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Digital simulators of the random processes

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Abstract

The proposed universal digital simulators of random processes based on their Markov models are considered as capable of generating sequences of samples of unlimited duration. It is shown that a simple Markov chain allows generating the random numbers with a specified two-dimensional probability distribution of the neighboring values while a doubly connected Markov model makes it possible to get the three-dimensional random numbers. The parameters of the model are determined from either a known probability density or experimental samples of the simulated random process. It is demonstrated that the simulation algorithms do not require complex mathematical transformations and that they can be implemented using a simple element base. To change the properties of the generated random processes one needs to reload the memory device with a pre-formed data array. The block diagrams of the simulators are studied and the probabilistic and correlation characteristics of the generated random processes are determined. It is established that with these simulators a high accuracy of convergence of the probability distributions of the selected model and the histograms of the generated sample sequences is ensured. In the common studies, one can hardly find the results that can surpass by their efficiency the ones that the proposed simulation algorithms demonstrate accounting for their non-problematic hardware implementation (the minimum computational costs) and the simplicity of reconfiguring the Markov model based simulators for generating new random processes. The introduced simulators can be used in the design, development and testing of the multi-purpose electronic equipment, with different meters and the devices for simulating radio paths.

Keywords: Random-number generator; Markov model; matrix of transition probabilities; probability density; statistical simulation

1. Introduction

Software or hardware generators of random information or interference signals with the specified probabilistic properties are used in solving various radio engineering problems, in the design, study and testing of equipment, to control devices for simulating radio channels with specific types of additive and multiplicative interferences, etc. (Law & Kelton, 2000;

Bardis, Markovskyi, Doukas, & Karadimas, 2009; Pchelintsev & Pergamenshchikov, 2019; Radchenko, Tokarev, Makarov, Gulmanov, & Melnikov, 2020).

In analog noise generators (Dobkin & Hamburger, 2014), thermal noise processes in electronic elements are used, their statistical properties are close to the Gaussian probability distribution. The disadvantages of such generators are the difficulty of providing the specified probabilistic characteristics (especially, multidimensional ones) with high accuracy and stability. Digital simulators can be designed based on the transformation of equiprobable random numbers (Devroye, 1986) obtained, for example, using a long *M*-sequence generator (Lee & Kim, 2002). However, in this case, there are computational difficulties while implementing nonlinear operations, especially for two-dimensional probability distributions.

The digital simulators of random processes based on their Markov models are largely devoid of the disadvantages mentioned above as it is described in (Glushkov, Menshikh, Khohlov, Bokova, & Kalinin, 2017; Glushkov, Kalinin, Litvinenko, & Litvinenko, 2020; Chernoyarov, Litvinenko, Matveev, Dachian, & Melnikov, 2020).

2. The Markov models of the random processes

Our study begins with focusing on the continuous random process $x(t)$ and the corresponding discrete random process with the samples z_n ($1 \le z_n \le M$) are considered. Here *n* ($n = 1, 2, \ldots, N$) is the current number of the sample, *N* is the sample size, $M = 2^m$ the number of quantization levels, *m* is the analog-to-digital converter (ADC) width. The process is the Markov one (Dynkin, 2006), if the current value z_n depends upon the previous values z_{n-1}, z_{n-2}, ..., z_{n-R} only. The value *R* is called the model connectivity. The Markov model is convenient for simulating various random processes and, in many cases, it is approximately applicable even when the simulated process is not the Markov one.

In a simple (simply connected) Markov chain, when $R = 1$, the value of the current sample $z_n = j$ depends upon the previous value $z_{n-1} = i$ ($1 \le i, j \le M$) only, so that the corresponding Markov model is described by the two-dimensional square matrix $\lfloor P_{ij} \rfloor$ of transition probabilities and the column matrix $[P_i]$ of the probabilities of the initial values $z_1 = i$.

