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A Note on Short-Time Response of Two-Dimensional Lattices During Dynamic Loading

S. Mastilovic *

Faculty of Construction Management, Union University

Cara Dusana 62-64, 11000, Belgrade, Serbia

ABSTRACT: The disordered two-dimensional lattices are used extensively to study damage evolution and fracture of inhomogeneous or multi-phase systems. The present note addresses their initial elastic response during dynamic loading. Namely, a transition from short-time values of modulus of elasticity and Poisson's ratio to respective long-time values, which is not accompanied by the corresponding change of stiffness tensor components. The study is performed on three two-dimensional truss-type lattices. It is demonstrated that the difference between the two sets of elastic properties is a result of combining effects of the initial lateral inertia and the disorder of the system.

KEY WORDS: lattice, disorder, short-time response, modulus of elasticity, inertial effects

INTRODUCTION

The disordered two-dimensional (2D) lattices are used extensively in the last two decades to study various aspects of damage evolution and failure of inhomogeneous or multi-phase systems (e.g., Hansen et al., 1989; Curtin and Scher, 1990; Monette and Anderson, 1994; Karihaloo et al., 2003; Liu et al., 2007). The recent statistical approach to damage mechanics relies crucially on the lattice simulations (e.g., Rinaldi et al., 2007, 2008). A thorough review of lattice models in micromechanics is presented by Ostoja-Starzewski (2002). The novelty of the present investigation is limited to an explanation of a detail of the dynamic response of the 2D triangular and square lattices: a transition from short-time (*st*) to long-time (*lt*) elastic properties, which is not accompanied by the corresponding change of stiffness tensor components.

Although the following discussion refers to the triangular lattice with the first-neighbor central interactions, two other 2D lattices considered in this investigation, the triangular lattice with the second-

* Tel.: +381 11 218-0287, e-mail: gmvv@eunet.yu.

neighbor central interactions and the square lattice with the second-neighbor central interaction, behave in the same manner.

LATTICE SIMULATIONS

The particular model used herein is described in a number of papers published over the last decade (see for instance, Mastilovic et al., 2008, and references therein); the only, but noteworthy, difference from those previous studies is that the present simulation setup does not include the application of the initial velocity field that cancels out the inertial effects. Thus, to put it as succinctly as possible, the lattice is formed by "continuum particles" located at lattice nodes (1000×1201) and connected by nonlinear springs characterized by a critical elongation defining rupture. The lattice is geometrically disordered since the equilibrium distances between particles (λ_0) are sampled from the Gaussian distribution within the range $\alpha\,\bar{\lambda} \leq \lambda_0 \leq (2-\alpha)\,\bar{\lambda}$, which takes into account, for instance, the distribution of grain sizes. The lattice is also structurally disordered. The link stiffnesses (k) are uniformly distributed within the range $\beta\,\bar{k} \leq k \leq (2-\beta)\,\bar{k}$ that encompasses the inherent or induced flaw structure of the material. (The model parameters $\bar{\lambda}$ and $\bar{k}$ are the mean values of the link lengths and stiffnesses.) For the purpose of the present note it is sufficient to introduce the geometrical-order parameter α, $(0 \leq \alpha \leq 1)$, and the structural-order parameter β, $(0 \leq \beta \leq 1)$, as measures of divergence from the ideal lattice ($\alpha = 1$, $\beta = 1$).

The natural (inhomogeneous) tension simulation mimics the corresponding laboratory experiment. The displacement-control mechanical load, $\dot{y} = \pm \dot{\varepsilon} L/2$, applied on the specimen boundaries ($y = \pm L/2$), is transferred by stress waves through the specimen, which results in the characteristic "staircase" time history of the local stress. Note that L is the sample length, $\dot{\varepsilon} = \dot{L}/L$ is the prescribed strain rate, and the coordinates refer to the centroidal coordinate system.

