Quantum origin of dark energy and the Hubble tension

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ABSTRACT

Local measurements of the Hubble parameter obtained from the distance ladder at low redshift are in tension with global values inferred from cosmological standard rulers. A key role in the tension is played by the assumptions on the cosmological history, in particular on the origin of dark energy. Here we consider a scenario where dark energy originates from the amplification of quantum fluctuations of a light field in inflation. We show that spatial correlations inherited from inflationary quantum fluctuations can reduce the Hubble tension down to one standard deviation, thus relieving the problem with respect to the standard cosmological model. Upcoming missions, like Euclid, will be able to test the predictions of models in this class.

1. Introduction

The standard cosmological model, known as ΛCDM, provides a description of the Universe in agreement with a large number of observations, including the cosmic microwave background (CMB), large-scale structure data from galaxy surveys and distance measurements of type-Ia supernovae (SNeIa). However, the values of some of the parameters in the model show tensions when comparing the results obtained by probing different scales. This is particularly relevant for the Hubble parameter, for which the tension has now overcome 4σ.

The Hubble tension within ΛCDM can be described as an incompatibility between H_0 measurements from cosmological probes at early times and values deduced from distance measurements at local scales. For simplicity, we will refer to these two classes of observations as global and local measurements, respectively. Global measurements include studies of CMB anisotropies by the *Planck* mission [1] and observations of the sound horizon scale imprinted by baryon acoustic oscillations (BAO) on the galaxy correlation function (see [2,3] for estimation of H_0 from BAO with a big bang nucleosynthesis prior on baryon density). It is also possible to study the variation in time of the Hubble parameter up to redshift $z \sim 2$ using the so-called cosmic chronometers [4,5], which are galaxies with known time evolution. Local measurements comprehend distances of SNeIa calibrated with Cepheids by the SH0ES collaboration [6] and time delays of multiple images of strongly lensed quasars studied by H0LiCOW [7]. Among

other local probes which have been considered, one can mention calibrating SNeIa with the tip of the red giant branch (TRGB) (finding a lower H_0 compared to Cepheids calibration, see [8]) or with Mira variables [9], measures of distances from surface brightness fluctuations [10] and observations of galaxies hosting megamasers [11]. The Hubble parameter estimated from *Planck* [1] is $H_0 = (67.4 \pm 0.5)$ km/s Mpc^{-1} while the SH0ES collaboration [6] finds $H_0 = (74.03 \pm 1.42)$ km/s Mpc^{-1}, thus resulting in a tension of more than 4σ.

Several ideas have been proposed in order to reconcile these two values. A possible approach consists in introducing new physics such that the Hubble parameter inferred from CMB is larger. Such modifications can affect the cosmological evolution at late times or even the early Universe. As an example of late-time proposal, it has been shown that a phantom dark energy equation of state shifts the Hubble parameter predicted from CMB towards higher values [12]. Other studies consider possible late-time solutions coming from interacting dark energy [13], metastable dark energy [14] or decaying dark matter [15]. Indeed these scenarios give higher values of H_0 when fitting CMB data only, but in general the other independent H_0 measurements coming from BAO and cosmic chronometers leave little room for such late-times modifications, see [16,17]. As for early-Universe proposals, an example is early dark energy [18,19] where a scalar field dilutes faster than matter starting from an initial state frozen by Hubble friction. Apart from fine-tuning difficulties (*e.g.* in the required potential of the scalar field, in its initial conditions or in making early dark energy appear and dilute at the right times), early dark energy still has troubles related to the σ_8 tension between large-scale structures and CMB [20]. Further early-Universe proposals try to reduce the sound speed of the primordial photon-baryon plasma

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before recombination or attempt to make recombination occur earlier [21]. While all these ideas deal with changing the prediction of H_0 from CMB, other approaches focus instead on local measurements by studying how the SNeIa calibration could be affected by new physics, like chameleon dark energy [22] or late-time changes in the effective Newton constant [23].

As an alternative line of thought, in this paper we want to explore the scenario where late-time dark energy originates from quantum fluctuations in the early Universe and we start a study of the consequences expected on the spatial correlations of dark energy today. The analysis will allow us to make quite general predictions on the relief of the Hubble tension within this framework. In such a scenario the Hubble parameter is a stochastic variable and the stochasticity source comes not only from (dark) matter fluctuations due to classical inhomogeneities, but also from the intrinsic quantum nature of dark energy. At time t and for every comoving coordinate $\vec{x}$ the effect of quantum dark energy is such that the expansion rate at that point is an operator satisfying the first Friedmann equation

$$3M_P^2 \hat{H}^2(t,\vec{x}) = \rho_C(t)\mathbb{1} + \hat{\rho}_Q(t,\vec{x}),\tag{1}$$

where $M_P \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass, $\rho_C(t)$ is the classical contribution to energy density, $\mathbb{1}$ is the identity operator and $\hat{\rho}_Q(t,\vec{x})$ is the quantum energy density operator,[1] which can depend on $\hat{H}^2(t,\vec{x})$.

A natural hypothesis for the form of the space dependence of dark-energy correlators is a power law behavior, which is in fact realized in the simple model that we discuss below.

In order to establish whether a dark energy of quantum origin could be enough to relieve the Hubble tension, we evaluate the conditional probability for local measurements (averaged over a spatial volume V_1) of the squared Hubble parameter to find a value above a threshold H_1^2, given that global measurements (averaged over a larger volume V_2 containing V_1) find a value below H_2^2. The square of the Hubble parameter is a natural fundamental variable to consider, as it appears in the Friedmann equation (1). We write the aforementioned conditional probability as

$$\mathrm{P}\left(\left[H^2\right]_{V_1} > H_1^2 \,\middle|\, \left[H^2\right]_{V_2} < H_2^2\right),\tag{2}$$

where $\left[H^2\right]_{V_1} \equiv (1/V_1)\int_{V_1} d^3x\, H^2(t_0,\vec{x})$ and similarly for V_2.[2] The notation t_0 indicates cosmological time today.

