A Hybrid Multi-Objective Teaching Learning-Based Optimization Using Reference Points and R2 Indicator

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ABSTRACT

Hybrid multi-objective evolutionary algorithms have recently become a hot topic in the domain of metaheuristics. Introducing new algorithms that inherit other algorithms' operators and structures can improve the performance of the algorithm. Here, we proposed a hybrid multi-objective algorithm based on the operators of the genetic algorithm (GA) and teaching learning-based optimization (TLBO) and the structures of reference point-based (from NSGA-III) and R2 indicators methods. The new algorithm (R2-HMTLBO) improves diversity and convergence by using NSGA-III and R2-based TLBO, respectively. Also, to enhance the algorithm performance, an elite archive is proposed. The proposed multi-objective algorithm is evaluated on 19 benchmark test problems and compared to four state-of-the-art algorithms. IGD metric is applied for comparison, and the results reveal that the proposed R2-HMTLBO outperforms MOEA/D, MOMBI-II, and MOEA/IGD-NS significantly in 16/19 tests, 14/19 tests and 13/19 tests, respectively. Furthermore, R2-HMTLBO obtained considerably better results compared to all other algorithms in 4 test problems, although it does not outperform NSGA-III on a number of tests.

CCS CONCEPTS

• **Theory of computation → Bio-inspired optimization**; **Evolutionary algorithms**.

KEYWORDS

optimization algorithm, multi-objective evolutionary algorithm (MOEA), teaching learning-based optimization (TLBO), NSGA-III, reference point-based method, R2 indicator

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1 INTRODUCTION

Optimization is a long-standing topic of importance in artificial intelligence (AI) research. Metaheuristics are well-known optimization methods following the principle of natural evolution. Compared to conventional methods (mathematical-based/gradient-based optimizers), metaheuristics perform better in non-deferential, noisy, and discontinuous environments. Evolutionary algorithms (EAs) and swarm intelligence (SI) are two branches of metaheuristics. Although they include a vast range of algorithms, such as the genetic algorithm (GA) [13] , differential evolution (DE) [22], and particle swarm optimization (PSO) [16], most of them have the same structure.

Optimization problems are typically classified into three categories depending on the number of objectives in the problem: single-objective (one objective), multi-objective (two or three objectives) and many-objective (more than three objectives). In general, real-world optimization problems consist of multi-objective problems (MOPs) or many-objective problems (MaOPs), and in many cases the objectives are contradictory. Multi-objective/many-objective evolutionary-based algorithms (MOEAs/MaOEAs) can be divided into three frameworks: 1) Domination-based, 2) Decomposition-based, and 3) Indicator-based methods. Domination-based algorithms apply two main outlines in their structure: Pareto dominance strategy and distance-based density strategy. The domination-based strategy is responsible for finding non-dominated solutions near the true pareto front (PF). On the other hand, the distance-based density strategy promotes the uniform solutions' distribution. Nondominated sorting genetic algorithm-II (NSGA-II) [5] and its modified version, NSGA-III [4], are classified in this group, and their structures follow an identical pattern. The basic idea behind them is sorting solutions based on their dominations and retaining their diversities based on the specific distribution approaches. The distribution approach helps the algorithm reach uniform solutions; the crowding distance operator [5] and a method based on the reference points [3] are devised for NSGA-II and NSGA-III, respectively. The multi-objective evolutionary algorithm based on decomposition (MOEA/D) [31] is the most well-known algorithm in the decomposition-based framework. In decomposition-based approaches, a scalarization function decomposes all the objectives into several single-objective sub-problems and then solves them. There are three popular scalarization functions: weighted sum (WS), Tchebycheff (TCH), and penalty-based intersection (PBI), of which TCH is most popular. Indicator based algorithms apply a criterion for evaluating and ranking the performance of the population members. These algorithms try to measure two criteria of convergence and diversity simultaneously. Hypervolume (HV) [8], Δ_p [21], and

R2 [1] are typical indicators. The Indicator-based evolutionary algorithm (IBEA) [35] is one of the first algorithms in this group.