If the two-dimensional probability density $w(x_1, x_2)$ of the process $x(t)$ is known, then, for the specified ADC quantization thresholds, one gets

$$g_m = \begin{cases} -\infty, & \text{if } m = 0, \\ d(m - M/2), & \text{if } 1 \le m \le M - 1, \\ \infty, & \text{if } m = M, \end{cases} \quad (1)$$

and thus the joint probability distribution $P(i, j)$ of the values $z_{n-1} = i$ and $z_n = j$ can be determined as follows

$$P(i, j) = \int_{g_{i-1}}^{g_i} \int_{g_{j-1}}^{g_j} w(x_1, x_2)\, dx_1 dx_2 . \quad (2)$$

Then, for the transition probabilities P_{ij} and the probabilities of the initial values P_i, one obtains

$$P_{ij} = P(i, j) \Big/ \sum_{j=1}^{M} P(i, j) \text{ or } P_i = \sum_{j=1}^{M} P(i, j). \quad (3)$$

If the experimental sampling $\{z_n, n = \overline{1, N}\}$ of the discrete random process values from the ADC output is observed, then under a large sample size *N*, the empirical simply connected model can be built. For this purpose, the numbers l_{ij} of transitions of the process values from the previous $z_{n-1} = i$ to the current $z_n = j$ ones should be calculated over the complete sampling $n = \overline{2, N}$. Then the estimates of the joint probabilities $P(i, j)$ are found in the following way $P(i, j) = l_{ij}/(N - 1)$, and the next step is to calculate the probabilities P_{ij}, P_i according to (3).

In the doubly connected Markov model ($R = 2$), the value of the current sample $z_n = j$ depends upon the values of the two previous samples $z_{n-2} = k$ and $z_{n-1} = i$, where $1 \le k, i, j \le M$. The model is described by the three-dimensional matrix $\lfloor P_{kij} \rfloor$ of transition probabilities and the column matrix $[P_k]$ of the probabilities of the initial values $z_1 = k$. With the known three-dimensional probability density $w(x_1, x_2, x_3)$ of the process $x(t)$ and the thresholds g_m (1), the joint probability distribution of the values $z_{n-2} = k$, $z_{n-1} = i$ and $z_n = j$ is determined as

$$P(k, i, j) = \int_{g_{k-1}}^{g_k} \int_{g_{i-1}}^{g_i} \int_{g_{j-1}}^{g_j} w(x_1, x_2, x_3)\, dx_1 dx_2 dx_3 , \quad (4)$$

while the transition probabilities P_{kij} and the probabilities of the initial values P_k – as

$$P_{kij} = \frac{P(k, i, j)}{\sum_{j=1}^{M} P(k, i, j)}, \quad P_k = \sum_{i=1}^{M} \sum_{j=1}^{M} P(k, i, j). \quad (5)$$

The empirical doubly connected Markov model can be built based on the experimental sampling of the random process samples z_n, $n = \overline{1, N}$ by calculating the numbers l_{kij} of the transitions of the process values from the previous $z_{n-2} = k$, $z_{n-1} = i$ to the current $z_n = j$ ones by means of determining both the joint probability $P(k, i, j) = l_{kij}/(N - 2)$ and the probabilities P_{kij}, P_k (5).

3. The simulator of the random processes based on the simple Markov chain

In order to implement the simulator based on a simple Markov model specified by the matrix of transition probabilities $\lfloor P_{ij} \rfloor$, the matrix of the two-dimensional probability distribution function is calculated as follows

$$F_{ij} = \sum_{v=1}^{j} P(i, v). \tag{6}$$

The block diagram of the simulator is shown in Figure 1. The clock generator (CG) producing the pulses with the frequency f_0 starts the random or pseudo-random number generator (RNG). The RNG forms a sequence of K-bit independent and equiprobable binary codes coming to the bus A_0 of the least significant bits of the storage device (SD) address. The most significant bits of the address bus A_1 determine the previous sample value $z_{n-1} = i$ from the register (RG) output.

