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An objective framework for determination of the air-entry value from the soil–water characteristic curve

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ABSTRACT

The air-entry value (AEV) suction, marking the transition between saturated and unsaturated soil mechanics, is arguably the most important parameter interpreted from the soil–water characteristic curve (SWCC); its accurate determination being essential for the prediction of unsaturated soil properties. The AEV is commonly obtained by a subjective and time-consuming graphical construction. This micro-article proposes an objective framework, developed based on a practical mathematical translation technique, for the AEV determination. Explicit equations for the AEV are derived based on eleven well-established SWCC fitting functions, covering a wide range of functional complexities. In addition to its objective nature (providing unique interpretations of the AEV), the proposed framework complements numerical implementations of unsaturated soil constitutive models.

1. Introduction

The soil–water characteristic curve (SWCC), defined as the relationship between the soil suction (i.e., soil–water matric potential ψ) and the amount of water held within the soil (often expressed in terms of degree of saturation S), is an integral part of the unsaturated soil mechanics framework; it provides the information needed to characterize unsaturated soil properties [1]. The air-entry value (AEV), recognized as a suction state where entrance of air to the largest soil pore is first permitted during desaturation of a saturated soil [2], is arguably the most important parameter interpreted from the SWCC. An accurate determination of the AEV suction (ψ_{AEV}) is essential for the prediction of unsaturated soil properties, such as shear strength, volume change potential and relative permeability [3–5].

Referring to Fig. 1; the AEV suction state is commonly interpreted by a graphical construction, which is implemented to the fitted SWCC function plotted in the semi-logarithmic space of S:$\log_{10}\psi$ [5]. Following the conventional graphical procedure, ψ_{AEV} (the abscissa of Point B) is defined as the intersection of the tangent line extended through the 'subjectively identified' inflection point of the SWCC (Point I) with the horizontal line of $S(\psi) = 1$. Aside from its apparent 'subjective' nature, this graphical procedure can be time-consuming. Since S:ψ measurements are routinely expressed in terms of continuous SWCC fitting functions [6–15], these drawbacks can be eliminated by way of mathematical translation, allowing ψ_{AEV} to be expressed as explicit equations, thereby complementing computational analyses and numerical simulations that use ψ_{AEV} as an input parameter. While attempts of this nature have been made by Zhai and Rahardjo [16] and the authors in Soltani et al. [17] for the Fredlund & Xing [14] and van Genuchten [10] SWCC models, respectively, other well-established SWCC fitting functions have not yet been considered. Accordingly, this micro-article aims at deriving explicit equations for the AEV suction, considering eleven well-established SWCC models.

2. Proposed framework

The general functional expression for the SWCC (fundamentally valid up to the residual state condition) can be given as follows [1,18]:

$$S(\psi) = S_{\mathrm{R}} + (1 - S_{\mathrm{R}})F(\psi) \tag{1}$$

where $S(\psi)$ = degree of saturation with respect to matric suction ψ ($\mathrm{ML^{-1}T^{-2}}$); S_{R} = residual degree of saturation (which can be fixed as zero or considered as an independent fitting parameter); and $F(\psi)$ =

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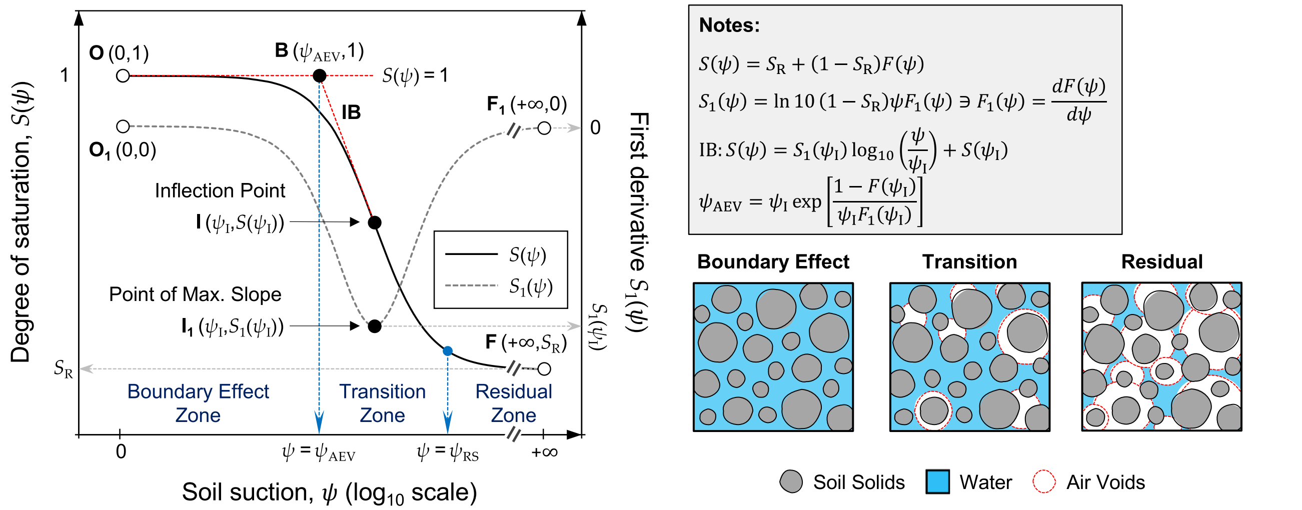

