Chapter

The Paradigm of Complex Probability and Quantum Mechanics: The Infinite Potential Well Problem – The Momentum Wavefunction and the Wavefunction Entropies

Abdo Abou Jaoudé

Abstract

The mathematical probability concept was set forth by Andrey Nikolaevich Kolmogorov in 1933 by laying down a five-axioms system. This scheme can be improved to embody the set of imaginary numbers after adding three new axioms. Accordingly, any stochastic phenomenon can be performed in the set $\mathcal{C}$ of complex probabilities which is the summation of the set $\mathcal{R}$ of real probabilities and the set $\mathcal{M}$ of imaginary probabilities. Our objective now is to encompass complementary imaginary dimensions to the stochastic phenomenon taking place in the "real" laboratory in $\mathcal{R}$ and as a consequence to calculate in the sets $\mathcal{R}$, $\mathcal{M}$, and $\mathcal{C}$ all the corresponding probabilities. Hence, the probability is permanently equal to one in the entire set $\mathcal{C} = \mathcal{R} + \mathcal{M}$ independently of all the probabilities of the input stochastic variable distribution in $\mathcal{R}$, and subsequently, the output of the random phenomenon in $\mathcal{R}$ can be determined perfectly in $\mathcal{C}$. This is due to the fact that the probability in $\mathcal{C}$ is calculated after the elimination and subtraction of the chaotic factor from the degree of our knowledge of the nondeterministic phenomenon. My innovative Complex Probability Paradigm (*CPP*) will be applied to the established theory of quantum mechanics in order to express it completely deterministically in the universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$.

Keywords: degree of our knowledge, chaotic factor, complex random vector, probability norm, complex probability set $\mathcal{C}$, momentum wavefunction, imaginary entropy, complex entropy

1. Introduction

1.1 The momentum wavefunction and *CPP*

1.1.1 The momentum wavefunction probability distribution and CPP

The probability density for finding a particle with a given momentum is derived from the wavefunction as $f(p) = |\phi(p)|^2$. As with position, the wavefunction momentum probability density function (*PDF*) for finding the particle at a given momentum depends upon its state, and is given by [1, 2]:

$$f(p) = |\phi(p)|^2 = \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]$$

Where $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant and $\text{sinc}(x) = \frac{\sin(x)}{x}$ is the cardinal sine *sinc* function.

Therefore, the wavefunction momentum cumulative probability distribution function (*CDF*) which is equal to $P_r(P)$ in $\mathcal{R}$ is:

$$P_r(P) = F\left(p_j\right) = P_{rob}\left(P \le p_j\right) = \int_{-\infty}^{p_j} |\phi(p)|^2 dp$$

$$= \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp$$

And the real complementary probability to $P_r(P)$ in $\mathcal{R}$ which is $P_m(P)/i$ is:

$$P_m(P)/i = 1 - P_r(P) = 1 - F\left(p_j\right) = 1 - P_{rob}\left(P \le p_j\right) = P_{rob}\left(P > p_j\right)$$

$$= 1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp = \int_{p_j}^{+\infty} |\phi(p)|^2 dp$$

$$= 1 - \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp$$

$$= \int_{p_j}^{+\infty} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp$$

Consequently, the imaginary complementary probability to $P_r(P)$ in $\mathcal{M}$ which is $P_m(P)$ is:

$$P_m(P) = i[1 - P_r(P)] = i\left[1 - F\left(p_j\right)\right] = i\left[1 - P_{rob}\left(P \le p_j\right)\right] = iP_{rob}\left(P > p_j\right)$$

$$= i\left[1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp\right] = i\int_{p_j}^{+\infty} |\phi(p)|^2 dp$$

$$= i\left[1 - \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]dp\right]$$

$$= i\int_{p_j}^{+\infty} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]dp$$

Furthermore, the complex random number or vector in $\mathcal{C} = \mathcal{R} + \mathcal{M}$ which is $Z(P)$ is:

$$Z(P) = P_r(P) + P_m(P) = P_r(P) + i[1 - P_r(P)] = F\left(p_j\right) + i\left[1 - F\left(p_j\right)\right]$$

$$= P_{rob}\left(P \le p_j\right) + i\left[1 - P_{rob}\left(P \le p_j\right)\right] = P_{rob}\left(P \le p_j\right) + iP_{rob}\left(P > p_j\right)$$

$$= \int_{-\infty}^{p_j} |\phi(p)|^2 dp + i\left[1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp\right] = \int_{-\infty}^{p_j} |\phi(p)|^2 dp + i\int_{p_j}^{+\infty} |\phi(p)|^2 dp$$

$$= \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]dp$$

$$+ i\left[1 - \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]dp\right]$$

$$= \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]dp$$

$$+ i\int_{p_j}^{+\infty} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]dp$$

Additionally, the degree of our knowledge which is $DOK(P)$ is:

$$
\begin{aligned}
DOK(P) &= [P_r(P)]^2 + [P_m(P)/i]^2 = [P_r(P)]^2 + [1 - P_r(P)]^2 = \left[F\left(p_j\right)\right]^2 + \left[1 - F\left(p_j\right)\right]^2 \\
&= \left[P_{rob}\left(P \le p_j\right)\right]^2 + \left[1 - P_{rob}\left(P \le p_j\right)\right]^2 = \left[P_{rob}\left(P \le p_j\right)\right]^2 + \left[P_{rob}\left(P > p_j\right)\right]^2 \\
&= \left[\int_{-\infty}^{p_j} |\phi(p)|^2 dp\right]^2 + \left[1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp\right]^2 = \left[\int_{-\infty}^{p_j} |\phi(p)|^2 dp\right]^2 + \left[\int_{p_j}^{+\infty} |\phi(p)|^2 dp\right]^2 \\
&= \left[\int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \operatorname{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp\right]^2 \\
&+ \left[1 - \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \operatorname{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp\right]^2 \\
&= \left[\int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \operatorname{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp\right]^2 \\
&+ \left[\int_{p_j}^{+\infty} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \operatorname{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp\right]^2
\end{aligned}
$$

Moreover, the chaotic factor which is $Chf(P)$ is:

$$
\begin{aligned}
Chf(P) &= 2iP_r(P)P_m(P) \\
&= 2iP_r(P) \times i[1 - P_r(P)] = -2P_r(P)[1 - P_r(P)] = -2F\left(p_j\right)\left[1 - F\left(p_j\right)\right] \\
&= -2P_{rob}\left(P \le p_j\right)\left[1 - P_{rob}\left(P \le p_j\right)\right] = -2P_{rob}\left(P \le p_j\right)P_{rob}\left(P > p_j\right) \\
&= -2\int_{-\infty}^{p_j} |\phi(p)|^2 dp \times \left[1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp\right] = -2\int_{-\infty}^{p_j} |\phi(p)|^2 dp \times \int_{p_j}^{+\infty} |\phi(p)|^2 dp \\
&= -2\int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \operatorname{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp \\
&\times \left[1 - \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \operatorname{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp\right] \\
&= -2\int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \operatorname{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp \\
&\times \int_{p_j}^{+\infty} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \operatorname{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp
\end{aligned}
$$

In addition, the magnitude of the chaotic factor which is $MChf(P)$ is:

$$\begin{aligned}
MChf(P) &= |Chf(P)| = -2iP_r(P)P_m(P) = -2iP_r(P) \times i[1 - P_r(P)] \\
&= 2P_r(P)[1 - P_r(P)] = 2F\left(p_j\right)\left[1 - F\left(p_j\right)\right] \\
&= 2P_{rob}\left(P \le p_j\right)\left[1 - P_{rob}\left(P \le p_j\right)\right] = 2P_{rob}\left(P \le p_j\right)P_{rob}\left(P > p_j\right) \\
&= 2\int_{-\infty}^{p_j} |\phi(p)|^2 dp \times \left[1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp\right] = 2\int_{-\infty}^{p_j} |\phi(p)|^2 dp \times \int_{p_j}^{+\infty} |\phi(p)|^2 dp \\
&= 2\int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp \\
&\times \left[1 - \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp\right] \\
&= 2\int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp \\
&\times \int_{p_j}^{+\infty} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp
\end{aligned}$$

Finally, the real probability in the complex probability universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$ which is $Pc(P)$ is:

$$\begin{aligned}
Pc^2(P) &= \{[P_r(P)] + [P_m(P)/i]\}^2 = \{[P_r(P)] + [1 - P_r(P)]\}^2 \\
&= \left\{\left[F\left(p_j\right)\right] + \left[1 - F\left(p_j\right)\right]\right\}^2 = \left\{P_{rob}\left(P \le p_j\right) + \left[1 - P_{rob}\left(P \le p_j\right)\right]\right\}^2 \\
&= \left\{P_{rob}\left(P \le p_j\right) + P_{rob}\left(P > p_j\right)\right\}^2 \\
&= \left\{\int_{-\infty}^{p_j} |\phi(p)|^2 dp + \left[1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp\right]\right\}^2 \\
&= \left\{\int_{-\infty}^{p_j} |\phi(p)|^2 dp + \int_{p_j}^{+\infty} |\phi(p)|^2 dp\right\}^2 = \left\{\int_{-\infty}^{+\infty} |\phi(p)|^2 dp\right\}^2 \\
&= \left\{\begin{array}{l} \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp \\ \qquad + \left[1 - \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp\right] \end{array}\right\}^2 \\
&= \left\{\begin{array}{l} \int_{-\infty}^{p_j} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp \\ \qquad + \int_{p_j}^{+\infty} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp \end{array}\right\}^2 \\
&= \left\{\int_{-\infty}^{+\infty} \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp\right\}^2 = 1^2 = 1 = Pc(P)
\end{aligned}$$

And, $Pc(P)$ can be computed using *CPP* as follows:

$$\begin{aligned}
Pc^2(P) &= DOK(P) - Chf(P) = [P_r(P)]^2 + [P_m(P)/i]^2 - 2iP_r(P)P_m(P) \\
&= [P_r(P)]^2 + [1 - P_r(P)]^2 + 2P_r(P)[1 - P_r(P)] = \{P_r(P) + [1 - P_r(P)]\}^2 \\
&= \left\{ \int_{-\infty}^{p_j} |\phi(p)|^2 dp + \left[1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp \right] \right\}^2 = \left\{ \int_{-\infty}^{p_j} |\phi(p)|^2 dp + \int_{p_j}^{+\infty} |\phi(p)|^2 dp \right\}^2 \\
&= \left\{ \int_{-\infty}^{+\infty} |\phi(p)|^2 dp \right\}^2 \\
&= 1^2 = 1 = Pc(P)
\end{aligned}$$

And, $Pc(P)$ can be computed using always *CPP* as follows also:

$$\begin{aligned}
Pc^2(P) &= DOK(P) + MChf(P) = [P_r(P)]^2 + [P_m(P)/i]^2 + [-2iP_r(P)P_m(P)] \\
&= [P_r(P)]^2 + [1 - P_r(P)]^2 + 2P_r(P)[1 - P_r(P)] = \{P_r(P) + [1 - P_r(P)]\}^2 \\
&= \left\{ \int_{-\infty}^{p_j} |\phi(p)|^2 dp + \left[1 - \int_{-\infty}^{p_j} |\phi(p)|^2 dp \right] \right\}^2 \\
&= \left\{ \int_{-\infty}^{p_j} |\phi(p)|^2 dp + \int_{p_j}^{+\infty} |\phi(p)|^2 dp \right\}^2 = \left\{ \int_{-\infty}^{+\infty} |\phi(p)|^2 dp \right\}^2 = 1^2 = 1 = Pc(P)
\end{aligned}$$

Hence, the prediction of all the wavefunction momentum probabilities of the random infinite potential well problem in the universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$ is permanently certain and perfectly deterministic.

1.1.2 The new model simulations

The following figures (**Figures 1**–**37**) illustrate all the calculations done above.

1.1.2.1 Simulations interpretation

In **Figures 1, 6, 11, 16, 21, 26, 31, 36, and 37** we can see the graphs of the probability density functions (*PDF*) of the wavefunction momentum probability distribution for this problem as functions of the random variable P for n = 1, 2, 3, 4, 5, 6, 7, 12, 100.

In **Figures 2, 7, 12, 17, 22, 27, and 32** we can see also the graphs and the simulations of all the *CPP* parameters (*Chf*, *MChf*, *DOK*, P_r, P_m/i, *Pc*) as functions of the random variable P for the wavefunction momentum probability distribution of the infinite potential well problem for n = 1, 2, 3, 4, 5, 6, 7. Hence, we can visualize all the new paradigm functions for this problem.

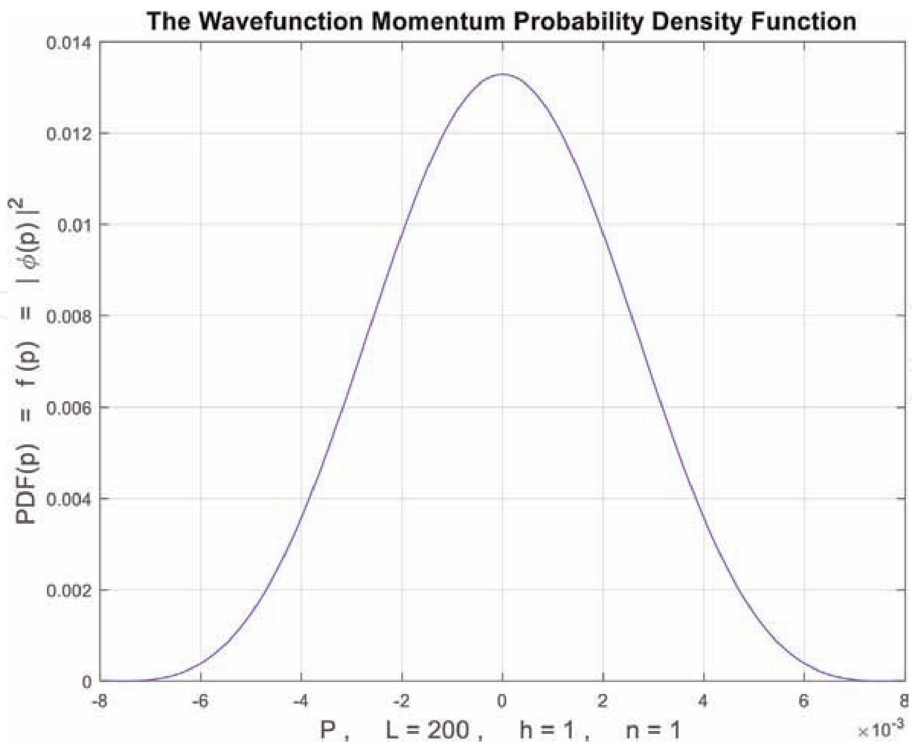

Figure 1.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = 1.

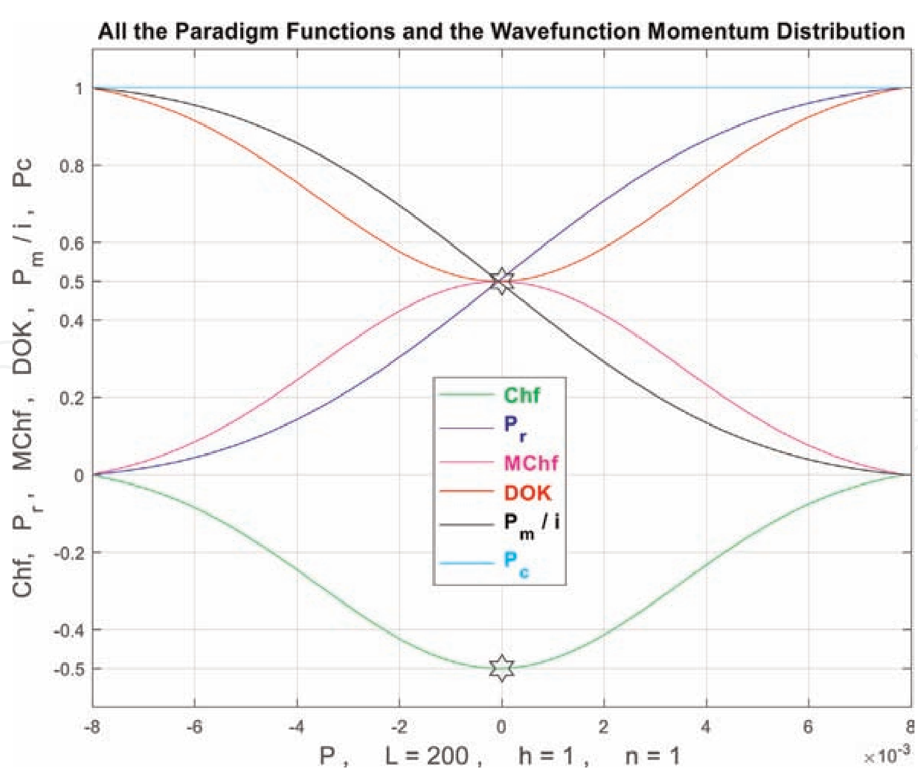

Figure 2.
The graphs of all the CPP *parameters as functions of the random variable* P *for the wavefunction momentum probability distribution for* n = 1.

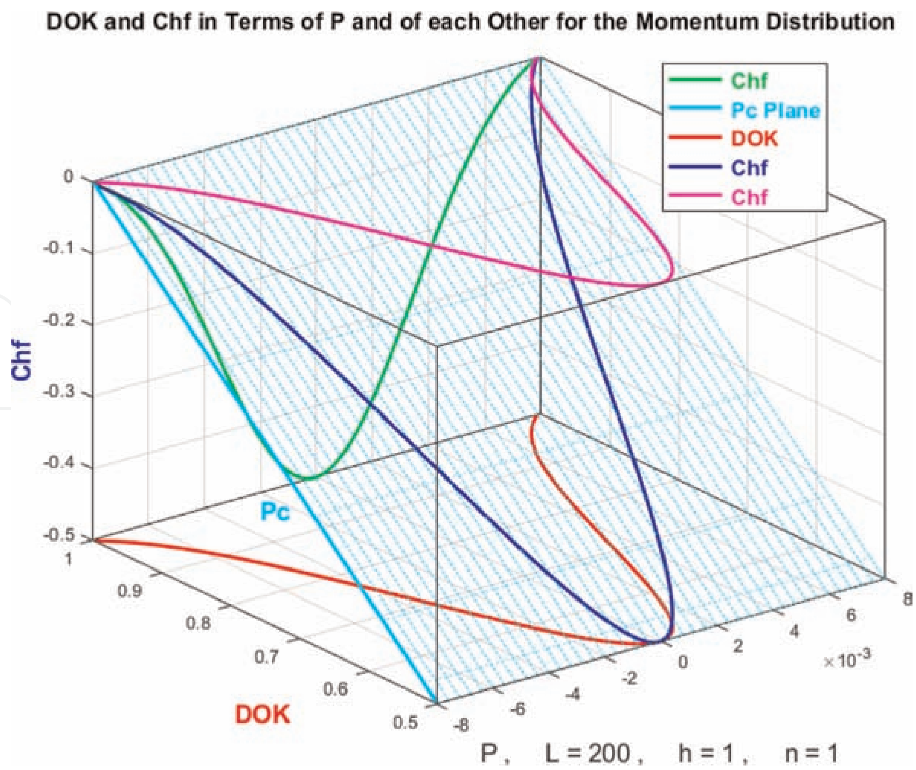

Figure 3.
The graphs of DOK *and* Chf *and the deterministic probability* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 1.