In the SD cells by the addresses $A = i\,2^K + v$, the precomputed minimum binary m-bit codes j ($0 \le j \le M - 1$) are saved, for which, while the values of the binary codes i ($0 \le i \le M - 1$) and v ($0 \le v \le 2^K - 1$) are specified, the inequality

$$v/2^K < F_{(i+1)(j+1)} \tag{7}$$

is satisfied. Here F_{ij} are determined according to (6).

By the next CG pulse, the value j from the SD output is poked into the RG and then it is fed both to the bus A_1 as the previous process value $z_{n-1} = i$ and then passes to the digital output of the simulator as the current value z_n for this cycle.

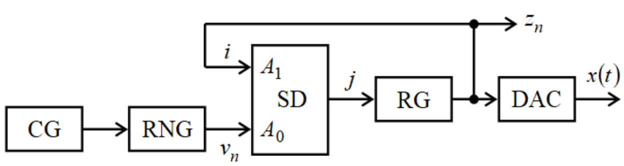


Figure 1. The block diagram of the simulator of the random signal with the specified two-dimensional probability distribution

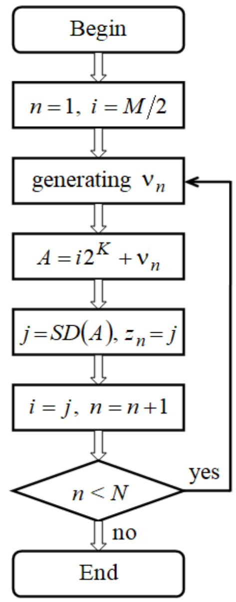


Figure 2. The algorithm for software implementation of the simulator based on the simple Markov chain

If the analog random process $x(t)$ is to be generated at the output of the simulator, then the value z_n should be passed through a digital-to-analog converter (DAC).

In Figure 2, the block diagram of the algorithm for the software implementation of the simulator is presented. Here the operation $SD(A)$ means reading data from the SD array at the address A, in accordance with (7).

As an example, one considers the Gaussian random process with the two-dimensional probability density (for two points in time t_1, t_2) of the form (Robinson, 1985)

$$w(x_1, x_2) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \times \exp\left[-\frac{(x_1-\bar{x})^2 + 2\rho(x_1-\bar{x})(x_2-\bar{x}) + (x_2-\bar{x})^2}{2\sigma^2(1-\rho^2)}\right]. \tag{8}$$

Here the notations are the following: $\bar{x}$ is the mean value (mathematical expectation), σ^2 is the dispersion, and $\rho = \langle [x(t_1) - \bar{x}][x(t_2) - \bar{x}] \rangle / \sigma^2$ is the correlation coefficient of the process $x(t)$.

In Figure 3a, the example of the joint probability distribution (2) calculated using the function (8) is presented for the case when $\bar{x} = 0$, $\sigma = 2$, $\rho = 0.8$, $M = 64$, and $d = 20/M$ in (1), while in Figure 3b and Figure 3c one can see the three-dimensional diagrams of the matrices of transition probabilities $\lfloor P_{ij} \rfloor$ (3) and probability distribution $\lfloor F_{ij} \rfloor$ (6).

The results of statistical simulation of the digital simulator of the Gaussian random process (8) can be seen in Figures 4. Here there are presented the generated time realization of random process samples (a); the estimation of the joint probability distribution of the neighboring pairs of samples (b); the histogram of the one-dimensional probability density of the simulated random process, which is drawn by vertical lines, and its corresponding theoretical values, which are marked by points (c); the experimental values $\rho_k = \rho^k$ of the correlation coefficient of the generated samples, which are traced by vertical lines, and the corresponding theoretical values, which are shown by dotted line (d).