Fig. 1. Typical illustration of the drying SWCC (with respect to sigmoid-type SWCC fitting functions). **Note:** Boundary Effect, Transition and Residual zones denote the different stages of soil desaturation; the water phase in the Residual zone is discontinuous.

SWCC model (or fitting function) that represents the effective degree of saturation — that is, $F(\psi) = [S(\psi) - S_R]/[1 - S_R]$.

The mathematical translation technique is employed to derive explicit equations for the AEV suction based on eleven well-established continuous–unimodal $F(\psi)$ models, listed as Equations (6)–(16) in Table 1 [6–15], covering a variety of different functional complexities. The proposed mathematical translation framework is elaborated in the following.

The inflection point of the SWCC in the S:$\log_{10}\psi$ space (Point I in Fig. 1) can be defined as the positive real root of the second derivative of Equation (1) with respect to $\log_{10}\psi$ [17,20]:

$$S_1(\psi) = \frac{dS(\psi)}{d\log_{10}\psi} = \frac{dS(\psi)}{d\psi} \times \psi \times \ln 10 = \ln 10\,\psi(1 - S_R)F_1(\psi) \tag{2}$$

$$S_2(\psi) = \frac{d^2S(\psi)}{d(\log_{10}\psi)^2} = \frac{dS_1(\psi)}{d\psi} \times \psi \times \ln 10 = (\ln 10)^2\psi(1 - S_R)[F_1(\psi) + \psi F_2(\psi)] \ni S_2(\psi_I) = 0 \tag{3}$$

where $S_1(\psi)$ and $S_2(\psi)$ = first and second derivatives of Equation (1) with respect to $\log_{10}\psi$; $F_1(\psi)$ and $F_2(\psi)$ = first and second derivatives of $F(\psi)$ with respect to ψ; and ψ_I = matric suction at the inflection point of the SWCC.

Note that the degree of saturation at the inflection point can be calculated by substituting $\psi = \psi_I$ into Equation (1); let this be $S(\psi_I)$. For the eleven SWCC models examined, $F_1(\psi)$ and $F_2(\psi)$ are listed in Table 1 as Equations (17)–(27) and (28)–(38), respectively.

The slope of the tangent line extended through the inflection point can be calculated by substituting $\psi = \psi_I$ into Equation (2); let this be $S_1(\psi_I)$. Making use of $S_1(\psi_I)$, along with Point I (for which its abscissa is listed as Equations (39)–(49) in Table 2), the tangent line IB, as shown in Fig. 1, can be expressed as follows:

$$\text{IB}: S(\psi) = S_1(\psi_I)\log_{10}\left(\frac{\psi}{\psi_I}\right) + S(\psi_I) \tag{4}$$

Finally, the intersection of lines IB and $S(\psi) = 1$ — shown as Point B in Fig. 1, with its abscissa representing the 'inferred' AEV suction (or ψ_{AEV}) — can be solved by equating Equation (4) to unity:

$$\psi_{AEV} = \psi_I \exp\left[\frac{1 - F(\psi_I)}{\psi_I F_1(\psi_I)}\right] \tag{5}$$

For the eleven SWCC models investigated, $F(\psi_I)$ and $F_1(\psi_I)$ are presented in Table 2 as Equations (50)–(60) and (61)–(71), respectively. Note that, for those SWCC models where $S_2(\psi) = 0$ could not be solved explicitly, which was the case for the McKee & Bumb (Equation (13)) and Fredlund & Xing (Equation (15)) models, the positive real root of $F_2(\psi) = 0$ (denoted as ψ_I^* and defined as the inflection point of the SWCC in the arithmetic space of S:ψ) can be used in lieu of ψ_I to avoid the need of implementing numerical root-finding algorithms. For the eleven SWCC models examined, the deduced AEV relationships are listed as Equations (72)–(82) in Table 2.

