

**THE UNIVERSITY OF MANCHESTER - APPROVED ELECTRONICALLY  
GENERATED THESIS/DISSERTATION COVER-PAGE**

Electronic identifier: 21987

Date of electronic submission: 10/07/2017

The University of Manchester makes unrestricted examined electronic theses and dissertations freely available for download and reading online via Manchester eScholar at <http://www.manchester.ac.uk/escholar>.

This print version of my thesis/dissertation is a TRUE and ACCURATE REPRESENTATION of the electronic version submitted to the University of Manchester's institutional repository, Manchester eScholar.

AN EFFICIENT RANKING ANALYSIS  
IN MULTI-CRITERIA DECISION MAKING

A THESIS SUBMITTED TO THE UNIVERSITY OF MANCHESTER  
FOR THE DEGREE OF DOCTOR PHILOSOPHY  
IN THE FACULTY OF SCIENCE & ENGINEERING

2017

By  
Nor Izzati Jaini  
School of Mechanical, Aerospace and Civil Engineering

# Contents

List of figures	5
List of tables	6
Nomenclature	8
Abstract	10
Declaration	11
Copyright	12
Acknowledgements	13
Author list of publications	15
<b>1 Introduction</b>	<b>18</b>
1.1 Background, motivation and scope	18
1.2 Multi-Criteria decision making method	21
1.3 Uncertainty analysis in multi-criteria decision making process	26
1.4 Critical criterion in multi-criteria decision making	27
1.5 Fuzzy multi-criteria decision making method	28
1.6 Research contributions and thesis structure	29
<b>2 Distance-based ranking methods in multi-criteria decision making</b>	<b>31</b>
2.1 Pareto optimality	33
2.2 Relative distance ranking	33
2.3 TOPSIS	35
2.4 Trade-off ranking method	37
2.5 Test cases: analysis and comparison	44

2.5.1	General Test Cases	45
2.5.2	Application Test Case	50
<b>3</b>	<b>Uncertainty and sensitivity analysis in multi-criteria decision making process</b>	<b>53</b>
3.1	Trade-off ranking modification	54
3.2	Uncertainty in the input data	59
3.2.1	Robust Set of Alternatives	61
3.2.2	Test Case: Two-bar Truss Structure	62
3.3	Uncertainty in the Decision Maker's preference	65
3.3.1	Computational Experiment	66
3.3.2	Objective Weights via the Trade-off Ranking Method	70
3.4	Critical criterion analysis in multi-criteria decision making	72
3.4.1	Methodology	73
3.4.2	Critical criterion analysis	74
<b>4</b>	<b>Fuzzy multi-criteria decision making method</b>	<b>80</b>
4.1	Multi-criteria decision making and fuzzy numbers	81
4.1.1	Arithmetic operations on triangular fuzzy numbers	85
4.2	Trade-off ranking method with defuzzification	87
4.3	Fuzzy trade-off ranking	89
4.4	Fuzzy TOPSIS	92
4.5	Fuzzy VIKOR	94
4.6	Analysis and comparison	96
<b>5</b>	<b>Conclusions</b>	<b>105</b>
5.1	Summary of research findings	105
5.2	Future research implications	108
	<b>References</b>	<b>110</b>
	<b>Appendix A</b>	<b>124</b>

<b>Appendix B</b>	<b>126</b>
<b>Appendix C</b>	<b>127</b>
Final Word Count: 28,735	

## List of Figures

2.1	Two sets of Pareto solutions	38
2.2	Practical examples for the trade-off ranking	41
2.3	Results for the top ranking with the TOPSIS, relative distance and trade-off methods	42
2.4	Results for the top ranking with the TOPSIS, relative distance and trade-off methods	43
2.5	Distance measures in TOPSIS and trade-off ranking	44
2.6	Results of the highest ranking for TNK	46
2.7	Results of the highest ranking for ZDT1	47
2.8	Results of the highest ranking for ZDT2	48
2.9	Results of the highest ranking for ZDT6	49
2.10	Results of the highest ranking for DTLZ5	49
3.1	Difference in distance measures between the trade-off ranking, TOPSIS and the relative distance method	58
3.2	Two-bar truss structure	63
3.3	Results of the top ranking for Possibilistic Mean and robust Pareto frontier	64
3.4	Top ranking for each method with each weight case for ZDT2	67
3.5	Top ranking for each method with each weight case for DTLZ5	69
3.6	Examples for the objective weights calculation	72
4.1	Membership functions of the linguistic variables	83
4.2	Triangular fuzzy number $\tilde{f} = (a, b, c)$	85
4.3	Triangular fuzzy numbers and their crisp values of each criterion for alternative $A_2$	100