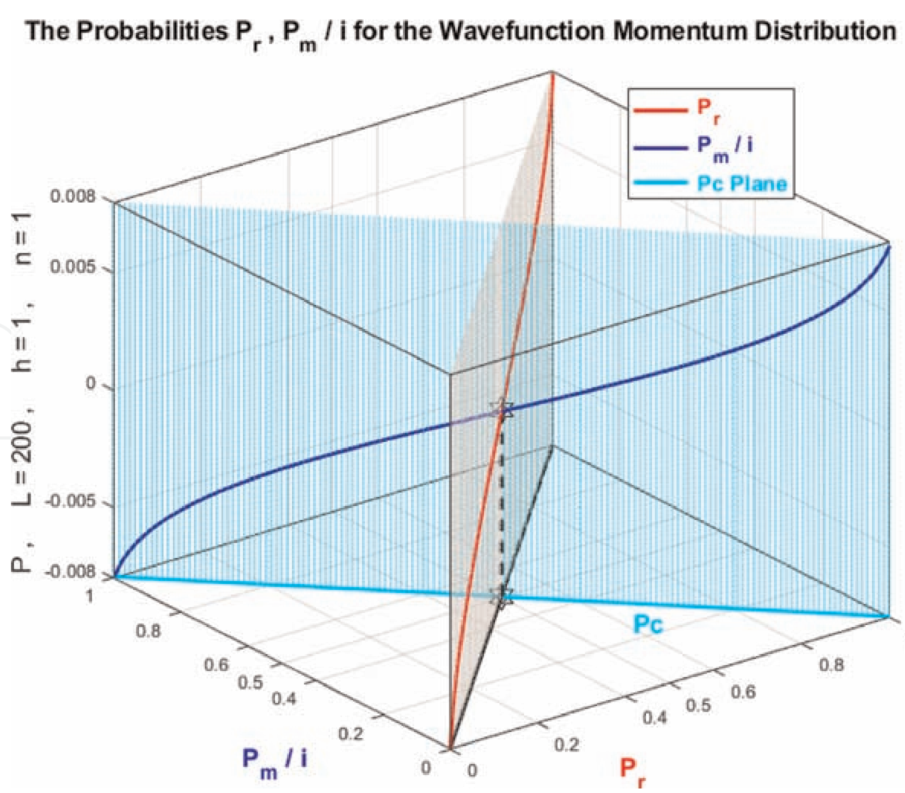

Figure 4.
The graphs of P_r *and* P_m/i *and* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 1.

DOI: http://dx.doi.org/10.5772/intechopen.107665

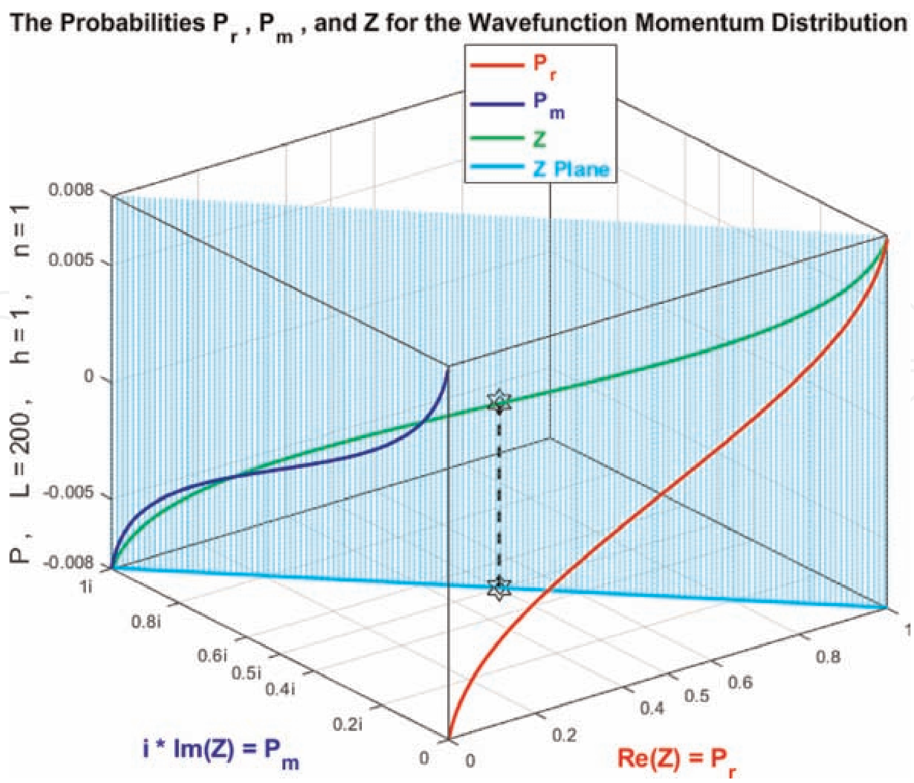

Figure 5.
The graphs of the probabilities P_r *and* P_m *and* Z *in terms of* P *for the wavefunction momentum probability distribution for* n = 1.

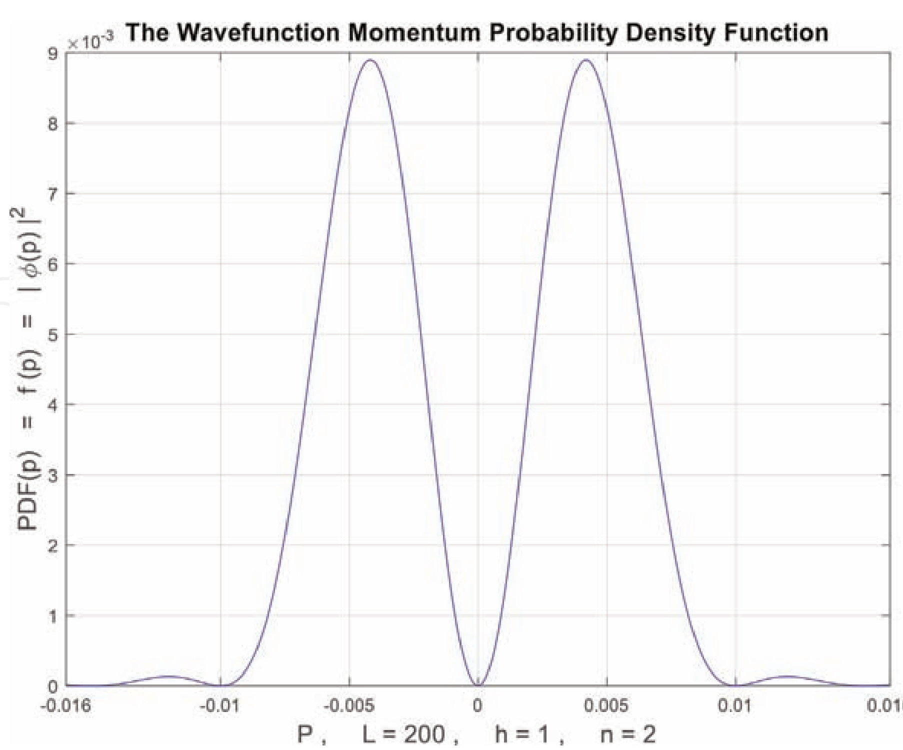

Figure 6.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = 2.

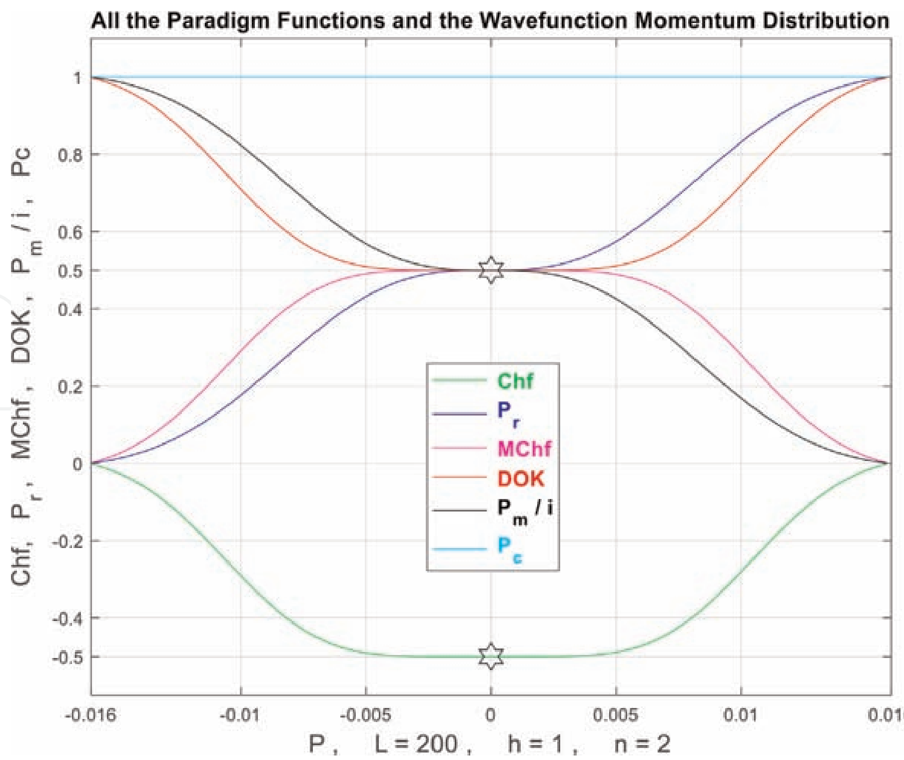

Figure 7.
The graphs of all the CPP *parameters as functions of the random variable* P *for the wavefunction momentum probability distribution for* n = 2.

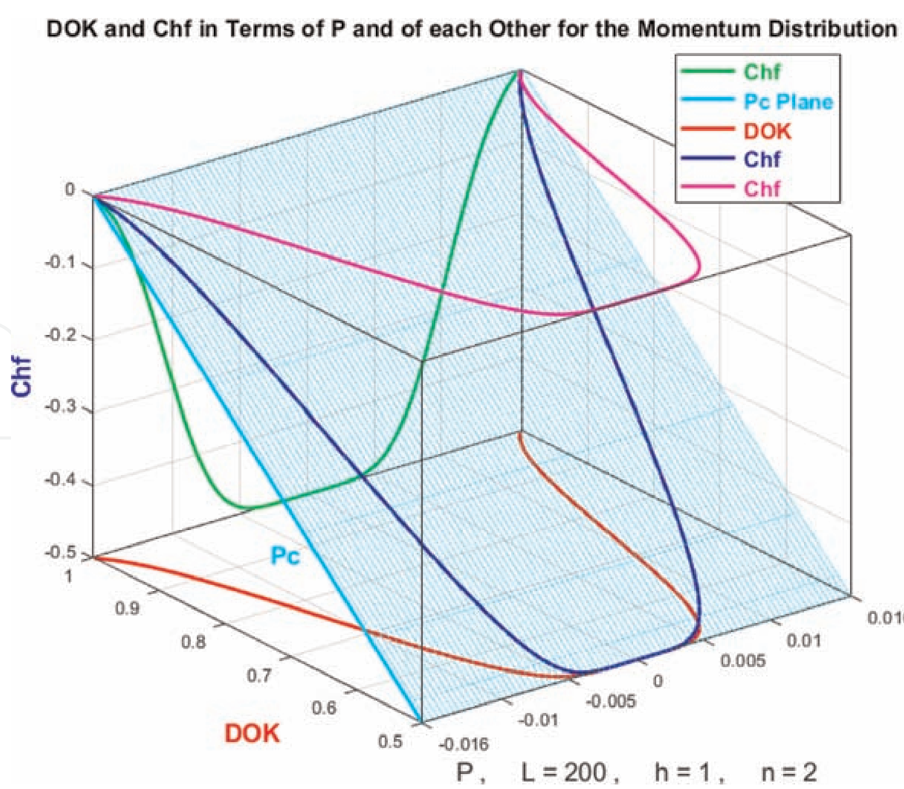

Figure 8.
The graphs of DOK *and* Chf *and the deterministic probability* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 2.

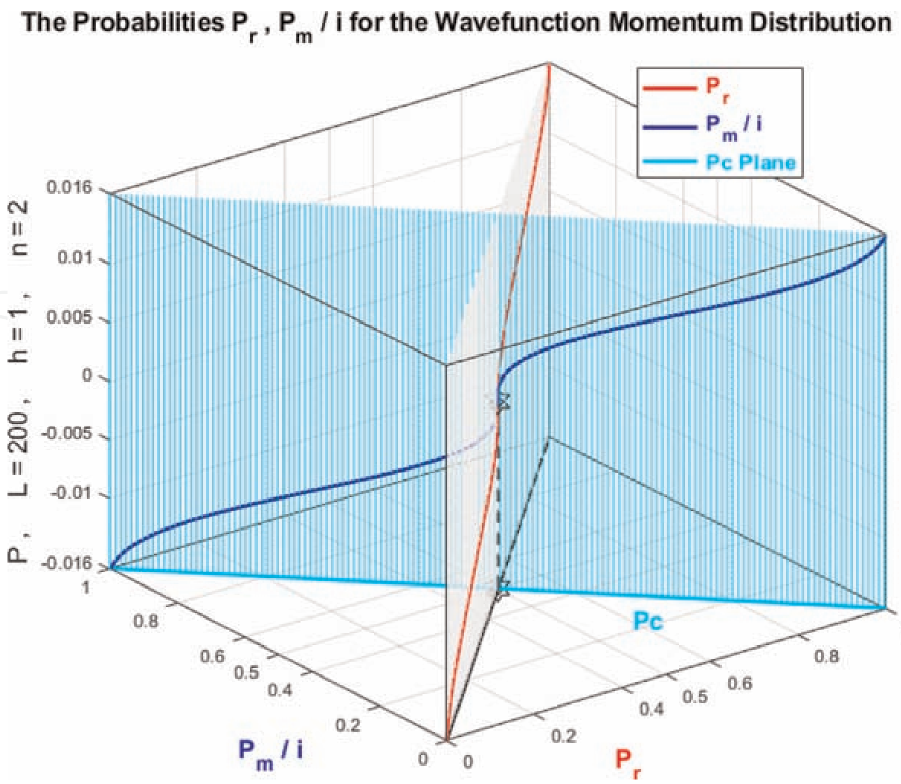

Figure 9.
The graphs of P_r *and* P_m/i *and* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 2.

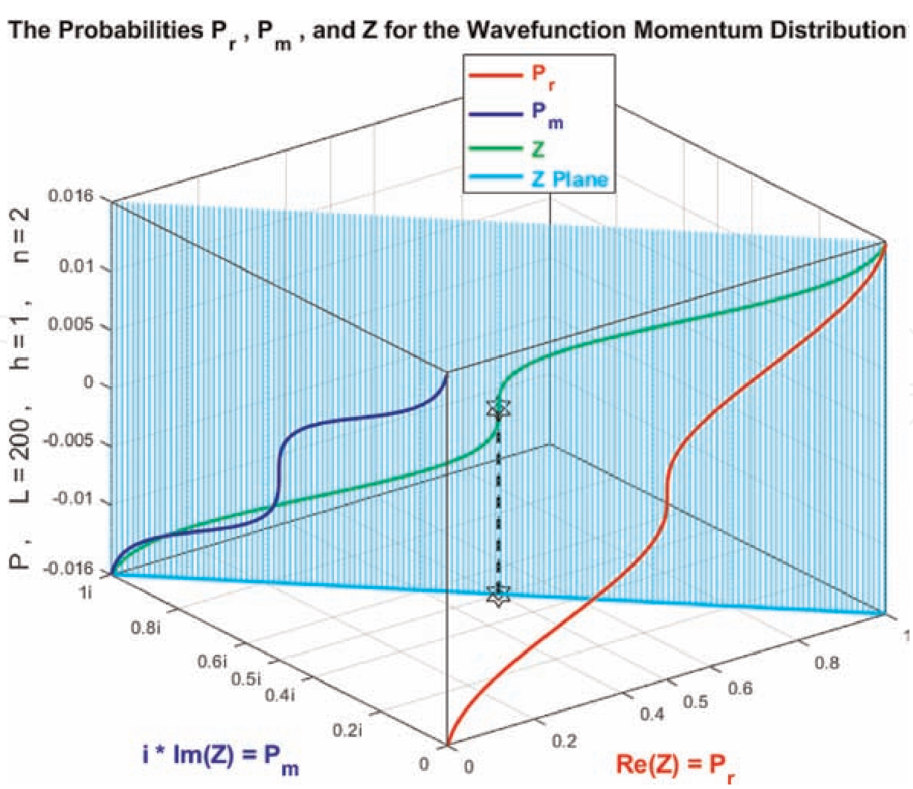

Figure 10.
The graphs of the probabilities P_r *and* P_m *and* Z *in terms of* P *for the wavefunction momentum probability distribution for* n = 2.