The time histories of the normal stress (in the loading direction) divided by the corresponding strain rate and the Poisson's ratio are presented in Figure 1. It is evident from Figure 1(a) that the two characteristic slopes are the short-time and long-time moduli ($E_{st}^{(\varepsilon)}$, $E_{lt}^{(\varepsilon)}$, respectively[1]) observed by Mastilovic et al. (2008), presented herein in a more convenient graph. The solid curve in Figure 1(a) is common for all loading rates that remain elastic for the given graph limits; departures from that curve mark onsets of the inelastic deformation that is characterized by damage nucleation and, depending on the loading rate, different levels of cooperative phenomena (Mastilovic et al., 2008). Thus, the only difference between the moderate and the high loading rates is that in the later case specimens fail before the

[1] The apparent plane-strain modulus of elasticity $E^{(\varepsilon)}$ and the apparent plane-strain Poisson's ratio $\upsilon^{(\varepsilon)}$ will be, for brevity, referred to as the modulus of elasticity and the Poisson's ratio, hereinafter.

$E_{st}^{(\varepsilon)} \rightarrow E_{lt}^{(\varepsilon)}$ elastic transition occurs, which explains the unchanged stiffness tensor components, in spite of the apparent change of the modulus of elasticity (and the Poisson's ratio).

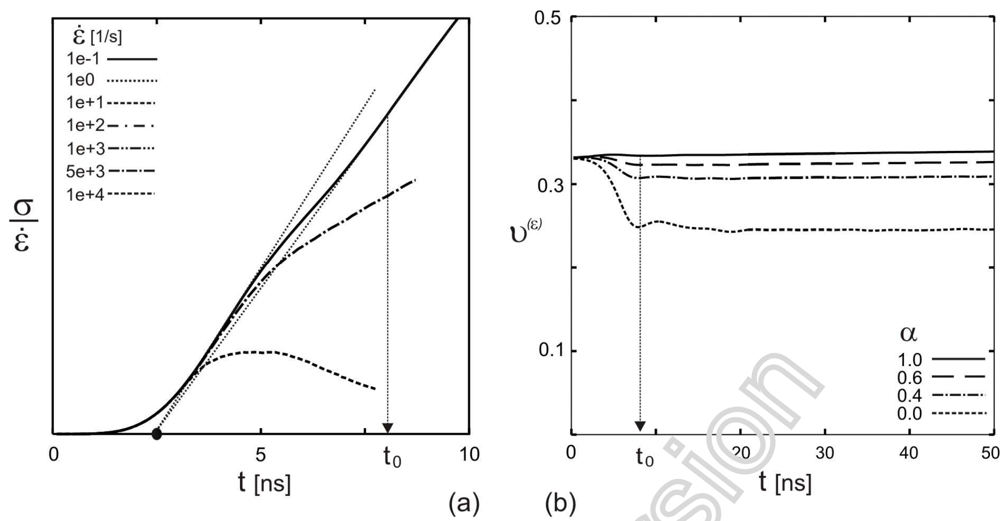

Figure 1. *Time history of: (a) the stress normalized by the corresponding strain rate, and (b) the Poisson's ratio for the tension test of the structurally and geometrically disordered sample.*

The influence that inertia exerts on the modulus of elasticity is reflected by $(E_{st}/E_{lt})^{(\varepsilon)} > 1$ even for the ideal lattice. The stiffness components, the elastic moduli, and the Poisson's ratio are weakly ($\approx 5\%$) affected by moderate geometrical disorder $(0.4 \leq \alpha \leq 1)$. On the other hand, the stiffness components and the Poisson's ratio are insensitive to the structural disorder (i.e., β-independent), while the effect on the moduli of elasticity is similar to that of the geometrical disorder.

The simulation results further indicate that for the ideal triangular lattice with the first-neighbor central interactions

$$E^{(\varepsilon)} = \begin{cases} E_{st}^{(\varepsilon)} = C_{11}^{(\varepsilon)} & t < t_0 \\ \\ E_{lt}^{(\varepsilon)} = 8\,C_{11}^{(\varepsilon)}/9 & t \geq t_0 \end{cases} \tag{1}$$

while for the moderately disordered lattice the expressions hold only approximately (hence "=" should be replaced by "≈" under the curly brace) with the degree of approximation driven by the level of disorder.[2] The expression for $E_{lt}^{(\varepsilon)} = 8\,C_{11}^{(\varepsilon)}/9$ corresponds to the analytical solution (Monette and Anderson, 1994) for the lattice in question.