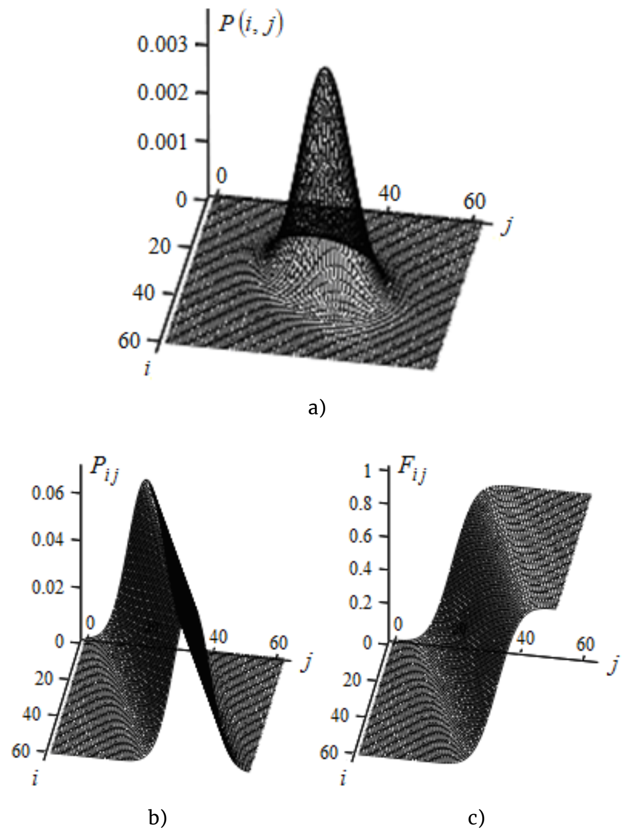
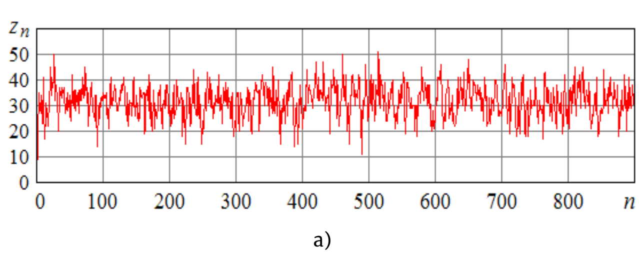


Figure 3. The probability characteristics of the Gaussian random process: a) the two-dimensional probability density; b) the diagram of the matrix of transition probabilities; c) the diagram of the matrix of the probability distribution function



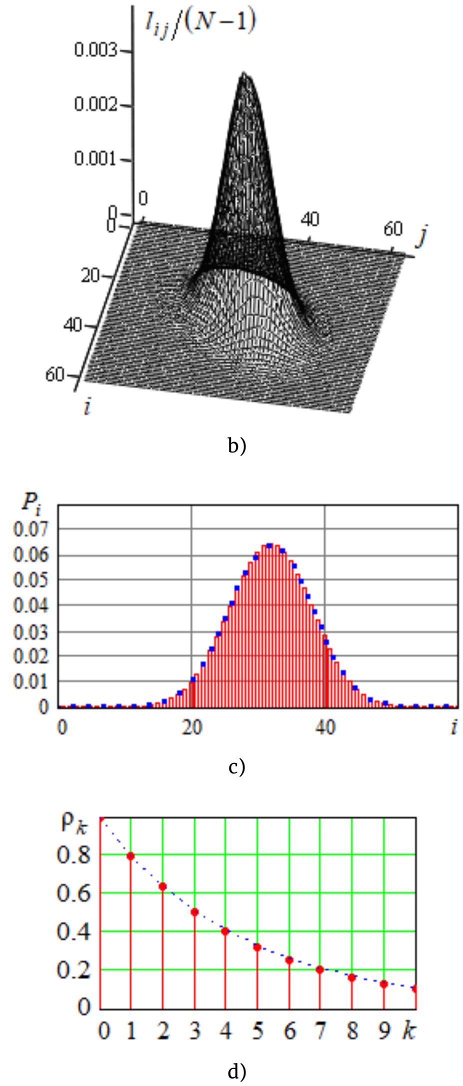


Figure 4. The results of the statistical simulation: a) the time realization of the random process; b) the estimate of the joint probability distribution of the neighboring pairs of samples; c) the histogram and the theoretical values of the one-dimensional probability density of the random process; d) the experimental and theoretical values of the correlation coefficient of the samples

In Figure 4a, the mean value of the samples z_n is equal to $M/2 = 32$ that, according to (1), corresponds to the zero quantization level of the signal.

