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Development of FWD based hybrid back-analysis technique for railway track condition assessment

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ABSTRACT

Track substructure is a key component of the railway transportation system. Similar to the built environment of other surface transportation systems, track substructures are subjected to aging and deterioration. This, frequently leads to the failure and collapse of the transportation systems, resulting in the imposition of costly repairs and maintenance. At the same time, the emergence of high-speed trains and heavier axle loads, together with a need for sustainable designs, has put additional pressure on asset owners. It has been shown that frequent condition assessments of railway substructures can considerably reduce the overall annual maintenance costs. Furthermore, limited knowledge of the substructure condition leads to the employment of inefficient, time-consuming, and expensive maintenance actions. Therefore, development of time- and cost-efficient techniques to frequently monitor existing railway track substructures is vital. The falling weight deflectometer (FWD) is recognised as an effective non-destructive test used to survey ballasted railway substructures through a back-analysis process. This paper presents a novel hybrid back-analysis technique that includes an artificial neural network (ANN) and genetic algorithm (GA) to estimate the substructure layer moduli of railway tracks using FWD testing data. To this aim, firstly a dynamic finite element (FE) model was developed and validated against experimental data from the literature. This FE model then were employed to generate a reliable dataset to train the ANN. In the next step, GA was employed as an optimisation tool within the back-analysis technique/framework to optimise the layer moduli (the ANN's input). A comparison study was performed to evaluate the performance of the developed technique. The results of this comparison revealed excellent performance and robustness of the developed technique.

Introduction

Railway track substructure also known as railway track foundation plays a vital role in supporting railway system; its mechanical properties directly affect the railway's operational performance [59]. Frequent condition assessments of railway systems (in particular the track substructure) are required for (i) early defect detection, (ii) identification of an effective maintenance method and consequently will result in significant maintenance costs saving [44,60].

Falling weight deflectometer (FWD) test as a non-destructive testing (NDT) method used to assess the condition of a subsurface transportation infrastructure. This method was originally developed for road and airfield pavements and then employed in the railway industry with some minor modifications [60]. For pavement condition assessment, deflection basins obtained from the FWD test are interpreted through a back-analysis technique to estimate the layer moduli [31]. Several studies have developed various back-analysis techniques to estimate the mechanical properties of pavement layers [4], [10,11,18,20]. However, less attention has been paid to the railway tracks application. A railway subsurface structure is fundamentally different from that of pavement and using pavement inverse analysis codes for a railway track can lead to errors. It should be noted that, currently, there are no commercially available back-analysis software for the application of FWD in railway condition assessments.

Back-analysis techniques consist of two main parts: (i) a forward model to analyse the problem; and (ii) the optimisation analysis, which is used to estimate the layer moduli. Available studies on the forward models based on the FWD testing method and optimisation analysis are reviewed in the following.

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Current practices in FWD forward models

Multi-layered elastic theory (MLET) and finite element method (FEM) based forward models are the main methods that have been used to develop back-analysis techniques [9,10,11,26,39,51]. MLET-based forward models are more time efficient but less accurate compared to FEM based models, due to a series of simplification assumptions in material models and loading conditions. On the other hand, FEM based forward models can be computationally costly and lead to an inefficient back-analysis technique, particularly for large problems with complex geometries [26,39,42,53]. To overcome the abovementioned problems, several artificial intelligence-based solutions, as an alternative, have been proposed [7,13,31,37,49,55]. These works include employing an artificial neural network (ANN) as a surrogate forward model to calculate ground surface deflections that occurs due to a drop load.

Available approaches for optimisation analysis

An optimisation analysis aims to find the best fit between the predicted deflections (computed through a forward model) and the measured deflections (obtained from the FWD tests) through optimisation and search methods [1,4,32,40,45]. For this purpose, various classic optimisation methods such as iterative and regression based technique have been utilised [27,34]. These methods suffer from various limitations, such as computational insufficiently and dependency on the seed modulus per each iteration. The latter can increase the possibility of trapping in local minima [14]. Moreover, although regression-based techniques have a fast calculation time, they can lead to inaccurate results.

Recently, different metaheuristic optimisation techniques such as genetic algorithms (GA) [43,46,46,57,62], particle swarm optimisation (PSO) [22,21,45], differential evolution (DE) [24], shuffled complex evolution (SCE) [22,21] and the Lévy ant colony optimisation (ACO_{RL}) [56] have been employed. Among these metaheuristic optimisation techniques, the GA has become a widely used method due to its robustness in global searches, overcoming the local minima problem. The GA can also use a parallel search method over the search space, which makes it comparable to the DE, PSO, and SCE methods.

Several studies have proposed a combined back-analysis technique that is based on integrating ANN and a metaheuristic optimisation method, to back-analyse layer moduli and thicknesses in pavements [12,13,19,21,23,25,38,48,48,54]. The results of these studies indicate that the combined technique can achieve more accurate results and offers a computationally efficient approach.

Despite various back-analysis techniques and studies that have been developed for pavement condition assessments, there have only been a couple of studies that directly focus on railway applications. Burrow et al. [9] employed FWD test data to back-calculate the layer moduli of railway track substructure layers. In their study, to identify the track substructures' layer moduli, a manual trial-and-error process was performed through a parametric study of layer moduli. Although they reported promising outcomes, their study cannot be extended to other railway subsurface scenarios. This is due to the computational demanding nature of their trial-and-error approach. Additionally, their method was highly relied on the user's experience to define seed moduli values. Recently, Haji Abdulrazagh et al. [26] developed an iterative back-analysis technique based on the rail falling weight test (RFWT) and ground falling weight test (GFWT). They used the MLET-based forward model for a limited number of track substructure layers (up to three layers). The accuracy of their results highly depends on seed modulus values per each iteration. This technique suffers from computational insufficiency due to the running of multiple forward simulations during the optimisation process (reported 900 s).

In this paper, a novel hybrid ANN–GA back-analysis technique was developed to estimate railway track substructures' layer moduli using FWD testing data. In this technique, the power of ANN including its ability to predict an accurate value for surface deflections combined with a robust optimisation search algorithm in order to overcome (i) the issues associated with local minima, (ii) computation time, (iii) dependency on the seed moduli and (iv) number of substructure layers.

To achieve this, a surrogate forward model using an ANN was developed to work as an alternative for finite element (FE) simulations (forward models). In the next step, a GA was employed to determine the optimal values of the ANN's inputs (i.e., layer moduli values). Validation and verification study phases were performed to evaluate the performance, robustness and accuracy of the proposed model.

Methodology

Overview

An ANN surrogate forward model was developed using a synthetic database generated by a series of FE simulations. The ANN model was chosen due to its computational efficiency and accuracy; it was used to map the deflection basin based on the corresponding layer moduli. To this effect, firstly FE model of a FWD test on a railway section near Leominster (UK) section was developed and validated against data from literature using COMSOL Multiphysics software. This site was chosen because it offered access to field data (core penetration test (CPT)) that can be used for initial layer moduli estimation as well as final validation of the technique. This FE model was then used as a basis to generate an extensive databank with various and wide ranges of layer moduli, Poisson's ratio, and the thickness of the layers.

In the next step, an optimisation problem was formulated where a GA was linked to the ANN to minimise the differences between ANN predictions and FWD test measurements and find the optimum values of the substructure's layer moduli. Two steps validation study was performed for the developed technique. First, the back-analysed layer modulus was compared with the CPT field data. Next, the back-analysed layer moduli were implemented in the FE model and the FE calculated deflections were compared with measured deflections from the experimental FWD test. Apart from this two steps validation study, the proposed technique's results were further validated against another back-analysed method from literature.

Track geometry and the FWD test

To develop the FE model, a track system consists of a loaded sleeper, two ballast layers, two clay layers (subgrades 1 and 2), and a sand and gravel layer (subgrade 3), was considered (see Fig. 1). The geometry of the railway track section and the FWD test data was taken from Brough et al. [8] and Burrow et al. [9], respectively.

In the FWD test employed in railways, a load is typically applied through a 1.1 m loading beam that applies a 125 kN load to both ends of the sleeper. This load is approximately equal to that of a single axle train passing at high speed [9]. A set of geophones, which are located on the sleeper and ballast surface at various distances from the centre of the loading beam, is used (Fig. 2).