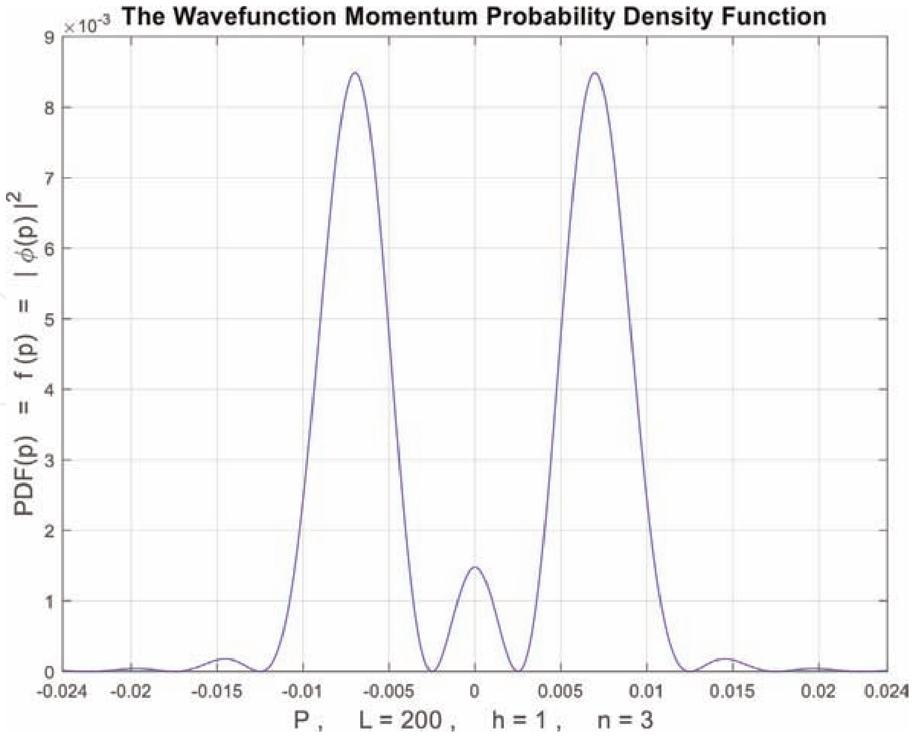

Figure 11.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = *3.*

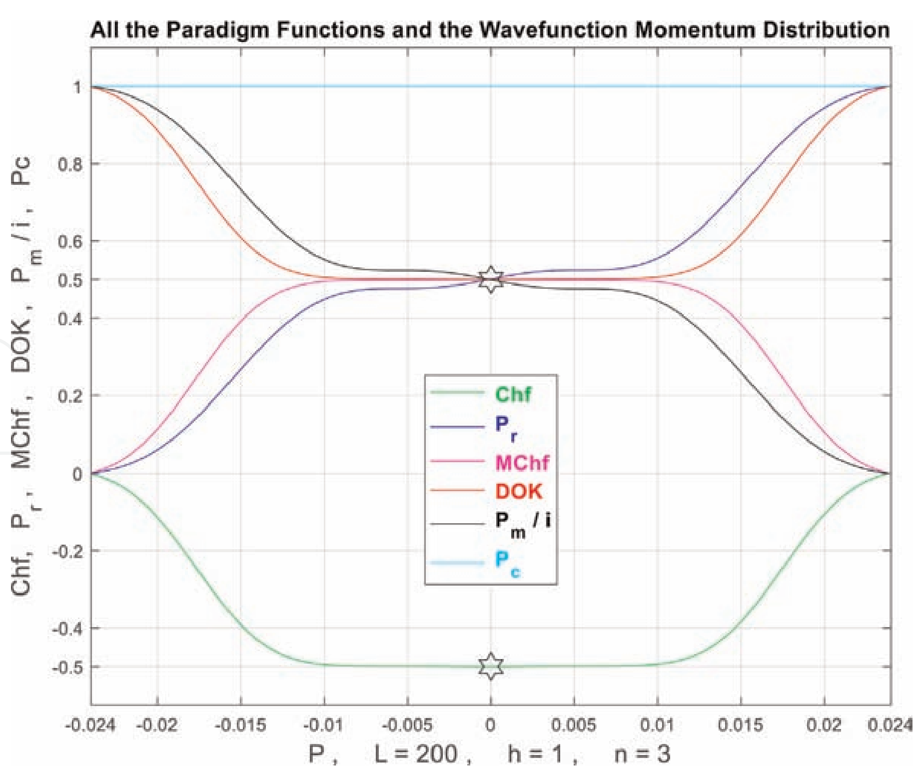

Figure 12.
The graphs of all the CPP *parameters as functions of the random variable* P *for the wavefunction momentum probability distribution for* n = *3.*

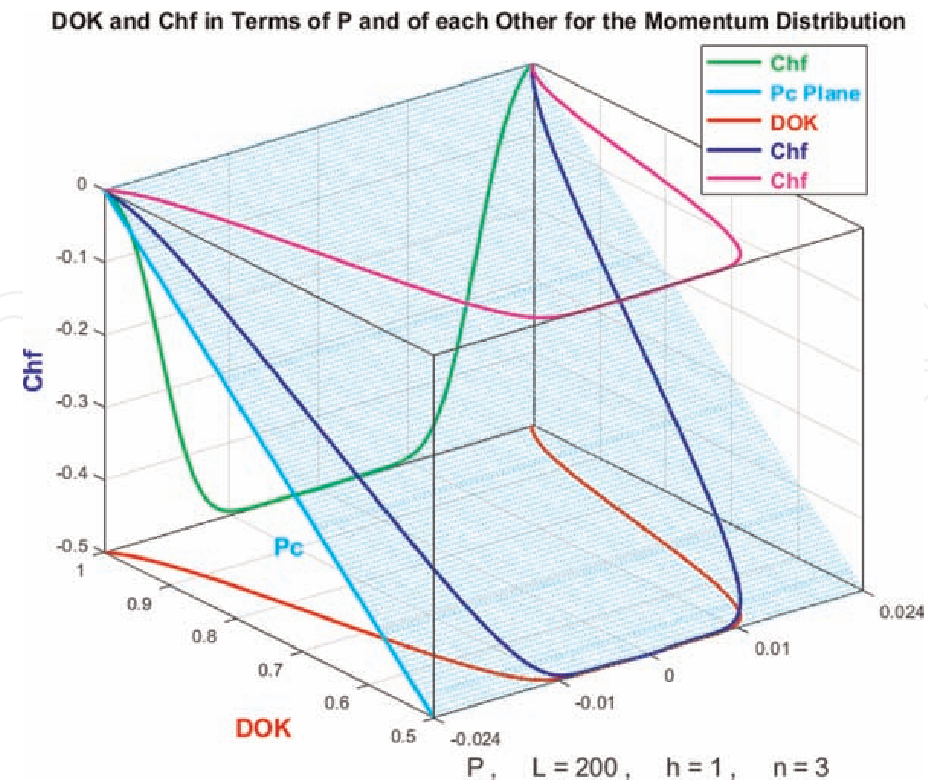

Figure 13.
The graphs of DOK *and* Chf *and the deterministic probability* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = *3*.

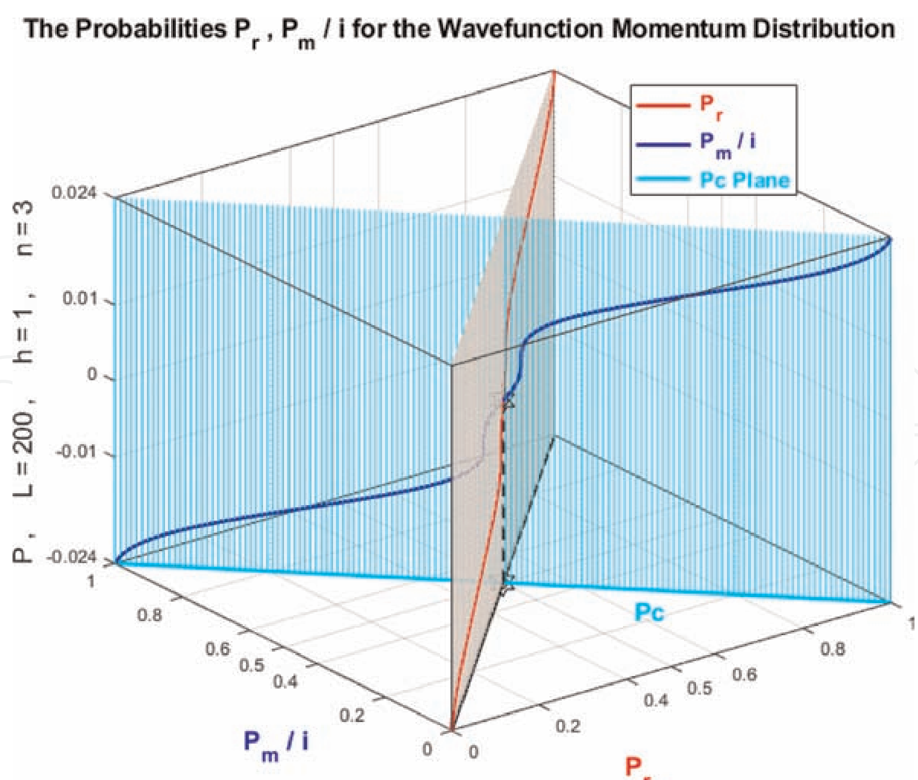

Figure 14.
The graphs of P_r *and* P_m/i *and* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = *3*.

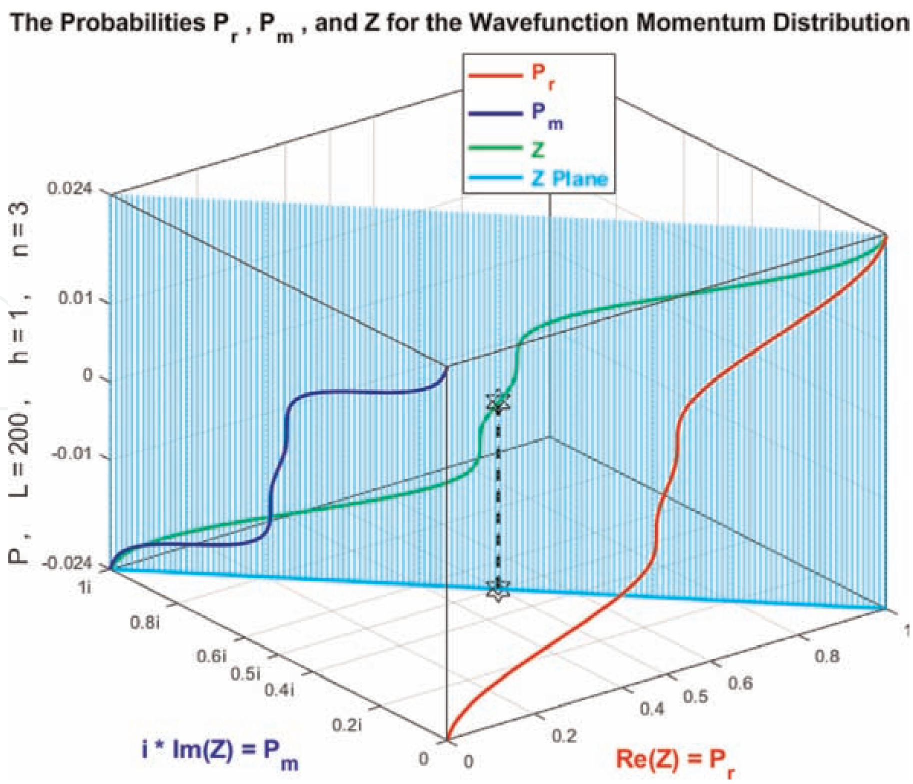

Figure 15.
The graphs of the probabilities P_r *and* P_m *and* Z *in terms of* P *for the wavefunction momentum probability distribution for* n = 3*.*

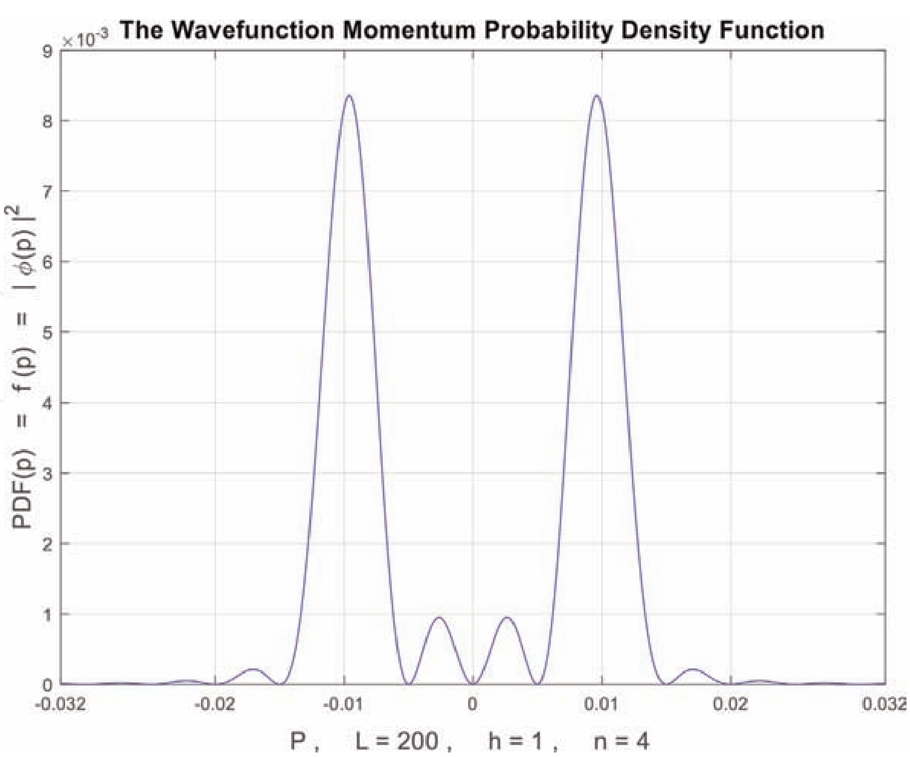

Figure 16.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = 4*.*

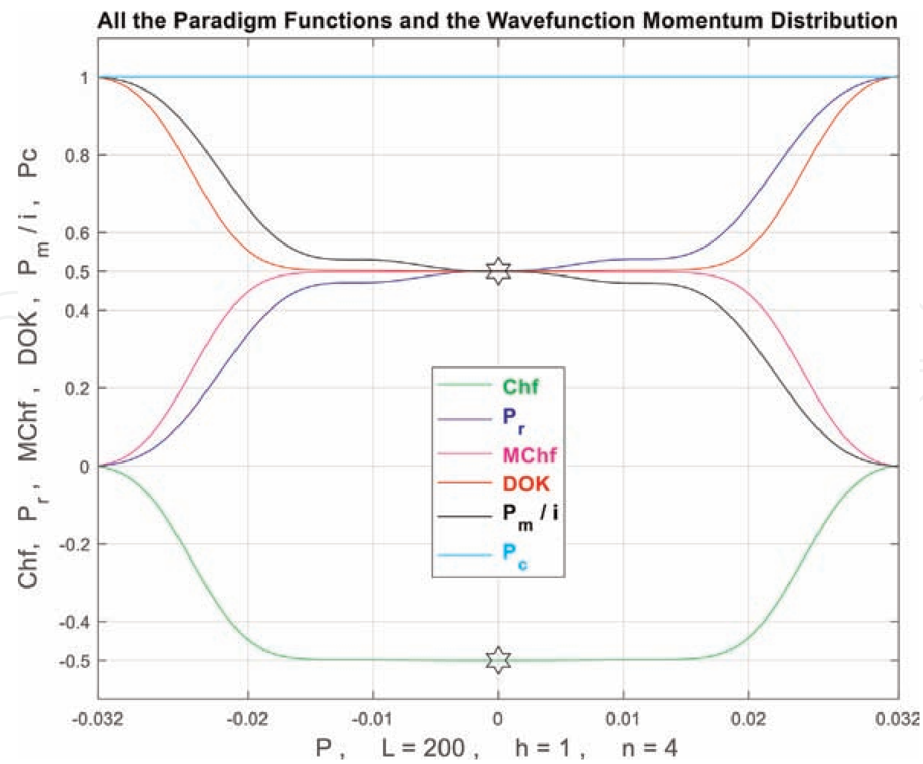

Figure 17.
The graphs of all the CPP *parameters as functions of the random variable* P *for the wavefunction momentum probability distribution for* n = 4.

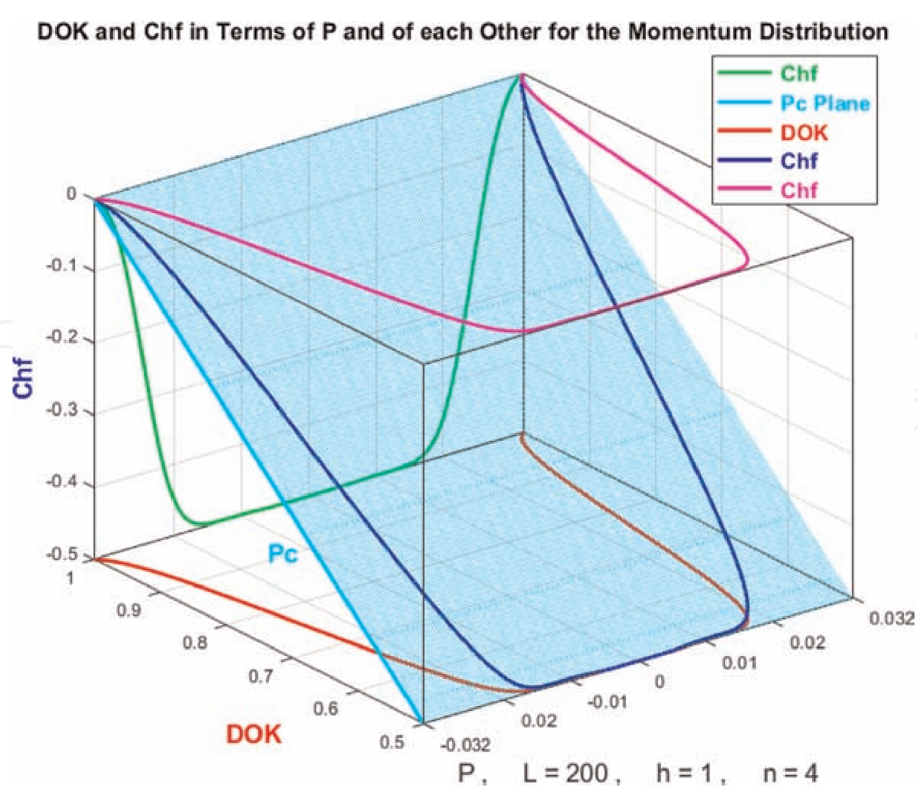

Figure 18.
The graphs of DOK *and* Chf *and the deterministic probability* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 4.

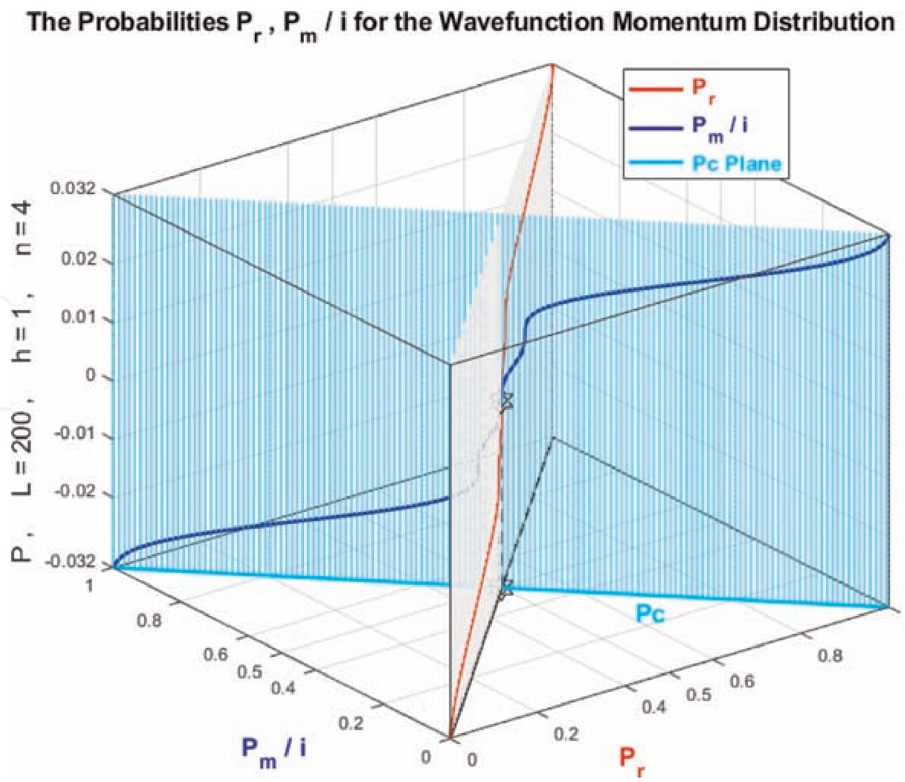

Figure 19.
The graphs of P_r *and* P_m/i *and* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 4.

