission to Conduct Research

Department of
Education & Training

2 Treasury Place
East Melbourne Victoria 3002
Telephone: 03 9637 2000
DX210083

2016_003211

Ms Heather Ernst
PO Box 3191
GIPPSLAND MAIL CENTRE 3841

Dear Ms Ernst

Thank you for your application of 6 October 2016 in which you request permission to conduct research in Victorian government schools titled *Thinking in senior secondary mathematics: a case study of probability.*

I am pleased to advise that on the basis of the information you have provided your research proposal is approved in principle subject to the conditions detailed below.

1. The research is conducted in accordance with the final documentation you provided to the Department of Education and Training.

2. Separate approval for the research needs to be sought from school principals. This is to be supported by the Department of Education and Training approved documentation and, if applicable, the letter of approval from a relevant and formally constituted Human Research Ethics Committee.

3. The project is commenced within 12 months of this approval letter and any extensions or variations to your study, including those requested by an ethics committee must be submitted to the Department of Education and Training for its consideration before you proceed.

4. As a matter of courtesy, you advise the relevant Regional Director of the schools or governing body of the early childhood settings that you intend to approach. An outline of your research and a copy of this letter should be provided to the Regional Director or governing body.

5. You acknowledge the support of the Department of Education Training in any publications arising from the research.

6. The Research Agreement conditions, which include the reporting requirements at the conclusion of your study, are upheld. A reminder will be sent for reports not submitted by the study's indicative completion date.

I wish you well with your research. Should you have further questions on this matter, please contact Youla Michaels, Project Support Officer, Insights and Evidence Branch, by telephone on (03) 9637 2707 or by email at michaels.youla.y@edumail.vic.gov.au.

Yours sincerely

Joyce Cleary
Director
Insights and Evidence

19/12/2016

VICTORIA
State
Government