Fig. 2 illustrates a side view of the loading plate and the geophones arrangement in the FWD test configuration for railway tracks. In this figure, the horizontal distance from the centre of the loading plate is shown by n (n is in mm). D_0, is the deflection at the first geophone which is located at the centre of the loading plate. D_{300}, D_{1000} and D_{1500} represent the deflection at 300 mm, 1000 mm and 1500 mm offset from the centre point which are referred as data points in this study as well. It should be noted that geophone 1 was located on the sleeper where the rest were located on the ballast surface. These geophones are used to measure movement velocities caused by the drop load. The measurement taken by the first geophone at the first data point (D_0) is the sleeper velocity and the rest of the geophones measure the ground velocity. These velocities are then converted into deflections by integration concerning time. Fig. 3 illustrates the loading plate and geophone

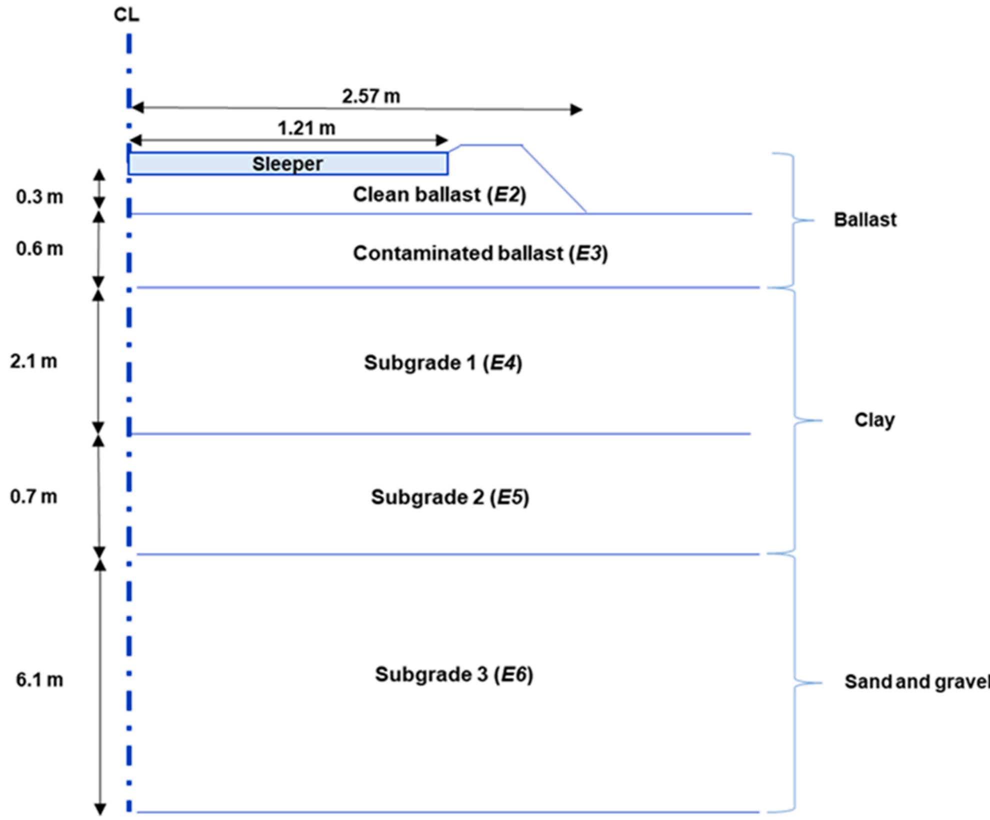


Fig. 1. A cross-section of the railway track section near Leominster station, UK.

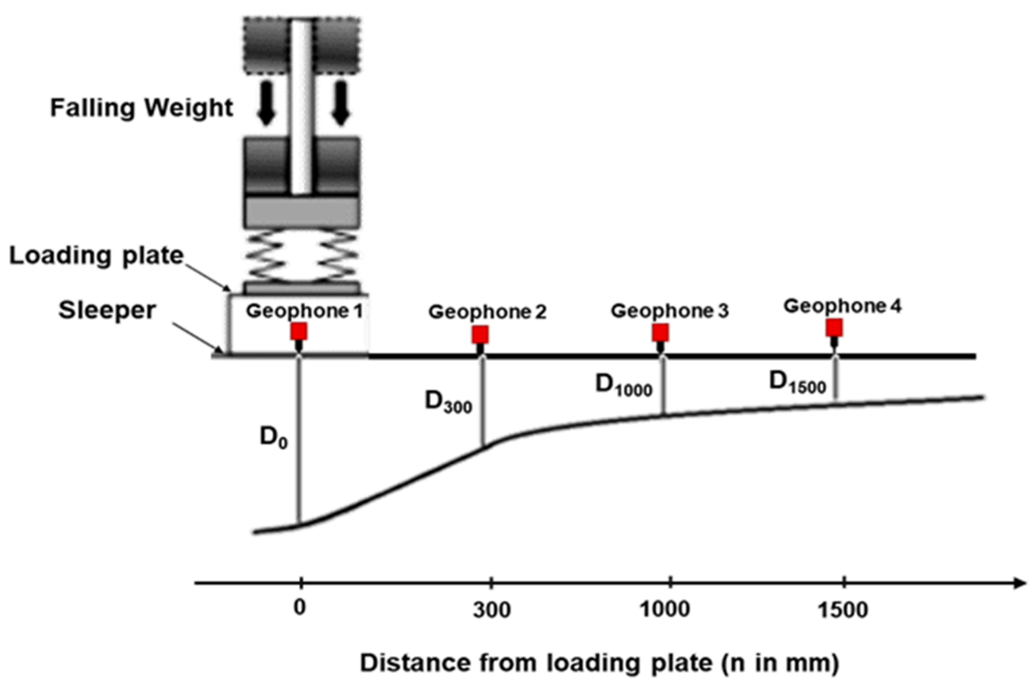


Fig. 2. Arrangement of the geophones in a FWD test for railway application [9].

arrangements in the FWD test configuration.

FE model of the railway track substructure

A three-dimensional model of the problem was developed using COMSOL Multiphysics software. Taking advantage of symmetrical nature of the problem, including the geometry, and loading conditions, only a quarter of the system was modelled (Fig. 4(a-b)).

Deflections in all three directions (x, y and z) were fixed at the bottom of the model. Symmetry boundary condition was applied to both the near-end perpendicular and left-side plane parallel to the track direction. In addition, the roller boundary condition was assigned to both the

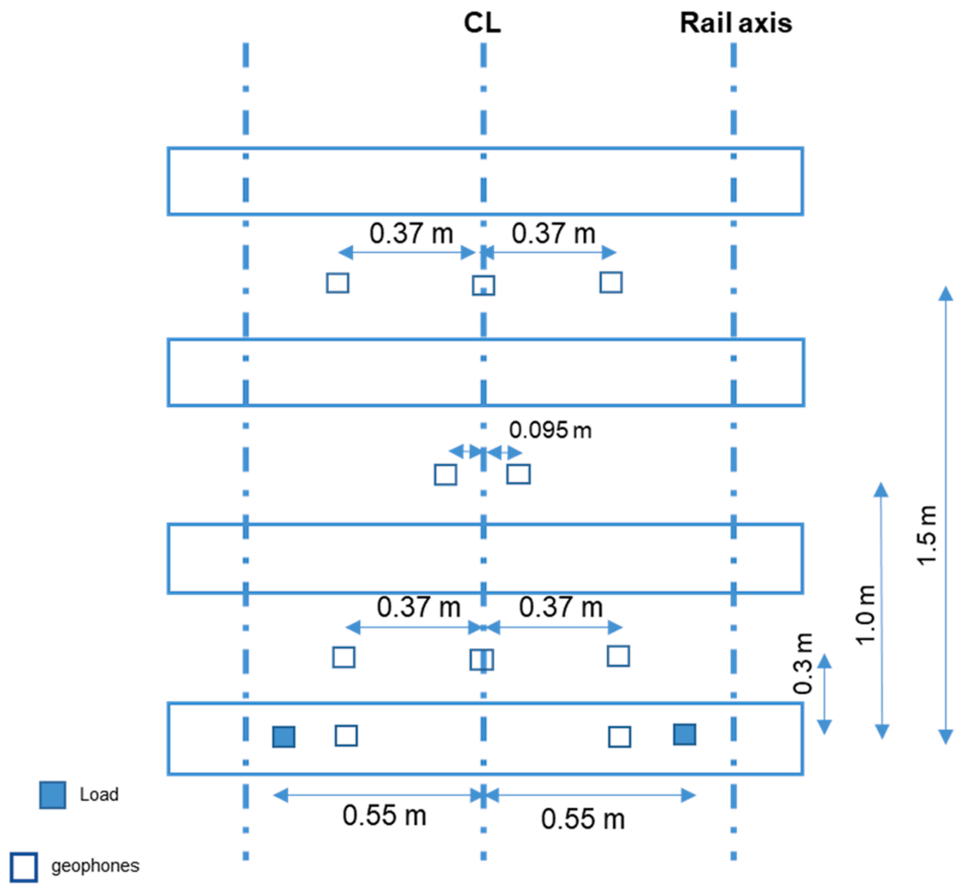


Fig. 3. FWD Loading and geophone configuration [9].

