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MULTI-SERVICE RESOURCE QUEUE WITH THE MULTY-COMPONENT POISSON ARRIVALS

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Abstract. In many papers devoted to telecommunication systems, multi-service queueing models with heterogeneous customer arrivals are given. As a rule, there is streaming traffic, corresponding to the audio and video conference, and elastic traffic, corresponding to the data and file transmission. Different types of traffic require different quality of service, therefore the multi-service systems are also differentiated service systems. Such papers are aimed at developing a framework for estimating the quality of service indicators in new generation multi-service networks. This paper considers a multi-service resource queueing system with three Poisson arrivals, one of which is splitting. The equation for the probabilities distribution of the total resource amounts on the system blocks is compiling by the dynamic screening method, and using the characteristic function of the stationary distribution, the solution is obtaining. The numerical characteristics of the system performance are obtained by the method of moments using characteristic function. A numerical example shows that arrivals intensity growing increases optimal total resource amounts on system blocks in the system with limited resources, and the splitting arrivals affect the correlation between total resource amounts.

1. Introduction

Modern multi-service networks are packet-based networks in which packet streams are generated by conventional Internet applications (i.e., services offered on the network). Such networks are subjected to the settings network traffic through various mechanisms such as dynamic resource reservation, call forwarding, priorities for selected call and service classes, etc. To implement the described mechanisms, the researcher needs to have an analytical framework that will allow calculating the main probabilistic and numerical characteristics of the system performance.

The study of combined models in queueing theory originates from the works of Cohen [3, 4, 5], which devoted to the problem of repeated calls in long-distance telephone systems. In modern scientific papers, the interest in the study of multi-service networks is observed again. In paper [10], a multi-service model of a queueing system with elastic and adaptive traffic presented and discussed. The

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presented model allows determining the characteristics of a multi-service queueing system, such as drop probability or the average queueing length. The model can be used for the analysis of modern network systems based on TCP/IP protocols, particularly the Internet. The papers [8, 9] present a multi-service resource queueing systems (RQSs) with state-dependent resource allocation for each call type. The proposed model allows estimating the averaged parameters of RQS for individual customer types that are arriving in the system.

The paper [2] considers a managed RQS and finds an optimal resource management problem in the context of a splitting multi-service cellular network. The papers [14, 15, 18] discuss models of multi-server RQSs with losses caused by a lack of resources required to service. During the service, each customer takes a random resource amount of several types. The random vectors describing resource requirements do not depend on the arrivals and service time. As with the Erlang problem, the task of calculates the drop probability of an arrival customer due to a resource lack was considered.

An approach to the study of multi-service heterogeneous RQS is also presented in papers [1, 16, 17]. The authors consider several variations of the RQS and propose to study them using dynamic screening and asymptotic analysis methods.

We have a similar task to obtain the main numerical characteristics of system performance. We consider a two-block RQS with three Poisson arrivals, one of which is with splitting. This paper is organized as follows. In Section 2, we describe in detail the mathematical model of the system. In Sections 3 and 4, we describe and apply the dynamic screening method to compose the Kolmogorov equations, then, in 5 and 6, we turn to the equation for the characteristic functions, find the solution and write it for a stationary distribution. In Section 7, using the method of moments, we obtain the main numerical characteristics, and in Section 8, we give a numerical example.

2. Mathematical Model

Consider the two-block RQS with an unlimited number of servers and resources shown in Fig. 1. Customers arrive in the system according to three Poisson processes with constant intensities λ_1, λ_2 and λ. The arrivals from λ_i process go to the i-th service block, $i = 1, 2$, and the arrivals from λ process are split and go to both service blocks.

The service times on the i-th block are the random variables ξ_i with distribution function $B_i(x) = P\{\xi_i < x\}$, $i = 1, 2$, and also taking random resources amount ν_i with distribution function $G_i(y) = P\{\nu_i < y\}$, $i = 1, 2$. Upon completion of service, each customer leaves the system, frees the server, and all used resources. The occupied resources amount and the service time are independent of each other.

The goal of this paper is to obtain the numerical characteristics of the total volumes of occupied resources in the system blocks. We study the system using the dynamic screening method, originally proposed in papers [12, 11], and method of moments [6].