The values of H_1 and H_2 are chosen as the mean values of the results from SH0ES and *Planck* mentioned before: $H_1 = 74.03$ km/s Mpc^{-1}, $H_2 = 67.4$ km/s Mpc^{-1}. For the volumes V_1 and V_2, we assume spheres centered around the observer with radii R_1 and R_2 that we specify below.

2. The model

Let us consider a light spectator field during inflation that, for the sake of simplicity, will be taken as a scalar Φ. The key ingredient of the class of theories that we want to study is the assumption that some mechanism determines the effective mass $M^2(t)$ of the spectator field in a cosmological background in such a way that, by the end of inflation, correlators reach super-Planckian

values and that these correlators are then translated into an effective dark energy at late times. A simple realization of such a mechanism is discussed in [24] (see also [25–27]) and consists in a non-minimally coupled massive scalar field with action

$$S[\Phi] = \int d^D x\sqrt{-g}\left\{-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}m^2\Phi^2 - \frac{1}{2}\xi R\Phi^2\right\},\tag{3}$$

where R is the Ricci curvature scalar of the metric $g_{\mu\nu}$ and $g = \det[g_{\mu\nu}]$. It is clear that here the effective mass of the scalar field, $M^2 = m^2 + \xi R$, receives contributions from the bare mass m and from a non-minimal coupling to the metric with coefficient ξ. As predicted in [24], in this model the field correlators of a light scalar grow during inflation and their enhancement is larger when the non-minimal coupling is negative and it dominates over the bare mass. Other mechanisms contributing to an effective mass are conceivable, like dynamical mass generation *via* interaction with another field acquiring a non-zero vacuum expectation value in a symmetry-breaking scenario. In the general case the effective mass is a field operator with a non-trivial derivative structure.

Notice that assuming a quadratic action for the spectator field at the classical level ensures a minimal amount of non-Gaussianity. However, at the level of quantum effective action, interactions with the metric perturbations (or in general with other fields) introduce additional non-Gaussianities in the evolution of the spectator scalar field, *e.g.* by generating effective self-interactions.

We consider a spatially flat FLRW Universe in $D = 4$ spacetime dimensions with scale factor $a(t)$, *i.e.* $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$, where t is cosmological time and $\vec{x}$ are comoving coordinates.

The spectator scalar field is $\Phi(x) = \Phi(t,\vec{x})$ and its action, computed by specializing (3) to the FLRW metric, is of the form

$$S[\Phi] \equiv \int dt d^3x\, \mathcal{L}_\Phi = \int dt d^3x\, a^3 \tag{4}$$
$$\times \left\{\frac{1}{2}\dot{\Phi}^2 - \frac{1}{2a^2}(\vec{\nabla}\Phi)^2 - \frac{1}{2}\left[m^2 + 6\xi(2-\epsilon)H^2\right]\Phi^2\right\},$$

where the dot (˙) denotes the derivative with respect to cosmological time d/dt and we introduced $\epsilon \equiv -\dot{H}/H^2$. After introducing the canonical momentum $\Pi(x) = a^3\dot{\Phi}(x)$, quantization of the model goes in the standard way (see *e.g.* [24]) by promoting field and momentum to operators $\hat{\Phi}$ and $\hat{\Pi}$ obeying canonical commutation relations.

The effect of the scalar field on cosmological evolution is determined by its energy-momentum tensor $T_{\mu\nu} = (-2/\sqrt{-g})\delta S[\Phi]/\delta g^{\mu\nu}$, which in its operator form reads,

$$\hat{T}_{\mu\nu} = \partial_\mu\hat{\Phi}\,\partial_\nu\hat{\Phi} - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\hat{\Phi}\,\partial_\beta\hat{\Phi} - \frac{m^2}{2}g_{\mu\nu}\hat{\Phi}^2$$
$$+ \xi\left[G_{\mu\nu} - \nabla_\mu\nabla_\nu + g_{\mu\nu}\Box\right]\hat{\Phi}^2,\tag{5}$$

where $G_{\mu\nu}$ is the Einstein curvature tensor.

Friedmann equations tell us that quantum fluctuations of the spectator field enter the expansion history of the Universe through expectation values of energy density $\rho_Q \equiv \langle\hat{\rho}_Q\rangle \equiv -\langle\hat{T}^0{}_0\rangle$ and pressure $p_Q\,\delta^i_j \equiv \langle\hat{T}^i{}_j\rangle$, evaluated on a homogeneous and isotropic state that we take to be the vacuum $|\Omega\rangle$. Neglecting spatial gradients of the field, the energy density operator from (5) simplifies to

$$\hat{\rho}_Q(t,\vec{x}) \equiv -\hat{T}^0{}_0(t,\vec{x}) = \frac{H^2}{2}\left\{\left[\left(\frac{m}{H}\right)^2 + 6\xi\right]\hat{\Phi}^2(t,\vec{x})\right.$$
$$\left. + \frac{6\xi}{a^3H}\left\{\hat{\Phi}(t,\vec{x}),\hat{\Pi}(t,\vec{x})\right\} + \frac{1}{a^6H^2}\hat{\Pi}^2(t,\vec{x})\right\}.\tag{6}$$

In [24], the relevant expectation values for Friedmann equations are coincident correlators, $\left\langle\hat{\Phi}^2(t,\vec{x})\right\rangle$, $\left\langle\hat{\Pi}^2(t,\vec{x})\right\rangle$ and $\left\langle\{\hat{\Phi}(t,\vec{x}),\right.$

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[1] In general, only a part of the full quantum energy density operator behaves as dark energy and other terms can act as additional matter contents.

[2] More precisely, we are implicitly defining a classical stochastic variable $H^2(t_0,\vec{x})$, whose statistical properties are the same as those of the quantum operator $\hat{H}^2(t_0,\vec{x})$.