Overall, each category has its pros and cons. For instance, domination-based approaches lack power to reach true PF, leading to poor convergence. Specifically, this issue degenerates the algorithms' convergence when the number of objectives is increased. On the other hand, regarding the solutions' distribution, it has better performance, improved further by introducing the concept of reference points in NSGA-III. In addition, choosing the proper scalarization function, weight vectors and the number of neighbors is a critical challenge for decomposition-based methods, and many modifications have been proposed in recent years to address these challenges [14, 15, 33]. Based on the type of indicator, most indicator-based methods simultaneously try to assess the convergence and diversity of solutions. However, the main drawback of this approach is its high computational cost, especially in MaOPs [28]. New indicators, such as the R2 indicator, have been introduced in an attempt to solve this issue. It is evident that there is no perfect and infallible algorithm for solving MOPs/MaOPs. To overcome this issue, researchers have recently considered hybrid algorithms where the deficiencies of one algorithm can be compensated by utilizing the structures or operators from other algorithms in hybridization. Hybridization can be applied to two sections: the algorithm's structure (popular in MOEAs/MaOEAs) [2, 19, 23, 24] or the algorithm's operators (popular in single-objective EAs) [17, 29, 30].

This paper introduces a hybrid MOEA based on the two algorithms of NSGA-III and teaching-learning based optimization(TLBO). The new algorithm (R2-HMTLBO) inherits the solution's diversity and convergence from the synergies of NSGA-III and TLBO, respectively. TLBO is a straightforward algorithm that does not need specific parameters and has powerful performance in combination with GA operators. Furthermore, the R2 indicator is incorporated in TLBO to convert TLBO to a multi-objective algorithm for the first time. For this purpose, an elite archive is created, which accelerates the algorithm's convergence. Finally, the algorithm's performance is evaluated by various functions from three well-known benchmark cost functions (ZDT, DTLZ and CEC09). Recently, hybrid algorithms have been proposed [18, 27], but most are restricted to popular SI and EA algorithms such as PSO and GA. Here we benchmark the proposed R2-HMTLBO algorithm and demonstrate that achieves SOTA performance when compared to NSGA-III, MOEA/D, MOMBI-II and MOEA/IGD-NS in nineteen test problems.

The remainder of this work is organized as follows: The preliminaries of MOPs and the fundamental structures of NSGA-III, TLBO, R2-indicator are explained in Section 2. Next, the structure of our proposed algorithm is presented in Section 3. Section 4 describes benchmark functions, performance indicator, and experimental design. The results of R2-HMTLBO are discussed Section 5. Finally, our conclusion and future plans are provided in Section 6.

2 BACKGROUND

2.1 Multi-Objective Optimization Problem

Generally, daily optimization problems include MOPs which can be described in Equation. 1:

$$\text{Minimize/Maximize} \quad F(x) = (f_1(x), ..., f_m(x))^T \qquad (1)$$
$$\forall x \in \Phi \, (\Phi \rightarrow R^m)$$

where Φ is the search space, and x is a decision variable. R^m, and m are objective space and number of objective functions, respectively. The goal of Equation. 1 is to minimize/maximize all objective functions $(f_1(x), ..., f_m(x))$ in one run and find Pareto optimal solutions and PF as follows: A vector $a = (a_1, a_2, ..., a_m)^T$ dominates vector $b = (b_1, b_2, ..., b_m)^T$ (represented as $a \prec b$) iff $\forall i \in 1, 2, ..., m, a_i \leq b_i$ and $a \neq b$. Moreover, an obtained solution $x^* \in \Phi$ is named a Pareto optimal solution if $F(y) \prec F(x^*)$ when iff $\forall y \nexists \Phi$. Pareto set (PS) is the set of Pareto optimal solutions, denoted as $PS = \{x \in \Omega \mid \nexists y \in \Omega, F(y) \prec F(x)\}$, and PF is the projection of PS into the objective space. PS is a subset in the variables space, whereas the PF is a subset in the objective space.

2.2 Reference point method

NSGA-III is one of the well-known domination-based algorithms that apply the reference points concept to improve diversity criterion. After applying selection, combination and mutation operators at the first stage, NSGA-III applies the non-dominated sorting operator (NSO) to produce non-dominated solutions. At the second stage, it utilizes reference points on the hyperplanes to distribute the solutions uniformly. The reference points are allocated on the hyperplane by Das and Dennis's method [3]. For instance, in a problem with m objectives and p divisions for each dimension, H reference points can be generated.

$$H = \frac{(m + p - 1)!}{p!(m - 1)!} \qquad (2)$$

The association operator then connects each individual to the nearest reference point and adds one number to that individual as the association population (by a counter). The connecting criterion is the perpendicular distance between each individual with the reference line (the reference line passes through the origin and reference points). Each reference point, which includes less associated population, will be chosen in the final stage regarding the Niche-preservation operation. This process continues until the population member equals the reference points.