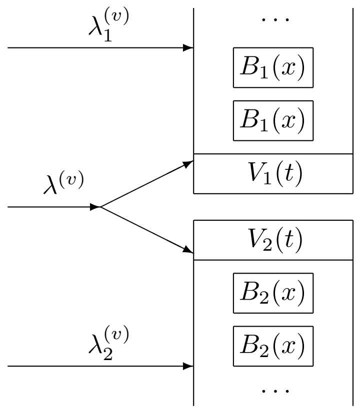

FIGURE 1. Mathematical model

3. Dynamic Screening Method

Let us denote the total amount of occupied resource on the first and second block at time t by $V_1(t)$ and $V_2(t)$, respectively. The goal is to find the stationary probability distribution of the two-dimensional random process $\{V_1(t), V_2(t)\}$. However, this process is non-Markovian, therefore, we will use the dynamic screening method for its investigation.

Let the system be empty at moment t_0, and let us fix a certain time moment $T > t_0$. The axis 0 shows all arrivals of customers (see Fig. 2). We generate the points of the screened processes (axes 1 and 2) from the moments of arrivals. Consider the probability that a customer arriving at time t will not finish its service until the moment T. Let us denote the probability of screened arrivals on axis i as $S_i(t) = 1 - B_i(T - t)$, $(i = 1, 2)$ and on both axes as $S_1(t) \cdot S_2(t) = (1 - B_1(T - t)) \cdot (1 - B_2(T - t))$.

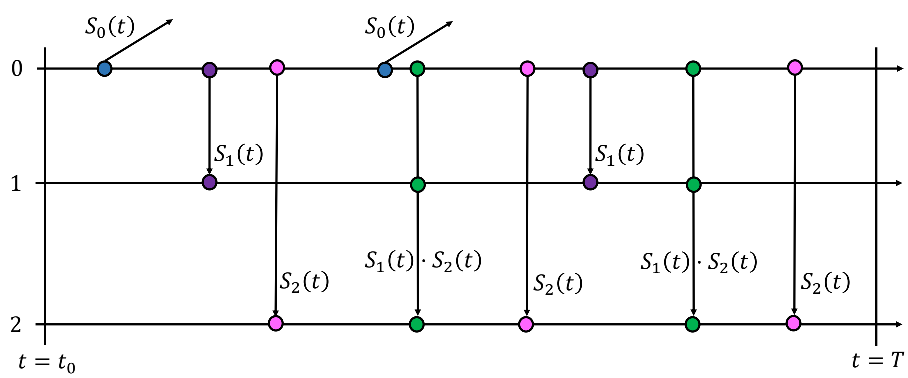

FIGURE 2. Screening of arrived of customers

Let us denote the total resource amounts occupied by screened arrivals in the interval $[t_0, t)$ by $W_1(t)$ and $W_2(t)$ on the first and second axes, respectively. At the time moment $t = T$, the probability distributions of random variables $\{V_1(t), V_2(t)\}$ and $\{W_1(t), W_2(t)\}$ coincide [13]:

$$P\{V_1(T) < y_1, V_2(T) < y_2\} = P\{W_1(T) < y_1, W_2(T) < y_2\}, \\ y_1 > 0, y_2 > 0. \tag{3.1}$$

4. Integro-Differential Equation

We denote the cumulative distribution function (CDF) of the 2-dimensional process $\{W_1(t), W_2(t)\}$ by $P(w_1, w_2, t)$, i.e. $P\{W_1(t) < w_1, W_2(t) < w_2\} = P(w_1, w_2, t)$. Then for this distribution we can write the following equality

$$\begin{gathered} P(w_1, w_2, t + \Delta t) = P(w_1, w_2, t)(1 - \lambda\Delta t)(1 - \lambda_1\Delta t)(1 - \lambda_2\Delta t) + \\ \lambda\Delta t P(w_1, w_2, t)(1 - S_1(t))(1 - S_2(t)) + \\ \lambda_1\Delta t P(w_1, w_2, t)(1 - S_1(t)) + \lambda_2\Delta t P(w_1, w_2, t)(1 - S_2(t)) + \\ \lambda\Delta t \left[S_1(t)(1 - S_2(t)) \int\limits_0^{w_1} P(w_1 - y_1, w_2, t)dG_1(y_1) + \right. \\ (1 - S_1(t))S_2(t) \int\limits_0^{w_2} P(w_1, w_2 - y_2, t)dG_2(y_2) + \\ \left. S_1(t)S_2(t) \int\limits_0^{w_1}\int\limits_0^{w_2} P(w_1 - y_1, w_2 - y_2, t)dG_1(y_1)dG_2(y_2) \right] + \\ \lambda_1\Delta t S_1(t) \int\limits_0^{w_1} P(w_1 - y_1, w_2, t)dG_1(y_1) + \\ \lambda_2\Delta t S_2(t) \int\limits_0^{w_2} P(w_1, w_2 - y_2, t)dG_2(y_2) + o(\Delta t), \quad w_1, w_2 > 0. \end{gathered}$$