$\hat{\Pi}(t, \vec{x})\}\Big)$, where $\{\hat{A}, \hat{B}\} \equiv \hat{A}\hat{B} + \hat{B}\hat{A}$. The result is that quantum fluctuations of the spectator field behave at late times as an effective *dark energy*.

Here we want to investigate the consequences of the spatial correlations inherited by dark energy. Therefore we turn our attention to 4-point functions built from field and momentum operators which are relevant for density-density correlators.

2.1. Stochastic formalism

The stochastic formalism of Alexey Starobinsky makes use of the fact that on super-Hubble scales non-conformal quantum fields $(\hat{\Phi}, \hat{\Pi})$ grow during inflation, such that their correlators can be accurately described by their classical stochastic equivalents $(\hat{\phi}, \hat{\pi})$. These infrared fields obey,

$$\frac{d}{dt}\hat{\phi}(t, \vec{x}) - a^{-3}\hat{\pi}(t, \vec{x}) = \hat{f}_\phi(t, \vec{x}), \quad (7)$$

$$a^{-3}\frac{d}{dt}\hat{\pi}(t, \vec{x}) + M^2(t)\hat{\phi}(t, \vec{x}) = a^{-3}\hat{f}_\pi(t, \vec{x}), \quad (8)$$

where $\hat{f}_\phi(t, \vec{x})$ and $\hat{f}_\pi(t, \vec{x})$ are stochastic sources, which arise because of coupling between the infrared and ultraviolet modes (see *e.g.* [24] for the precise definition of stochastic sources).

2.2. IR correlators

We can now proceed to extend the treatment of [24] to study spatial correlations of long modes. Let us recall that, in eqs. (30–32) of that paper, three 2-point functions were introduced because they appear in the expectation value of the energy-momentum tensor (conveniently rescaled with a power of $a^3(t)H(t)$ for each occurrence of a momentum operator), namely $\Delta_{\phi\phi}(t) \equiv \langle\hat{\phi}^2(t, \vec{x})\rangle$, then $\Delta_{\phi\pi}(t) \equiv \langle\{\hat{\phi}(t, \vec{x}), \hat{\pi}(t, \vec{x})\}\rangle/[a^3(t)H(t)]$ and $\Delta_{\pi\pi}(t) \equiv \langle\hat{\pi}^2(t, \vec{x})\rangle/[a^6(t)H^2(t)]$. Here we are interested in spatial fluctuations, therefore the relevant quantity is the density-density correlator at two different comoving coordinates $\vec{x}_1$ and $\vec{x}_2$ at the same cosmological time t: $\langle\hat{\rho}_Q(t, \vec{x}_1)\hat{\rho}_Q(t, \vec{x}_2)\rangle$, where the energy density operator is $\hat{\rho}_Q = -\hat{T}^0_0$. Since each $\hat{T}^0_0$ is quadratic in field/momentum operators, we infer that $\langle\hat{\rho}(t, \vec{x}_1)\hat{\rho}(t, \vec{x}_2)\rangle$ depends on 4-point functions of fields/momenta. Due to the homogeneity and isotropy of the FLRW background, comoving coordinates $\vec{x}_1$ and $\vec{x}_2$ only appear in correlators through their relative comoving distance $r \equiv \|\vec{x}_2 - \vec{x}_1\|$.

The relevant correlators are six symmetric combinations (see [28]), Δ_{ϕ^2,ϕ^2}, $\Delta_{\phi^2,\phi\pi}$, $\Delta_{\phi\pi,\phi\pi}$, Δ_{ϕ^2,π^2}, $\Delta_{\phi\pi,\pi^2}$, Δ_{π^2,π^2}. The simplest one is

$$\Delta_{\phi^2,\phi^2}(t, r) \equiv \langle\hat{\phi}^2(t, \vec{x}_1)\hat{\phi}^2(t, \vec{x}_2)\rangle. \quad (9)$$

We can study the time evolution of 4-point functions and identify their stochastic noise sources induced by the stochastic forces $\hat{f}_\phi(t, \vec{x})$ and $\hat{f}_\pi(t, \vec{x})$ of (7)–(8), to obtain a system of six coupled differential equations. Here we give two equations,

$$\frac{d}{dN}\Delta_{\phi^2,\phi^2} - \Delta_{\phi^2,\phi\pi} = n_{\phi^2,\phi^2}, \quad (10)$$

$$\frac{d}{dN}\Delta_{\pi^2,\pi^2} + 4(3 - \epsilon)\Delta_{\pi^2,\pi^2} + \left(\frac{M}{H}\right)^2\Delta_{\phi\pi,\pi^2} = n_{\pi^2,\pi^2}, \quad (11)$$

where we have traded the time t for the number of e-foldings $N \equiv \ln a(t)$, and the source in Eq. (10) is,

$$n_{\phi^2,\phi^2}(t, r) = \frac{1}{H(t)}\Big\langle\{\hat{f}_\phi(t, \vec{x}_1), \hat{\phi}(t, \vec{x}_1)\}\hat{\phi}^2(t, \vec{x}_2)$$

$$+ \hat{\phi}^2(t, \vec{x}_1)\{\hat{f}_\phi(t, \vec{x}_2), \hat{\phi}(t, \vec{x}_2)\}\Big\rangle \quad (r = \|\vec{x}_1 - \vec{x}_2\|). \quad (12)$$

The complete set of equations and sources is found in [28]. The stochastic sources can be computed in terms of the mode function $\varphi(t, k)$ of $\hat{\Phi}$. The source in (12) is thus,

$$n_{\phi^2,\phi^2} = \frac{1}{2\pi^4}(\mu a H)^3(1 - \epsilon)\left[|\varphi(t, k)|^2\right]_{k=\mu a H}$$

$$\times \int_{k_0}^{\mu a H} dk\, k^2|\varphi(t, k)|^2\left[1 + 2j_0(\mu a H r)j_0(kr)\right], \quad (13)$$

where $j_0(kr) \equiv \sin(kr)/(kr)$ is the spherical Bessel function and $\mu a H$ ($\mu \ll 1$) is an UV cut-off.