2.3 R2 Indicator method

The R2 indicator and its family (R1 and R3) were proposed in [10] for the first time, trying to assess the two sets' relative quality. The utility function (scalarization function) plays a crucial role in R2 and R3 calculations [10]. WS, TCH, PBI are the most popular utility functions in recent publications. For example, to evaluate the quality of an arbitrary set ($\mathcal{S}$) compared to reference point z^* regarding the TCH utility function, the R2 indicator is calculated as shown in Equation. 3:

$$R2(\mathcal{S}, U, z^*) = \frac{1}{|U|} \sum_{u \in U} \min_{x \in \mathcal{S}} \left\{ \max_{i=1,2,...,m} \frac{|f_i(x) - z^*|}{u_i} \right\} \qquad (3)$$

where U is a distributed weight vector, and the probability distribution on U is considered as $\frac{1}{U}$. The best result is related to a smaller value of R2, indicating a smaller distance between reference point z^* and the arbitrary set $\mathcal{S}$.

2.4 Teaching Learning-Based Optimization

Teaching Learning-Based Optimization [20] is a population-based EA derived from the interaction between teachers and students in a class. Recently it has undergone many changes, but its basic form includes two phases of Teaching and Learning. As the best solution among all individuals (in one generation), in the teacher phase, the teacher tries to guide other students according to Equation. 4:

$$\overline{x}_{new} = \overline{x}_{old} + \overline{r}\left(\overline{tc} - t_f \overline{x}_{mean}\right) \tag{4}$$

$$\overline{x}_{mean} = \frac{1}{N}\sum_{i=1}^{N} \overline{x}_i$$

where N is the total number of individuals in a generation, and $\overline{r}$ is a vector of uniformly distributed random numbers between 0 to 1. $\overline{tc}$ is teacher, and t_f is known as the teaching factor, a value between 1 or 2 selected randomly (with equal probability). In the learning phase, students (Individuals) improve their performance by communicating and sharing their knowledge. These activities result in better solutions and improve the algorithm's exploration. Therefore, the learning phase can be formulated as Equation. 5:

$$\overline{x}_{new} = \begin{cases} \overline{x}_{old} + \overline{r}\left(\overline{x}_j - \overline{x}_k\right) & \text{if} \quad f(x_j) < f(x_k) \\ \overline{x}_{old} + \overline{r}\left(\overline{x}_k - \overline{x}_j\right) & \text{if} \quad f(x_k) < f(x_j) \end{cases} \tag{5}$$

where j and k are randomly selected numbers between 1 to N, and $f(.)$ is the algorithm's fitness function.

3 PROPOSED ALGORITHM FRAMEWORK - R2-HMTLBO

We propose a new algorithm framework, R2-HMTLBO, that employs both the NSGA-III and TLBO algorithms to improve the diversity and convergence of solutions, respectively. To apply TLBO for MOPs, a scheme is devised based on the R2-indicator. Compared to WS, TCH, and PBI, recently, achievement scalarization function (ASF) has received increasing attention compared to other methods as promising results have been obtained for MaOPs. Therefore, we have applied ASF, as shown in Equation. 6 in our framework:

$$ASF(\mathcal{S}|U, z^*) = \max_{1 \le i \le m} \left\{ \frac{\left|f_i(x) - z_i^*\right|}{U_i} \right\} \tag{6}$$

where U is a weight vector ($U_i \ge 0$ for $1 \le i \le m$ and $\sum_{i=1}^{m} U_i = 1$), and z^* is a reference vector. As TLBO is an elitist based MOEA, an external archive is considered to select teachers among all non-dominated individuals, collected from the first generation onwards. The archive's members are ranked and sorted using (Algorithm. 1) [12]. The sorting criteria (in ascending order), based on the priority, includes ASF output and the 2-norm of the objectives, respectively.

The main structure of R2-HMTLBO is presented in Algorithm. 2. Firstly, the population (N individuals) is initialized and evaluated. Also, at this stage, reference points are generated based on the Riesz s-energy method [11]. NSO defines the non-dominated individuals. Non-dominated individuals are ranked and sorted by the R2-rank operator and stored in the archive. Then the algorithm continues until it reaches the termination criterion. In each generation, GA operators (simulated binary crossover [SBX] and polynomial mutation [PM]) and TLBO operators are applied to the population in parallel. Regarding the teaching phase, teachers are selected from 20% of the best archive's R2-ranked individuals. After concatenating the populations, normalization, association and niche-preservation operators from NSGA-III are applied, separating N individuals. NSO is then be applied to the populations, and non-dominated individuals are added to the archive. At the end of each generation, the R2-ranking operator is applied to the archive's members and N Individuals will be sorted and selected if their numbers exceed N.