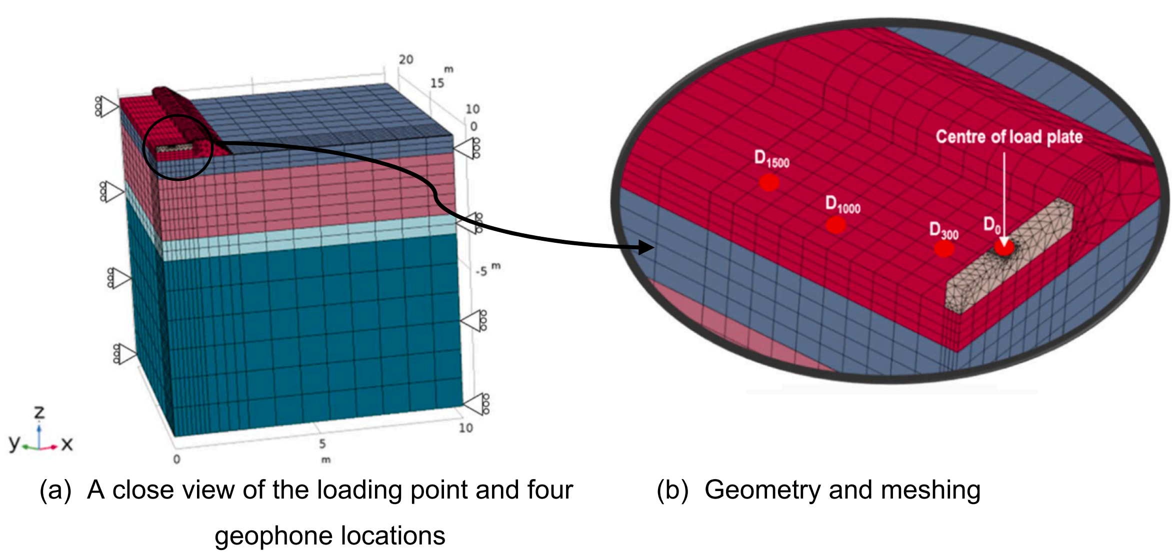


Fig. 4. (a-b): (a) Geometry and meshing of the FE model; (b) A close view of the loading point and the geophones.

right side and the far end of the model boundaries [16].

This model is considered sufficiently large to address the shear wave reflection from the outer boundaries that occurs due to the dynamic loading condition in this analysis. The size of the model was calculated by considering the subgrade's shear wave velocity and the analysis time. Based on this, a geometry of 10 m × 10 m × 10 m was modelled which is a suitable dimension to prevent the wave from being reflected by the outer boundaries [17].

A dynamic FWD pulse load was defined by a haversine function with a duration of 40 ms (ms). It was applied at the same location as the geophone 1 (see Fig. 2). The magnitude of the load was considered to be 31.25 kN, which is a quarter of the FWD load magnitude defined in the UK standard. The load was applied to the sleeper through a 30 cm diameter circular load plate, and the centre of the plate was modelled at 0.55 m from the sleeper centre (Fig. 3).

Fig. 4 also shows the typical finite element mesh employed to model the system. Quadratic brick elements were used to represent ballast, subballast, and subgrade layers, excluding the slope side of the ballast in which tetrahedral elements were employed.

In this study, a linear elastic material model was employed to describe the soil layers. This assumption is reasonable due to the small deflection values recorded by the geophones, as such the material behaviour remains in the elastic zone [52]. The available FWD and CPT data from a railway track section near Leominster station in Herefordshire, UK, have been used in the development the FE model [8,9]. The CPT data were used in the development of the FE model to identify the thicknesses of layers (Fig. 1) as well as layers moduli (Table 1). More details regarding the mechanical properties of the track materials that were used to develop the FE model is shown in Table 1.

FE model results and validation

To validate the developed FE model, the deflection basins calculated by the FE model were compared with the FWD testing data acquired at the railway track section near Leominster station, Herefordshire, UK and reported by Burrow et al. [9]. The results of this validation are shown in Fig. 5a-d.

It can be observed that the calculated (FE model) deflection-time history follows a similar trend to that of the measured data (Fig. 5(a-d)). Moreover, the deflection values predicted by the FE model shows a close agreement with the FWD experimental data for all four data points (D_0, D_{300}, D_{1000} and D_{1500}), with percentage error less than 10 %.

Both peak deflections calculated by the FE model and recorded by the FWD experiment were compared and their difference were presented using error (%) (Table 2). This error at data point 1, data point 2, data point 3 and data point 4 were 6.961 %, 9.473 %, 7.821 % and 5.378 %, respectively. In the scale of this study, these error values (which are less than 10 % and have an average of 7.4 %) confirm the accuracy and significant agreement of the FE model predictions with the experimental data.

Table 1
Mechanical properties of the sleeper and track substructure used in the FE model [8,9].

Layer	Property	Value	Reference
Sleeper	Layer modulus, E (GPa)	20.7	Brough et al. [8]
	Poisson's ratio, ν	0.15	Burrow et al. [9]
	Density, ρ (kg/m^3)	2,500	Burrow et al. [9]
	Damping ratio	0.05	Haji Abdulrazagh et al. [26]
Clean ballast (ballast 1)	Layer modulus, E (MPa)	110	Brough et al. [8]
	Poisson's ratio, ν	0.2	Burrow et al. [9]
	Density, ρ (kg/m^3)	1,700	Burrow et al. [9]
	Damping ratio	0.05	Wehbi et al. [66]
Contaminated ballast (ballast 2)	Layer modulus, E (MPa)	32.5	Brough et al. [8]
	Poisson's ratio, ν	0.49	Burrow et al. [9]
	Density, ρ (kg/m^3)	1,800	Burrow et al. [9]
	Damping ratio	0.05	Wehbi et al. [66]
Subgrade 1	Layer modulus, E (MPa)	71.83	Brough et al. [8]
	Poisson's ratio, ν	0.49	Burrow et al. [9]
	Density, ρ (kg/m^3)	1,900	Burrow et al. [9]
	Damping ratio	0.05	Wehbi et al. [66]
Subgrade 2	Layer modulus, E (MPa)	33.96	Brough et al. [8]
	Poisson's ratio, ν	0.49	Burrow et al. [9]
	Density, ρ (kg/m^3)	1,900	Burrow et al. [9]
	Damping ratio	0.2	Wehbi et al. [66]
Subgrade 3	Layer modulus, E (MPa)	362.1	Brough et al. [8]
	Poisson's ratio, ν	0.49	Burrow et al. [9]
	Density, ρ (kg/m^3)	1,800	Burrow et al. [9]
	Damping ratio	0.2	Wehbi et al. [66]

Parametric analysis

Layer moduli are the most significant parameters which are used to analyse the railway track surface deflections under loadings[33]. Thus, understanding the extent of their effect on the surface deflections is crucial for development of a deflection-based back-analysis technique. To this aim, a parametric analysis using the developed FE model were carried out to investigate which layer modulus/moduli have more impact on the deflections.

A uniformly distributed non-random range of values for all five-substructure layers moduli (i.e., E_2, E_3, E_4, E_5 and E_6) was considered. The range of moduli considered for each layer for the parametric analysis in addition to all initial modulus (used in the FE model- see section 2.3) are presented in Table 3. The presented ranges in Table 3 varies from 2 % to 8 % difference (by increment of 2 %) with respect to the initial layer modulus. For each set of the layer moduli variation, the railway track deflections at the four data points (see Fig. 4a) (i.e., D_0, D_{300}, D_{1000} and D_{1500}) were calculated using the FE model.