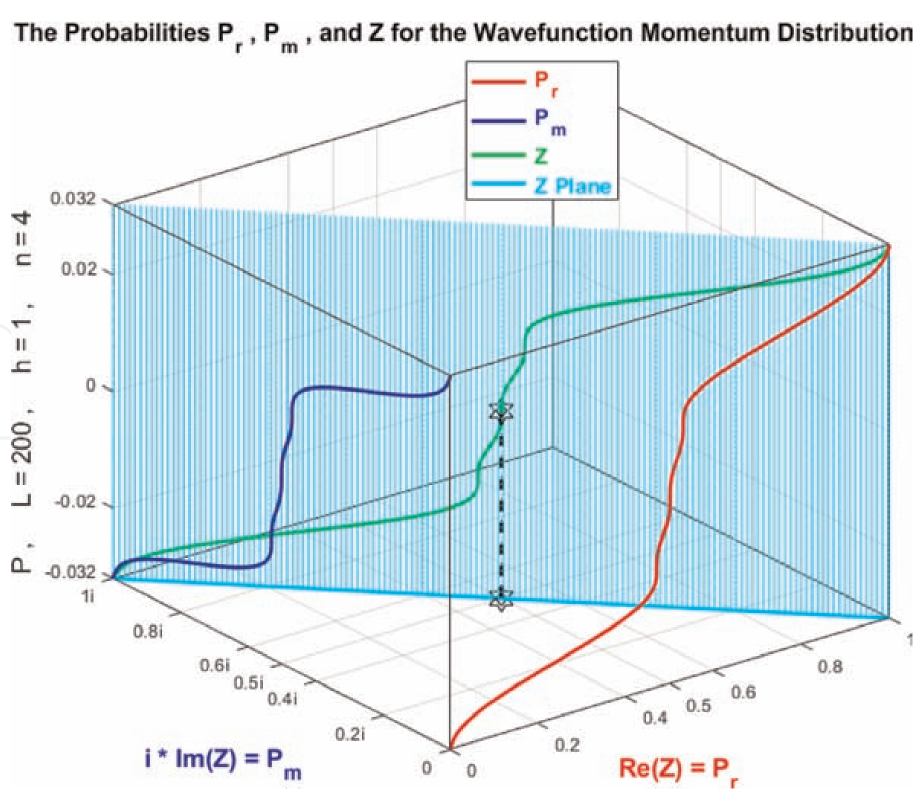

Figure 20.
The graphs of the probabilities P_r *and* P_m *and* Z *in terms of* P *for the wavefunction momentum probability distribution for* n = 4.

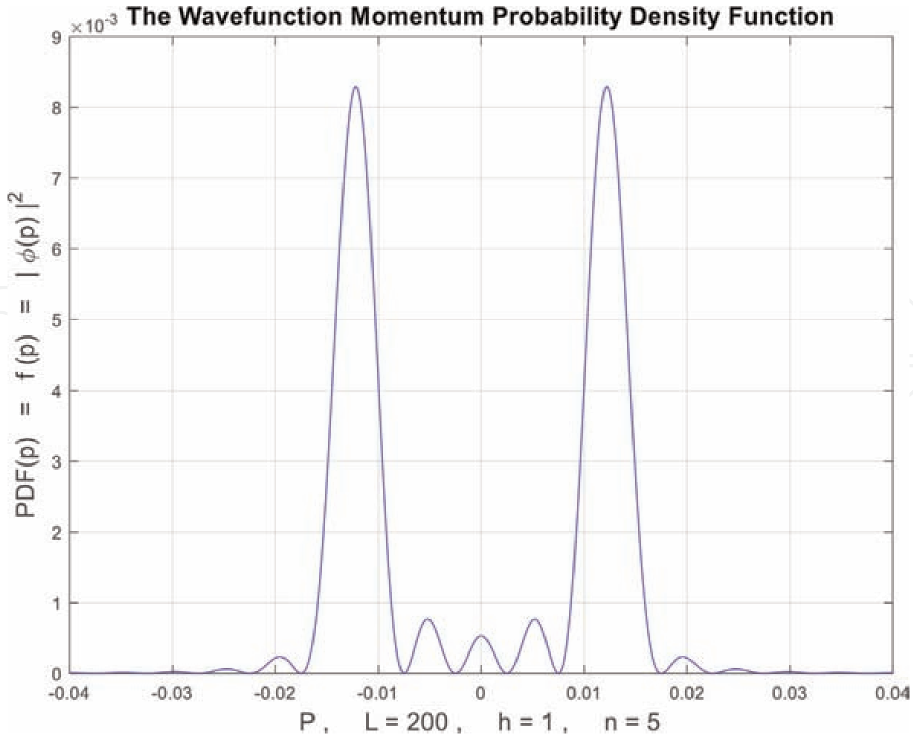

Figure 21.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = 5.

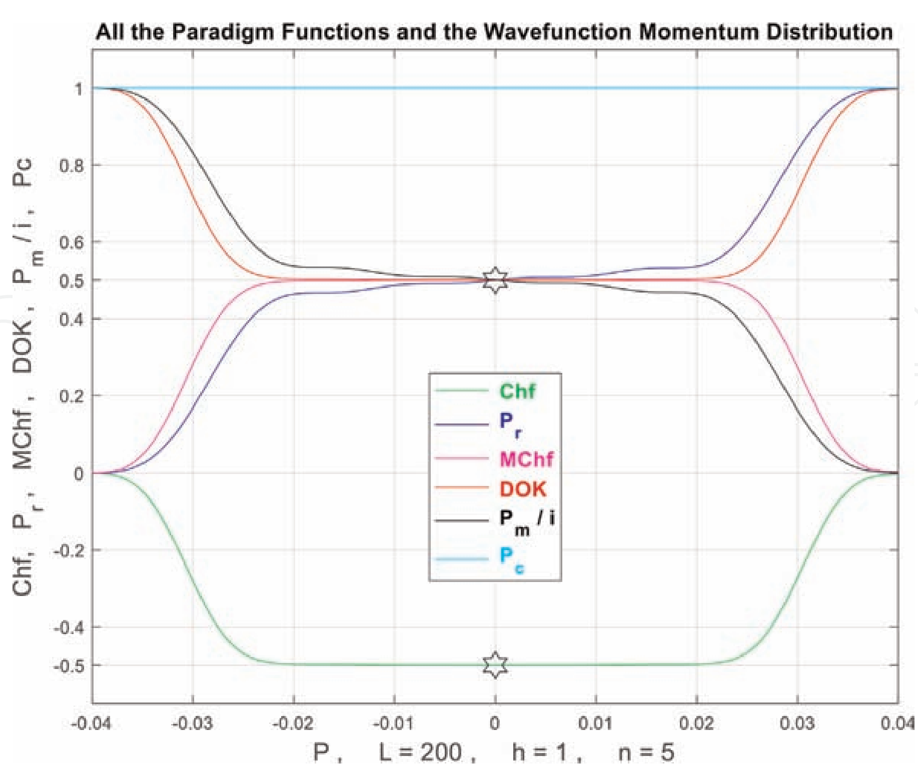

Figure 22.
The graphs of all the CPP *parameters as functions of the random variable* P *for the wavefunction momentum probability distribution for* n = 5.

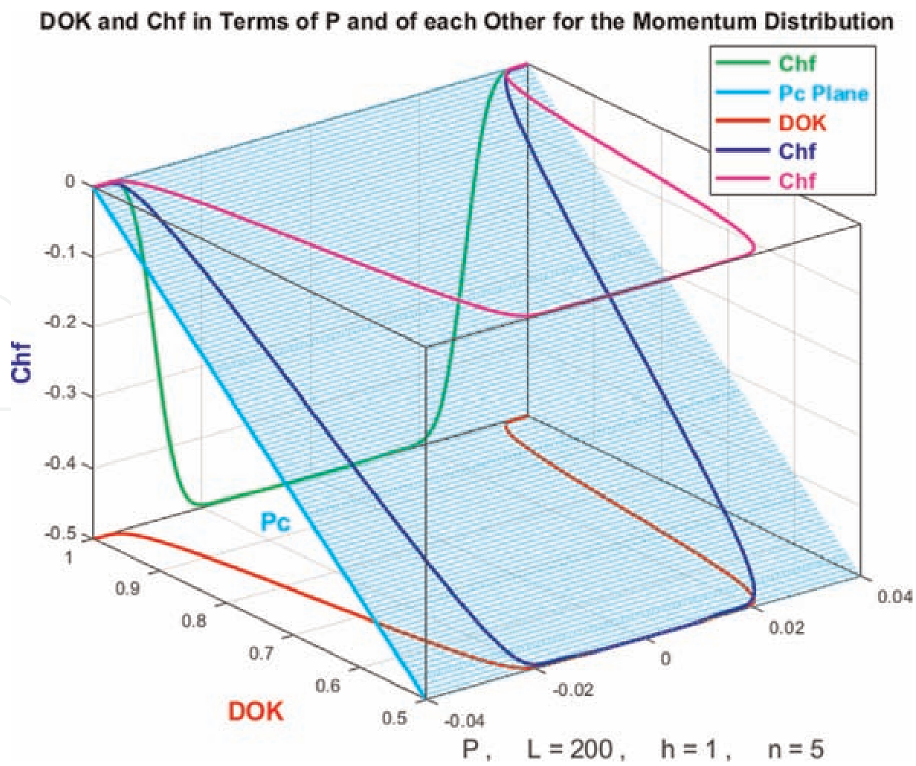

Figure 23.
The graphs of DOK *and* Chf *and the deterministic probability* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 5.

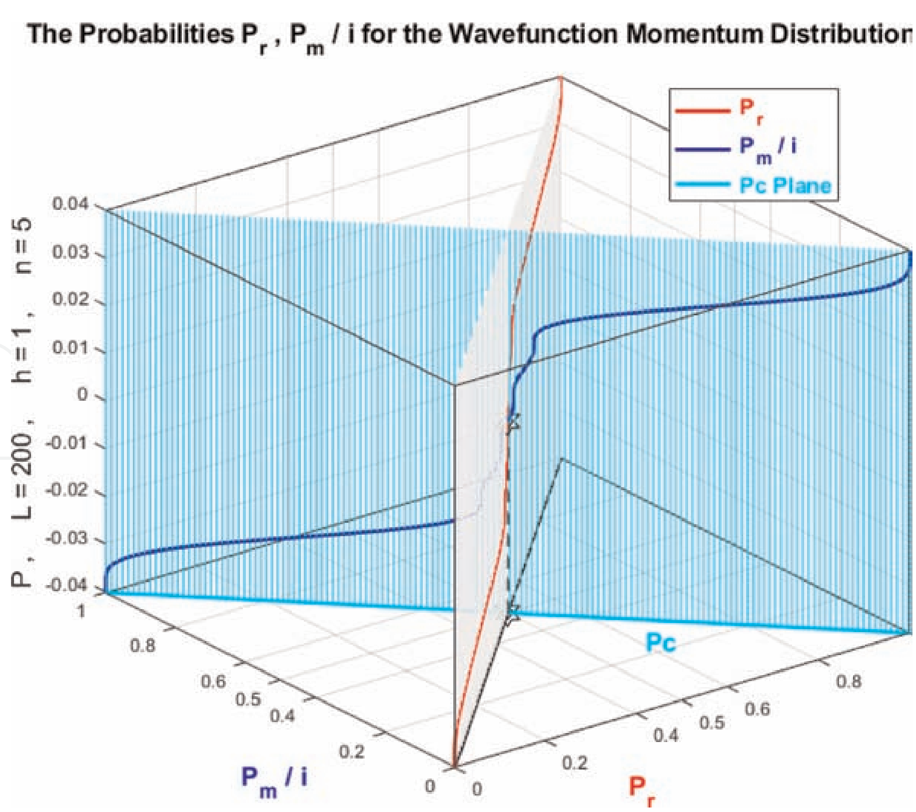

Figure 24.
The graphs of P_r *and* P_m/i *and* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 5.

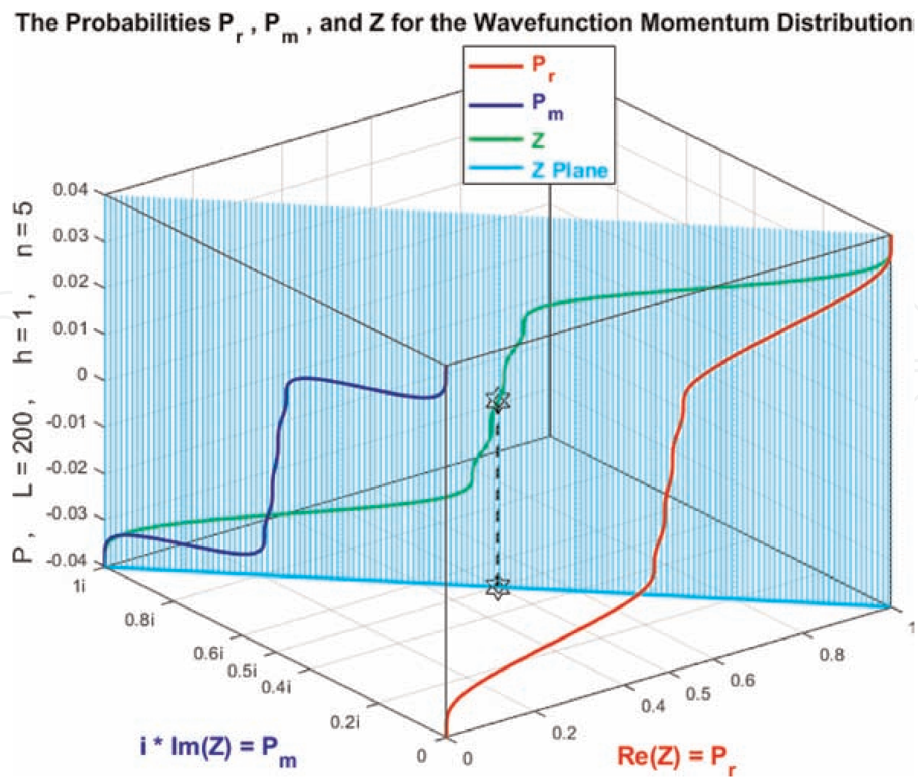

Figure 25.
The graphs of the probabilities P_r *and* P_m *and* Z *in terms of* P *for the wavefunction momentum probability distribution for* n = 5.

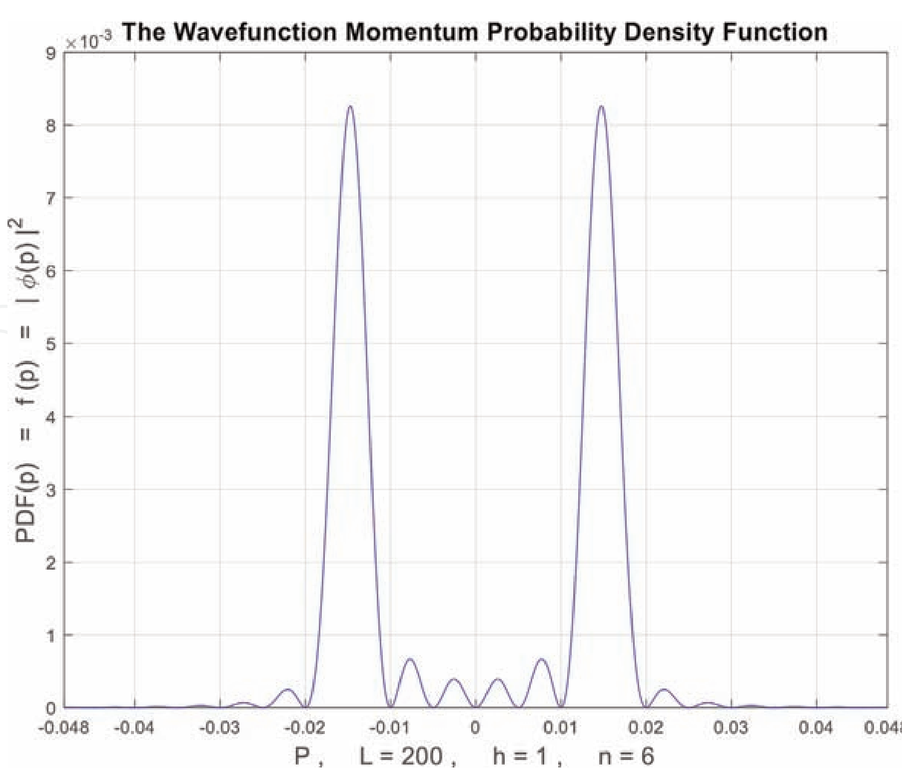

Figure 26.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = 6.

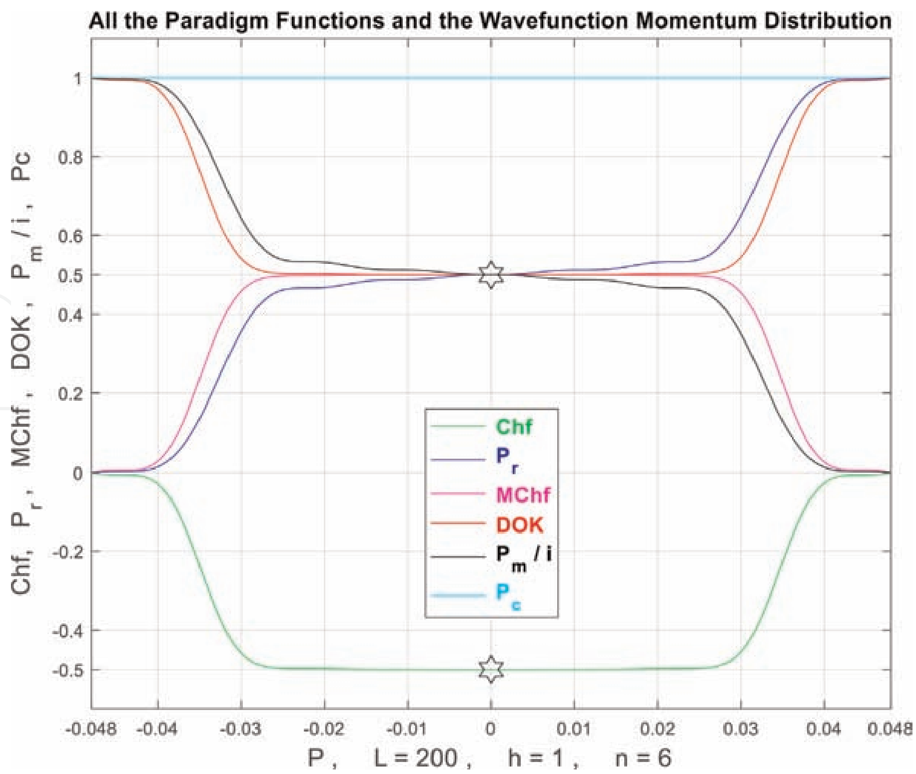

Figure 27.
The graphs of all the CPP *parameters as functions of the random variable* P *for the wavefunction momentum probability distribution for* n = 6.

