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COMBINED RECOIL AND THRESHOLD RESUMMATION FOR HARD SCATTERING CROSS SECTIONS

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We discuss the simultaneous resummation of threshold and recoil enhancements to partonic cross sections due to soft radiation. Our method is based on a refactorization of the parton cross section near its partonic threshold. It avoids double counting, conserves the flow of partonic energy, and reproduces either threshold or recoil resummation when the other enhancements are neglected.

1 Introduction

A large class of hard-scattering cross sections in QCD may be factorized into convolutions of parton distributions and fragmentation functions with hard-scattering functions. [1] The latter are computed in perturbation theory. Intermediate infinities associated with virtual and emitted soft gluons cancel in their higher-order corrections, but finite remnants, assuming the form of plus-distributions and delta-functions, may lead to large enhancements when integrated against the smooth functions in the convolutions. In physical terms, soft gluon radiation affects the hard-scattering process by reweighting the relative importance of near-threshold partonic subprocesses to the physical cross section, and by providing the final state with overall transverse momentum Q_T through its recoil. When the conservation of energy is taken into account, the sign of the cumulative effect is *a priori* unclear: enhancement effects of recoil may be offset by the suppression due to the extra energy required to produce the desired final state plus recoil radiation. A combined and consistent analysis is required [2] and in what follows we sketch our approach to this problem [3,4].[a]

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2 Electroweak annihilation

These processes are characterized at lowest order by $ab \to V$, with a, b partons and V an electroweak final state of mass Q and transverse momentum $\mathbf{Q}_T$. Because of their relative simplicity and phenomenological interest, these processes have been studied intensely for resummation purposes. Partonic threshold is at $z \equiv Q^2/\hat{s} = 1$ and $\mathbf{Q}_T = 0$, and the singular functions to be resummed are plus-distributions in $1 - z$ and Q_T/Q. Our method generalizes that of Ref. [5], and organizes these distributions in the cross section. This is done by refactorizing partonic cross sections into short-distance functions, $\sigma^{(H)}_{ab}$, sensitive only to the hardest scale, Q, and matrix elements of appropriate operators that absorb all dependence on $(1-z)Q$ and Q_T, to leading power in $1 - z$.

$$\frac{d\sigma_{ab\to V}}{dQ^2 d^2\mathbf{Q}_T} = \frac{1}{S}\,\sigma^{(H)}_{ab\to V}(Q^2, \alpha_s(Q^2))$$
$$\times \int dx_a d^2\mathbf{k}_a\, R_{a/a}(x_a, \mathbf{k}_a, Q)$$
$$\times \int dx_b d^2\mathbf{k}_b\, R_{b/b}(x_b, \mathbf{k}_b, Q)$$
$$\times \int dw_s d^2\mathbf{k}_s\, U_{ab}(w_s, Q, \mathbf{k}_s)$$
$$\times \delta(1 - Q^2/S - (1-x_a) - (1-x_b) - w_s)$$
$$\times\, \delta^2\left(\mathbf{Q}_T - \mathbf{k}_a - \mathbf{k}_b - \mathbf{k}_s\right) + Y_{\rm j}\,, \qquad (1)$$

where

$$R_{f/f}(x, \mathbf{k}, 2p_0) =$$
$$\frac{1}{4\sqrt{2}N_c}\int \frac{d\lambda}{2\pi}\,\frac{d^2\mathbf{b}}{(2\pi)^2}{\rm e}^{-i\lambda x p_0 + i\mathbf{b}\cdot\mathbf{k}}$$
$$\times\langle f(p)|\bar{q}_f\left(\lambda^+, \mathbf{b}, 0^-\right)\gamma^+ q_f(0)|f(p)\rangle \quad (2)$$

is a partonic quark density at fixed energy and transverse momentum, and U_{ab} a purely eikonal function, depending on parton velocities β, and on the soft radiation's energy $w_s\,Q$. The x's and $\mathbf{k}$'s are defined by (2). The delta functions in the triple convolution of (1) relate the singular behavior of the various functions, and Y_j represents the matching to finite order. The all-order behavior of the above functions and the cross section can be analyzed through their eikonalized equivalents. The eikonal cross section can be expressed as an exponent, E_{ab}, an integral over diagrammatically-defined functions referred to as "webs", [6] w_{ab},

$$E_{ab} = 2\int^Q \frac{d^{4-2\epsilon}k}{\Omega_{1-2\epsilon}}$$
$$\times\, w_{ab}\left(k^2, \frac{k\cdot\beta k\cdot\beta'}{\beta\cdot\beta'}, \mu^2, \alpha_s(\mu^2), \epsilon\right)$$
$$\times \left({\rm e}^{-N(k_0/Q) - i\mathbf{k}\cdot\mathbf{b}} - 1\right)\,. \qquad (3)$$

The integral is over the energy and transverse momentum contributed by each web to the final state. The k-dependence of w_{ab} follows from the invariance of the eikonal cross section under rescalings of β and β'.