In order to presents parametric analysis results, a parameter called percentage discrepancy of the surface deflection was introduced. This parameter is defined to describe the difference between the peak deflections at four different data points obtained from the FE using initial layers moduli and other variations of layer moduli.

Fig. 6a–d presents the percentage discrepancy of the surface deflections caused by substructure layer moduli variations corresponding to 2 %, 4 %, 6 % and 8 %. For example, $E_2 + 2$ % scenario means that all layer moduli from E_3 to E_6 are equal to their initial values except for E_2 which has higher value.

Generally, layer modulus (E_4) (subgrade 1) variations have the highest impact on the track surface deflections, while variations in E_5 and *E6* have less percentage deflection discrepancy. Moreover, this parametric analysis revealed that the peak deflections obtained from the first two geophones (corresponding to the data points of D_0 and D_{300}) are more affected by the moduli of the layers which are close to the surface.

Development of a hybrid ANN–GA back-analysis technique

Synthetic database generation

There are very limited FWD test data relevant to railway track applications. Due to this limitation, the developed FE model was employed to generate a total of 536 sets of deflection basin data. Due to the importance of the layer moduli compared to other parameters in railway track substructure condition assessment [33], only the substructure's layer moduli (E_2 to E_6) were considered as variables. Moreover, other studies showed that Poisson's ratio has negligible impact on the pavement system response [36,64,68]. Therefore, constant values of Poisson's ratio were assumed for each layer [28] (Table 4). The range of layer moduli for each layer was determined based on recommendations from the literature [33]. To achieve an efficient computational time for data generation, the LiveLink COMSOL with MATLAB was used. This database contains 100 uniformly distributed random values over the defined range of layer moduli for each layer (including the maximum and minimum values – see Table 4), in addition to a set of uniformly distributed non-random values obtained from the available CPT data.

The peak surface deflections at four different data points (see Fig. 2) served as the network outputs. Details of each layer including thickness, Poisson's ratio, and layer moduli range used to generate the database, are presented in Table 4.

Data division and pre-processing

In order to avoid over-fitting problems and improve the data generation ability of the ANN, a cross-validation method was employed. Also, the database was divided into three subsets: training data, testing data, and validation data [41,61]. Accordingly, 80 %, 10 % and 10 % of the 536 sets of synthetic data were considered for the training, validation and testing processes, respectively. The training dataset was used to

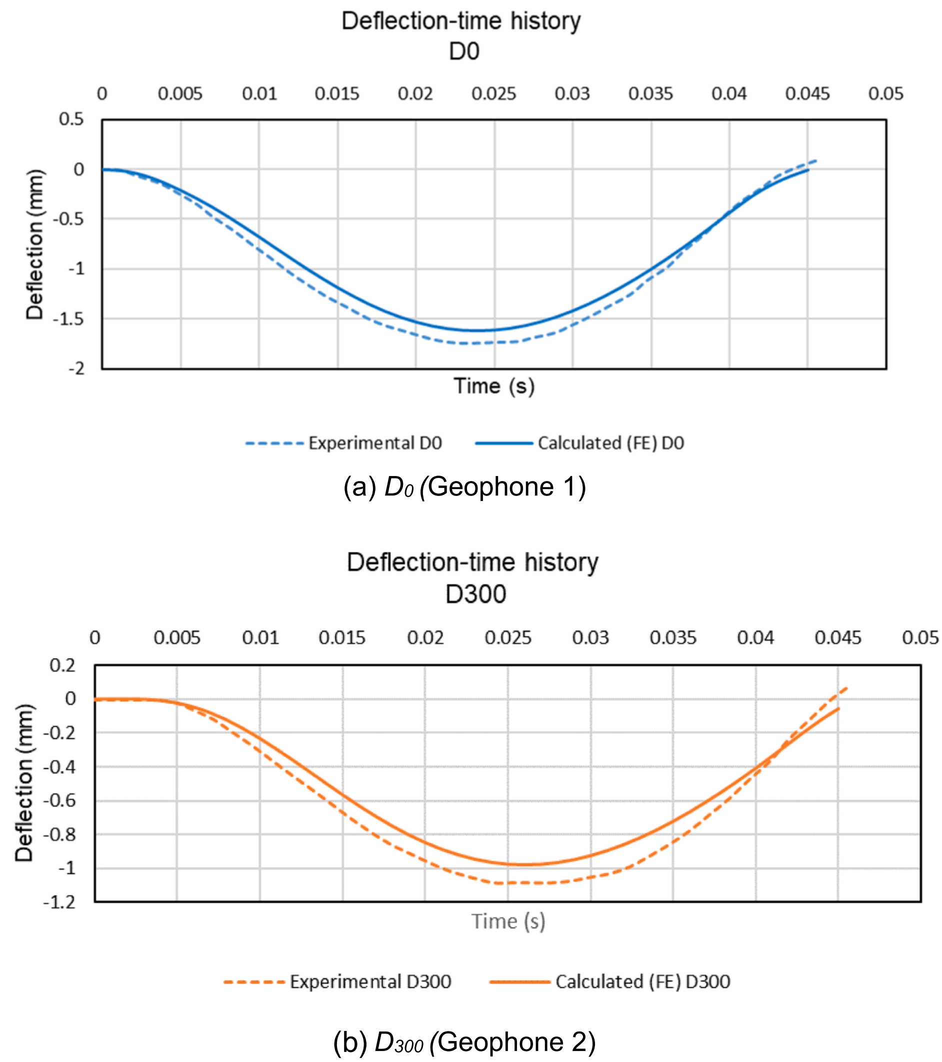


Fig. 5. (a-d): Experimental FWD deflection-time histories versus calculated deflection-time histories at (a) *Geophone 1*; (b) *Geophone 2*; (c) *Geophone 3*; and (d) *Geophone 4*.

adjust the weights and biases, while the testing dataset which contains the unseen data, was employed to evaluate the performance and accuracy of the final model. Once the training was completed, the performance of the trained network was examined using the validation dataset. Upon data division, all the input–output values were normalised into the [-1,1] range to train the network to pay equal attention to all variables. This normalisation increases the efficiency of the training process [5,30].

ANN architecture

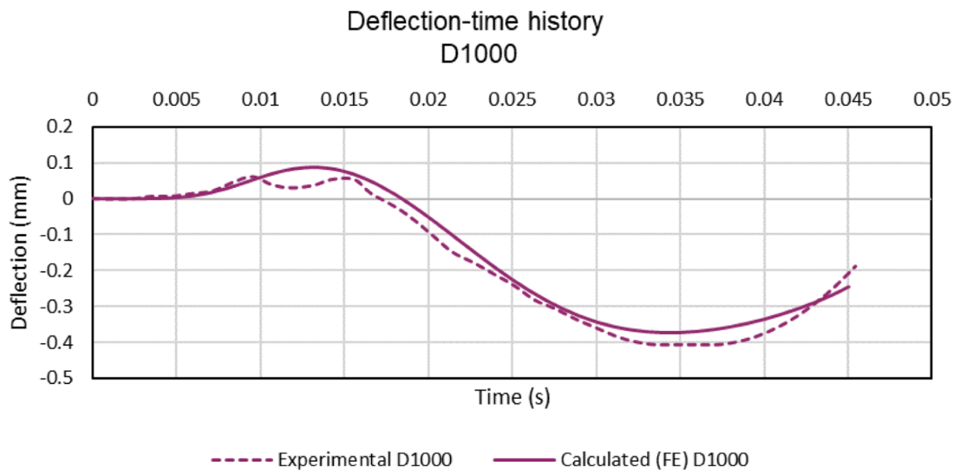
From various available ANN architectures, the multi-layer perceptron (MLP) feedforward neural network was chosen. This type of ANN is the most commonly used ANN in civil and pavement engineering applications [2,3,67].