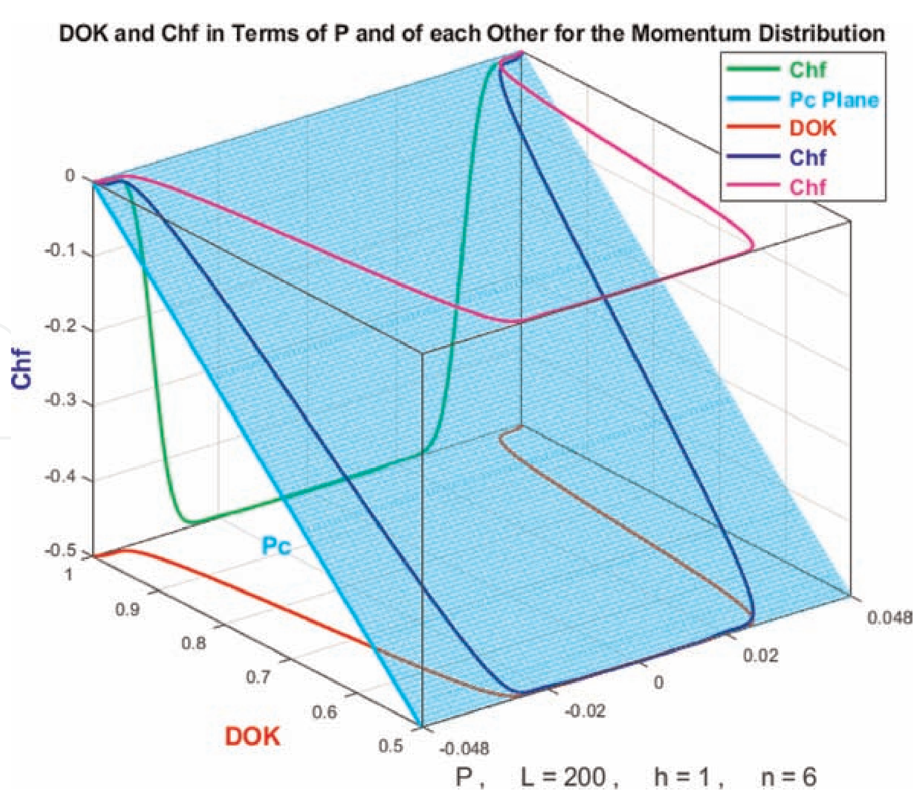

Figure 28.
The graphs of DOK *and* Chf *and the deterministic probability* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 6.

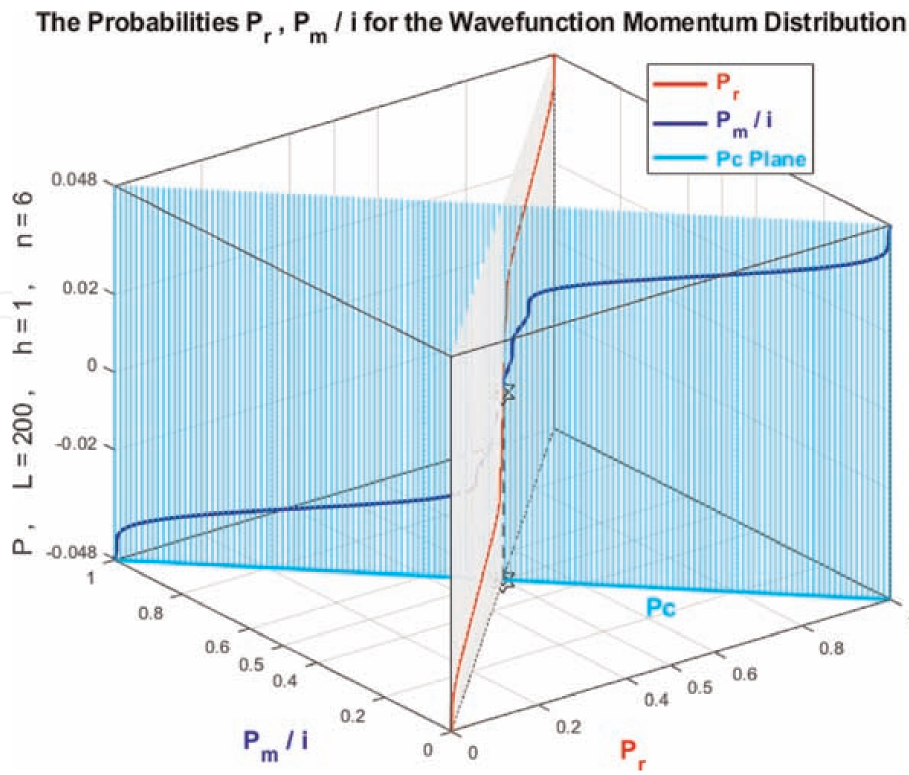

Figure 29.
The graphs of P_r *and* P_m/i *and* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 6.

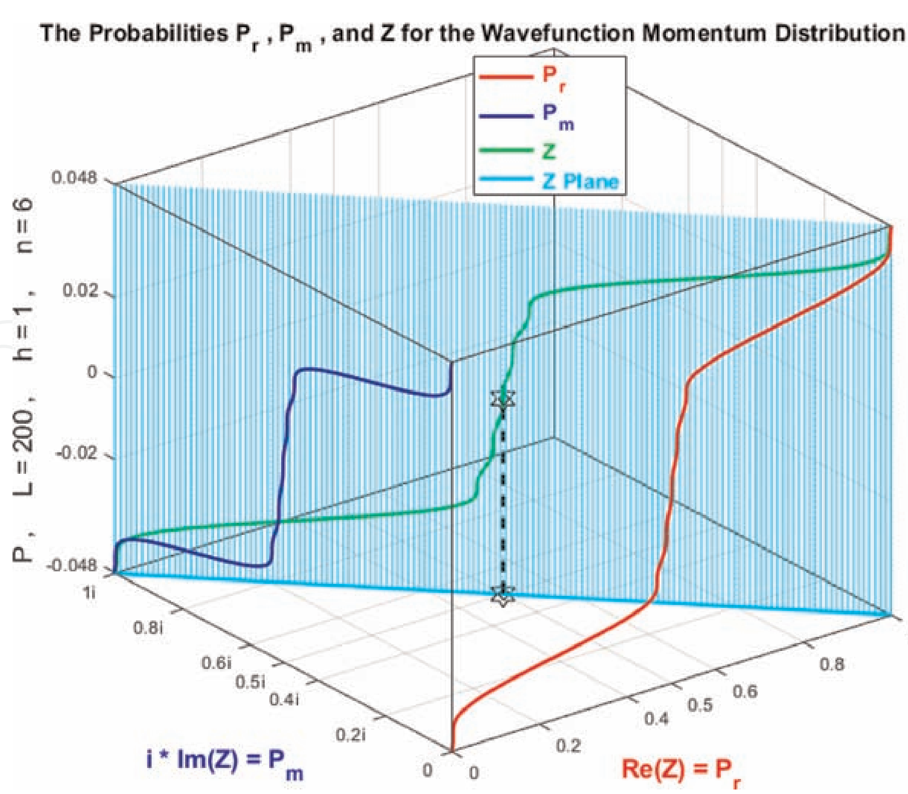

Figure 30.
The graphs of the probabilities P_r *and* P_m *and* Z *in terms of* P *for the wavefunction momentum probability distribution for* n = 6.

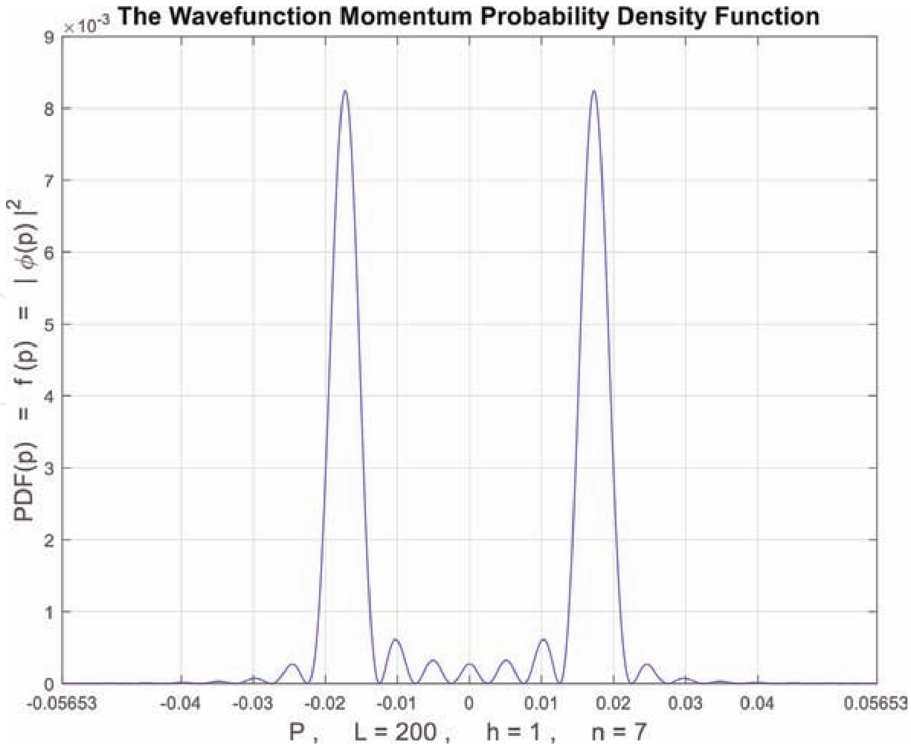

Figure 31.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = *7.*

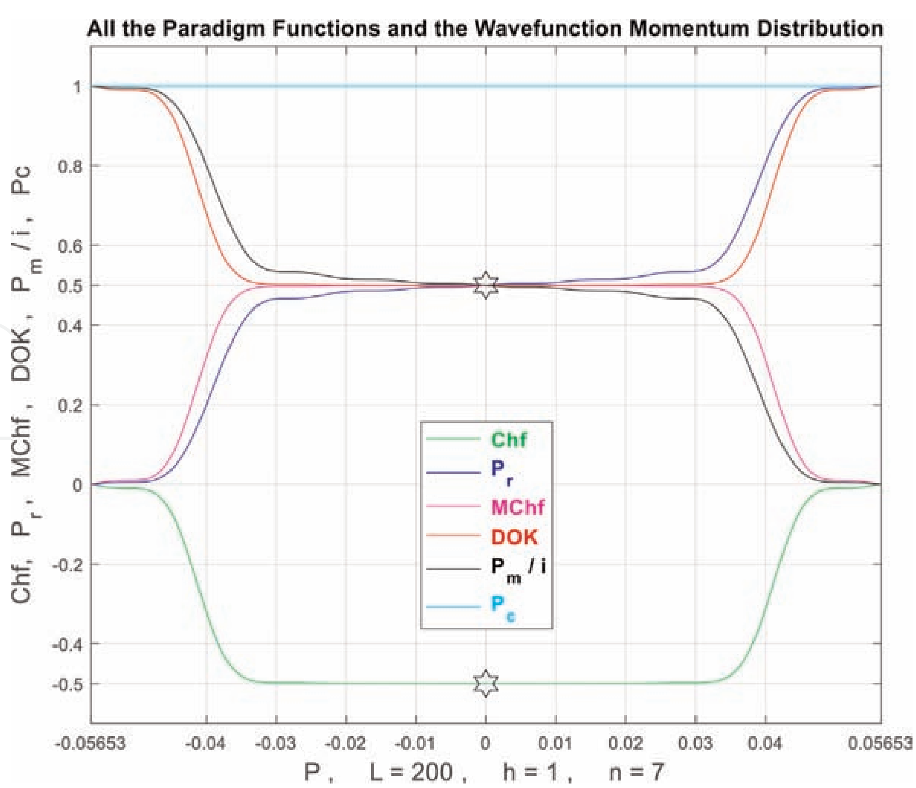

Figure 32.
The graphs of all the CPP *parameters as functions of the random variable* P *for the wavefunction momentum probability distribution for* n = *7.*

DOI: http://dx.doi.org/10.5772/intechopen.107665

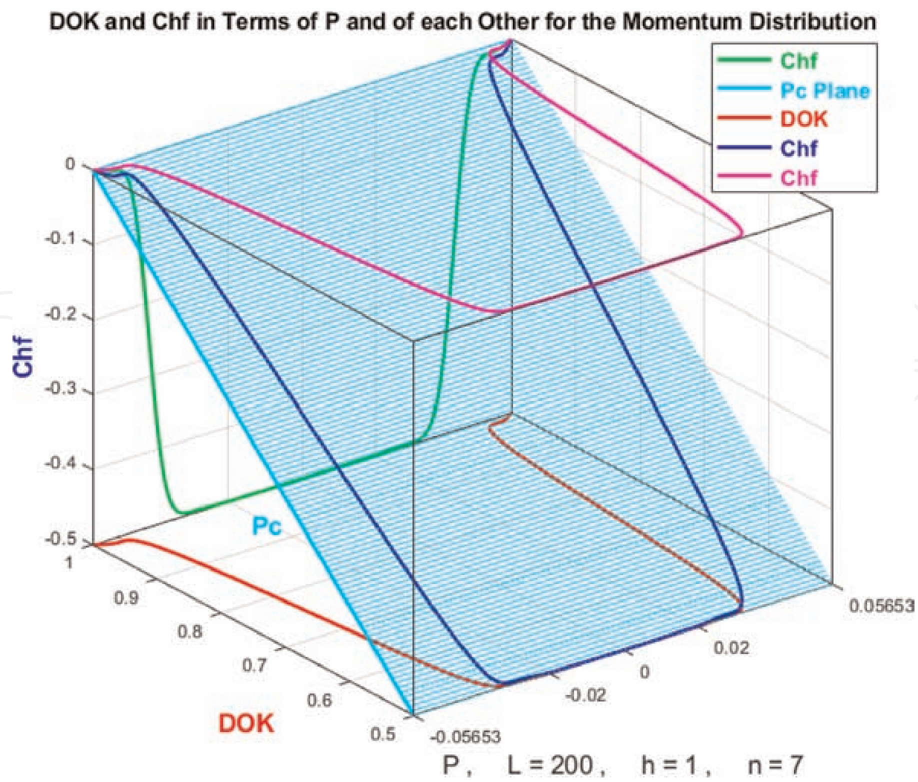

Figure 33.
The graphs of DOK *and* Chf *and the deterministic probability* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 7.

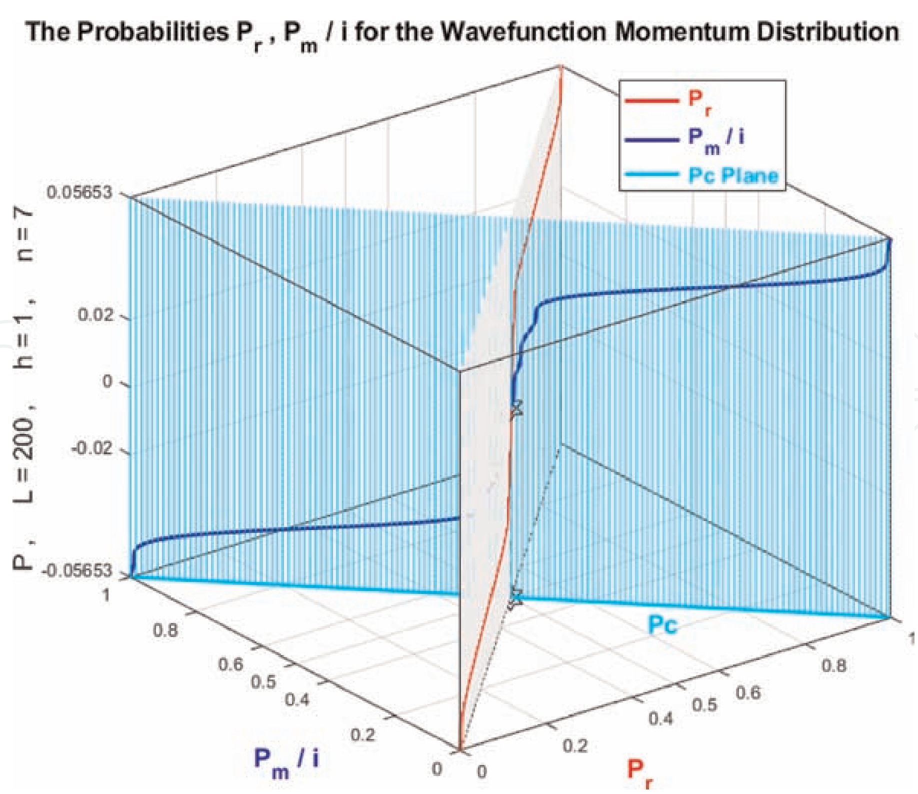

Figure 34.
The graphs of P_r *and* P_m/i *and* Pc *in terms of* P *and of each other for the wavefunction momentum probability distribution for* n = 7.

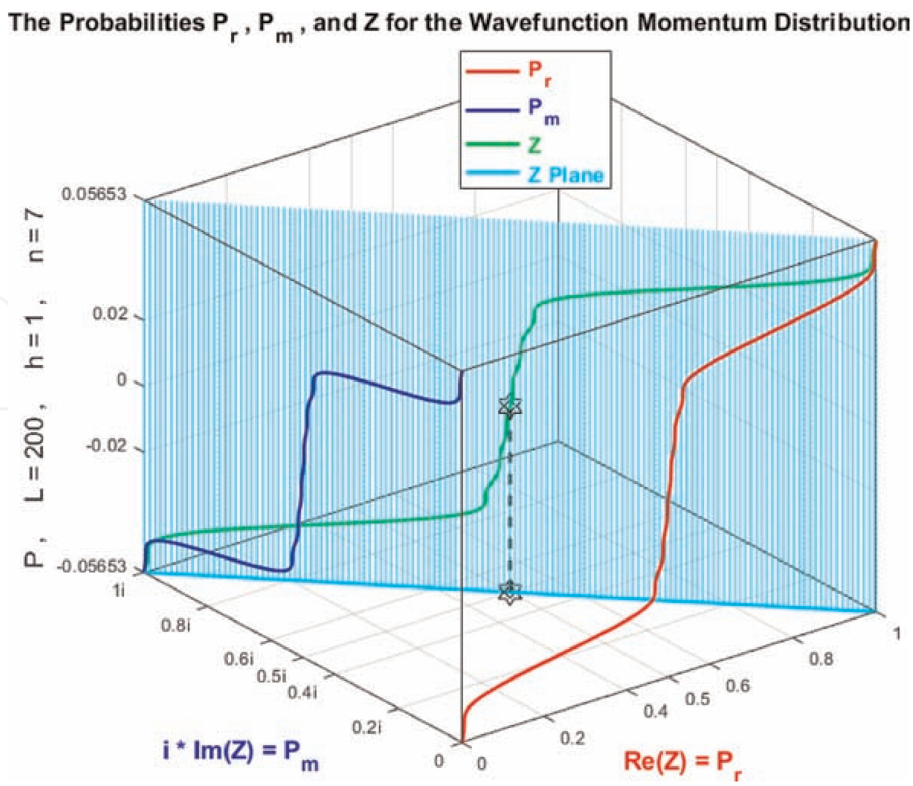

Figure 35.
The graphs of the probabilities P_r *and* P_m *and* Z *in terms of* P *for the wavefunction momentum probability distribution for* n = *7.*