We divide obtained equality by Δt and use limit condition $\Delta t \to 0$, we have the Kolmogorov integro-differential equation in the form

$$\frac{\partial P(w_1, w_2, t)}{\partial t} = P(w_1, w_2, t)\left[-\lambda_1 - \lambda_2 - \lambda + \lambda(1 - S_1(t))(1 - S_2(t)) + \right.$$

$$\left. \lambda_1(1 - S_1(t)) + \lambda_2(1 - S_2(t))\right] + \lambda \left[S_1(t)(1 - S_2(t)) \int\limits_0^{w_1} P(w_1 - y_1, w_2, t)dG_1(y_1) + \right.$$

$$
\begin{gathered}
(1-S_1(t))S_2(t)\int\limits_0^{w_2} P(w_1,w_2-y_2,t)dG_2(y_2)+\\
\left. S_1(t)S_2(t)\int\limits_0^{w_1}\int\limits_0^{w_2} P(w_1-y_1,w_2-y_2,t)dG_1(y_1)dG_2(y_2)\right]+\\
\lambda_1 S_1(t)\int\limits_0^{w_1} P(w_1-y_1,w_2,t)dG_1(y_1)+\lambda_2 S_2(t)\int\limits_0^{w_2} P(w_1,w_2-y_2,t)dG_2(y_2),\\
w_1,w_2>0,
\end{gathered}
\tag{4.1}
$$

with the initial condition

$$
P(w_1,w_2,t_0)=\begin{cases} 1, & w_1=w_2=0,\\ 0, & \text{otherwise.}\end{cases}
$$

5. Characteristic Function

We introduce the characteristic function $h(u_1,u_2,t)$ for the distribution $P(w_1,w_2,t)$ in the form

$$
h(u_1,u_2,t)=\int\limits_0^{\infty}\int\limits_0^{\infty} e^{ju_1w_1}e^{ju_2w_2}P(dw_1,dw_2,t),\quad j=\sqrt{-1}.
$$

Then, we rewrite the Equation (4.1) for the characteristic function $h(u_1,u_2,t)$

$$
\begin{gathered}
\frac{\partial h(u_1,u_2,t)}{\partial t}=h(u_1,u_2,t)\left[S_1(t)(\lambda+\lambda_1)(G_1^*(u_1)-1)+\right.\\
\left.S_2(t)(\lambda+\lambda_2)(G_2^*(u_2)-1)+\lambda S_1(t)S_2(t)(G_1^*(u_1)-1)(G_2^*(u_2)-1)\right],
\end{gathered}
\tag{5.1}
$$

where we introduced the notation

$$
G_i^*(u_i)=\int\limits_0^{\infty} e^{ju_iy}dG_i(y),
$$

with the initial condition

$$
h(u_1,u_2,t_0)=1. \tag{5.2}
$$

The Equation (5.1) is the separable differential equation, therefore, we rewrite as

$$
\begin{gathered}
\frac{dh(u_1,u_2,t)}{h(u_1,u_2,t)}=\left[S_1(t)(\lambda+\lambda_1)(G_1^*(u_1)-1)+S_2(t)(\lambda+\lambda_2)(G_2^*(u_2)-1)+\right.\\
\left.\lambda S_1(t)S_2(t)(G_1^*(u_1)-1)(G_2^*(u_2)-1)\right]dt,
\end{gathered}
$$

and the solution to this equation takes the form

$$h(u_1,u_2,t) = C\exp\Bigg\{\int\limits_{t_0}^{t}\Big[S_1(\tau)(\lambda+\lambda_1)(G_1^*(u_1)-1)+$$

$$S_2(\tau)(\lambda+\lambda_2)(G_2^*(u_2)-1)+\lambda S_1(\tau)S_2(\tau)(G_1^*(u_1)-1)(G_2^*(u_2)-1)\Big]d\tau\Bigg\}.$$