3. Applications and future research

Worked examples demonstrating the implementation of the proposed framework to a typical experimental S:ψ dataset reported in Uchaipichat and Khalili [21] are provided in Figure S1 of the **Supplementary Material**. Aside from its objective nature (providing unique interpretations of the AEV) and its apparent benefits for numerical implementations (of unsaturated soil constitutive models), the proposed framework can be adopted to critically (and accurately) examine the effects of the selected $F(\psi)$ SWCC fitting function on the deduced AEV; an important aspect that has not yet been examined in the literature.

Table 1

Summary of the investigated SWCC models (or fitting equations), along with their first and second derivatives. **Note:** For details regarding the physical meaning of the model/fitting parameters (α, n and m), the reader is referred to the original source of the SWCC model.

SWCC Reference	SWCC Function $F(\psi)$	First Derivative $F_1(\psi)$	Second Derivative $F_2(\psi)$
Burdine [6]	$\frac{1}{[1+(\alpha\psi)^n]^{\frac{n-2}{n}}}$ (6)	$\frac{-(n-2)(\alpha\psi)^n}{\psi[1+(\alpha\psi)^n]^{\frac{2n-2}{n}}}$ (17)	$\frac{(n-1)(n-2)(\alpha\psi)^n[(\alpha\psi)^n-1]}{\psi^2[1+(\alpha\psi)^n]^{\frac{3n-2}{n}}}$ (28)
Gardner [7]	$\frac{1}{1+\alpha\psi^n}$ (7)	$\frac{-n\alpha\psi^n}{\psi(1+\alpha\psi^n)^2}$ (18)	$\frac{n\alpha\psi^n[(n+1)\alpha\psi^n-(n-1)]}{\psi^2(1+\alpha\psi^n)^3}$ (29)
Brutsaert [8]	$\frac{1}{1+\left(\frac{\psi}{\alpha}\right)^n}$ (8)	$\frac{-n\left(\frac{\psi}{\alpha}\right)^n}{\psi\left[1+\left(\frac{\psi}{\alpha}\right)^n\right]^2}$ (19)	$\frac{n\left(\frac{\psi}{\alpha}\right)^n\left[(n+1)\left(\frac{\psi}{\alpha}\right)^n-(n-1)\right]}{\psi^2\left[1+\left(\frac{\psi}{\alpha}\right)^n\right]^3}$ (30)
Mualem [9]	$\frac{1}{[1+(\alpha\psi)^n]^{\frac{n-1}{n}}}$ (9)	$\frac{-(n-1)(\alpha\psi)^n}{\psi[1+(\alpha\psi)^n]^{\frac{2n-1}{n}}}$ (20)	$\frac{(n-1)(\alpha\psi)^n\{1+n[(\alpha\psi)^n-1]\}}{\psi^2[1+(\alpha\psi)^n]^{\frac{3n-1}{n}}}$ (31)
van Genuchten [10]	$\frac{1}{[1+(\alpha\psi)^n]^m}$ (10)	$\frac{-nm(\alpha\psi)^n}{\psi[1+(\alpha\psi)^n]^{m+1}}$ (21)	$\frac{nm(\alpha\psi)^n[(nm+1)(\alpha\psi)^n-(n-1)]}{\psi^2[1+(\alpha\psi)^n]^{m+2}}$ (32)