2.3. Time evolution of correlators

Analogously as it was done in [24], one can solve the equations of motion for the correlators (10)–(11) from an early inflation, through radiation and matter era. Even though the correct evolution of the correlators in matter era requires taking account of the dark energy backreaction, for simplicty here we model the late time universe by non-relativistic matter and cosmological constant. This introduces a few percent error [24], but has the advantage that one can solve the evolution of the relevant four-point functions exactly. The result is [28]

$$\Delta_{(4)}(N_0, r) \equiv \left(\Delta_{\phi^2,\phi^2}, \Delta_{\phi^2,\phi\pi}, \Delta_{\phi\pi,\phi\pi}, \Delta_{\phi^2,\pi^2}, \Delta_{\phi\pi,\pi^2}, \Delta_{\pi^2,\pi^2}\right)$$

$$\simeq \frac{H_I^4 e^{16|\xi|N_I}e^{8\zeta N_0}}{1024\pi^4\xi^2}s(r)(1, 8\zeta, 16\zeta^2, 8\zeta^2, 32\zeta^3, 16\zeta^4), \quad (14)$$

where $N_0 = \ln\left(\frac{\Omega_M}{\Omega_R}\right) \simeq 8.1$ e-foldings since matter-radiation equality,

$$\zeta \equiv |\xi| - \frac{\frac{1}{6\Omega_\Lambda}\left(\frac{m}{H_0}\right)^2 - |\xi|}{\frac{3}{2}\frac{\ln(\Omega_R)}{\ln(\Omega_M)} - 1}, \quad (15)$$

and $s(r)$ contains spatial dependence, which is well approximated by,

$$s(r) \simeq \begin{cases} 3 & \text{if } 0 \leq \mu a_{in}H_I r < e^{-N_I} \\ 3 - 2\,(\mu a_{in}H_I r)^{16|\xi|} & \text{if } e^{-N_I} < \mu a_{in}H_I r < 1 \\ 1 & \text{if } \mu a_{in}H_I r \geq 1, \end{cases} \quad (16)$$

where a_{in} is the scale factor at the beginning of inflation. Therefore the comoving length scale ruling spatial variations is the quantity $(\mu a_{in}H_I)^{-1}$, *i.e.* the comoving Hubble horizon at the beginning of inflation (up to the μ^{-1} factor). Furthermore, Ω_R, Ω_M and $\Omega_\Lambda \simeq 1 - \Omega_M$ are the energy density fractions in radiation and matter today, taken to be $\Omega_R = 9.1 \cdot 10^{-5}$ and $\Omega_M = 0.3$.

The quantum backreaction of the scalar field contributes to energy density and pressure during matter domination as in [24],

$$\rho_Q = \frac{H_I^2}{32\pi^2|\xi|}e^{8|\xi|N_I}\left(\frac{m^2}{2} - 3|\xi|H^2\right),$$

$$p_Q = -\frac{H_I^2}{32\pi^2|\xi|}e^{8|\xi|N_I}\frac{m^2}{2}. \quad (17)$$

Matching the cosmological constant-like term to its value today Ω_Λ determines the number of inflationary e-foldings N_I in terms of the model parameters m and $|\xi|$,

$$N_I = \frac{1}{8|\xi|}\ln\left[24\pi|\xi|\left(\frac{m_P}{H_I}\right)^2\left(\frac{H_{DE}}{m}\right)^2\right], \quad (18)$$

where $m_P \equiv G^{-1/2}$ is the Planck mass, $H_{\mathrm{DE}} \equiv H_0\sqrt{\Omega_\Lambda}$ and H_0 is the Hubble parameter today.

Dark energy eventually leads the cosmological expansion provided that the cosmological constant-like term dominates over the matter-like term in the quantum backreaction until late times, *i.e.* we require

$$|\xi| < \frac{1}{6}\left(\frac{m}{H_{\mathrm{DE}}}\right)^2 \quad \text{and} \quad \xi < 0. \tag{19}$$

We also need $m/H_{\mathrm{DE}} < 1$ so that the scalar field stays light throughout the cosmological expansion.

The spatial dependence in (14) is the one inherited from inflation through the function $s(r)$ in (16). Before discussing the Hubble tension problem, it is useful to work on the comoving length scale $(\mu a_{\mathrm{in}} H_I)^{-1}$ associated to $s(r)$ in order to relate its physical size today to the model parameters. Assuming an instantaneous reheating after inflation, we find the following ratio between the physical length scale of spatial variations today and the current Hubble horizon:

$$\frac{(\mu a_{\mathrm{in}} H_I)^{-1} a_0}{H_0^{-1}} = \mu^{-1} e^{N_I} \left(\frac{H_I}{H_0}\right)^{-\frac{1}{2}} \Omega_R^{-\frac{1}{4}}. \tag{20}$$

Spatial variations are relevant for the Hubble tension problem discussed in section 3 only if they are significant at sub-Hubble scales. Therefore, we require that the ratio appearing in the left-hand side of (20) is at most of order one, which then limits the number of inflationary e-foldings. For $H_I \simeq 10^{13}$ GeV, $H_0 \simeq 10^{-33}$ eV (and μ not too small), this gives $N_I \lesssim 60$. The condition on N_I translates into a requirement on the parameters of the model, because N_I is determined by $|\xi|$ and m via the combination in (18).

3. Relieving the Hubble tension

Note that the local expansion rate operator $\hat{H}^2(t,\vec{x})$ in (1) can be recast as,

$$\hat{H}^2(t_0,\vec{x}) = \frac{1}{3M_P^2} \frac{\rho_C(t_0)\mathbb{1} + \frac{m^2}{2}\hat{\phi}^2(t_0,\vec{x})}{\mathbb{1} + \frac{|\xi|}{M_P^2}\hat{\phi}^2(t_0,\vec{x})}, \tag{21}$$

where we made use of $\hat{\rho}_Q$ from (6) and neglected contributions from of $\{\hat{\phi},\hat{\pi}\}$ and $\hat{\pi}^2$, which is a reasonable assumption [28]. Next, we assume that $H_0 \simeq H_2 = 67.4$ km/s Mpc^{-1}, meaning that the statistical average value H_0 is estimated very well by the result of global measurements.