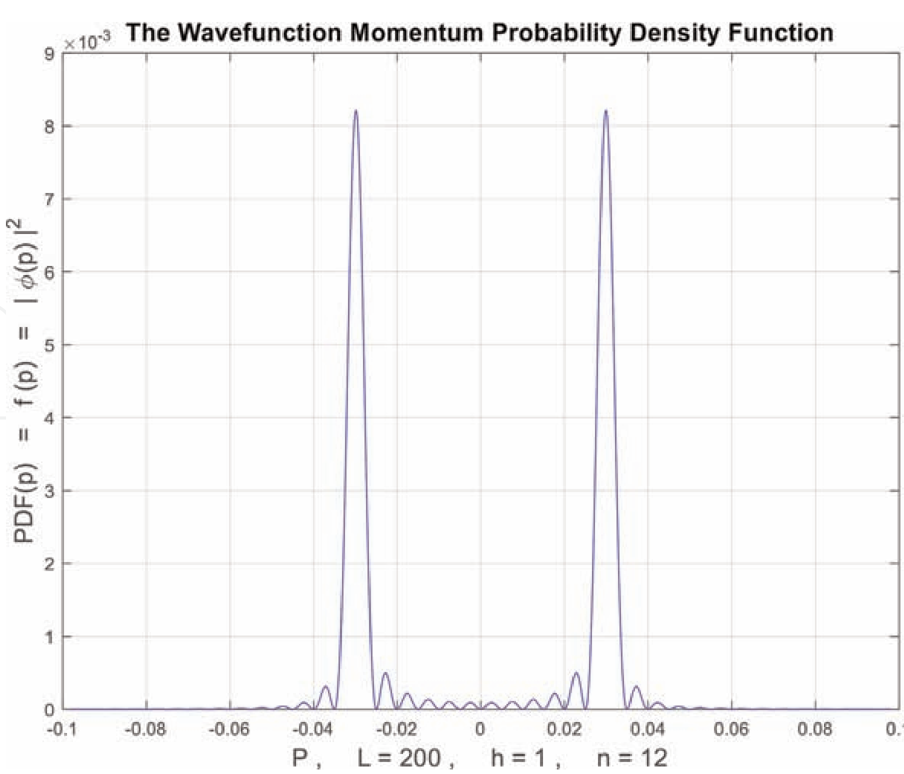

Figure 36.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = *12.*

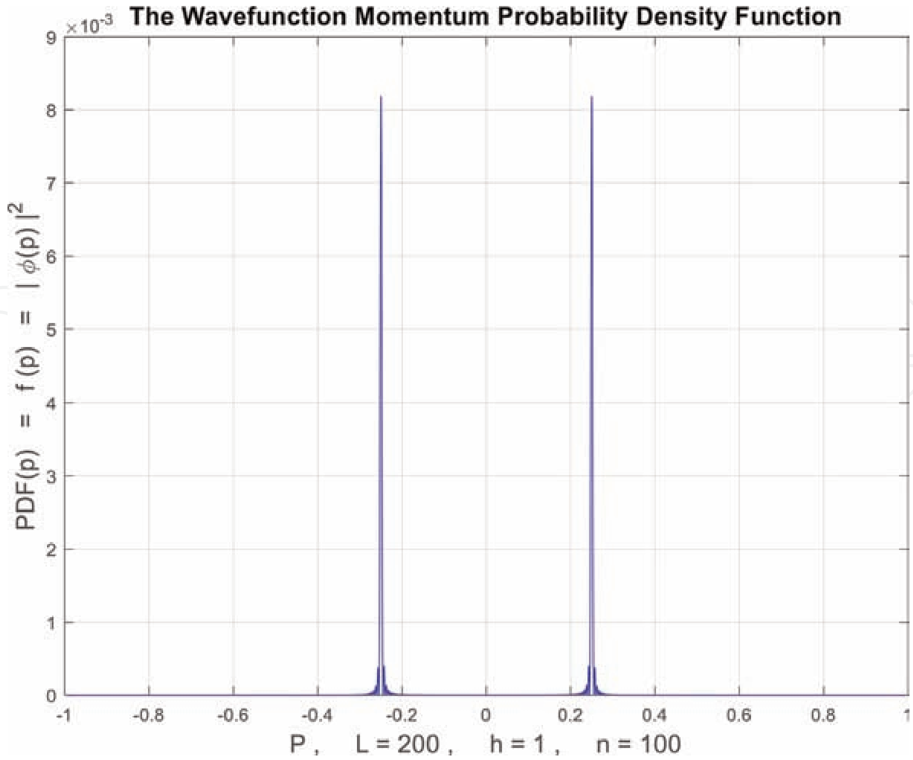

Figure 37.
The graph of the PDF *of the wavefunction momentum probability distribution as a function of the random variable* P *for* n = *100.*

In the cubes (**Figures 3, 8, 13, 18, 23, 28**, and **33**), the simulation of DOK and Chf as functions of each other and the random variable P for the infinite potential well problem wavefunction momentum probability distribution can be seen. The thick line in cyan is the projection of the plane $Pc^2(P) = DOK(P) - Chf(P) = 1 = Pc(P)$ on the plane $P = L_b$ = lower bound of P. This thick line starts at the point ($DOK = 1$, $Chf = 0$) when $P = L_b$, reaches the point ($DOK = 0.5$, $Chf = -0.5$) when $P = 0$, and returns at the end to ($DOK = 1$, $Chf = 0$) when $P = U_b$ = upper bound of P. The other curves are the graphs of $DOK(P)$ (red) and $Chf(P)$ (green, blue, pink) in different simulation planes. Notice that they all have a minimum at the point ($DOK = 0.5$, $Chf = -0.5$, $P = 0$). The last simulation point corresponds to ($DOK = 1$, $Chf = 0$, $P = U_b$).

In the cubes (**Figures 4, 9, 14, 19, 24, 29**, and **34**), we can notice the simulation of the real probability $P_r(P)$ in $\mathcal{R}$ and its complementary real probability $P_m(P)/i$ in $\mathcal{R}$ also in terms of the random variable P for the infinite potential well problem wavefunction momentum probability distribution. The thick line in cyan is the projection of the plane $Pc^2(P) = P_r(P) + P_m(P)/i = 1 = Pc(P)$ on the plane $P = L_b$ = lower bound of P. This thick line starts at the point ($P_r = 0$, $P_m/i = 1$) and ends at the point ($P_r = 1$, $P_m/i = 0$). The red curve represents $P_r(P)$ in the plane $P_r(P) = P_m(P)/i$ in light gray. This curve starts at the point ($P_r = 0$, $P_m/i = 1$, $P = L_b$ = lower bound of P), reaches the point ($P_r = 0.5$, $P_m/i = 0.5$, $P = 0$), and gets at the end to ($P_r = 1$, $P_m/i = 0$, $P = U_b$ = upper bound of P). The blue curve represents $P_m(P)/i$ in the plane in cyan $P_r(P) + P_m(P)/i = 1 = Pc(P)$. Notice the importance of the point which is the intersection of the red and blue curves at $P = 0$ and when $P_r(P) = P_m(P)/i = 0.5$.

In the cubes (**Figures 5, 10, 15, 20, 25, 30**, and **35**), we can notice the simulation of the complex probability $Z(P)$ in $\mathcal{C} = \mathcal{R} + \mathcal{M}$ as a function of the real probability $P_r(P)$ = Re(Z) in $\mathcal{R}$ and of its complementary imaginary probability $P_m(P) = i \times \text{Im}(Z)$ in $\mathcal{M}$, and this in terms of the random variable P for the infinite potential well problem wavefunction momentum probability distribution. The red curve represents $P_r(P)$ in the plane $P_m(P) = 0$ and the blue curve represents $P_m(P)$ in the plane $P_r(P) = 0$. The green curve represents the complex probability $Z(P) = P_r(P) + P_m(P) = \text{Re}(Z) + i \times \text{Im}(Z)$ in the plane $P_r(P) = iP_m(P) + 1$ or $Z(P)$ plane in cyan. The curve of $Z(P)$ starts at the point ($P_r = 0$, $P_m = i$, $P = L_b$ = lower bound of P) and ends at the point ($P_r = 1$, $P_m = 0$, $P = U_b$ = upper bound of P). The thick line in cyan is $P_r(P = L_b) = iP_m(P = L_b) + 1$ and it is the projection of the $Z(P)$ curve on the complex probability plane whose equation is $P = L_b$. This projected thick line starts at the point ($P_r = 0$, $P_m = i$, $P = L_b$) and ends at the point ($P_r = 1$, $P_m = 0$, $P = L_b$). Notice the importance of the point corresponding to $P = 0$ and $Z = 0.5 + 0.5i$ when $P_r = 0.5$ and $P_m = 0.5i$.

1.1.3 The characteristics of the momentum probability distribution

In quantum mechanics, the average, or expectation value of the momentum of a particle is given by: $\langle p \rangle = \int_{-\infty}^{+\infty} p|\phi(p)|^2 dp = \int_{-\infty}^{+\infty} p \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right] dp.$

For the steady state particle in a box, it can be shown that the average momentum is always $\langle p \rangle = 0$ regardless of the state of the particle. In the probability set and universe $\mathcal{R}$, we have:

$$\langle p \rangle_R = \langle p \rangle = 0$$

The variance in the momentum is a measure of the uncertainty in momentum of the particle, so in the probability set and universe $\mathcal{R}$, we have:

$$\begin{aligned}\text{Var}_{p,R} = \text{Var}(p) = \langle p^2 \rangle_R - \langle p \rangle_R^2 = \int_{-\infty}^{+\infty} p^2 |\phi(p)|^2 dp - 0 \\ = \int_{-\infty}^{+\infty} p^2 \left\{ \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]\right\} dp = \left(\frac{\hbar n\pi}{L}\right)^2\end{aligned}$$

In the probability set and universe $\mathcal{M}$, we have:

$$\begin{aligned}\langle p \rangle_M &= \int_{-\infty}^{+\infty} p\left\{i\left[1 - |\phi(p)|^2\right]\right\} dp = i\int_{-\infty}^{+\infty} p\left\{1 - \frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]\right\} dp \\ &= i\left\{\int_{-\infty}^{+\infty} p dp - \int_{-\infty}^{+\infty} p\left\{\frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi + pL/\hbar}\right)^2 \text{sinc}^2\left[\frac{1}{2}(n\pi - pL/\hbar)\right]\right\} dp\right\} \\ &= i\left\{\left[\frac{p^2}{2}\right]_{-\infty}^{+\infty} - \langle p \rangle_R\right\} = i\left\{\left[\frac{p^2}{2}\right]_{-\text{U}_b}^{\text{U}_b} - \langle p \rangle_R\right\} = i\{0 - 0\} = 0\end{aligned}$$

$$
\begin{aligned}
\mathrm{Var}_{p,M} &= \langle p^2\rangle_M - \langle p\rangle_M^2 \\
&= \int_{-\infty}^{+\infty} p^2\left\{i\left[1-|\phi(p)|^2\right]\right\}dp - 0 \\
&= i\int_{-\infty}^{+\infty} p^2\left\{1-\frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi+pL/\hbar}\right)^2 \mathrm{sinc}^2\left[\frac{1}{2}(n\pi-pL/\hbar)\right]\right\}dp \\
&= i\left\{\int_{-\infty}^{+\infty} p^2dp - \int_{-\infty}^{+\infty} p^2\left\{\frac{L}{\pi\hbar}\left(\frac{n\pi}{n\pi+pL/\hbar}\right)^2 \mathrm{sinc}^2\left[\frac{1}{2}(n\pi-pL/\hbar)\right]\right\}dp\right\} \\
&= i\left\{\int_{-\infty}^{+\infty} p^2dp - \mathrm{Var}_{p,R}\right\} = i\left\{\left[\frac{p^3}{3}\right]_{-\infty}^{+\infty} - \mathrm{Var}_{p,R}\right\} \to i\left\{+\infty - \left(\frac{\hbar n\pi}{L}\right)^2\right\} \\
&\to +\infty
\end{aligned}
$$

In the probability set and the universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$, we have from *CPP*:

$$
\begin{aligned}
\langle p\rangle_C &= \int_{-\infty}^{+\infty} p[z(p)]dp = \int_{-\infty}^{+\infty} p\left\{|\phi(p)|^2 + i\left[1-|\phi(p)|^2\right]\right\}dp \\
&= \int_{-\infty}^{+\infty} p|\phi(p)|^2dp + \int_{-\infty}^{+\infty} pi\left[1-|\phi(p)|^2\right]dp \\
&= \langle p\rangle_R + \langle p\rangle_M = 0 + i(0) = 0
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Var}_{p,C} &= \langle p^2\rangle_C - \langle p\rangle_C^2 = \left[\int_{-\infty}^{+\infty} p^2[z(p)]dp\right] - \left[\langle p\rangle_R + \langle p\rangle_M\right]^2 \\
&= \left[\int_{-\infty}^{+\infty} p^2\left\{|\phi(p)|^2 + i\left[1-|\phi(p)|^2\right]\right\}dp\right] - \left[\langle p\rangle_R + \langle p\rangle_M\right]^2 \\
&= \left[\int_{-\infty}^{+\infty} p^2|\phi(p)|^2dp + \int_{-\infty}^{+\infty} p^2i\left[1-|\phi(p)|^2\right]dp\right] - \left[\langle p\rangle_R + \langle p\rangle_M\right]^2 \\
&= \left[\langle p^2\rangle_R + \langle p^2\rangle_M\right] - \left[\langle p\rangle_R + \langle p\rangle_M\right]^2 \\
&= \left[\langle p^2\rangle_R + \langle p^2\rangle_M\right] - \left[\langle p\rangle_R^2 + \langle p\rangle_M^2 + 2\langle p\rangle_R\langle p\rangle_M\right] \\
&= \left[\langle p^2\rangle_R - \langle p\rangle_R^2\right] + \left[\langle p^2\rangle_M - \langle p\rangle_M^2\right] - 2\langle p\rangle_R\langle p\rangle_M \\
&= \mathrm{Var}_{p,R} + \mathrm{Var}_{p,M} - 2\langle p\rangle_R\langle p\rangle_M \\
&\to \left(\frac{\hbar n\pi}{L}\right)^2 + \infty - 2(0)(0) \\
&\to +\infty
\end{aligned}
$$

Momentum distribution characteristics	$L = 200, h = 1, n = 1$
$\langle p \rangle_R$	0
$\text{Var}_{p,R}$	6.2500e−06
$\langle p \rangle_M$	0
$\text{Var}_{p,M}$	$+\infty$
$\langle p \rangle_C = \langle p \rangle_R + \langle p \rangle_M$	$0 + i(0)$
$\text{Var}_{p,C} = \text{Var}_{p,R} + \text{Var}_{p,M} - 2\langle p \rangle_R \langle p \rangle_M$	$+\infty$

Table 1.
The momentum distribution characteristics for $L = 200$, $h = 1$, and $n = 1$.

Momentum distribution characteristics	$L = 200, h = 1, n = 2$
$\langle p \rangle_R$	0
$\text{Var}_{p,R}$	2.500e−05
$\langle p \rangle_M$	0
$\text{Var}_{p,M}$	$+\infty$
$\langle p \rangle_C = \langle p \rangle_R + \langle p \rangle_M$	$0 + i(0)$
$\text{Var}_{p,C} = \text{Var}_{p,R} + \text{Var}_{p,M} - 2\langle p \rangle_R \langle p \rangle_M$	$+\infty$

Table 2.
The momentum distribution characteristics for $L = 200$, $h = 1$, and $n = 2$.

Momentum distribution characteristics	$L = 200, h = 1, n = 8$
$\langle p \rangle_R$	0
$\text{Var}_{p,R}$	4.0000e−04
$\langle p \rangle_M$	0
$\text{Var}_{p,M}$	$+\infty$
$\langle p \rangle_C = \langle p \rangle_R + \langle p \rangle_M$	$0 + i(0)$
$\text{Var}_{p,C} = \text{Var}_{p,R} + \text{Var}_{p,M} - 2\langle p \rangle_R \langle p \rangle_M$	$+\infty$

Table 3.
The momentum distribution characteristics for $L = 200$, $h = 1$, and $n = 8$.

The following tables (**Tables 1–4**) compute the momentum distribution characteristics for $L = 200$, $h = 1$, and $n = 1,2,8,10000$.

For $n \gg 1$ (large n) we get: $\text{Var}_{p,R} = \left(\frac{\hbar n \pi}{L}\right)^2 \to +\infty$.

2. Heisenberg uncertainty principle in $\mathcal{R}$, $\mathcal{M}$, and $\mathcal{C}$

The uncertainties in the probability set and universe $\mathcal{R}$ in position and momentum (Δx_R and Δp_R) are defined as being equal to the square root of their respective variances in $\mathcal{R}$, so that:

Momentum distribution characteristics	$L = 200, h = 1, n = 10{,}000$
$\langle p \rangle_R$	0
$\text{Var}_{p,R}$	625
$\langle p \rangle_M$	0
$\text{Var}_{p,M}$	$+\infty$
$\langle p \rangle_C = \langle p \rangle_R + \langle p \rangle_M$	$0 + i(0)$
$\text{Var}_{p,C} = \text{Var}_{p,R} + \text{Var}_{p,M} - 2\langle p \rangle_R \langle p \rangle_M$	$+\infty$

Table 4.
The momentum distribution characteristics for L = 200, h = 1, and n = 10,000.

$$\Delta x_R \times \Delta p_R = \sqrt{\text{Var}_{x,R}} \times \sqrt{\text{Var}_{p,R}} = \sqrt{\frac{L^2}{12}\left(1 - \frac{6}{n^2\pi^2}\right)} \times \sqrt{\frac{\hbar^2 n^2 \pi^2}{L^2}} = \frac{\hbar}{2}\sqrt{\frac{n^2\pi^2}{3} - 2}$$

This product increases with increasing n, having a minimum value for $n = 1$. The value of this product for $n = 1$ is about equal to $0.568\ \hbar$ which obeys the Heisenberg uncertainty principle, which states that:

$$\Delta x \times \Delta p \geq \frac{\hbar}{2} \Leftrightarrow \forall n \geq 1 : \Delta x_R \times \Delta p_R \geq \frac{\hbar}{2}$$

The uncertainties in the probability set and universe $\mathcal{M}$ in position and momentum (Δx_M and Δp_M) are defined as being equal to the square root of their respective variances in $\mathcal{M}$, so that:

$$\Delta x_M \times \Delta p_M = \sqrt{\text{Var}_{x,M}} \times \sqrt{\text{Var}_{p,M}} \to \sqrt{i\left\{\frac{L^2}{12}\left[L - \left(1 - \frac{6}{n^2\pi^2}\right)\right]\right\}} \times \sqrt{+\infty} \to +\infty$$

$\Leftrightarrow \forall n \geq 1 : \Delta x_M \times \Delta p_M \geq \frac{\hbar}{2}$, in accordance with the Heisenberg uncertainty principle.

The uncertainties in the probability set and universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$ in position and momentum (Δx_C and Δp_C) are defined as being equal to the square root of their respective variances in $\mathcal{C}$, so that:

$$\Delta x_C \times \Delta p_C = \sqrt{\text{Var}_{x,C}} \times \sqrt{\text{Var}_{p,C}}$$
$$\to \sqrt{\frac{L^2}{12}\left(1 - \frac{6}{n^2\pi^2}\right) + i\left\{\frac{L^2}{12}\left[L - \left(1 - \frac{6}{n^2\pi^2}\right)\right]\right\}} \times \sqrt{+\infty} \to +\infty$$

$\Leftrightarrow \forall n \geq 1 : \Delta x_C \times \Delta p_C \geq \frac{\hbar}{2}$, in accordance with the Heisenberg uncertainty principle.

Consequently, the Heisenberg uncertainty principle is verified in the universe $\mathcal{R}$, in the universe $\mathcal{M}$, and the complex universe $\mathcal{C}$.