Algorithm 1: R2 Ranking structure

```
input  : Archive Members (P_arc), and weight vector (U)
output : Ranked Archive's members (NewP_arch)
for ū_arc ∈ U do
    for p_arc ∈ P_arc do
        p.Sval ← ASF(p.Obj‾|0, ū)   # p.Sval: scalarization
            value,  p.Obj‾: objectives
    NewP_arch = Sort(P_arch, p.Sval^1st, 2 − norm^2nd)   #1st:
        first priority,  2nd: second priority
```

Algorithm 2: R2-HMTLBO structure

```
input  : Number of individuals (N)
output : Final Population
Initialize first population (P);
Generate reference points;
Apply NSO;
Rank non-dominated solutions and archive them in A_rc;
for i to Iteration_max do
    Q_i1 ← apply SBX and PM on P_i;
    Q_i2 ← apply TLBO operators on P_i;
    R_i = Q_i1 ∪ Q_i2;
    Normalize R_i members;
    Apply Association operator;
    Apply Niche-preservation operator and create P'_i;
    Apply NSO;
    Rank and sort P'_i and archive it in A_rc
```

4 EXPERIMENTAL DESIGN

In order to evaluate the proposed algorithm R2-HMTLBO, three benchmark test functions of ZDT [34], DTLZ [6] and CEC09 (UF) [32] are applied. The results obtained are compared with two well-known algorithms of NSGA-III and MOEA/D, as well as two new indicator-based algorithms of MOMBI-II [12] and MOEA/IGD-NS [26]. The applied test problems' features are shown in Table. 1. The number of objectives is considered two for ZDT and UF1-UF7 test functions and three for UF8-UF10 and DTLZ1-DTLZ4 test functions. We use inverse generational distance (IGD) as the performance

Table 1: ZDT, UF, and DTLZ characteristics

Test Problem	Objective	Dimension	features
ZDT1	2	30	Convex PF
ZDT2	2	30	Concave PF
ZDT3	2	30	Discrete PS & PF
ZDT4	2	30	Multifrontal PF
ZDT6	2	30	Concave PF
UF1	2	30	Concave PF & Complex PS
UF2	2	30	Concave PF & Complex PS
UF3	2	30	Concave PF & Complex PS
UF4	2	30	Convex PF & Complex PS
UF5	2	30	Discrete PF & Complex PS
UF6	2	30	Discrete PF & Complex PS
UF7	2	30	Complex PS
UF8	3	30	Concave and Parabolic PF & Complex PS
UF9	3	30	Discrete and Planar PF & Complex PS
UF10	3	30	Concave and Parabolic PF
DTLZ1	3	7	Linear
DTLZ2	3	12	Concave
DTLZ3	3	12	Concave & Multimodal
DTLZ4	3	12	Concave & Biased

metric. IGD (Equation. 7) is a well-known metric for evaluating an algorithms' convergence and diversity simultaneously.

$$IGD(\mathcal{A}, \mathcal{P}) = \frac{\sum_{x \in \mathcal{P}} ED(x, \mathcal{A})}{|\mathcal{P}|} \tag{7}$$

where $\mathcal{A}$ is an estimated set, and $\mathcal{P}$ is related to the true PF. $\frac{1}{|\mathcal{P}|}$ is the total number of points in $\mathcal{P}$, and the Euclidean distance between x and its nearest point in $\mathcal{P}$ is calculated by the $ED(.)$. Each test function ran twenty times independently to evaluate performance across multiple randomly initialized starting points. For ZDT test problems 10k are chosen as the maximum number of function evaluations (NFE). In addition, for other bi-objective and three-objective test problems (CEC09 and DTLZ), 60k and 100k are considered as the maximum NFE, respectively. In all algorithms, 100 is selected as the number of the population. The source code for NSGA-III and MOEA/D are available in [9] and the source code for MOMBI-II and MOEA/IGD-NS are available in [25].

5 DISCUSSION

The performance (mean and standard deviation of IGD metric for 20 runs) of each algorithm is shown in Table. 2, and the best results (the smallest mean value) are highlighted and presented in boldface. The Tukey HSD test [7] with a 5% significant level is applied for each test problem to perform statistical analysis. For example, in Table. 2, the symbols of +, −, and ≈ represent that the other algorithms work significantly better ($p < 0.05$), worse, and similar, respectively, compared R2-HMTLBO.