The translation of a probability statement for $\hat{H}^2$ like that in (2) into an equivalent statement for $\hat{\phi}^2$ depends on whether these two quantities are related by a growing or decreasing function. A simple analysis of (21) shows that $\frac{d(\hat{H}^2)}{d(\hat{\phi}^2)} \propto \frac{m^2}{2} - \frac{|\xi|}{M_P^2}\rho_C(t_0) \propto \left[1 - 6|\xi|\left(\frac{H_0}{m}\right)^2\right]$, such that $\hat{H}^2$ is a growing (decreasing) function of $\hat{\phi}^2$ when $|\xi| < \frac{1}{6}(m/H_0)^2$ ($|\xi| > \frac{1}{6}(m/H_0)^2$). For example, with the choice $\xi = -\frac{1}{6}(m/H_{\mathrm{DE}})^2$ that saturates the constraint (19), the quantity $1 - 6|\xi|(H_0/m)^2 = 1 - \Omega_\Lambda^{-1} < 0$ and $\hat{H}^2$ is a decreasing function of $\hat{\phi}^2$.

In what follows, we introduce some simplifications, whose accuracy is discussed in detail in Ref. [28]. Firstly, we shall assume that the field $\hat{\psi}(t_0,\vec{x}) \equiv \hat{\phi}^2(t_0,\vec{x})$ is a classical stochastic variable $\psi(t_0,\vec{x})$ which is Gaussian distributed. Secondly, we shall assume that the stochastic properties of ψ can be modeled by a Gaussian probability distribution,

$$P[\psi] \propto \exp\left\{-\frac{1}{2}\int d^3x \int d^3y \left[\psi(t_0,\vec{x}) - \psi_0\right]\right.$$
$$\left.\times\, C^{-1}(t_0,\|\vec{x}-\vec{y}\|)\left[\psi(t_0,\vec{y}) - \psi_0\right]\right\}, \tag{22}$$

where the position-independent mean value is $\psi_0 = \Delta_{\phi\phi}(t_0)$, while the covariances are

$$C(t_0,\|\vec{x}-\vec{y}\|) = \Delta_{\phi^2,\phi^2}(t_0,\|\vec{x}-\vec{y}\|) - \Delta_{\phi\phi}^2(t_0). \tag{23}$$

As for the domain of definition, the probability distribution $P[\psi]$ is assumed to be normalized in the global volume V_2 (which includes V_1).

Using the usual relation between conditional probabilities and joint probabilities, the problem of finding (2) amounts to study the joint probability distribution of the mean variables $[\psi]_{V_1} \equiv (1/V_1)\int_{V_1} d^3x\, \psi(t_0,\vec{x})$ and $[\psi]_{V_2} \equiv (1/V_2)\int_{V_2} d^3x\, \psi(t_0,\vec{x})$. These are still Gaussian variables and one can evaluate their (2×2 and symmetric) covariance matrix

$$\mathcal{M} = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{12} & \mathcal{M}_{22} \end{bmatrix}, \quad \mathcal{M}_{ij} = \frac{1}{V_i V_j}\int_{V_i} d^3x \int_{V_j} d^3y\, C(t_0,\|\vec{x}-\vec{y}\|).$$

An important quantity is the correlation coefficient ρ defined as $\rho = \mathcal{M}_{12}/\sqrt{\mathcal{M}_{11}\mathcal{M}_{22}}$. We can evaluate the covariance matrix $\mathcal{M}$ and the correlation coefficient ρ by using for the volumes V_1 and V_2 two spheres of radii R_1 and R_2 centered around the observer. The conditional probability (2) can be reduced to,

$$\mathrm{P}\left(\left[H^2\right]_{V_1} > H_1^2 \,\bigg|\, \left[H^2\right]_{V_2} < H_2^2\right) \tag{24}$$
$$= 2\,T\left(\sqrt{2}\delta\chi, \frac{-\rho}{\sqrt{1-\rho^2}}\right) + \frac{1}{2}\left[1 \mp \mathrm{erf}(\delta\chi)\right],$$

where $T(h,a) = \frac{1}{2\pi}\int_0^a dx \exp\left(-\frac{h^2}{2}(1+x^2)\right)/(1+x^2)$ is Owen's T function and

$$\delta\chi = \frac{\frac{H_1^2}{H_0^2} - 1}{\sqrt{2}J_{11}} \frac{(m/H_{\mathrm{DE}})^2 + 6|\xi|}{(m/H_0)^2 - 6|\xi|\frac{H_1^2}{H_0^2}}, \quad J_{11} = I_{11}e^{8\zeta N_0} - 1, \tag{25}$$

where $I_{11} = \int_0^2 dx\left(\frac{3}{16}x^5 - \frac{9}{4}x^3 + 3x^2\right)s_1(x)$ and

$$s_1(x) = \begin{cases} 3 & \text{if } 0 \leq x \leq k_1 e^{-N_I} \\ 3 - 2\left(\frac{x}{k_1}\right)^{16|\xi|} & \text{if } k_1 e^{-N_I} < x \leq k_1 \\ 1 & \text{if } x \geq k_1 \end{cases} \tag{26}$$

with $k_1 = e^{N_I}\left(\frac{H_I}{H_0}\right)^{-\frac{1}{2}}\Omega_R^{-\frac{1}{4}}(\mu H_0 R_1)^{-1}$.