## List of Tables

2.1	Trade-off ranking for Pareto frontier FH	40
2.2	Trade-off ranking for Pareto frontier FJ	40
2.3	Input data for car selection example and the results of ranking	50
2.4	Preference ranking for each criterion in the car example	51
3.1	Weight cases for DTLZ5	68
3.2	Data of the problem	74
3.3	Current ranking for each method	74
3.4	New ranking for each method with weight change in $w_1$	75
3.5	New ranking for each method with weight change in $w_2$	76
3.6	New ranking for each method with weight change in $w_3$	77
3.7	New ranking for each method with weight change in $w_4$	78
4.1	Fuzzy numbers for the importance weight of each criterion	82
4.2	Linguistic variables for the alternative performance	83
4.3	Criteria weight by decision makers	97
4.4	Alternatives ratings by decision makers	97
4.5	The fuzzy decision matrix for the personnel selection problem	98
4.6	Defuzzified decision matrix	98
4.7	Normalized defuzzified decision matrix by the trade-off ranking	99
4.8	Ranking by fuzzy trade-off	99
4.9	Results by fuzzy trade-off	99
4.10	Ranking by fuzzy VIKOR	101
4.11	Results by fuzzy VIKOR	101
4.12	Ranking and results by fuzzy TOPSIS	102
4.13	New criteria weight by decision makers	102
4.14	New fuzzy and defuzzified weights	102
4.15	Results by the fuzzy MCDM methods with the new criteria weights	103
5.1	Data for TNK problem $(x_1, x_2)$	127
5.2	Data for ZDT1 problem $(F_1, F_2)$	127

5.3	Data for ZDT2 problem ( $F_1, F_2$ )	128
5.4	Data for DTLZ5 problem ( $F_1, F_2, F_3$ )	128



## Nomenclature

$A_i$	Alternative $i$
$Y_j$	Criterion $j$
$Y_{ij}$	Performance of alternative $i$ in terms of criterion $j$
$w_j$	Weight of criterion $j$
$w^*$	New weight
$w'$	Normalized weight
$m$	Number of criteria/objective functions
$q$	Number of alternatives/Pareto solutions
$I^+$	Ideal solution
$I^-$	Anti-ideal solution
$A_k^*$	$k$ -th extreme solution
$\sim$	Uncertain parameter
$\sigma_Y^2$	Variance of variable $Y$
$n$	number of parameters which vary
$\delta$	Value of change in the current weight
$mp$	Possibilistic mean
$K$	Number of Decision Makers
DM	Decision Maker
MCDM	Multi-criteria decision making
AHP	Analytic Hierarchy Process
ANP	Analytic Network Process
MAUT	Multi-Attribute Utility Theory

ELECTRE	Elimination and Choice Expressing Reality
PROMETHEE	Preference Ranking Organization Method for Enrichment Evaluations
GA	Genetic algorithm
PSO	Partial swarm optimization
DSD	Directed Search Domain
TOPSIS	Technique for order preference by similarity to ideal solution
VIKOR	Vise Kriterijumska Optimizacija I Kompromisno Resenje

# **An Efficient Ranking Analysis in Multi-criteria Decision Making**

Nor Izzati Jaini, 2017

PhD in Mechanical Engineering

The University of Manchester

## **Abstract**

This study is conducted with the aims to develop a new ranking method for multi-criteria decision making problem with conflicting criteria. Such a problem has a set of Pareto solutions, where the act of improving a value of one solution will result in depreciating some of the others. Thus, in this type of problem, there is no unique solution. However, out of many available options, the Decision Maker eventually has to choose only one solution. With this problem as the motivation, the current study develops a compromise ranking algorithm, namely a trade-off ranking method. The trade-off ranking method able to give a trade-off solution with the least compromise compared to other choices as the best solution. The properties of the algorithm are studied in the thesis on several test cases. The proposed method is compared against several multi-criteria decision making methods with ranking based on the distance measure, which are the TOPSIS, relative distance and VIKOR. The sensitivity analysis and uncertainty test are carried out to examine the methods robustness. A critical criteria analysis is also done to test for the most critical criterion in a multi-criteria problem. The decision making method is considered further in a fuzzy environment problem where the fuzzy trade-off ranking is developed and compared against existing fuzzy decision making methods.

**Keywords:** Trade-off, ranking, multi-objective optimization, multi-criteria decision making, Pareto optimal solution, directed search domain algorithm