It follows from these Figures that the simulation results are in good agreement with the specified theoretical random process model.

4. The simulators of the random processes based on the doubly connected Markov model

The block diagram of the simulator of random signals based on the doubly connected Markov model is shown in Figure 5.

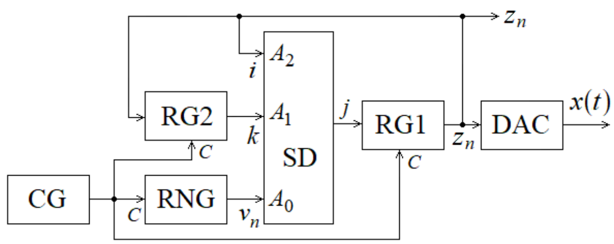


Figure 5. The block diagram of the simulator of the random signal with the specified three-dimensional probability distribution

According to the signals with the frequency f_0 from the CG with a frequency, the RNG generates binary K-bit independent random numbers v_n with a uniform probability distribution. These numbers determine the least significant bits of the SD address bus A_0.

The joint probability distribution of the three neighboring samples $P(k, i, j)$ (4) is used to calculate the matrix of the three-dimensional probability distribution function:

$$F_{kij} = \sum_{v=1}^{j} P(k, i, v).$$

The SD cells with the addresses $A = i2^{K+m} + k2^K + v$ contain the smallest values of j ($0 \le j \le M - 1$) which are pre-calculated for all $v = \overline{0, 2^K - 1}$ and for each pair k, i and satisfy the inequality

$$v/2^K < F_{(k+1)(i+1)(j+1)}.$$

The values $z_{n-2} = k$ saved in the register RG2 and $z_{n-i} = i$ occupying the high-order bits of the SD address bus form the code j at the SD output. This code is poked into the register RG1 by the CG pulse and it is now both the output signal of the simulator and the code i that is transmitted through the bus A_2 and is also recorded in the register RG2 forming the code k for the next cycle. Then the process is repeated.

The block diagram of the algorithm for the software implementation of the simulator of the random process based on the doubly connected Markov model is shown in Figure 6.

As an example, one considers the simulation of the Gaussian random process $x(t)$ with the specified three-dimensional probability density (for the three points in time t_1, t_2, t_3) (Robinson, 1985)

$$w(x_1, x_2, x_3) = \frac{1}{\sigma^3 \sqrt{(2\pi)^3 R}} \times$$

$$\times \exp\left[-\frac{1}{2\sigma^2} \sum_{m=1}^{3} \sum_{n=1}^{3} H_{mn}(x_m - \overline{x})(x_n - \overline{x})\right],$$

where $\overline{x}$ is the mean value, σ^2 is the dispersion of the simulated random process, while $R = \left\|r_{mn}\right\|$ is the determinant of the normalized correlation matrix

$$\left[r_{mn}\right] = \begin{bmatrix} 1 & r_{12} & r_{13} \\ r_{21} & 1 & r_{23} \\ r_{31} & r_{32} & 1 \end{bmatrix}, \qquad (9)$$

$$r_{mn} = r_{nm} = \langle [x(t_m) - \overline{x}][x(t_n - \overline{x})] \rangle / \sigma^2 .$$

The joint probability distribution of the three neighboring samples is calculated according to (4) and for its graphical imaging in three-dimensional space it is advisable to present the matrix $P(k, i, j)$ as the discrete function of two coordinates $P(k2^m + i, j)$.