Taking into account the Equation (5.2), we can conclude that $C = 1$ and

$$h(u_1,u_2,t) = \exp\left\{\left[(\lambda+\lambda_1)(G_1^*(u_1)-1)\int\limits_{t_0}^{t}S_1(\tau)d\tau+\right.\right.$$

$$(\lambda+\lambda_2)(G_2^*(u_2)-1)\int\limits_{t_0}^{t}S_2(\tau)d\tau+$$

$$\left.\left.\lambda(G_1^*(u_1)-1)(G_2^*(u_2)-1)\int\limits_{t_0}^{t}S_1(\tau)S_2(\tau)d\tau\right]\right\}.$$

6. Steady-State Regime

Further, we obtain the characteristic function of the 2D process $\{V_1(t),V_2(t)\}$ in the steady-state regime. To this aim, we put $t_0\to-\infty, t=T$ and use the main formula of the dynamic screening method (3.1). Thus, we obtain

$$h(u_1,u_2,T) = \exp\left\{(\lambda+\lambda_1)(G_1^*(u_1)-1)\int\limits_{-\infty}^{T}(1-B_1(T-\tau))d\tau+\right.$$

$$(\lambda+\lambda_2)(G_2^*(u_2)-1)\int\limits_{-\infty}^{T}(1-B_2(T-\tau))d\tau+ \tag{6.1}$$

$$\left.\lambda(G_1^*(u_1)-1)(G_2^*(u_2)-1)\int\limits_{-\infty}^{T}(1-B_1(T-\tau))(1-B_2(T-\tau))d\tau\right\}.$$

We consider the integrals that are in the exponent of (6.1)

$$\int\limits_{-\infty}^{T}(1-B_i(T-\tau))d\tau = \int\limits_{0}^{\infty}(1-B_i(w))dw \triangleq b_i,\quad i=1,2,$$

$$\int\limits_{-\infty}^{T}(1-B_1(T-\tau))(1-B_2(T-\tau))d\tau = \int\limits_{0}^{\infty}(1-B_1(w))(1-B_2(w))dw \triangleq b_{12},$$

therefore, we get the characteristic function $h(u_1,u_2)$ in the form

$$h(u_1,u_2)=\exp\{(\lambda+\lambda_1)(G_1^*(u_1)-1)b_1+\\(\lambda+\lambda_2)(G_2^*(u_2)-1)b_2+\lambda(G_1^*(u_1)-1)(G_2^*(u_2)-1)b_{12}\}.$$

7. Method of Moments

At the next step, we find the numerical characteristics by the method of moments. According to the method of moments, we know that the means $m_1^{(i)}$ of processes $V_i(t)$ for $i=1,2$ can be calculate as

$$m_1^{(i)}=-j\cdot\left.\frac{\partial h(u_1,u_2)}{\partial u_i}\right|_{u_1=u_2=0}.$$

Primarily, we consider the following derivatives

$$\frac{\partial h(u_1,u_2)}{\partial u_1}=\exp\Big\{(\lambda+\lambda_1)(G_1^*(u_1)-1)b_1+(\lambda+\lambda_2)(G_2^*(u_2)-1)b_2+\\ \lambda(G_1^*(u_1)-1)(G_2^*(u_2)-1)b_{12}\Big\}\cdot\Big[(\lambda+\lambda_1)G_1^{*'}(u_1)b_1+\\ \lambda G_1^{*'}(u_1)(G_2^*(u_2)-1)b_{12}\Big],$$

and similarly

$$\frac{\partial h(u_1,u_2)}{\partial u_2}=\exp\Big\{(\lambda+\lambda_1)(G_1^*(u_1)-1)b_1+(\lambda+\lambda_2)(G_2^*(u_2)-1)b_2+\\ \lambda(G_1^*(u_1)-1)(G_2^*(u_2)-1)b_{12}\Big\}\cdot\Big[(\lambda+\lambda_2)G_2^{*'}(u_2)b_2+\\ \lambda(G_1^*(u_1)-1)G_2^{*'}(u_2)b_{12}\Big],$$

we note, that

$$G_i^*(u_i)\big|_{u_i=0}=\left.\int_0^\infty e^{ju_iy_i}dG_i(y_i)\right|_{u_i=0}=1,$$

$$\left.G_i^{*'}(u_i)\right|_{u_i=0}=\left.\int_0^\infty jy_ie^{ju_iy_i}dG_i(y_i)\right|_{u_i=0}=\\ j\int_0^\infty y_idG_i(y_i)=ja_1^{(i)},$$