For definiteness we choose the radius R_2 of the global volume such that $\mu R_2 = H_0^{-1} \simeq 4400$ Mpc and the radius R_1 of the local volume such that $\mu R_1 = 100$ Mpc (we set $\mu = 1$). The conditional probability (24) depends on the two parameters of the model ξ and m. Solid lines in Fig. 1 show its behavior as a function of $|\xi|$ for constant values of the ratio $\alpha \equiv \frac{1}{6|\xi|}(m/H_{\mathrm{DE}})^2$. Notice that the condition (19) imposes to choose $\alpha > 1$. Furthermore the condition of light field $m/H_{\mathrm{DE}} < 1$ means that, for a given constant α, only values of $|\xi|$ with $|\xi| < 1/(6\alpha)$ have to be considered (this is automatically true for the values of α and the plot range chosen in Fig. 1). The number of inflationary e-foldings throughout the range of parameters in Fig. 1 is of order $N_I \sim 60$ and is therefore compatible with the approximate constraint $N_I \lesssim 60$.
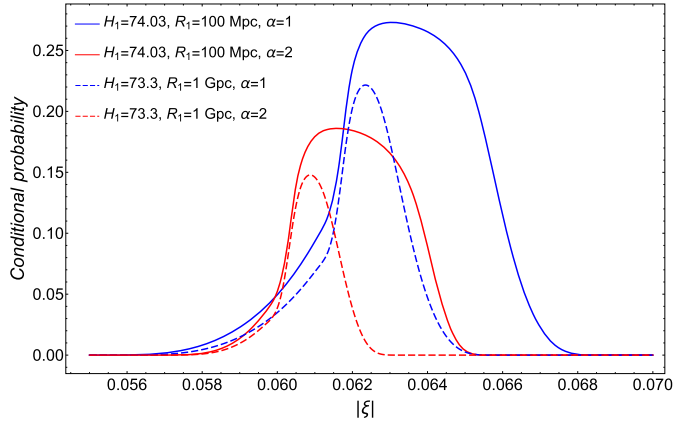
Fig. 1. The conditional probability in (24), as a function of the non-minimal coupling $|\xi|$, for constant values of $\alpha = \frac{1}{6|\xi|} (m/H_{DE})^2$. The blue curves correspond to the limit case $\alpha = 1$, which is its minimum possible value. The parameter ξ is mildly tuned to a rather natural value, since the width of the region of ξ values relevant for relieving the Hubble tension (to less than 2σ) can reach $\Delta\xi/|\xi| \sim \mathcal{O}(10\%)$ in the best case.
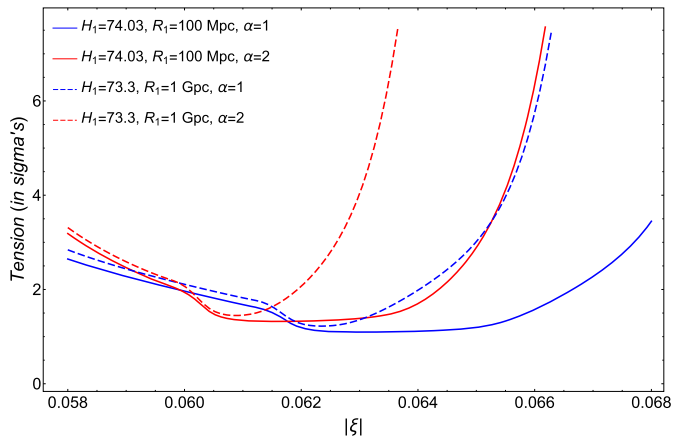
Fig. 2. Hubble tension in standard deviations σ as a function of the non-minimal coupling $|\xi|$, in the same conditions for m/H_{DE} as in Fig. 1. The plot range is slightly zoomed in with respect to that in Fig. 1. The tension can be reduced down to $\simeq 1\sigma$ (solid lines) and $\simeq 1.2\sigma$ (dashed lines) for local measurements in $R_1 = 100$ Mpc and $R_1 = 1$ Gpc, respectively.

Fig. 2 shows the corresponding Hubble tension zooming slightly on a more central region. As can be seen from the plot, the model is able to reduce the tension down to almost 1σ which is a great improvement with respect to the $> 4\sigma$ tension in ΛCDM. This has been possible thanks to spatial fluctuations of dark energy inherited from inflation.

In the limit of zero local volume one gets the largest conditional probability that can be reached, which can be obtained from (24) to be $\frac{1}{2} - \frac{1}{\pi}\arctan\left(\frac{1}{\sqrt{2}}\right) \simeq 30.4\%$, corresponding to a "tension" of only 1.03σ. This is in agreement with the solid lines in Figs. 1-2, where the ratio $R_1/R_2 \simeq 0.02$ is small but finite. To mimic H0LiCOW measurements [7] we increase $R_1 \simeq 1$ Gpc and set $H_1 = 73.3$ km/s Mpc^{-1}. The corresponding results are shown by the dashed lines in Figs. 1-2. Our model relieves the Hubble tension, but now the probability peaks at about 20%, corresponding to about 1.2σ tension.

Another major problem in ΛCDM is the σ_8 tension. A full quantitative discussion of our model with regard to this issue goes beyond the scope of the paper. However, it is interesting to have a qualitative understanding of it. In our model, a higher local value of the Hubble parameter (with respect to the global one) means a positive local fluctuation in dark energy. Assuming for simplicity

that there is no local curvature effect so that the sum of the local matter density fraction and local dark energy density fraction is still 1, this would correspond to a reduced value of the local matter density fraction (Ω_M) with respect to its global value. Smaller Ω_M means less growth of structures (e.g. assuming that the growth rate f obeys the widely used power law $f = \Omega_M^\gamma$ with $\gamma \approx 0.55$), thus going in the direction of reducing the amplitude of local matter perturbations. This is in agreement with current data, see e.g. Fig. 2 in [31].