The last row of Table. 2 indicates how many times other algorithms perform better, worse, and similar to our algorithm. Statistically, R2-HMTLBO outperforms MOEA/D, MOMBI-II, and MOEA/IGD-NS significantly 16/19 tests, 14/19 tests and 13/19 tests, respectively. Furthermore, in ZDT4, UF5, UF7, and DTLZ1, R2-HMTLBO obtained the best results compared to all other algorithms. There is considerable overlap between R2-HMTLBO and NSGA-III performance. On the other hand, regarding the IGD mean, R2-HMTLBO defeated other algorithms in all ZDT test problems. Also, it outperformed the other algorithms in CEC09 and DTLZ test problems such as UF1, UF2, UF5, UF6, UF7, UF10, DTLZ1, and DTLZ4. In UF3, UF4, UF8, UF9, and DTLZ2 NSGA-III and in DTLZ3 MOMBI-II achieved better IGD than the other algorithms. In DTLZ3 R2-HMTLBO has the worst IGD. It is more apparent that our algorithm did not have proper performance in UF9, which contains planar and discrete PF. The DTLZ3 is a difficult multimodal problem, and it has been shown that R2-HMTLBO could not achieve acceptable solution. Also, although DTLZ1 is more complicated than DTLZ2, R2-HMTLBO produced better results in DLLZ1 regarding its linearity. Overall, based on the results, R2-HMTLBO, compared to all algorithms, produced the smallest average of IGDs across all the test problems. Also, in two-objective problems, it performs specifically better than three-objective problems.

6 CONCLUSION AND FUTURE WORKS

In this paper, a hybrid multi-objective algorithm based on the two algorithms of TLBO and GA is proposed. The results demonstrated that the concatenation of the GA and TLBO operators leads to a more robust algorithm. Furthermore, as two subjects of diversity and convergence have a crucial impact on MOPs, two reference points and R2 indicator structures are applied to improve both issues in our algorithm. Finally, the proposed algorithm is tested on three well-known benchmark test problems (ZDT, DTLZ, and CEC09) and compared with two well-known algorithms of NSGA-III and MOEA/D. Results are also compared against MOMBI-II and MOEA/IGD-N, which are devised based on the R2 and IGD indicators. Based on the obtained results, R2-HMTLBO revealed notable performance in two-objective problems, and compared to the other algorithms converged well. On the other hand, regarding the three-objective test problems, it had a competitive performance. We plan to investigate the synergy of different EA operators and MOEA structures more as part of our future work. In three-objective problems, the algorithm does not perform as successfully as bi-objective problems in reaching the solutions. Therefore, there is a need to search for alternative indicator-based methods or devise a better scheme for the elite archive to address this problem.

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Table 2: ZDT, UF, and DTLZ test problem results based on the IGD metric