The MLP feed-forward network has one input, one output layer and at least one hidden layer [19]. Each layer consists of basic particles called artificial neurons, which are set up in different layers. These elements are interconnected, and any of these connections have a specific synaptic weight. The number of neurons in the input layer corresponds to the number of mechanical properties of each layer (such as layer moduli, Poisson's ratio), layer thickness and the FWD loading magnitude. The number of output layer neurons corresponds to the number of predefined geophones at different offsets from the load point in the FWD test (Fig. 2). The optimal network architecture of 19–6–5–4–4 was selected for a five-layer track system. This network structure was chosen through a trial-and-error process, based on the lowest value of the mean squared error (MSE). It is worth mentioning that the minimum number of neurons in the hidden layers improves the performance of the developed ANN surrogate forward model by decreasing both the computation time and the probability of overfitting in the network [58]. However, there is no precise method used to define the number of neurons in the hidden layer.

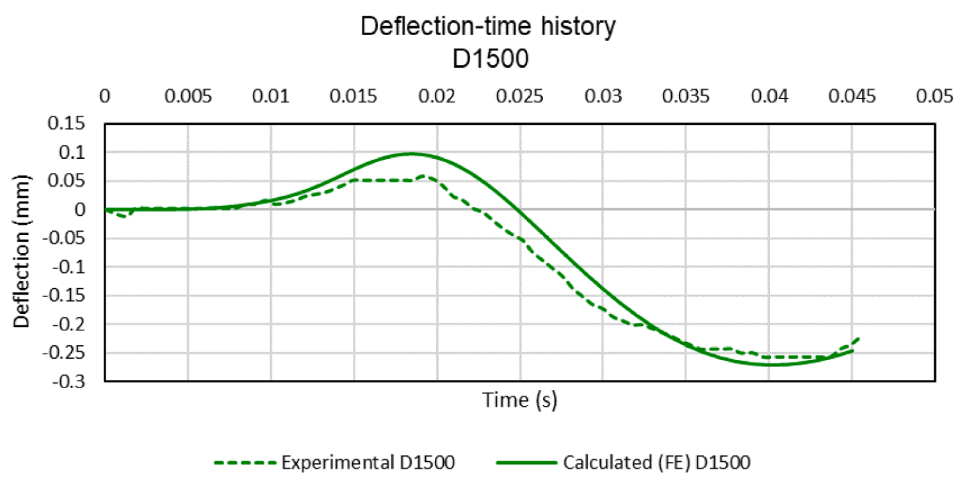
The next step, after assigning the number of network layers and the number of neurons in each layer, is to determine the transfer function in the ANN model. In this study, the tan-sigmoid transfer function was used between hidden layers and a linear function was employed to transfer data to the output layer. These two types of transfer functions are typical functions that have been highly recommended and widely used in ANN training [30,38,49,65].

ANN training

The Levenberg–Marquardt (LVM) algorithm was adopted for the selected network. This method is one of the most popular search methods, due to its speed and stable convergence and its efficient implementation in MATLAB [6,15,63]. The initial synaptic weights and biases for ANN training were chosen randomly by the LVM



(c) D_{1000} (Geophone 3)



(d) D_{1500} (Geophone 4)

Fig. 5. (*continued*).

Table 2
Comparisons between the peak deflections measured by FWD and predicted by FE model.

Data point	Maximum surface deflection (mm)		Error (%)
	Measured (experiment)	Calculated (FE model)	
D_0	−1.737	−1.616	6.961
D_{300}	−1.083	−0.980	9.473
D_{1000}	−0.406	−0.374	7.821
D_{1500}	−0.257	−0.270	5.378

Table 3
Tested layer moduli of the railway substructure for parametric analysis.

Layer	Layer moduli	Initial layer moduli (MPa)	Layer moduli values used in parametric study (MPa)
Clean ballast (ballast 1)	E_2	110	112.20, 114.40, 116.60, 118.80
Contaminated ballast (ballast 2)	E_3	32.5	33.15, 33.80, 34.45, 35.10
Subgrade 1	E_4	71.83	73.27, 74.70, 76.14, 77.58
Subgrade 2	E_5	33.96	34.64, 35.31, 36.00, 36.70
Subgrade 3	E_6	362.1	369.34, 376.58, 383.83, 391.07

backpropagation method. During this process, the ANN was trained to predict deflections by adjusting weights and biases. In the ANN training via the backpropagation method, the value of the MSE was checked to assess the network performance. The best validation performance observed was 1.034e-7 at epoch 600. Once the training process was completed, its performance was checked through Root Mean Square Error (RMSE), MSE and coefficient of correlation (R). Figs. 7a–d to 10a–d show the predicted deflection results for the ANN surrogate forward model at four different data points (Fig. 2). R, in Fig. 7b, 8b, 9b and 10b is the correlation between predicted deflections (ANN outputs) and measured FWD deflections, which is approximately equal to 1 for all outputs. The ANN surrogate forward model for D_0, D_{300}, D_{1000} and D_{1500} has the MSE values of 1.2271e-07,1.7628e-07, 6.2147e-08 and 6.3088e-08, respectively. In addition, for data pints of D_0, D_{300}, D_{1000} and D_{1500}, the value of RMSE were 3.503e-04, 4.1986e-04, 2.4929e-04 and 2.5117e-04, respectively (Figs. 7c, 8c, 9c and 10c). The high value of R in addition to the low values of MSE and RMSE confirm the great performance and accuracy of the 19–6–5–4–4 ANN model configuration. Although the ANN model presented in this study was developed using an extensive set of databases and have shown to perform well with a high accuracy, it should be noted that the model is limited to the range of data

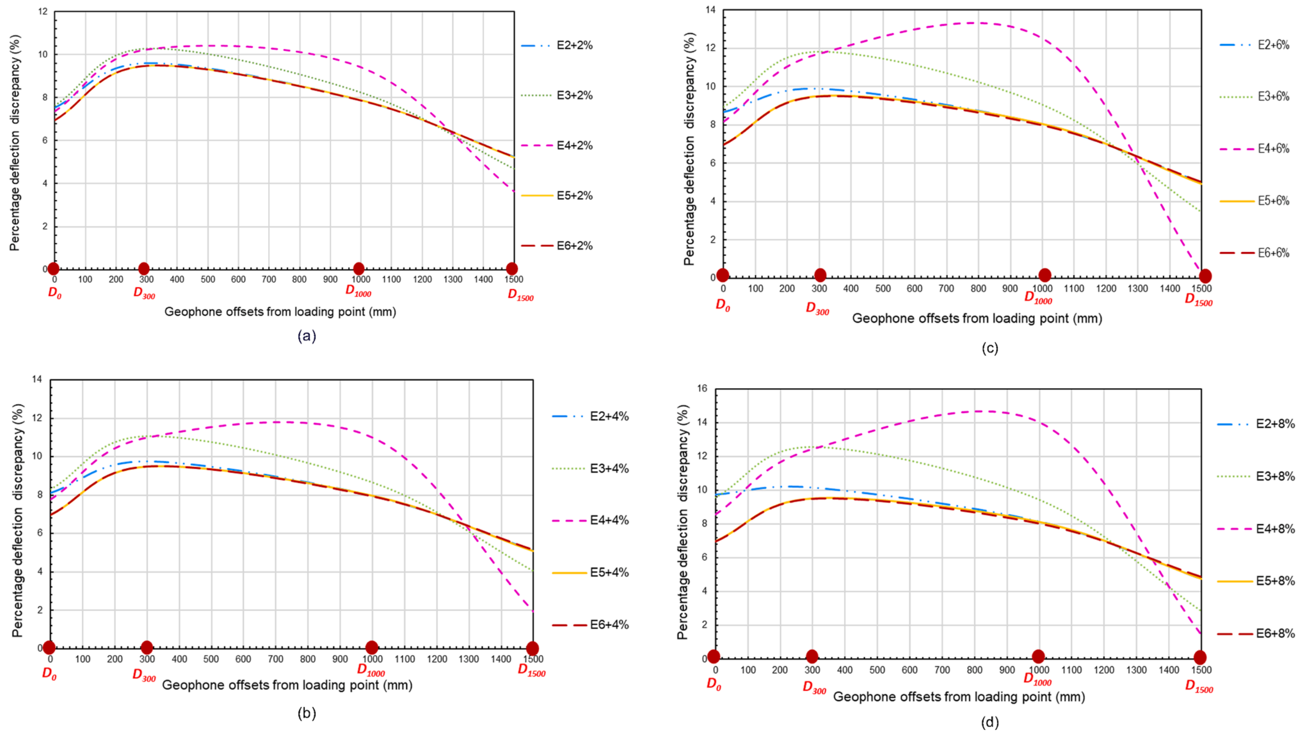


Fig. 6. (a-d): Percentage deflection discrepancy for various geophone offsets (a) + 2 % variation in E_2 to E_6; (b) + 4 % variation in E_2 to E_6; (c) + 6 % variation in E_2 to E_6; and (d) + 8 % variation in E_2 to E_6.