More work is needed to reveal and test the deviations of the model proposed here from ΛCDM. Some plots on the evolution of the Hubble rate and the dark energy equation of state in recent cosmological times were presented in Figs. 2, 3, 4, 5 of [24] for a few choices of the parameters ξ and m; the expected deviations are within reach of Euclid and Vera Rubin Observatory. In [29] the authors test the same quantum dark energy model against cosmological data and compare its performance to ΛCDM, finding that the model is slightly favored, although not at a statistically significant level. We are currently investigating the form of luminosity distance correlators for type Ia supernovae induced by spatially correlated dark energy, which provide a test for deviations from ΛCDM. This is just an example of physical observables that could be used to test the features predicted by the model. It is also worth mentioning that, within a different approach using cosmological perturbations, dark energy with very large-scale inhomogeneities has been considered in [30] and their amplitude is constrained with CMB anisotropies data.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

Appendix A. Four-point functions calculation

Here we provide some details on the calculation leading to the result (14) by following the evolution of correlators in the three cosmological epochs.

A.1. Inflation

The mode function $\varphi(t, k)$ to be used in exact de Sitter inflation ($\epsilon = 0$) is the Chernikov-Tagirov-Bunch-Davies (CTBD) mode function

$$\varphi(t, k) = \sqrt{\frac{\pi}{4a^3 H_I}} H_{\nu_I}^{(1)}\left(\frac{k}{aH_I}\right), \qquad \nu_I \equiv \sqrt{\frac{9}{4} - \left(\frac{M}{H_I}\right)^2}, \qquad (A.1)$$

where H_I is the constant inflationary Hubble rate, $M^2 = m^2 + 12\xi H_I^2$ is the constant squared effective mass and $H_{\nu_I}^{(1)}$ is the Hankel function of the first kind. The CTBD mode function is needed to specify the sources in the system of first-order differential equations including (10)–(11). Since we are dealing with long-wavelength modes in the integral of (13), it is sufficient to consider an IR approximation of the mode function

$$\varphi(t, k) \approx \frac{-i}{\sqrt{\pi}} 2^{\nu_I - 1} \Gamma(\nu_I) a^{\nu_I - 3/2} H_I^{\nu_I - 1/2} k^{-\nu_I}. \qquad (A.2)$$

As initial conditions, we assume correlators to be zero at the beginning of inflation.[3] During de Sitter inflation, the evolution with the number of e-foldings N of correlators $\Delta_{(4)}(N, r)$ defined by $\Delta_{(4)}(N, r) \equiv \left(\Delta_{\phi^2, \phi^2}, \Delta_{\phi^2, \phi\pi}, \Delta_{\phi\pi, \phi\pi}, \Delta_{\phi^2, \pi^2}, \Delta_{\phi\pi, \pi^2}, \Delta_{\pi^2, \pi^2} \right)$ can be read from equations like (10)–(11) with $\epsilon = 0$, resulting in the system of equations

$$\frac{d}{dN} \Delta_{(4)}(N, r) + A \, \Delta_{(4)}(N, r) = n(N, r), \tag{A.3}$$

where A is the following 6×6 constant matrix containing $X \equiv (M/H_I)^2 = (m/H_I)^2 + 12\xi$,

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 4X & 3 & -2 & -2 & 0 & 0 \\ 0 & 2X & 6 & 0 & -2 & 0 \\ 0 & X & 0 & 6 & -1 & 0 \\ 0 & 0 & 2X & 2X & 9 & -4 \\ 0 & 0 & 0 & 0 & X & 12 \end{bmatrix}, \tag{A.4}$$

and the source vector $n(N, r)$ can be evaluated from the mode function. For[4] $\xi < 0$ and $(m/H_I)^2 \ll |\xi| \ll 1$, working at the lowest order in the non-minimal coupling ξ, we get (recalling that the scale factor is $a(N) = e^N$)

$$n(N, r) = \left(\frac{H_I}{2\pi} \right)^4 \left[1 + 2 j_0(\mu e^N H_I r) \right]$$
$$\times \left(\frac{1}{4|\xi|}, 4, 16|\xi|, 8|\xi|, 64\xi^2, 64|\xi|^3 \right). \tag{A.5}$$

The differential problem (A.3) is conveniently solved by diagonalizing the matrix A. Denoting by λ_j the eigenvalues of A, by B the change-of-basis matrix (matrix whose columns are the eigenvectors of A) and by B^{-1} its inverse, the i-th component of the correlators vector $\Delta_{(4)}(N, r)$ is given by

$$\Delta_{(4),i}(N, r) = \sum_j B_{ij} e^{-\lambda_j N} \int_0^N dN' \, e^{\lambda_j N'} \sum_k (B^{-1})_{jk} \, n_k(N', r). \tag{A.6}$$

The integral can be evaluated by approximating the spherical Bessel function $j_0(x) = \sin(x)/x$ appearing in (A.5) as $j_0(x) \simeq \theta(1 - x)$ where θ is the Heaviside step function. Furthermore, in the limit of small X it can be shown that the sum over eigenvalues in (A.6) is dominated by the smallest eigenvalue of A which is $6 - 2\sqrt{9 - 4X} = 4/3 \, X + \mathcal{O}(X^2)$. With these simplifications one finds that, at the end of inflation (lasting N_I e-foldings), the 4-pt correlators are

$$\Delta_{(4)}(N_I, r) \simeq \frac{H_I^4}{1024\pi^4\xi^2} e^{16|\xi|N_I} s(r)$$
$$\times \left(1, 16|\xi|, 64\xi^2, 32\xi^2, 256|\xi|^3, 256\xi^4 \right), \tag{A.7}$$

where the function $s(r)$ encoding spatial dependence was defined in (16).

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[3] The effect of non-zero initial conditions for 2-point functions (and therefore for 4-point functions as well) inherited from a pre-inflationary epoch will be discussed in [28].

[4] As already shown in [24], this hierarchy choice for the parameters gives the best enhancement of correlators in inflation, which also means that a smaller number of inflationary e-foldings is required to match the dark energy content of the Universe today.

A.2. Radiation epoch

After the end of inflation, the correlators will evolve in radiation era starting from the initial conditions (A.7) inherited from inflation. Similarly to the case of 2-pt functions studied in [24], we can safely neglect stochastic sources after inflation because their contribution will be irrelevant compared to that of initial conditions. This is true due to the enhancement factor $e^{16|\xi|N_I}/\xi^2$, which is present in (A.7) but not in the stochastic sources of radiation epoch.