where

$$\int_0^\infty y_idG_i(y_i)\triangleq a_1^{(i)}.$$

Therefore, we obtain that

$$m_1^{(1)} = -j \left. \frac{\partial h(u_1,u_2)}{\partial u_1} \right|_{u_1=u_2=0} = -j \cdot (\lambda+\lambda_1) b_1 j a_1^{(1)} = (\lambda+\lambda_1) b_1 a_1^{(1)},$$

and

$$m_1^{(2)} = -j \left. \frac{\partial h(u_1,u_2)}{\partial u_2} \right|_{u_1=u_2=0} = -j \cdot (\lambda+\lambda_2) b_2 j a_1^{(2)} = (\lambda+\lambda_2) b_2 a_1^{(2)}.$$

Secondly, we know that the second initial moments can be calculated as

$$m_2^{(i)} = -\left. \frac{\partial^2 h(u_1,u_2)}{\partial u_i^2} \right|_{u_1=u_2=0},$$

therefore, we consider the following derivatives

$$\frac{\partial^2 h(u_1,u_2)}{\partial u_1^2} = \exp\Big\{(\lambda+\lambda_1)(G_1^*(u_1)-1)b_1 + (\lambda+\lambda_2)(G_2^*(u_2)-1)b_2 + \lambda(G_1^*(u_1)-1)(G_2^*(u_2)-1)b_{12}\Big\} \cdot \Big[(\lambda+\lambda_1)G_1^{*'}(u_1)b_1 + \lambda G_1^{*'}(u_1)(G_2^*(u_2)-1)b_{12}\Big]^2 + \exp\Big\{(\lambda+\lambda_1)(G_1^*(u_1)-1)b_1 + (\lambda+\lambda_2)(G_2^*(u_2)-1)b_2 + \lambda(G_1^*(u_1)-1)(G_2^*(u_2)-1)b_{12}\Big\} \cdot \Big[(\lambda+\lambda_1)G_1^{*''}(u_1)b_1 + \lambda G_1^{*''}(u_1)(G_2^*(u_2)-1)b_{12}\Big],$$

and symmetrically for $\dfrac{\partial^2 h(u_1,u_2)}{\partial u_2^2}$. Here, we note that

$$G_i^{*''}(u_i) = \left. \int_0^\infty (jy_i)^2 e^{ju_i y_i} dG_i(y_i) \right|_{u_i=0} = -\int_0^\infty y_i^2 dG_i(y_i) = -a_2^{(i)},$$

where

$$\int_0^\infty y_i^2 dG_i(y_i) \triangleq a_2^{(i)}.$$

Thus, we write

$$m_2^{(1)} = -\left. \frac{\partial^2 h(u_1,u_2)}{\partial u_1^2} \right|_{u_1=u_2=0} = \Big[(\lambda+\lambda_1)b_1 a_1^{(1)}\Big]^2 + (\lambda+\lambda_1) b_1 a_2^{(1)}$$

and

$$m_2^{(2)} = -\left. \frac{\partial^2 h(u_1,u_2)}{\partial u_2^2} \right|_{u_1=u_2=0} = \Big[(\lambda+\lambda_2)b_2 a_1^{(2)}\Big]^2 + (\lambda+\lambda_2) b_2 a_2^{(2)}.$$

Then, the variations can be calculated as $\sigma_i^2 = m_2^{(i)} - \left(m_1^{(i)}\right)^2$, and we obtain

$$\sigma_1^2 = (\lambda+\lambda_1) a_2^{(1)} b_1, \quad \sigma_2^2 = (\lambda+\lambda_2) a_2^{(2)} b_2.$$

Finally, consider the mixed derivative to obtain the covariance

$$\frac{\partial^2 h(u_1,u_2)}{\partial u_1 \partial u_2} = \exp\Big\{(\lambda+\lambda_1)(G_1^*(u_1)-1)b_1 + (\lambda+\lambda_2)(G_2^*(u_2)-1)b_2 +$$

$$(\lambda+\lambda_2)(G_2^*(u_2)-1)b_2 + \lambda(G_1^*(u_1)-1)(G_2^*(u_2)-1)b_{12}\Big\}\cdot$$

$$\Big\{\Big[(\lambda+\lambda_2)G_2^{*'}(u_2)b_2 + \lambda(G_1^*(u_1)-1)G_2^{*'}(u_2)b_{12}\Big]\cdot$$

$$\Big[(\lambda+\lambda_1)G_1^{*'}(u_1)b_1 + \lambda(G_2^*(u_2)-1)G_1^{*'}(u_1)b_{12}\Big]+$$

$$\lambda G_1^{*'}(u_1)G_2^{*'}(u_2)b_{12}\Big\},$$

then,

$$K_{12} = -\left.\frac{\partial^2 h(u_1,u_2)}{\partial u_1 \partial u_2}\right|_{u_1=u_2=0} - m_1^{(1)} m_1^{(2)} = \lambda a_1^{(1)} a_1^{(2)} b_{12}.$$