Tani [11]	$\left(1+\frac{\psi}{\alpha}\right)\exp\left(-\frac{\psi}{\alpha}\right)$ (11)	$\frac{-\psi}{\alpha^2}\exp\left(-\frac{\psi}{\alpha}\right)$ (22)	$\left(\frac{\psi-\alpha}{\alpha^3}\right)\exp\left(-\frac{\psi}{\alpha}\right)$ (33)
McKee & Bumb [12]	$\exp\left(-\frac{\psi}{\alpha}\right)$ (12)	$\frac{-1}{\alpha}\exp\left(-\frac{\psi}{\alpha}\right)$ (23)	$\frac{1}{\alpha^2}\exp\left(-\frac{\psi}{\alpha}\right)$ (34)
McKee & Bumb [12]	$\frac{1}{1+\exp\left(\frac{\psi-\alpha}{n}\right)}$ (13)	$\frac{-\exp\left(\frac{\psi-\alpha}{n}\right)}{n\left[1+\exp\left(\frac{\psi-\alpha}{n}\right)\right]^2}$ (24)	$\frac{\exp\left(\frac{\psi-\alpha}{n}\right)\left[\exp\left(\frac{\psi-\alpha}{n}\right)-1\right]}{n^2\left[1+\exp\left(\frac{\psi-\alpha}{n}\right)\right]^3}$ (35)
Russo [13]	$\left[\left(1+\frac{\alpha\psi}{2}\right)\exp\left(-\frac{\alpha\psi}{2}\right)\right]^{\frac{2}{n+2}}$ (14)	$\frac{-\alpha^2\psi\left[\left(1+\frac{\alpha\psi}{2}\right)\exp\left(-\frac{\alpha\psi}{2}\right)\right]^{\frac{2}{n+2}}}{(n+2)(\alpha\psi+2)}$ (25)	$\frac{\alpha^2[(\alpha\psi)^2-2(n+2)]\left[\left(1+\frac{\alpha\psi}{2}\right)\exp\left(-\frac{\alpha\psi}{2}\right)\right]^{\frac{2}{n+2}}}{(n+2)^2(\alpha\psi+2)^2}$ (36)
Fredlund & Xing [14]	$\frac{1}{\left\{\ln\left[e+\left(\frac{\psi}{\alpha}\right)^n\right]\right\}^m}$ (15)	$\frac{-nm\left(\frac{\psi}{\alpha}\right)^n}{\psi\left[e+\left(\frac{\psi}{\alpha}\right)^n\right]\left\{\ln\left[e+\left(\frac{\psi}{\alpha}\right)^n\right]\right\}^{m+1}}$ (26)	$\frac{nm\left(\frac{\psi}{\alpha}\right)^n\left\{n(m+1)\left(\frac{\psi}{\alpha}\right)^n+\left[\left(\frac{\psi}{\alpha}\right)^n-e(n-1)\right]\ln\left[e+\left(\frac{\psi}{\alpha}\right)^n\right]\right\}}{\psi^2\left[e+\left(\frac{\psi}{\alpha}\right)^n\right]^2\left\{\ln\left[e+\left(\frac{\psi}{\alpha}\right)^n\right]\right\}^{m+2}}$ (37)
Kosugi [15]	$\frac{1}{2}\mathrm{erfc}\left[\frac{\ln\left(\frac{\psi}{\alpha}\right)}{\sqrt{2}n}\right]$ (16)	$\frac{-\exp\left\{-\left[\frac{\ln\left(\frac{\psi}{\alpha}\right)}{\sqrt{2}n}\right]^2\right\}}{\sqrt{2\pi}n\psi}$ (27)	$\frac{\left[n^2+\ln\left(\frac{\psi}{\alpha}\right)\right]\exp\left\{-\left[\frac{\ln\left(\frac{\psi}{\alpha}\right)}{\sqrt{2}n}\right]^2\right\}}{\sqrt{2\pi}n^3\psi^2}$ (38)