3 Single-particle inclusive

For definiteness we consider prompt photon production, but the method sketched below is more general [3]. A similar refactorization as in (1) holds for single-particle inclusive cross sections at high p_T. In particular, it contains the same $R_{f/f}$ functions. The arguments supporting this refactorization are somewhat more involved than for (1), but reveal that only initial state radiation contributes to Q_T (the transverse momentum of the $2 \to 2$ parton cms frame) [7]. In the inclusive cross section, final-state interactions require threshold resummation only. In contrast to electroweak annihilation, $\mathbf{Q}_T$ is an unobserved variable, akin to $1 - z$. Thus, the jointly resummed prompt photon p_T spectrum may be written as an integral over $\mathbf{Q}_T$ of a "profile function" [3]

$$P_{ij}(N, \mathbf{Q}_T, Q) = \int d^2\mathbf{b}\, e^{-i\mathbf{b}\cdot\mathbf{Q}_T}\, e^{E_{ij\to\gamma k}} \quad (4)$$

where the exponential exhibits the joint resummation, as

$$\frac{p_T^3 d\sigma^{(\rm resum)}_{AB\to\gamma}}{dp_T} =$$

$$
\begin{aligned}
&\times \sum_{ij} \frac{p_T^4}{8\pi S^2} \int_{\mathcal{C}} \frac{dN}{2\pi i}\, \tilde{\phi}_{i/A}(N)\tilde{\phi}_{j/B}(N) \\
&\times \int_0^1 d\tilde{x}_T^2 \left(\tilde{x}_T^2\right)^N \frac{|M_{ij}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} \\
&\times C_\delta^{(ij\to\gamma k)}(\alpha_s, \tilde{x}_T^2) \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\, \Theta\left(\bar{\mu} - Q_T\right) \\
&\times \left(\frac{S}{4\mathbf{p}_T'^2}\right)^{N+1} P_{ij}(N, \mathbf{Q}_T, Q) \qquad (5)
\end{aligned}
$$

with $\tilde{x}_T^2 = 4|\mathbf{p}_T - \mathbf{Q}_T/2|^2/\hat{s}$, $\tilde{\phi}_{i/A}(N)$ Mellin moments of the parton distributions, $|M_{ij}|^2$ the Born amplitudes, and $\bar{\mu}$ a cut-off restricting $\mathbf{Q}_T$ to sufficiently small values for resummation to be relevant. The $C_\delta^{(ij\to\gamma k)}$ are infrared safe coefficient functions, which include short-distance dynamics at the scale Q. For notational simplicity we have suppressed all factorization and renormalization scale dependence. Cross sections computed on the basis of (5) are shown in Fig. 1 as functions of Q_T at fixed p_T. The kinematics are those of the E706 experiment; [9] see [3] for details of the calculation, in particular those regarding the evaluation of the b-integral in (4). The dashed lines are $d\sigma^{(\mathrm{resum})}_{\mathrm{pN}\to\gamma X}/dQ_T dp_T$, with recoil neglected by fixing $\mathbf{p}_T' = \mathbf{p}_T$, thus showing how each Q_T contributes to threshold enhancement. The solid lines show the same, but now including the true recoil factor $(S/4\mathbf{p}_T'^2)^{N+1}$. The resulting enhancement is clearly substantial. For small p_T, the enhancement simply grows with Q_T, while for p_T above 5 GeV it has a dip at about $Q_T = 5$ GeV, which remains substantially above zero. This makes it difficult to determine $\bar{\mu}$ on this basis alone.

So far, the numerical results given in [3] are primarily to be regarded as illustrations, rather than quantitative predictions. This applies in particular to the resummed Q_T-integrated cross section, shown in Fig. 2 for $p_T \geq 3.5$ GeV and $\bar{\mu} = 5$ GeV. These figures demonstrate, however, the size of the additional enhancement that recoil can produce, and its potential phenomenological impact.

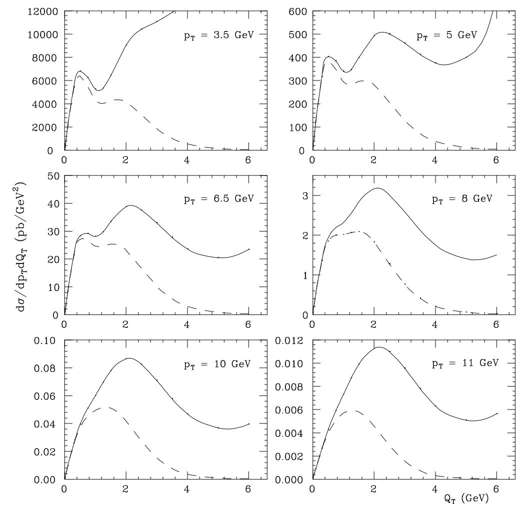

Figure 1. $d\sigma_{\mathrm{pN}\to\gamma X}/dQ_T dp_T$ at $\sqrt{s} = 31.5$ GeV, as a function of Q_T for various values of photon p_T. Dashed lines are computed without recoil ($\mathbf{p}_T' = \mathbf{p}_T$ in (5)), solid lines are with recoil.

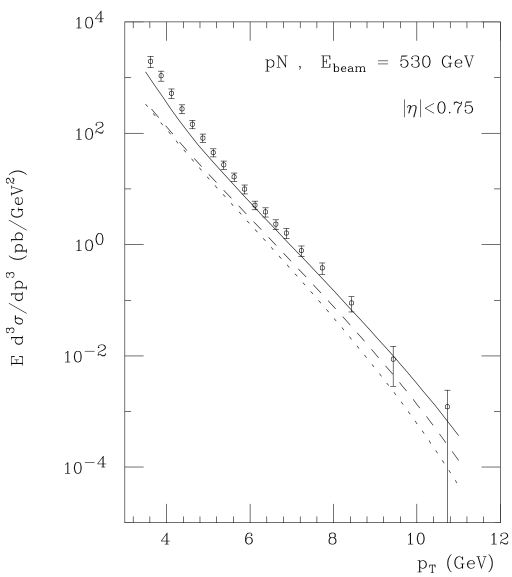

Figure 2. $Ed^3\sigma_{\mathrm{pN}\to\gamma X}/dp^3$ for pN collisions at $\sqrt{s} = 31.5$ GeV. The dotted line represents the full NLO calculation, while the dashed and solid lines respectively incorporate pure threshold resummation [8] and the joint resummation described in this paper. Data have been taken from [9].

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