In radiation epoch ($\epsilon = 2$) the Ricci scalar is null and we can also neglect $(m/H(t))^2$ (due to the assumption of very light scalar field, this term only becomes relevant at recent cosmological times, when matter and cosmological constant dominate the evolution of the Universe). This leads to a simplified evolution during radiation era, of the form

$$\frac{d}{dN} \Delta_{(4)}(N, r) + A \, \Delta_{(4)}(N, r) = 0, \tag{A.8}$$

where A is the constant matrix

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}. \tag{A.9}$$

If $N = 0$ is the beginning of radiation epoch, then the system is easily solved as $\Delta_{(4)}(N, r) = e^{-A \, N} \Delta_{(4)}(0, r)$, where $\Delta_{(4)}(0, r)$ is the initial condition taken from the end of inflation (A.7). The exponential of the matrix $-A$ is computed by diagonalizing A. At leading order in $|\xi|$ we find that, by the end of radiation epoch lasting $N_R \approx 50$ e-foldings,

$$\Delta_{(4)}(N_R, r) \simeq \frac{H_I^4 e^{16|\xi|N_I}}{1024\pi^4\xi^2} s(r)$$
$$\times \left(1, 16|\xi| e^{-N_R}, 64\xi^2 e^{-2N_R}, 32\xi^2 e^{-2N_R}, \right.$$
$$\left. 256|\xi|^3 e^{-3N_R}, 256\xi^4 e^{-4N_R} \right). \tag{A.10}$$

Since $N_R \gg 1$, we can say that, up to contributions suppressed by powers of e^{-N_R}, at the end of radiation epoch

$$\Delta_{(4)}(N_R, r) \simeq \frac{H_I^4}{1024\pi^4\xi^2} e^{16|\xi|N_I} s(r)(1, 0, 0, 0, 0, 0). \tag{A.11}$$

This is the initial condition that will be used in the final stage of cosmological evolution after matter-radiation equality. We remark that the spatial dependence is still that imprinted by inflation. This is a consequence of the smallness of stochastic sources after inflation, so that the post-inflationary evolution does not affect the form of $s(r)$.

A.3. Matter + cosmological constant epoch

After matter-radiation equality, non-relativistic matter becomes the dominant component of the Universe leading its expansion. We know from eq. (90) of [24] that in matter period, the 2-pt functions of the scalar field evolve in such a way to develop a dark energy component (more precisely they have a matter-like contribution and a CC-like contribution). We are interested here in 4-pt functions, that should be evolved numerically in a background containing the backreaction of the scalar field. However we can obtain a simplified analytical treatment by approximating the background evolution as an exact matter+CC Universe.

We set $N = 0$ at matter-radiation equality and the current (today) time corresponds to $N_0 = \ln\left(\frac{\Omega_M}{\Omega_R}\right) \simeq 8.1$. As usual, we denote by Ω_M and $\Omega_\Lambda = 1 - \Omega_M$ the matter and cosmological constant fractions of energy density today. Then $\epsilon(N)$ needed in (11) evolves in time as

$$\epsilon(N) = \frac{3}{2}\frac{1}{1 + \frac{\Omega_\Lambda}{\Omega_M}e^{3(N-N_0)}}. \tag{A.12}$$

It is convenient to trade N for the variable $x \equiv \left[1 + \frac{\Omega_\Lambda}{\Omega_M}e^{3(N-N_0)}\right]^{-1}$ so that $\epsilon(x) = (3/2)x$, and with simple algebra the quantity $M(t)/H(t)$ appearing in (11) becomes, in the x variable,

$$\left(\frac{M(x)}{H(x)}\right)^2 = \left(\frac{m}{H_{DE}}\right)^2(1-x) - 12|\xi|\left(1 - \frac{3}{4}x\right), \tag{A.13}$$

where H_{DE} was already defined right after (18). The initial time $N = 0$ (matter-radiation equality) corresponds to $x_{eq} = \left(1 + \frac{\Omega_\Lambda}{\Omega_M}e^{-3N_0}\right)^{-1}$ while the current time N_0 gives $x_0 = \Omega_M$. Using $dx/dN = 3x(x-1)$ the 4-pt functions obey a system of the form (again we can neglect stochastic sources)

$$\frac{d}{dx}\Delta_{(4)}(x,r) + B(x)\,\Delta_{(4)}(x,r) = 0, \tag{A.14}$$

In the equation above B(x) is the time-dependent matrix $B(x) = b_1(x)B_1 + b_2(x)B_2 + b_3(x)B_3$, where the time dependence is encoded in functions

$$b_1(x) = \frac{2-x}{2x(x-1)}, \qquad b_2(x) = -\frac{\left(\frac{m}{H_{DE}}\right)^2}{3x} - |\xi|\frac{4-3x}{x(x-1)},$$

$$b_3(x) = \frac{1}{3x(x-1)}, \tag{A.15}$$

while B_1, B_2 and B_3 are the following constant matrices

$$B_1 = \mathrm{diag}(0,1,2,2,3,4), \qquad B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.16}$$

The exact solution of (A.14) is

$$\Delta_{(4)}(x,r) = T\exp\left[-\int_{x_{eq}}^{x} dx'\, B(x')\right]\Delta_{(4)}(x_{eq},r), \tag{A.17}$$

where $T\exp$ denotes the time-ordered exponential, which is there because $B(x)$ matrices at different x's do not commute. However we checked numerically that neglecting the time ordering gives accurate results at a few percent level, with the advantage of displaying simpler formulas.[5] Then the integrals can be solved analytically and the matrix exponential can be also evaluated analytically with

a symbolic computation software. Further simplifications arise by the assumptions of small $|\xi|$ and $\left(\frac{m}{H_{DE}}\right)^2$, leading after a few steps to the result (14) at the current epoch N_0.

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