In Figures 7, the three-dimensional diagrams $P(k2^m + i, j)$ are drawn for the different matrices of the correlation coefficients (9): $r_{12} = r_{23} = 0.1$ and $r_{13} = 0.5$ (Figure 7a) or $r_{13} = 0.8$ (Figure 7b).

In Figures 8a and 8b, the corresponding level lines are plotted.

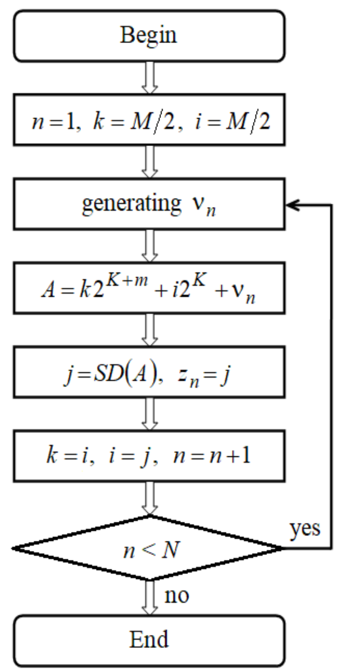
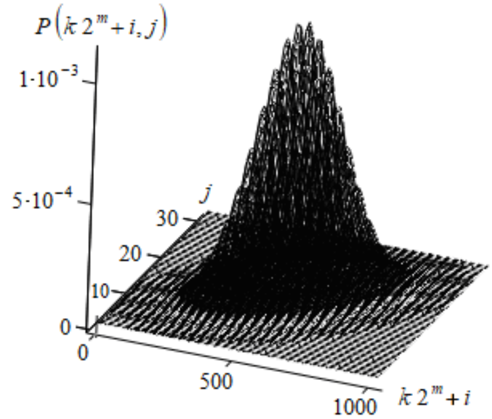
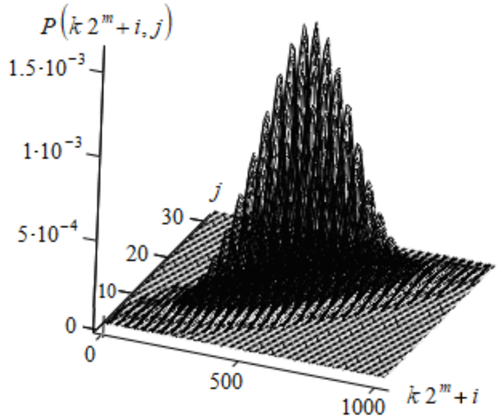


Figure 6. The algorithm for software implementation of the simulator based on a doubly connected Markov model

As it can be seen, these are multimodal surfaces, the width and slope of which in the plane of variables is determined by the correlation matrix (9). Finally, in Figures 9, for the two indicated cases the results of the statistical simulation are presented of the simulator operation (the realizations of simulated process samples and the estimates of the correlation matrices).

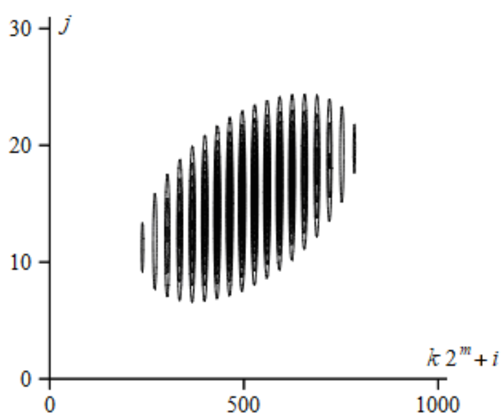


a)