Note: exp = exponential function (i.e., $\exp[x] = e^x$ where e is Napier's constant); ln = natural logarithm function (i.e., $\ln[x] = \log_e[x]$); erfc = complementary error function (i.e., $\mathrm{erfc}[x] = 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$); the correction factor for the Fredlund & Xing SWCC model (Equation (15)) is assumed as $C(\psi) = 1$, as recommended by Leong and Rahardjo [19].

Table 2
Explicit equations for AEV determination.

SWCC Reference	Inflection Point ψ_I		$F(\psi_I)$		$F_1(\psi_I)$		Air-Entry Value ψ_{AEV}	
Burdine [6]	$\psi_I = \frac{1}{\alpha}\left(\frac{n}{n-2}\right)^{\frac{1}{n}}$	(39)	$\left(\frac{n-2}{2n-2}\right)^{\frac{n-2}{n}}$	(50)	$-\frac{n}{\psi_I}\left(\frac{n-2}{2n-2}\right)^{\frac{2n-2}{n}}$	(61)	$\psi_I \exp\left\{\frac{2n-2}{n(n-2)}\left[1-\left(\frac{2n-2}{n-2}\right)^{\frac{n-2}{n}}\right]\right\}$	(72)
Gardner [7]	$\psi_I = \left(\frac{1}{\alpha}\right)^{\frac{1}{n}}$	(40)	$\frac{1}{2}$	(51)	$-\frac{n}{4\psi_I}$	(62)	$\psi_I \exp\left(-\frac{2}{n}\right)$	(73)
Brutsaert [8]	$\psi_I = \alpha$	(41)	$\frac{1}{2}$	(52)	$-\frac{n}{4\psi_I}$	(63)	$\psi_I \exp\left(-\frac{2}{n}\right)$	(74)
Mualem [9]	$\psi_I = \frac{1}{\alpha}\left(\frac{n}{n-1}\right)^{\frac{1}{n}}$	(42)	$\left(\frac{n-1}{2n-1}\right)^{\frac{n-1}{n}}$	(53)	$-\frac{n}{\psi_I}\left(\frac{n-1}{2n-1}\right)^{\frac{2n-1}{n}}$	(64)	$\psi_I \exp\left\{\frac{2n-1}{n(n-1)}\left[1-\left(\frac{2n-1}{n-1}\right)^{\frac{n-1}{n}}\right]\right\}$	(75)
van Genuchten [10]	$\psi_I = \frac{1}{\alpha}\left(\frac{1}{m}\right)^{\frac{1}{n}}$	(43)	$\left(\frac{m}{m+1}\right)^{m}$	(54)	$-\frac{n}{\psi_I}\left(\frac{m}{m+1}\right)^{m+1}$	(65)	$\psi_I \exp\left\{\frac{m+1}{nm}\left[1-\left(\frac{m+1}{m}\right)^{m}\right]\right\}$	(76)
Tani [11]	$\psi_I = 2\alpha$	(44)	$\frac{3}{e^2}$	(55)	$-\frac{4}{e^2\psi_I}$	(66)	$\psi_I \exp\left(\frac{3-e^2}{4}\right)$	(77)
McKee & Bumb [12]	$\psi_I = \alpha$	(45)	$\frac{1}{e}$	(56)	$-\frac{1}{e\psi_I}$	(67)	$\psi_I \exp(1-e)$	(78)
McKee & Bumb [12]	$\psi_I^* = \alpha$	(46)	$\frac{1}{2}$	(57)	$-\frac{1}{4n}$	(68)	$\psi_I^* \exp\left(-\frac{2n}{\alpha}\right)$	(79)
Russo [13]	$\psi_I = \frac{2G_n}{\alpha}$ $\ni G_n = \frac{n+2+\sqrt{(n+2)(n+18)}}{4}$	(47)	$\left[\frac{1+G_n}{\exp(G_n)}\right]^{\frac{2}{n+2}}$	(58)	$-\frac{2G_n^{\,2}}{\psi_I(n+2)(G_n+1)}\left[\frac{1+G_n}{\exp(G_n)}\right]^{\frac{2}{n+2}}$	(69)	$\psi_I \exp\left(\frac{(n+2)(G_n+1)\left\{\left[\frac{1+G_n}{\exp(G_n)}\right]^{\frac{2}{n+2}}-1\right\}}{2G_n^{\,2}\left[\frac{1+G_n}{\exp(G_n)}\right]^{\frac{2}{n+2}}}\right)$	(80)
Fredlund & Xing [14]	$\psi_I^* = \alpha$	(48)	$\frac{1}{[\ln(e+1)]^m}$	(59)	$-\frac{nm}{\psi_I^*(e+1)[\ln(e+1)]^{m+1}}$	(70)	$\psi_I^* \exp\left(\frac{(e+1)\ln(e+1)\{1-[\ln(e+1)]^m\}}{nm}\right)$	(81)
Kosugi [15]	$\psi_I = \alpha$	(49)	$\frac{1}{2}$	(60)	$-\frac{1}{\sqrt{2\pi}n\psi_I}$	(71)	$\psi_I \exp\left(-\sqrt{\frac{\pi}{2}}n\right)$	(82)

Note: exp = exponential function (i.e., $\exp[x] = e^x$ where e is Napier's constant); ln = natural logarithm function (i.e., $\ln[x] = \log_e[x]$).

Credit Author Statement

A. Soltani: Conceptualization, Methodology, Formal Analysis, Writing—Original Draft Preparation, Visualization, Supervision; M. Azimi: Validation, Formal Analysis, Writing—Original Draft Preparation, Visualization; A. Boroomandnia: Validation, Formal Analysis, Writing—Review and Editing; B.C. O'Kelly: Conceptualization, Writing—Review and Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.rineng.2021.100298.

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