Therefore, the correlation has the form:

$$r_{12} = \frac{K_{12}}{\sigma_1 \cdot \sigma_2} = \frac{\lambda a_1^{(1)} a_1^{(2)} b_{12}}{\sqrt{(\lambda+\lambda_1)a_2^{(1)} b_1 \cdot (\lambda+\lambda_2)a_2^{(2)} b_2}} =$$

$$= \frac{\lambda}{\sqrt{(\lambda+\lambda_1)\cdot(\lambda+\lambda_2)}} \frac{a_1^{(1)} \cdot a_1^{(2)}}{\sqrt{a_2^{(1)} \cdot a_2^{(2)}}} \frac{b_{12}}{\sqrt{b_1 \cdot b_2}}.$$

8. Numerical Example

8.1. About total resource amounts. Let us consider a numerical example. Let the intensities $\lambda_1 = 2$, $\lambda_2 = 5$, $\lambda \in [0; 50]$; $B_1(x)$, $B_2(x)$ are Gamma CDFs with parameters $\alpha_1 = 2$, $\beta_1 = 5$ and $\alpha_2 = 3$, $\beta_2 = 5$, respectively; and $G_1(y)$, $G_2(y)$ are Poisson CDFs with parameters $\mu_1 = 4$, $\mu_2 = 2$, respectively.

Then, we have $m_1^{(1)} = 40(2+\lambda)$ and $m_1^{(2)} = 30(5+\lambda)$, $\sigma_1^2 = 200(2+\lambda)$ and $\sigma_2^2 = 90(5+\lambda)$.

Using the Three Sigma Rule [7] and the obtained functions of λ, we can find the optimal required resource amount on the system blocks for the system with the limited resource by the formula

$$R_{opt}^{(i)} = m_1^{(i)} + 3\sigma_i. \tag{8.1}$$

Let us show the means required resource amount and their optimal values from (8.1) for both system blocks graphically in the figure 3, as a function of the parameter $\lambda \in [0; 50]$. It is clear, that with an increase in the arrival intensity, the means of the total occupied resource amounts increases.

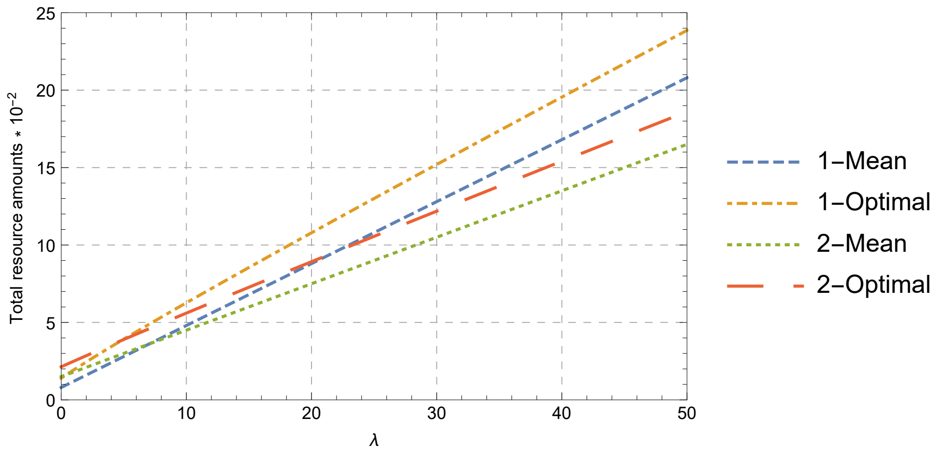

FIGURE 3. Means and optimal values

8.2. About correlation. Obviously, the correlation value is influenced by the parameters of service times, arrival processes and required resource amounts. The following is an effects analysis.