Table 4
The input values and range (used to generate the ANN training database).

Railway track system	Layers	Thickness (mm)	Poisson's ratio	Range of layer moduli (MPa)
Five-layer system	Clean ballast (E_2)	300	0.2	70–170
	Contaminated ballast (E_3)	600	0.49	20–50
	Subgrade 1 (E_4)	2,100	0.49	50–100
	Subgrade 2 (E_5)	700	0.49	15–50
	Subgrade 3 (E_6)	6,100	0.49	100–400

that were included within the training and any attempt to use the model for data outside such range may lead to inaccuracy/errors in results.

Development of a hybrid ANN–GA technique

A single-objective GA, as an optimisation tool, was integrated with the ANN surrogate forward model. The use of a GA as an optimisation technique can eliminate the need to define layer seed moduli at the beginning of each iteration. The GA was employed to find the optimum value for each substructure layer moduli. Integration of a GA and ANN has been proven to offer a robust solution to complex optimisation problems [29]. Defining the objective function is a crucial part of solving this optimisation problem. A deflection-based objective function, shown in Equation (1), was chosen to solve the current optimisation problem [19,24]:

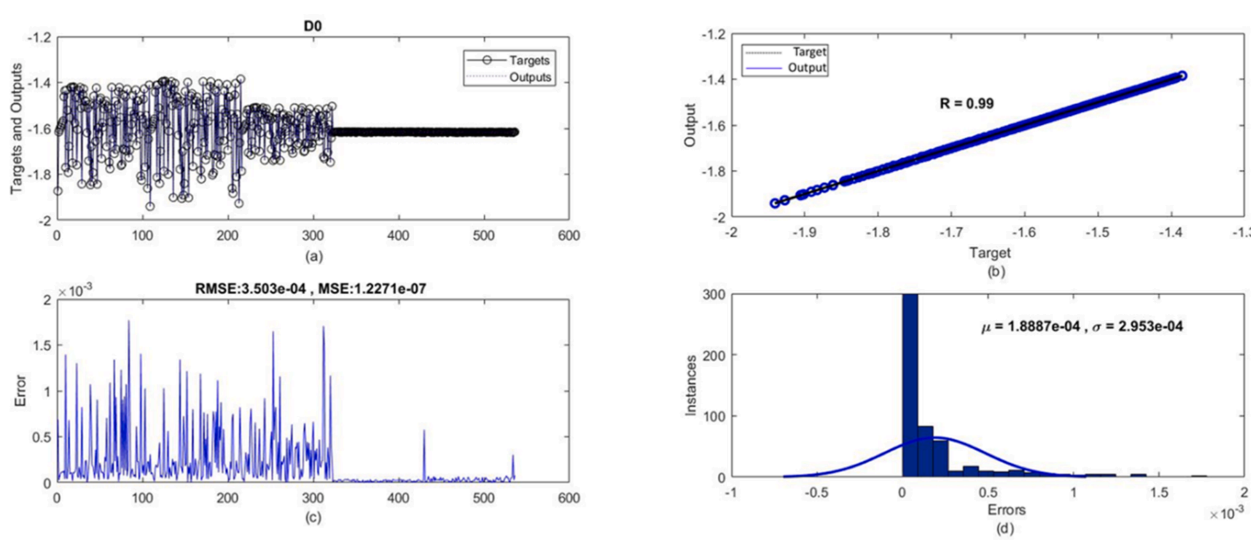


Fig. 7. (a-d): ANN prediction accuracy for track surface deflection at data point *D0*.

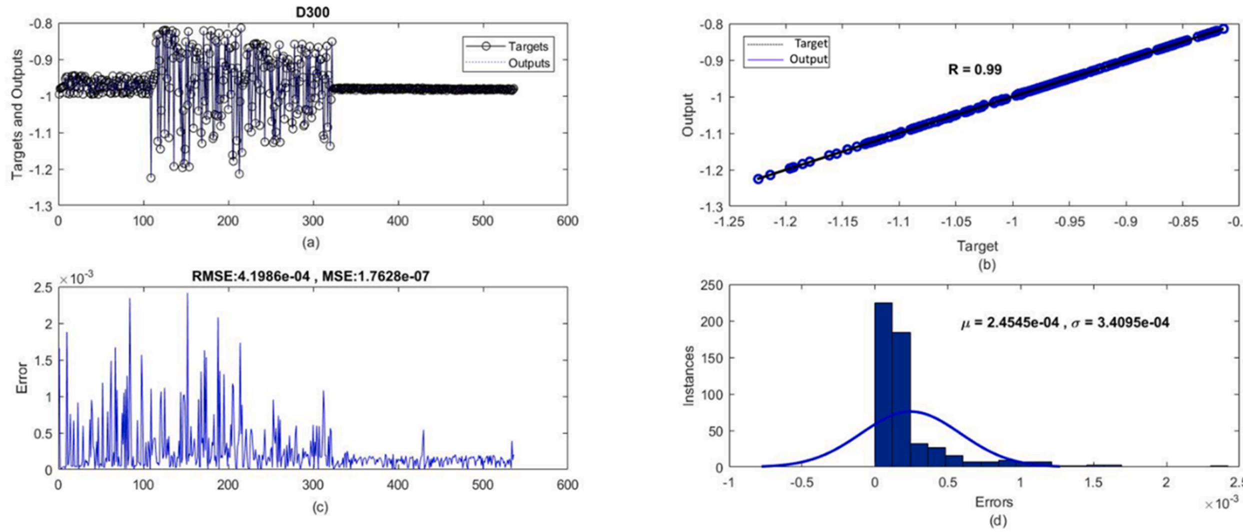


Fig. 8. (a-d): ANN prediction accuracy for track surface deflection at data point D_{300}.

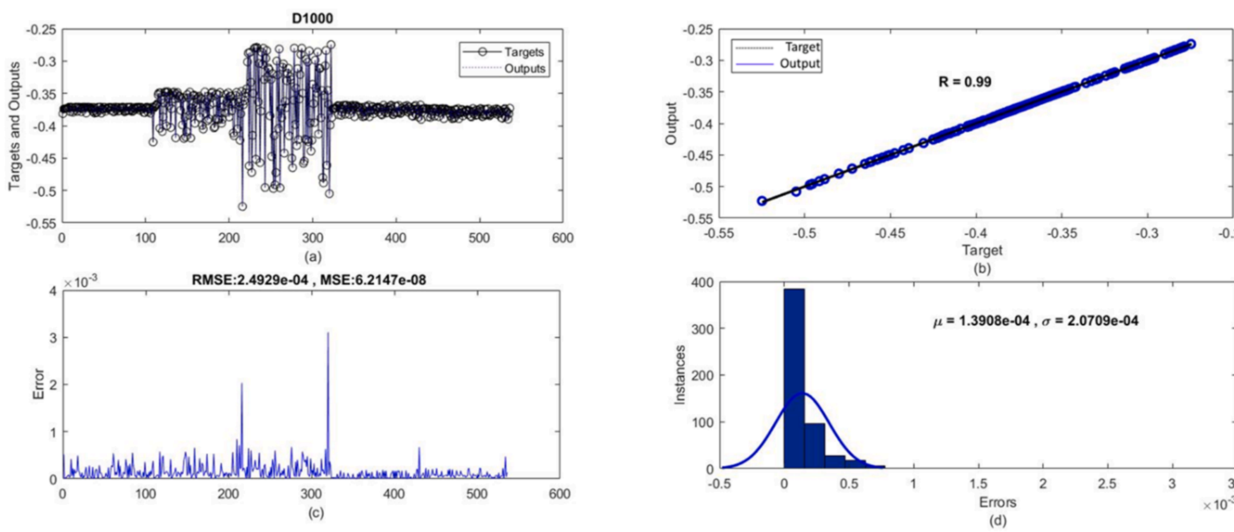


Fig. 9. (a-d): ANN prediction accuracy for track surface deflection at data point D_{1000}.