The $E_{st}^{(\varepsilon)} \rightarrow E_{lt}^{(\varepsilon)}$ transition time is

[2] The first expression in Equation (1) is obviously valid only away from the two transient phases depicted in Figure 1(a).

$$t_o \approx 2\pi \frac{l_c}{C_0} = T_0 \qquad (2)$$

where T_0 is the period of harmonic undamped oscillations corresponding to the natural frequency $\omega_0 = C_0/l_c$ of the discrete system, $l_c = \overline{\lambda}$ the model resolution length, and C_0 the velocity of the elastic longitudinal wave propagation. The transition time is independent of the aspect ratio and size of the specimen used in simulations.

According to Figure 1(b), the similar elastic transition is observed for the Poisson's ratio: it changes from the analytical value $1/3$, characteristic of the ideal triangular lattice with the first-neighbor central interactions (Monette and Anderson, 1994), to a long-time value dependent on the geometrical disorder. Again, the dependence is relatively moderate for moderately geometrically disordered lattice: the difference in $\upsilon^{(\varepsilon)}$ is $\approx 5\%$ for the change in α from 1 to 0.4. The more substantial $\upsilon^{(\varepsilon)}$ reduction is observed for the strongly geometrically disordered lattice ($\alpha = 0.02$); the lower bound of $\upsilon_{lt}^{(\varepsilon)}$ settles at $1/4$. On the other hand, the structural disorder, defined by parameter β, has no effect on $\upsilon^{(\varepsilon)}$.

CONCLUSIONS

At first look, the $E_{st}^{(\varepsilon)} \rightarrow E_{lt}^{(\varepsilon)}$ elastic transition might appear reminiscent of the difference between the adiabatic and the isothermal moduli of elasticity, but one should recall that the $E_{st}^{(\varepsilon)} \rightarrow E_{lt}^{(\varepsilon)}$ elastic transition occurs at all loading rates investigated herein, reaffirming that all physical events are dynamic on some scale. Similarly, it is well known that the dynamic modulus of elasticity measured by, for instance, the Ultrasonic Pulse Method, somewhat exceeds the quasi-static counterpart. The Ultrasonic Pulse Method is based on the relationship $E = \rho C_0^2$ between the modulus of elasticity and velocity of longitudinal wave propagation under 1D stress conditions. For the ideal triangular lattice with the first-neighbor central interactions $(\alpha = 1, \beta = 1)$[3], this relation yields the analytical result $E^{(\varepsilon)} = 57.7$ (Monette and Anderson, 1994), which agrees remarkably well with $E_{lt}^{(\varepsilon)}$ obtained in the simulations.

In view of this, two sources of discrepancy between the elastic properties can be identified:

- the lateral inertia of the system, which in the case of natural (inhomogeneous) simulation setup and the moderately disordered lattice governs the observed elastic transition $E_{st}^{(\varepsilon)} \approx C_{11}^{(\varepsilon)} \rightarrow E_{lt}^{(\varepsilon)}$.
- The geometrical and structural disorder, which influences the $E_{st}^{(\varepsilon)} \rightarrow E_{lt}^{(\varepsilon)}$ elastic transition (including the ratio $E_{st}^{(\varepsilon)}/E_{lt}^{(\varepsilon)}$) independently of the inertial effects.

[3] The ideal triangular lattice is characterized by $\rho = 2m/\sqrt{3}\,l_c^2 = 2/\sqrt{3}$ and $C_0 = 7.07$; the latter is obtained from a simulation of the low-velocity Taylor (rigid-anvil) test with 7×181 lattice representing a slender rod.

Finally, it is demonstrated that the lattice behavior is equivalent to the behavior of a solid under the plane-strain conditions only as long as the lattice is ideal (that is, in absence of disorder). As soon as the disorder is introduced, the behavior of a solid could only be approximated by the behavior of the 2D systems, with the degree of accuracy that decreases with the level of disorder "quenched" within the lattice.

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