Test Problem	R2-HMTLBO	NSGA-III	MOEAD	MOMBI-II	MOEA/IGD-NS
ZDT1	**3.4980E-3 ± 6.9000E-5**	1.2970E-2 ± 2.4660E-3 (≈)	6.3064E-2 ± 1.9113E-2 (−)	5.8522E-2 ± 5.4020E-2 (−)	1.8554E-2 ± 5.1080E-3 (−)
ZDT2	**3.6490E-3 ± 6.8000E-5**	2.4248E-2 ± 1.6111E-1 (−)	2.6808E-1 ± 1.8547E-2 (−)	1.3736E-1 ± 1.2649E-2 (−)	6.4300E-2 ± 7.7360E-2 (≈)
ZDT3	**5.3010E-3 ± 1.4200E-5**	1.5776E-2 ± 1.6888E-2 (≈)	7.0039E-2 ± 2.3990E-2 (−)	1.7309E-2 ± 1.2429E-2 (≈)	1.8676E-2 ± 5.7220E-3 (−)
ZDT4	**3.4700E-3 ± 2.7000E-5**	4.6259E-1 ± 2.2945E-1 (−)	8.0349E-1 ± 7.6720E-1 (−)	3.4504E-1 ± 1.6524E-1 (−)	4.7288E-1 ± 2.2013E-1 (−)
ZDT6	**1.7058E-1 ± 1.1500E-4**	9.2324E-1 ± 8.3029E-2 (−)	6.2133E-1 ± 1.3369E-1 (−)	5.1522E-2 ± 2.3780E-2 (−)	2.0145E-1 ± 6.8613E-2 (≈)
UF1	**6.2471E-2 ± 1.8898E-2**	9.0994E-2 ± 2.5818E-2 (≈)	1.3840E-1 ± 5.9939E-2 (−)	1.1333E-1 ± 3.3444E-2 (−)	1.2125E-1 ± 3.6193E-2 (−)
UF2	**2.8347E-2 ± 1.4620E-3**	3.4976E-2 ± 8.0130E-3 (≈)	6.2177E-2 ± 3.2276E-2 (−)	4.6555E-2 ± 8.5870E-3 (−)	4.7135E-2 ± 6.3410E-3 (−)
UF3	1.2748E-1 ± 1.4573-2	**1.0381E-1 ± 2.5687E-2** (≈)	1.7016E-1 ± 4.7597E-2 (−)	2.4948E-1 ± 5.5551E-2 (−)	2.3950E-1 ± 5.2926E-2 (−)
UF4	4.7389E-2 ± 6.7300E-4	**4.6581E-2 ± 7.9700E-4** (≈)	6.0119E-2 ± 4.3580E-3 (−)	4.9119E-2 ± 2.7780E-3 (≈)	5.0113E-2 ± 1.6680E-3 (−)
UF5	**9.4608E-2 ± 3.5075E-2**	2.8592E-1 ± 5.9337E-2 (−)	3.8003E-1 ± 7.3909E-2 (−)	3.1048E-1 ± 9.6874E-2 (−)	2.6684E-1 ± 6.1080E-2 (−)
UF6	**1.1308E-1 ± 4.7015E-2**	1.7407E-1 ± 8.7382E-2 (≈)	3.3660E-1 ± 1.4270E-1 (−)	1.7919E-1 ± 1.3136E-1 (≈)	1.7980E-1 ± 9.1991E-2 (≈)
UF7	**3.9900E-2 ± 4.7540E-3**	1.6243E-1 ± 1.3405E-1 (−)	3.5603E-1 ± 1.4325E-1 (−)	1.9037E-1 ± 1.6987E-1 (−)	1.7146E-1 ± 1.4254E-1 (−)
UF8	1.7369E-1 ± 6.7322E-2	**1.7199E-1 ± 6.5957E-2** (≈)	2.1263E-1 ± 1.7940E-1 (≈)	4.6871E-1 ± 9.8168E-2 (−)	2.4927-1 ± 1.1205E-2 (≈)
UF9	1.6174E-1 ± 8.7934E-2	**1.4985E-1 ± 6.8612E-2** (≈)	1.9476E-1 ± 1.0253E-1 (≈)	3.8101E-1 ± 8.4204E-2 (−)	2.5676E-1 ± 8.3413E-2 (−)
UF10	**2.9766E-1 ± 1.0957E-2**	3.6350E-1 ± 8.4059E-2 (≈)	5.7772E-1 ± 2.1179E-1 (−)	4.9464E-1 ± 1.0506E-1 (−)	3.2839E-1 ± 9.9105E-2 (≈)
DTLZ1	**8.7780E-3 ± 5.6000E-5**	1.0667E-2 ± 4.5130E-3 (−)	2.8708E-2± 5.4500E-4 (−)	1.3784E-2 ± 4.2000E-5 (−)	1.9270E-2 ± 1.2294E-4 (−)
DTLZ2	2.7560E-2 ± 6.6000E-5	**3.7430E-3 ± 3.1700E-4** (+)	1.0614E-1 ± 3.3530E-3 (−)	3.6485E-2 ± 2.3000E-5 (−)	5.1396E-2 ± 5.5300E-4 (−)
DTLZ3	1.0542 ± 1.1682	4.8244E-2 ± 9.3120E-3 (+)	1.0019E-1 ± 5.9810E-3 (+)	**3.7222E-2 ± 5.1300E-4** (+)	5.0662E-2 ± 4.0041E-4 (+)
DTLZ4	**1.1255E-2 ± 1.45E-4**	4.4416E-2 ± 1.1189E-1 (≈)	2.5684E-1 ± 2.7505E-1 (−)	3.6549E-2 ± 5.7000E-5 (≈)	2.2281E-1 ± 2.3942E-1 (−)
Average of IGDs	**1.281E-1**	1.647E-1	2.530E-1	1.693E-1	2.249E-1
+/−/≈		2/6/11	1/16/2	1/14/4	1/13/5

Note: −, ≈, and + besides each algorithm indicate that R2-HMTLBO's performance is significantly better than, equivalent to, and worse than the mentioned algorithm

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