3. The Wavefunction Entropies in $\mathcal{R}$,$\mathcal{M}$,and $\mathcal{C}$

Another measure of uncertainty in position is the information entropy of the probability distribution H_x which is the entropy in $\mathcal{R}$ and is equal to:

$$H_x = -\sum_{x=-\infty}^{x=+\infty} |\psi(x)|^2 Ln\left[|\psi(x)|^2 x_0\right] = -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} |\psi(x)|^2 Ln\left[|\psi(x)|^2 x_0\right] = H_x^R = Ln\left(\frac{2L}{e x_0}\right)$$

where x_0 is an arbitrary reference length [1, 2]. Take $x_0 = 1$:

$$\Leftrightarrow H_x^R = -\sum_{x=x_c-\dfrac{L}{2}}^{x=x_c+\dfrac{L}{2}} |\psi(x)|^2 Ln\left[|\psi(x)|^2\right]$$
$$= Ln\left(\frac{2L}{e}\right) = Ln(2L) - Ln(e) = Ln(2L) - 1 = Ln(2 \times 200) - 1 = 4.991464547\ldots$$

$\Leftrightarrow \forall x : x_c - \frac{L}{2} \leq x \leq x_c + \frac{L}{2}$,we have : $d\left[H_x^R\right] \geq 0$, that means that H_x^R is a nondecreasing series with x and converging to $Ln\left(\frac{2L}{e}\right)$ and that also in $\mathcal{R}$, chaos and disorder are increasing with x.

The negative real entropy corresponding to H_x^R in $\mathcal{R}$ is $NegH_x^R$ and is the following:

$$NegH_x^R = -H_x^R = \sum_{x=-\infty}^{x=+\infty} |\psi(x)|^2 Ln\left[|\psi(x)|^2\right] = \sum_{x=x_c-\dfrac{L}{2}}^{x=x_c+\dfrac{L}{2}} |\psi(x)|^2 Ln\left[|\psi(x)|^2\right] = -Ln\left(\frac{2L}{e}\right)$$
$$= 1 - Ln(2L) = 1 - Ln(2 \times 200) = -4.991464547\ldots$$

$\Leftrightarrow \forall x : x_c - \frac{L}{2} \leq x \leq x_c + \frac{L}{2}$,we have : $d\left[NegH_x^R\right] \leq 0$, which means that $NegH_x^R$ is a nonincreasing series with x and converging to $-Ln\left(\frac{2L}{e}\right)$. Therefore, if H_x^R measures in $\mathcal{R}$ the amount of disorder, of uncertainty, of chaos, of ignorance, of unpredictability, and of information gain in a random system then since $NegH_x^R = -H_x^R$, that means the opposite of H_x^R, $NegH_x^R$ measures in $\mathcal{R}$ the amount of order, of certainty, of predictability, and of information loss in a stochastic system.

The complementary real entropy to H_x^R in $\mathcal{R}$ is $\overline{H}_x^R$ and is the following:

$$\overline{H}_x^R = -\sum_{x=-\infty}^{x=+\infty} \left[1 - |\psi(x)|^2\right] Ln\left[1 - |\psi(x)|^2\right] = -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} \left[1 - |\psi(x)|^2\right] Ln\left[1 - |\psi(x)|^2\right] = 1$$

In the complementary real probability set to $\mathcal{R}$, we denote the corresponding real entropy by $\overline{H}_x^R$.

The meaning of $\overline{H}_x^R$ is the following: it is the real entropy in the real set $\mathcal{R}$ and which is related to the complementary real probability $P_m/i = 1 - P_r$.

$\Leftrightarrow \forall x : x_c - \frac{L}{2} \leq x \leq x_c + \frac{L}{2}$,we have : $d\left[\overline{H}_x^R\right] \geq 0$, that means that $\overline{H}_x^R$ is a nondecreasing series with x and converging to 1 and that also means that in the complementary real probability set to $\mathcal{R}$, chaos and disorder are increasing with x.

In the complementary imaginary probability set $\mathcal{M}$ to the set $\mathcal{R}$, we denote the corresponding imaginary entropy by H_x^M. The meaning of H_x^M is the following: it is the imaginary entropy in the imaginary set $\mathcal{M}$ and which is related to the complementary imaginary probability $P_m = i(1 - P_r)$. The complementary entropy to H_x^R in $\mathcal{M}$ is H_x^M and is computed as follows:

$$
\begin{aligned}
H_x^M &= -\sum_{x=-\infty}^{x=+\infty} i\left[1-|\psi(x)|^2\right]Ln\left\{i\left[1-|\psi(x)|^2\right]\right\}\\
&= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} i\left[1-|\psi(x)|^2\right]Ln\left\{i\left[1-|\psi(x)|^2\right]\right\}\\
&= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} i\left[1-|\psi(x)|^2\right]\left\{Lni+Ln\left[1-|\psi(x)|^2\right]\right\}\\
&= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} i\left\{Lni+Ln\left[1-|\psi(x)|^2\right]-\left[|\psi(x)|^2\right]Lni-\left[|\psi(x)|^2\right]Ln\left[1-|\psi(x)|^2\right]\right\}\\
&= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} iLni+iLn\left[1-|\psi(x)|^2\right]-i\left[|\psi(x)|^2\right]Lni-i\left[|\psi(x)|^2\right]Ln\left[1-|\psi(x)|^2\right]\\
&= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} iLni\left[1-|\psi(x)|^2\right]+i\left[1-|\psi(x)|^2\right]Ln\left[1-|\psi(x)|^2\right]\\
&= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} iLni\left[1-|\psi(x)|^2\right]-i\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}}\left[1-|\psi(x)|^2\right]Ln\left[1-|\psi(x)|^2\right]\\
&= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} iLni\left[1-|\psi(x)|^2\right]+i\overline{H}_x^R = -iLni\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}}\left[1-|\psi(x)|^2\right]+i\overline{H}_x^R\\
&= -iLni\left\{\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}}1-\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}}|\psi(x)|^2\right\}+i\overline{H}_x^R\\
&= -iLni\left\{\left[\left(x_c+\frac{L}{2}\right)-\left(x_c-\frac{L}{2}\right)+1\right]-1\right\}+i\overline{H}_x^R \quad \text{since} \quad \sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}}|\psi(x)|^2=1\\
&= -(iLni)L+i\overline{H}_x^R
\end{aligned}
$$

From the properties of logarithms, we have: $\theta Lnx = Ln\left(x^{\theta}\right)$ then $iLni = Lni^{i}$.

Moreover, Leonhard Euler's formula for complex numbers gives: $e^{i\theta} = \cos\theta + i\sin\theta$.

Take $\theta = \pi/2 + 2k\pi \Leftrightarrow e^{i(\pi/2+2k\pi)} = \cos(\pi/2 + 2k\pi) + i\sin(\pi/2 + 2k\pi) = 0 + i(1) = i$, then:

$i^{i} = \left(e^{i(\pi/2+2k\pi)}\right)^{i} = e^{i^{2}(\pi/2+2k\pi)} = e^{-(\pi/2+2k\pi)}$ since $i^{2} = -1$, therefore:

$-iLni = -Lni^{i} = -Ln\left[e^{-(\pi/2+2k\pi)}\right] = \pi/2 + 2k\pi$ since $Ln[e] = 1$ and where k belongs to the set of integer numbers Z.

Consequently,

$$H_x^M = -(iLni)L + i\overline{H}_x^R = (\pi/2 + 2k\pi)L + i\overline{H}_x^R$$

That means that H_x^M is a complex number where:

the real part is: $\mathrm{Re}\left(H_x^M\right) = (\pi/2 + 2k\pi)L$, and the imaginary part is: $\mathrm{Im}\left(H_x^M\right) = \overline{H}_x^R$.

For $k = -1$ then

$\mathrm{Re}\left(H_x^M\right) = (-3\pi/2)L = -4.71238898L = -942.4777961\ldots \text{ for } L = 200.$

For $k = 0$ then $\mathrm{Re}\left(H_x^M\right) = (\pi/2)L = 1.570796327L = 314.1592654\ldots \text{ for } L = 200.$

For $k = 1$ then

$\mathrm{Re}\left(H_x^M\right) = (5\pi/2)L = 7.853981634L = 1570.796327\ldots \text{ for } L = 200,$

etc.

Finally, the entropy H_x^C in $\mathcal{C} = \mathcal{R} + \mathcal{M}$ is the following:

$$\begin{aligned} H_x^C &= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} Pc(x)Ln[Pc(x)] \\ &= -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} 1 \times Ln[1] = -\sum_{x=x_c-\frac{L}{2}}^{x=x_c+\frac{L}{2}} (1 \times 0) = 0 \\ &= H_x^R + NegH_x^R \end{aligned}$$

$\Leftrightarrow \forall x : x_c - \frac{L}{2} \le x \le x_c + \frac{L}{2}$, we have: $d\left[H_x^C\right] = 0$, that means that H_x^C is a constant series with x and is always equal to 0. That means also and most importantly, for the wavefunction position distribution and in the probability set and universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$, we have complete order, no chaos, no ignorance, no uncertainty, no disorder, no randomness, no information loss or gain but a conservation of information, and no unpredictability since all measurements are completely and perfectly deterministic ($Pc(x) = 1$ and $H_x^C = 0$).

Similarly, we can determine another measure of uncertainty in momentum which is the information entropy of the probability distribution H_p and which is [1, 2]:

$$H_p = -\sum_{p=-\infty}^{p=+\infty} |\phi(p)|^2 Ln\left[|\phi(p)|^2 p_0\right] = Ln\left(\frac{4\pi\hbar e^{2(1-\gamma)}}{Lp_0}\right) = \lim_{n\to+\infty} H_p(n)$$

DOI: http://dx.doi.org/10.5772/intechopen.107665

Where γ is Euler's constant and is equal to: 0.577215664901532 …

For $p_0 = 1$ we can compute all the defined entropies in $\mathcal{R}$, $\mathcal{M}$, and $\mathcal{C}$ and which are [1–30]:

$$H_p^R = -\sum_{p=-\infty}^{p=+\infty} |\phi(p)|^2 Ln\left[|\phi(p)|^2\right] = Ln\left(\frac{4\pi\hbar e^{2(1-\gamma)}}{L}\right) = \lim_{n\to+\infty} H_p(n)$$

$$NegH_p^R = \sum_{p=-\infty}^{p=+\infty} |\phi(p)|^2 Ln\left[|\phi(p)|^2\right] = -Ln\left(\frac{4\pi\hbar e^{2(1-\gamma)}}{L}\right) = -\lim_{n\to+\infty} H_p(n)$$

$$\overline{H}_p^R = -\sum_{p=-\infty}^{p=+\infty} \left[1 - |\phi(p)|^2\right] Ln\left[1 - |\phi(p)|^2\right]$$

$$H_p^M = -\sum_{p=-\infty}^{p=+\infty} i\left[1 - |\phi(p)|^2\right] Ln\left\{i\left[1 - |\phi(p)|^2\right]\right\}$$

$$H_p^C = -\sum_{p=-\infty}^{p=+\infty} Pc(p) Ln[Pc(p)] = -\sum_{p=-\infty}^{p=+\infty} 1 \times Ln[1] = -\sum_{p=-\infty}^{p=+\infty} (1 \times 0) = 0 = H_p^R + NegH_p^R$$

That means also and most importantly, for the wavefunction momentum distribution and in the probability set and universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$, we have complete order, no chaos, no ignorance, no uncertainty, no disorder, no randomness, no information loss or gain but a conservation of information, and no unpredictability since all measurements are completely and perfectly deterministic ($Pc(p) = 1$ and $H_p^C = 0$).

The quantum mechanical entropic uncertainty principle states that for $x_0 p_0 = \hbar$ then:

$H_x^R + H_p^R(n) \geq Ln(e\pi) \cong 2.144729886 \ldots$ nats, (base e in Ln gives the "natural units" nat).

For $x_0 p_0 = \hbar$, the sum of the position and momentum entropies yields:

$H_x^R + H_p^R(\infty) = Ln(8\pi e^{1-2\gamma}) \cong 3.069740098 \ldots$ nats, (base e in Ln gives the "natural units" nat).

which satisfies the quantum entropic uncertainty principle.

The following figures (**Figures 38–51**) illustrate all the computations done above.

4. Conclusion and perspectives

In the current research work, the original extended model of eight axioms (*EKA*) of A. N. Kolmogorov was connected and applied to the infinite potential well problem in quantum mechanics theory. Thus, a tight link between quantum mechanics and the novel paradigm (*CPP*) was achieved. Consequently, the model of "Complex Probability" was more developed beyond the scope of my 19 previous research works on this topic.

Additionally, as it was proved and verified in the novel model, before the beginning of the random phenomenon simulation and at its end we have the chaotic factor (*Chf* and *MChf*) is zero and the degree of our knowledge (*DOK*) is one since the stochastic fluctuations and effects have either not started yet or they have terminated

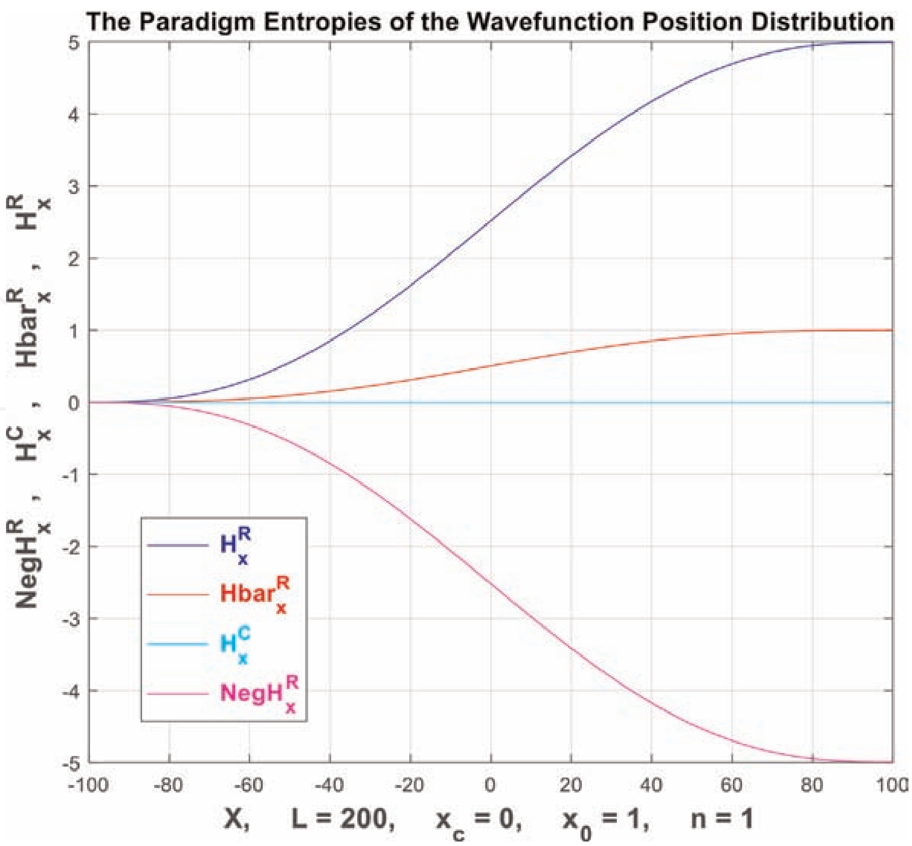

Figure 38.
The graphs of $H_x^R, \overline{H}_x^R, H_x^C, NegH_x^R$ *as functions of* X *for* $n = 1$.

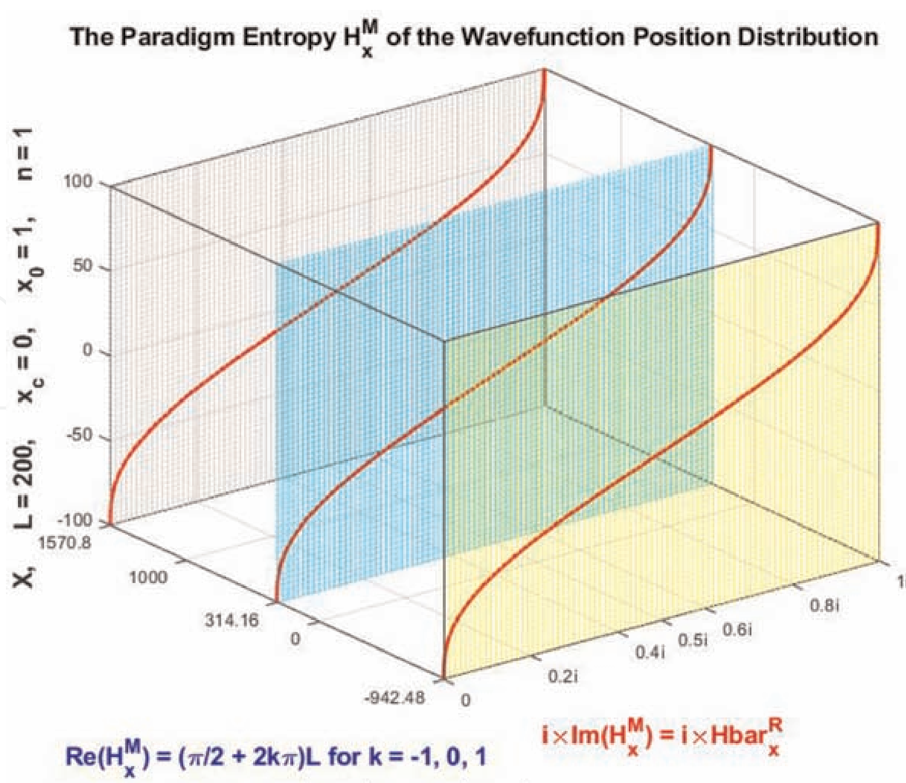

Figure 39.
The graph of $H_x^M = \text{Re}\left(H_x^M\right) + i\text{Im}\left(H_x^M\right)$ *in red as functions of* X *for* $n = 1$ *and for* $k = -1, 0, 1$ *in the planes in yellow, in cyan, and in light gray, respectively.*

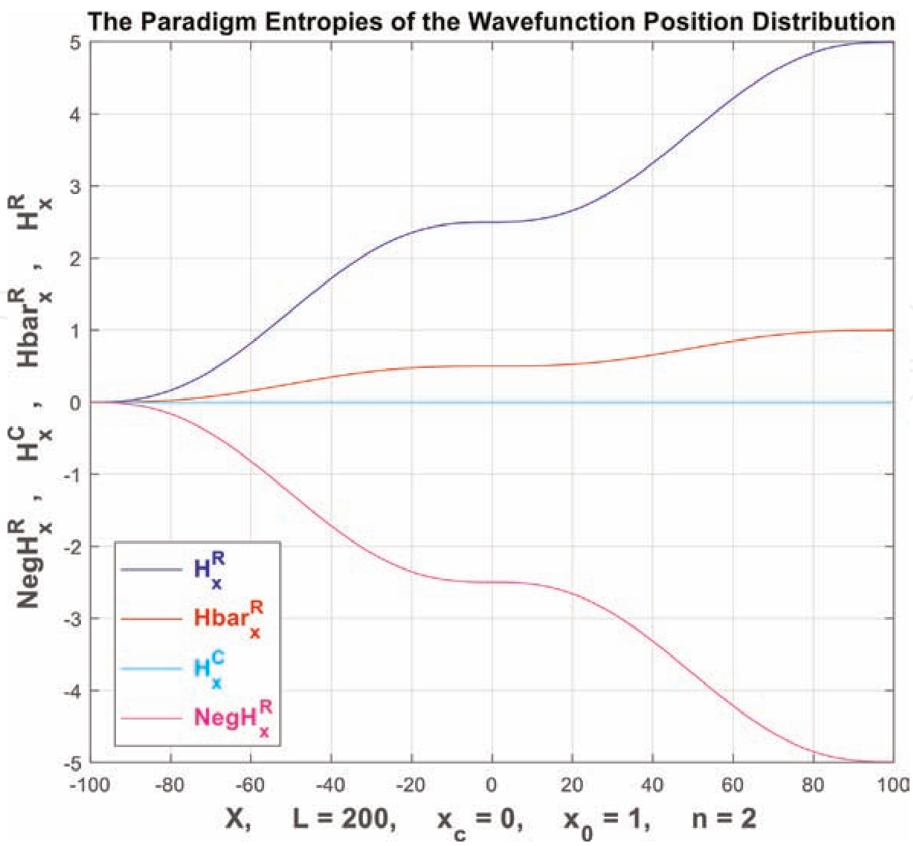

Figure 40.
The graphs of H_x^R,$\overline{H}_x^R$,H_x^C,$NegH_x^R$ *as functions of* X *for* $n = 2$.