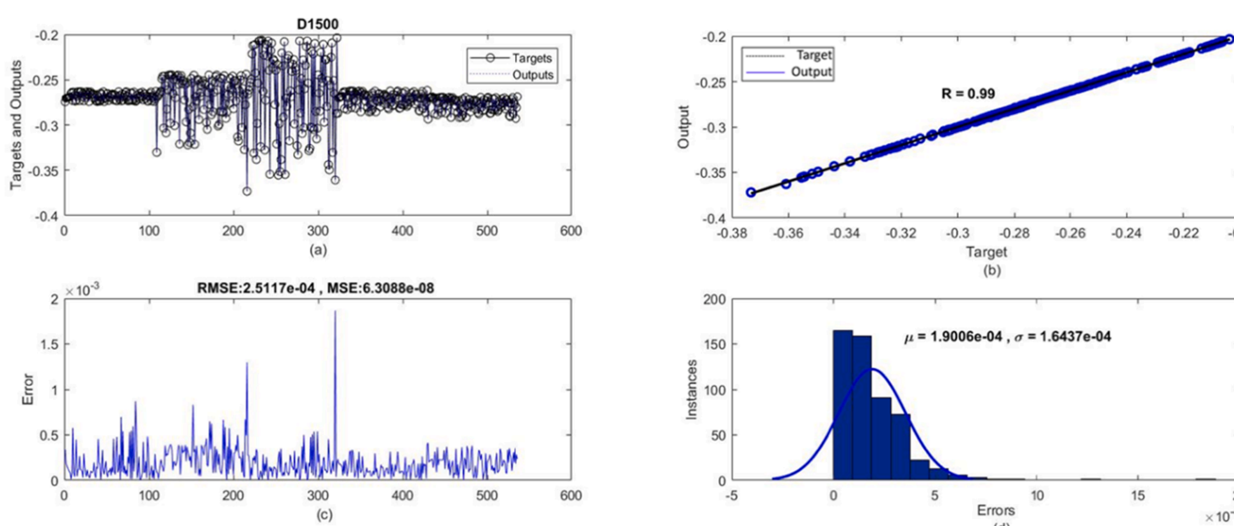


Fig. 10. (a-d): ANN prediction accuracy for track surface deflection at data point D_{1500}.

$$Cost\ \ Function = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left((D_i - d_i)/d_i\right)^2} \times 100 \quad (1)$$

Where D_i and d_i are the predicted and measured values of the deflections, respectively, and n is the number of measurement points. To adjust the population and generation size as GA parameters, various population sizes and numbers of generations were investigated to find the optimum values for these parameters in the current problem. The result of this analysis is presented in Fig. 11. This figure shows that 200 generations and a population size of 350 were selected for the current problem.

Mutation and crossover probabilities are two other parameters of GA optimisation; these were defined based on the literature [50]. The adjusted GA parameter values for this study are presented in Table 5.

The details of the developed ANN–GA back-analysis technique are given in a flowchart (Fig. 12). The GA optimisation process starts with generating an initial set of random estimations for the variables of the problem, i.e., parent generation or layer moduli. Each parent generation will be passed to the trained ANN surrogate forward model to calculate the surface deflections. The values of the surface deflection are returned to the GA to evaluate the objective function (see Equation (1)). Through the defined GA's operators (crossovers and mutations) the first generation of offspring values are then produced from the first parent. At the end of each step of the offspring generation process, these solutions are evaluated through the ANN surrogate forward model and defined objective function to check the termination criteria which was assumed maximum allowable RMSE of 10 %. Through this iterative process of fitting predicted and measured track surface deflections, the GA optimisation algorithm finds the optimum layer moduli and maps relationships between inputs (layer moduli) and outputs (maximum deflection at four offsets from the load applying point 0, 300, 1000 and 1500).

The back-analysis technique results and validation

Table 6 presents the back-analysed results of layer moduli for a five-layer track substructure, as well as provides the target layer moduli obtained from the CPT data. Details of the percentage error in this table, will be employed to further evaluate the developed ANN-GA technique, and will be presented later.

The error values revealed that the back-analysis technique underestimates the elastic modulus of the clean ballast, contaminated

Table 5
GA parameters.

Parameter	Value
Population size	350
Generation size	200
Crossover probability	0.85
Mutation probability	0.01

ballast and top subgrade layer (subgrade 1 – see Fig. 1) by 5.75 %, 14.97 % and 4.66 %, respectively. In addition, the minimum error value obtained for subgrade 1 (i.e., 4.66 %) confirms the accuracy of the ANN–GA technique. This result is in good agreement with the parametric study carried out in section 2.5. Moreover, the estimated layer moduli values for subgrades 2 and 3 were overestimated by 41 % and 7.39 %, respectively. However, based on the parametric analysis, the high value of the percentage error corresponding to the subgrade 2 layer modulus (E_5) could be due to the lower sensitivity of surface deflection to the subgrade 2 layer modulus.

A validation study was conducted to further check the performance and accuracy of the developed technique. To this aim, the back-calculated layer moduli (Table 6) were implemented in the FE model of the problem to calculate deflections at different offsets (data points). Both the calculated deflections (FE model results) using back-analysed data and measured deflections (FWD test data) in addition to their errors are presented in Table 7.

The higher value of percentage error for D_{1500} (Table 7) is consistent with the results of the parametric analysis which *geophone 4 (located at 1500 mm from the loading point)* was the least affected geophone by the substructure's layer moduli variation (i.e., there was a weaker correlation). In other words, the back-analysis showed that the impact of layer moduli is more pronounced on D_0, D_{300} and D_{1000} and have negligible effect on D_{1500}. This may be due to the greater distance between D_{1500} and the loading point, which shows that more meaningful information is provided by deflections closer to the loading point. A potential point for consideration is that an FWD experiment for railway substructure condition assessment can be performed using only first three geophones (i.e., D_0, D_{300} and D_{1000} data points). The results show that the average error between the predicted deflections using the back-analysed data and the measured data from the FWD test for all sensors is around 4 %. Moreover, the RMSE was 6.1 %. This error value is acceptable and

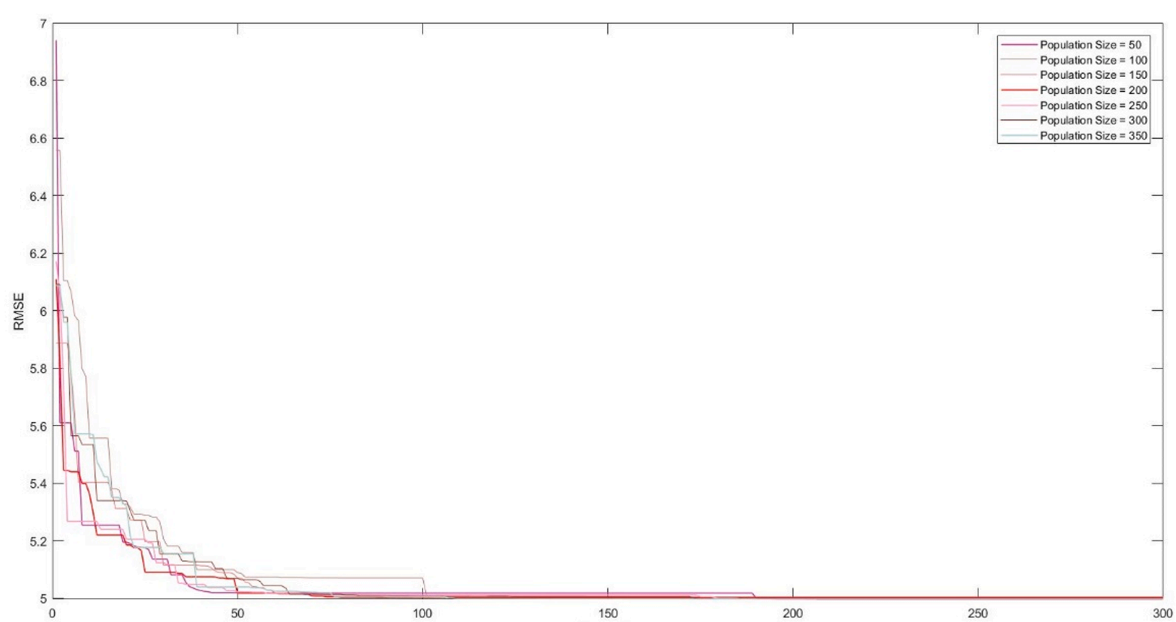


Fig. 11. Pre-analysis of the population size parameter in the GA optimisation for the developed ANN–GA back-analysis technique.