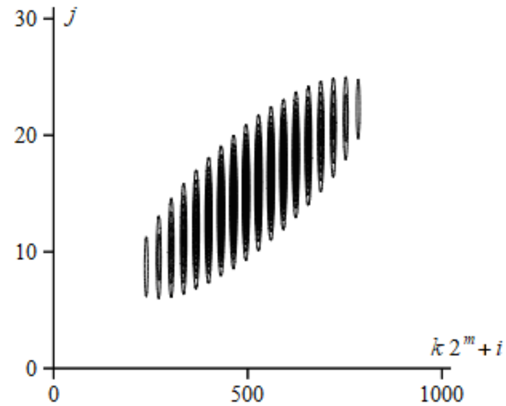


b)

Figure 7. The three-dimensional diagrams of the joint probability distribution of the simulated process samples for specified correlation coefficients r_{12}=r_{23}=0.1 and r_{13}=0.5 (a) or r_{13}=0.8 (b)

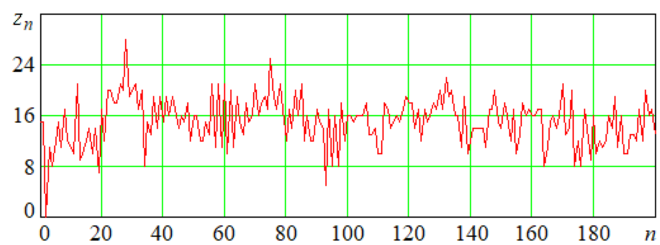


a)



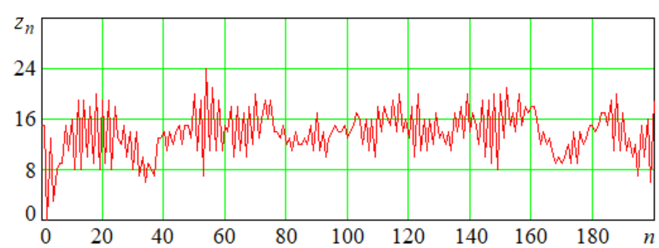
b)

Figure 8. The level lines of the joint probability distribution of the simulated process samples for specified correlation coefficients r_{12}=r_{23}=0.1 and r_{13}=0.5 (a) or r_{13}=0.8 (b)



$$[r_{mn}] = \begin{bmatrix} 1 & 0.1 & 0.497 \\ 0.1 & 1 & 0.1 \\ 0.497 & 0.1 & 1 \end{bmatrix}$$

a)



$$[r_{mn}] = \begin{bmatrix} 1 & 0.098 & 0.796 \\ 0.098 & 1 & 0.098 \\ 0.796 & 0.098 & 1 \end{bmatrix}$$

b)

Figure 9. The realizations of simulated process samples and the estimates of the correlation matrices for specified correlation coefficients r_{12}=r_{23}=0.1 and r_{13}=0.5 (a) or r_{13}=0.8 (b)

When $M = 32$, the mean value of the samples z_n shown in Figures 8 is equal to 16 that in this case corresponds to the zero quantization level of the signal in (1).

It follows from these Figures that the simulation results are in good agreement with the specified theoretical random process model.

The transition from a simple Markov model to a doubly connected one expands the possibilities of reproducing the probabilistic properties of the simulated random signals based on either theoretical expressions for a multidimensional probability density or according to the results of the random process experimental sampling processing. This does not increase complexity of the software or hardware implementation of the simulator.

If necessary, the Markov models of higher connectivity can be similarly used.

5. Conclusions

The possibilities of software and hardware implementation of the simulators of the random processes based on their simple and doubly connected Markov model are considered. It is shown that the introduced simulators provide a high speed of discrete sample generation due to the small specific number of the simple arithmetic operations required. The model for simulating the random process is changed by reloading the storage device with a previously calculated data array. In addition, the realizations of the random processes of unlimited duration can also be simulated.

The simulators based on the multiply connected Markov models allow reproducing the fine structure of the probabilistic links of the simulated random processes. The possibility of building the models by means of the experimental realizations of the random processes is also provided.

It should be noted that, strictly speaking, the proposed simulators make it possible to generate random processes that can be considered as Markov processes. Otherwise, only an approximate simulation of a random process with the specified statistical characteristics can be implemented.

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