The expression

$$\frac{a_1^{(1)} \cdot a_1^{(2)}}{\sqrt{a_2^{(1)} \cdot a_2^{(2)}}} = \frac{a_1^{(1)} \cdot a_1^{(2)}}{\sqrt{\left(\sigma_1^2 + \left(a_1^{(1)}\right)^2\right) \cdot \left(\sigma_2^2 + \left(a_1^{(2)}\right)^2\right)}} = C \leq 1$$

is obviously taking the largest value at zero variance, i.e. for a deterministic variable.

Figure 4 (pink line) shows the change in the correlation by the intensity $\lambda \in [0; 50]$. Obviously, when $\lambda = 0$, the system is a set of independently functioning blocks and therefore $r_{12} = 0$. As the intensity increases $\lambda \gg \lambda_1, \lambda_2$, the correlation increases and takes the greatest value

$$r_{12} = \frac{a_1^{(1)} \cdot a_1^{(2)}}{\sqrt{a_2^{(1)} \cdot a_2^{(2)}}} \frac{b_{12}}{\sqrt{b_1 \cdot b_2}}$$

and depends on the service parameters.

Let us consider the change in the correlation values by the service intensity. For clarity, we consider the example of exponential service with the parameters μ_1, μ_2. Then

$$r_{12} = \frac{a_1^{(1)} \cdot a_1^{(2)}}{\sqrt{a_2^{(1)} \cdot a_2^{(2)}}} \frac{\dfrac{\lambda}{\mu_1 + \mu_2}}{\sqrt{\dfrac{(\lambda + \lambda_1)}{\mu_1} \dfrac{(\lambda + \lambda_2)}{\mu_2}}} \xrightarrow[\lambda \gg \lambda_1, \lambda_2]{} C \frac{\sqrt{\mu_1 \mu_2}}{\mu_1 + \mu_2}.$$

We denote $\mu_2 = \gamma\mu_1$, where γ is an arbitrary non-negative number, then correlation has the form

$$r_{12} = C\frac{\sqrt{\gamma\mu_1^2}}{\mu_1(1+\gamma)} = C\frac{\sqrt{\gamma}}{(1+\gamma)}.$$

Let the resource parameters and arrival intensities be the same, the service times has an exponential distribution with the parameters $\mu_1 = 2$ and $\mu_2 = 2\gamma$ and the intensity $\lambda = 500$. Figure 4 (blue line) shows that the greatest dependence of the processes is achieved when $\gamma = 1$, i.e. with the same parameters of the service time, the correlation coefficient, in this case, is 0.36. It is obvious, that with an increase in the difference between the service times, the dependence of the processes in blocks decreases.

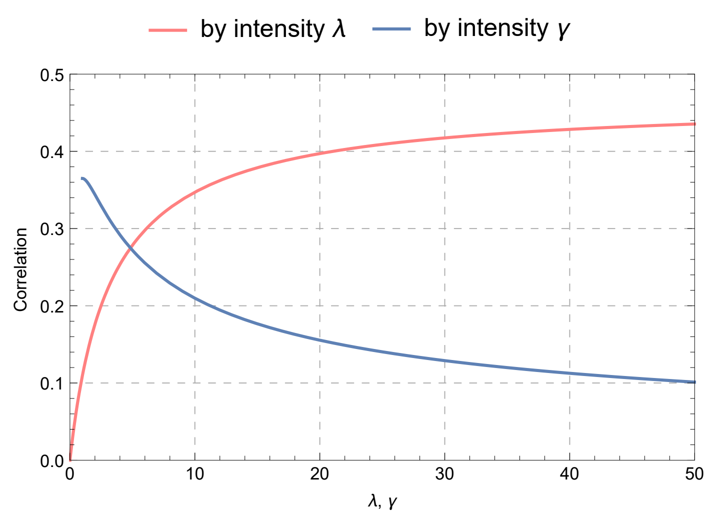

FIGURE 4. Change in correlation with arrival intensity λ and service intensity γ

9. Conclusion

The paper considers a multi-service resource queueing system with three Poisson arrivals, one of which is splitting. The equation for the probabilities distribution of the total resource amounts on the system blocks is compiled by the dynamic screening method. Using the equation the characteristic function of the stationary distribution was obtained. The numerical characteristics of the system performance were obtained by the method of moments from characteristic function. A numerical example is presented, it shows that arrivals growing leads to increasing the optimal total resource amounts on system blocks in the system with limited resources, and the splitting arrivals affect the correlation between total resource amounts.

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