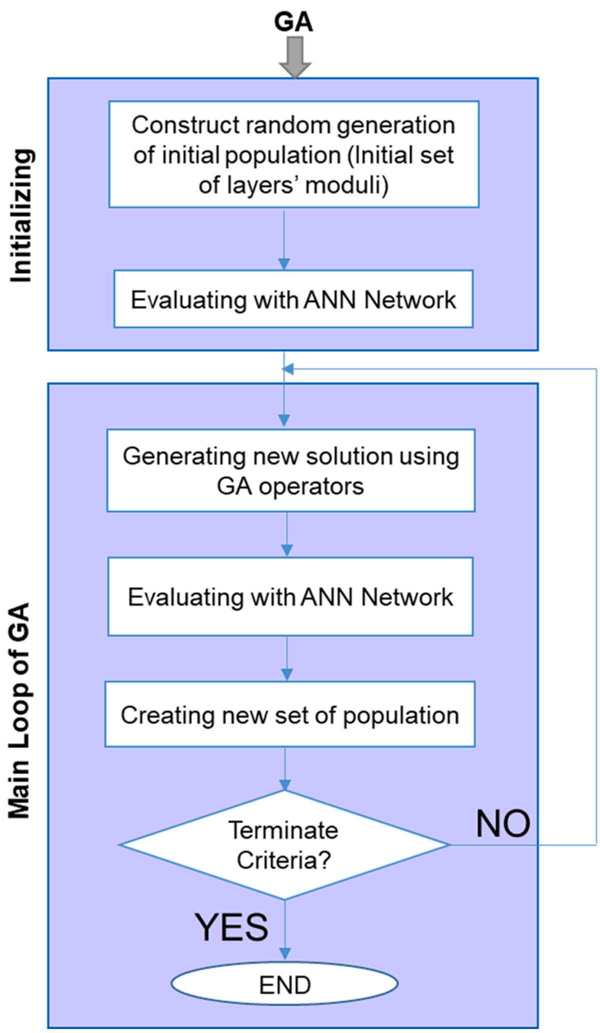


Fig. 12. Developed ANN–GA back-analysis technique flowchart.

Table 6
ANN–GA back-analysed layer moduli for a railway section near Leominster station, UK.

Layers	Back-analysed layer modulus (MPa)	Target layer modulus (MPa)	Error (%)
Clean ballast (E_2)	103.671	110.000	5.75
Contaminated ballast (E_3)	27.635	32.500	14.97
Subgrade 1 (E_4)	68.485	71.829	4.66
Subgrade 2 (E_5)	47.905	33.956	41.08
Subgrade 3 (E_6)	388.849	362.100	7.39

Table 7
deflection obtained from the FE model using back-analysed data vs FWD data.

Data points	Measured deflection (mm)	FE deflection based on ANN–GA back-analysis output (mm)	Error (%)
D_0	−1.737	−1.763	1.543
D_{300}	−1.083	−1.073	0.954
D_{1000}	−0.406	−0.399	1.628
D_{1500}	−0.257	−0.288	12.075

comparable with the maximum variation criterion in pavement structures' back-analysis technique which is 10 % and shows high accuracy for the proposed technique [35].

The method was further validated, and the predicted layer moduli were compared with the estimated values reported by Burrow et al. [9]. Table 8 presents these values in addition to errors. These errors were calculated for both reported data from the literature Burrow et al. [9] ($Error_1$) and predicted by the proposed method in this study ($Error_2$) with respect to the reported CPT test. It should be noted that, in Burrow et al. [9]'s study for modelling purposes, the configuration of substructure layers was idealised and they considered one more subgrade (clay layer) called extra subgrade in Table 8. However, the layer configuration assumed in this study is based on the CPT results (soil profile) reported by Brough et al. [8] performed on the same location as Burrow et al. [9]'s study, at railway track section near Leominster station. The results in Table 8 show that, the average error in the predictions by developed method is significantly lower (about 50 % less) than the layer moduli's estimations by Burrow et al. [9].

Summary and conclusions

In this research, a time-efficient, systematic, and accurate hybrid back-analysis technique for condition assessment of railway track substructure layers was developed based on FWD test. This technique has no dependency on the seed moduli and the number of track substructure layers. To develop this technique, firstly a finite element model of a railway track section near Leominster station, UK, under FWD loading condition was developed and validated. By considering the back-analysis problem as an optimisation problem, a hybrid back-analysis technique, consisting of a well-trained ANN and GA, was developed. The ANN was employed to map the relationship between substructure layer moduli and surface deflection at four different offsets (0 mm, 300 mm, 1000 mm and 1500 mm) from the FWD loading point. The results of the FE model were then used to generate a database for the ANN surrogate forward model training. In addition, a GA was integrated into the ANN surrogate forward model to determine the optimum values of the layer moduli as ANN inputs and thereby optimise the deflection-based objective function.

Incorporating an ANN surrogate forward model with GA as a *meta*-heuristic optimisation algorithm to find the optimal set of layer moduli values for track substructure layers produces an efficient and robust technique. In addition, the proposed technique has no dependency on the seed moduli and number of track substructure layers, making it suitable for employment in railway track systems. The validation results of the developed technique confirms its excellent performance. It offers high accuracy in prediction of layer moduli particularly for the clean ballast layer, subgrade 1 and subgrade 3, with less than a 10 % error. Also, the validation study through the comparison of measured deflections and calculated ones based on the back-analysed layers' moduli shows the RMSE value of less than 10 %.

Moreover, the developed hybrid back-analysis technique designed for a five-layer railway substructure, unlike previously developed techniques that can only accommodate a limited layer substructure. It is worth mentioning that the developed technique can address another limitation of the conventional back-analysis technique: namely, the computation time. Using ANN as a surrogate forward model and

Table 8
Further validation of the developed technique.

Layer	CPT data	Burrow's results	$Error_1$ (%)	This study results	$Error_2$ (%)
Ballast 1	110	130.96	19.05	103.67	5.75
Ballast 2	32.50	26.19	19.42	27.64	14.97
Subgrade 1	71.829	65	9.50	68.49	4.66
Subgrade 2	33.956	15.85	53.32	47.91	41.08
Extra Subgrade	N/A	3.17	N/A	N/A	N/A
Subgrade 3	362.1	224.5	38.00	388.85	7.38
Ave error (%)			27.86		14.77

metaheuristic optimisation algorithm for the hybrid technique, resulted in the back-analysis process being completed in 458.9 s. This computation time is considerably less than that the required time for other techniques in the literature. The following conclusions are highlighted:

- The ANN model with low values of RMSE, MSE, and high R values was demonstrated to be a reliable forward model for analysis of railway track sections under FWD testing conditions and to calculate surface deflections.
- The developed ANN-GA hybrid back-analysis technique provides a systematic framework to interpret the FWD data and estimate substructure layers' moduli.
- The developed back-analysis technique is a computationally efficient method which estimates railway substructure layers' moduli in 458.9 s, with no dependency on the seed moduli.
- The back-analysed data obtained from the ANN-GA technique, show low percentage errors (less than 10 %) for clean ballast, subgrade 1, and subgrade 3 layers' moduli.
- The estimated modulus of subgrade 1 with a minimum percentage error of 4.7 % shows the robustness of the developed ANN-GA technique. As it is consistent with the high sensitivity of the surface deflections (calculated by the FE model) to the top subgrade layer modulus variations.
- The result of the developed technique was found to be effective in matching the peak deflections with the developed FE model by an average percentage error and RMSE values of 4 % and 6.1 % (less than 10 %) respectively.
- The results of the parametric analysis suggest that an FWD experiment for railway substructure condition assessment can be performed using only three geophones (D_0, D_{300} and D_{1000}). The reason is layer moduli variations showed negligible impact on the deflections obtained from the geophone 4 which is located at 1500 mm distance from the loading point.

CRediT authorship contribution statement

Shadi Fathi: Conceptualization, Methodology, Software, Validation, Visualization, Writing – original draft. **Moura Mehravar:** Conceptualization, Methodology, Supervision, Funding acquisition, Validation, Writing – review & editing. **Mujib Rahman:** Visualization, Investigation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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