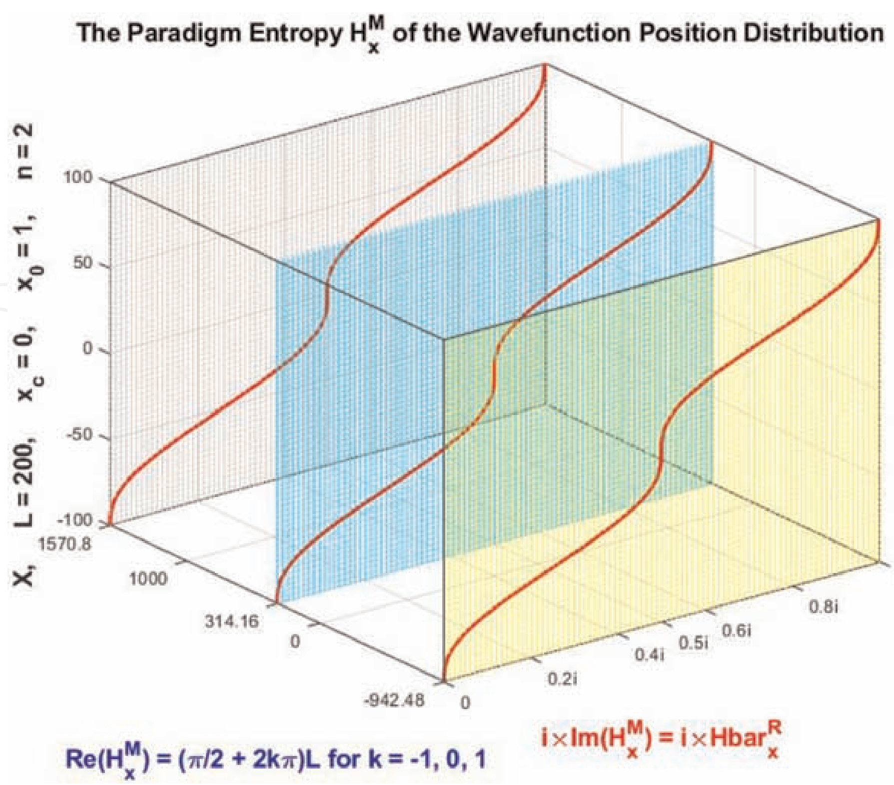

Figure 41.
The graph of $H_x^M = \text{Re}\left(H_x^M\right) + i\text{Im}\left(H_x^M\right)$ *in red as functions of* X *for* $n = 2$ *and for* $k = -1,0,1$ *in the planes in yellow, in cyan, and in light gray, respectively.*

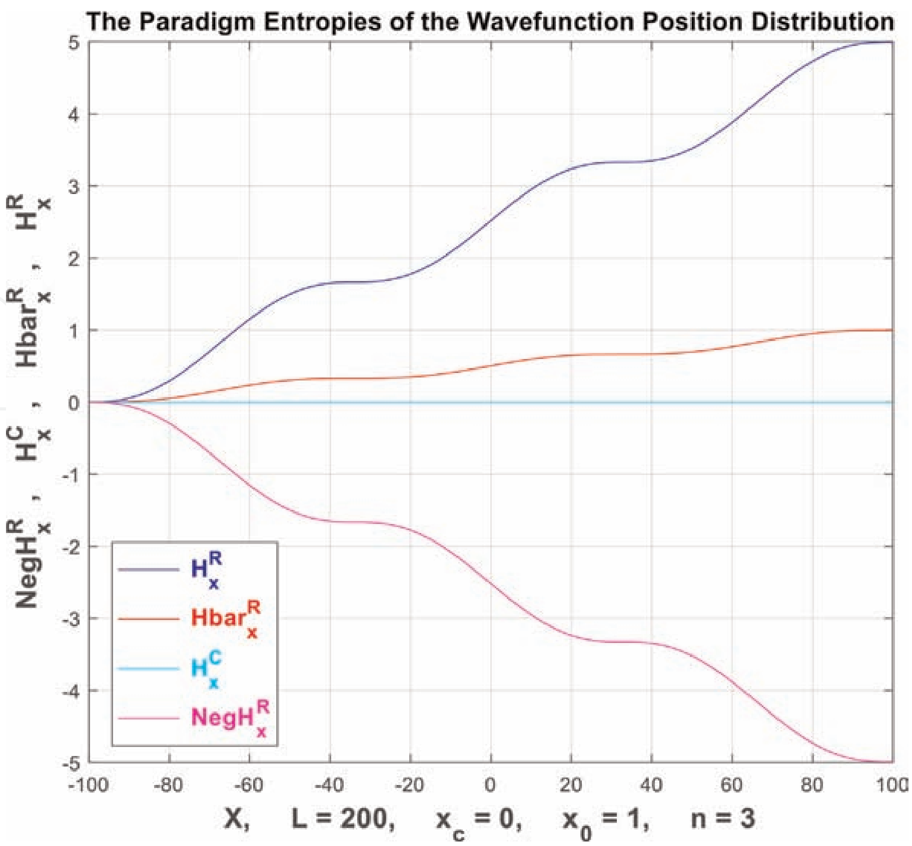

Figure 42.
The graphs of $H_x^R, \overline{H}_x^R, H_x^C, NegH_x^R$ *as functions of* X *for* $n = 3$.

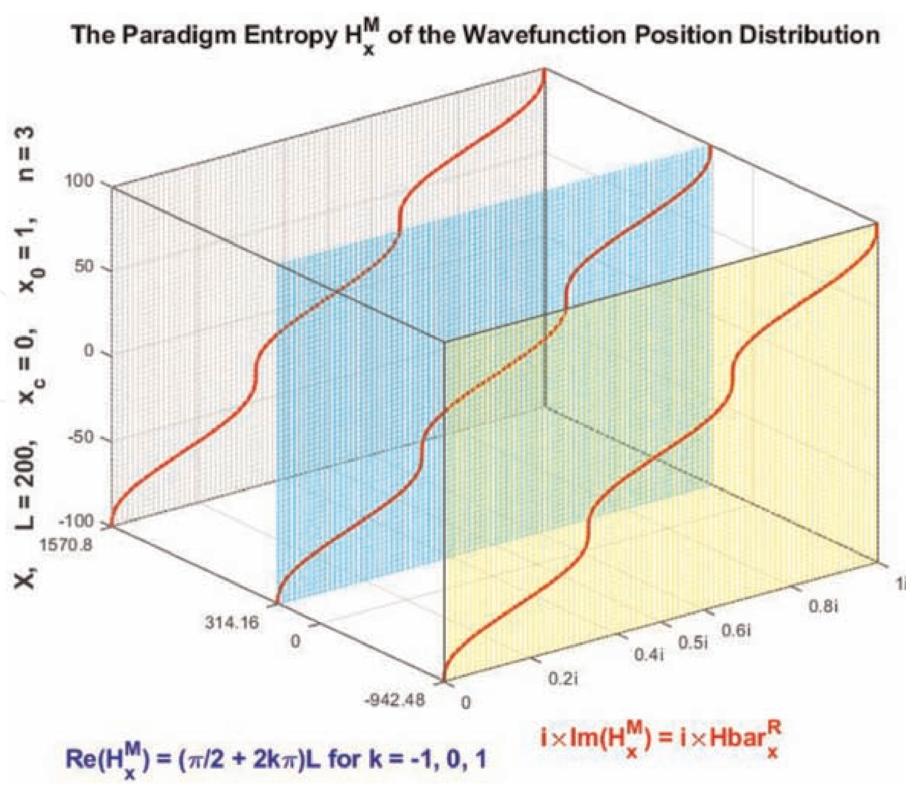

Figure 43.
The graph of $H_x^M = \mathrm{Re}\left(H_x^M\right) + i\mathrm{Im}\left(H_x^M\right)$ *in red as functions of* X *for* $n = 3$ *and for* $k = -1,0,1$ *in the planes in yellow, in cyan, and in light gray, respectively.*

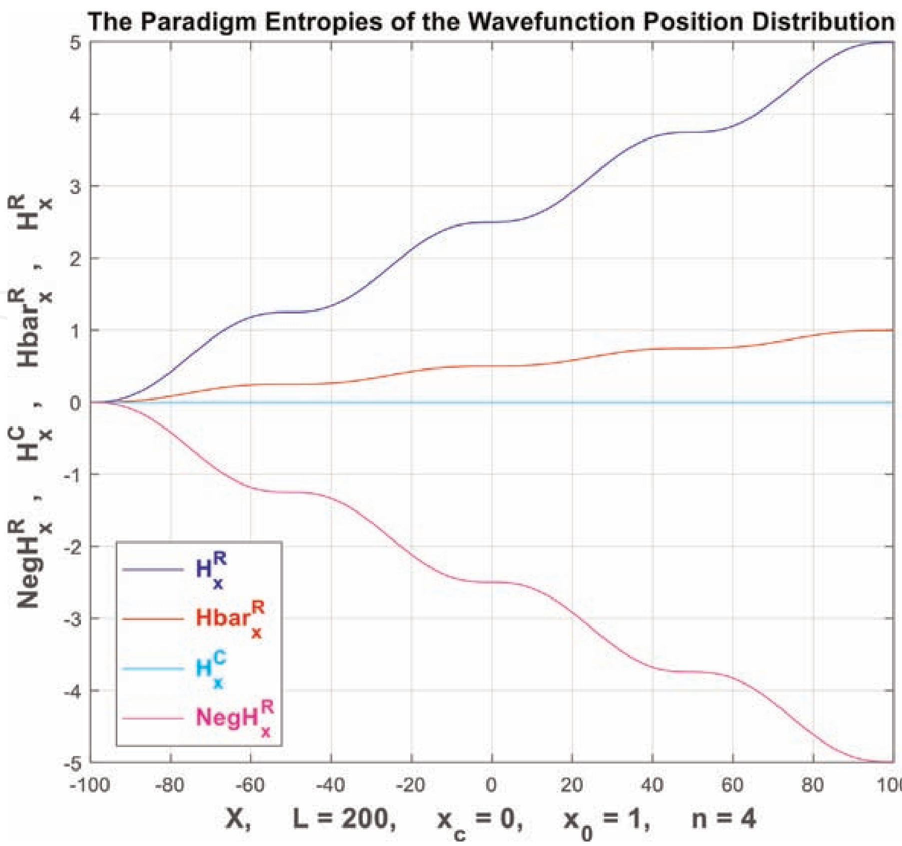

Figure 44.
The graphs of $H_x^R, \overline{H}_x^R, H_x^C, NegH_x^R$ *as functions of* X *for* $n = 4$.

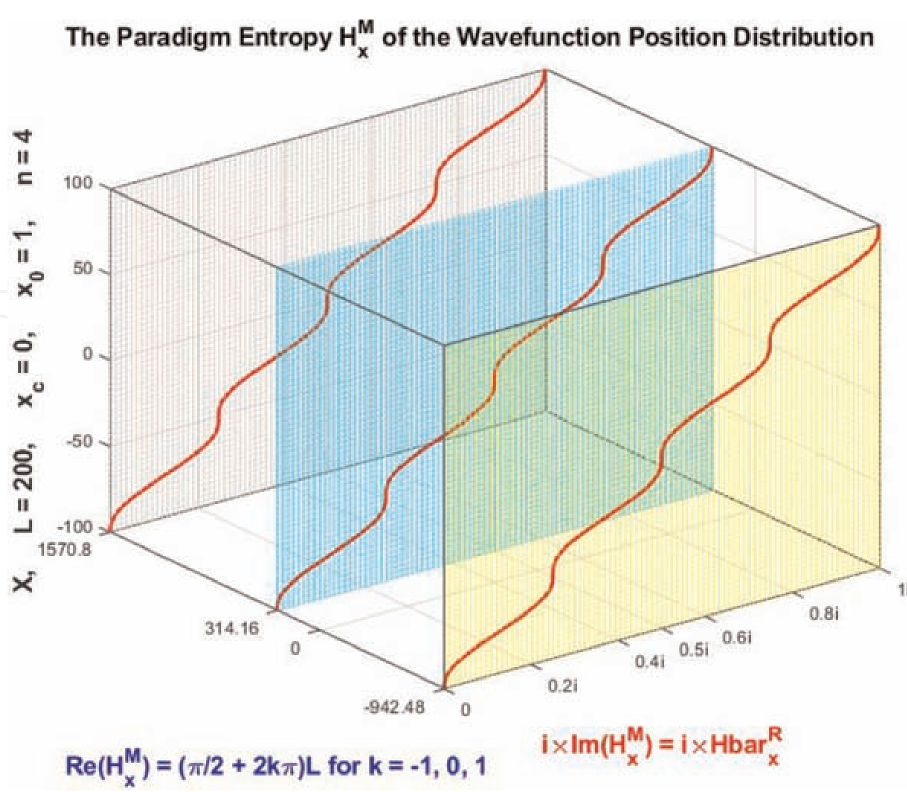

Figure 45.
The graph of $H_x^M = \text{Re}\left(H_x^M\right) + i\text{Im}\left(H_x^M\right)$ *in red as functions of* X *for* $n = 4$ *and for* $k = -1,0,1$ *in the planes in yellow, in cyan, and in light gray, respectively.*

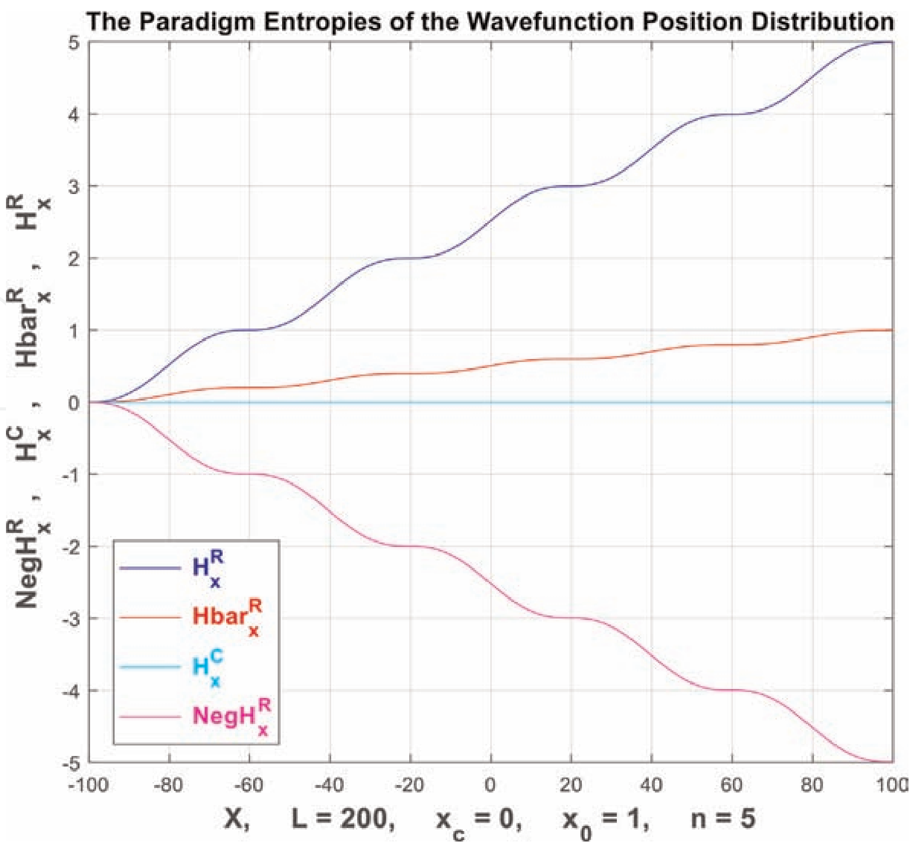

Figure 46.
The graphs of $H_x^R, \overline{H}_x^R, H_x^C, NegH_x^R$ *as functions of* X *for* $n = 5$.

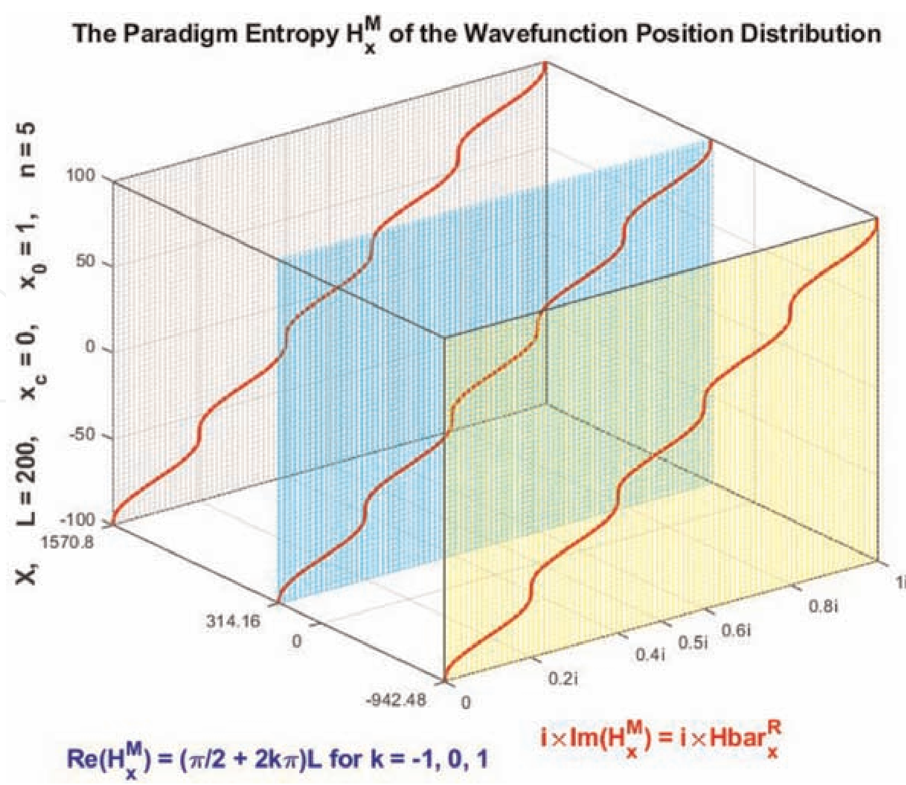

Figure 47.
The graph of $H_x^M = \text{Re}\left(H_x^M\right) + i\text{Im}\left(H_x^M\right)$ *in red as functions of* X *for* $n = 5$ *and for* $k = -1,0,1$ *in the planes in yellow, in cyan, and in light gray, respectively.*

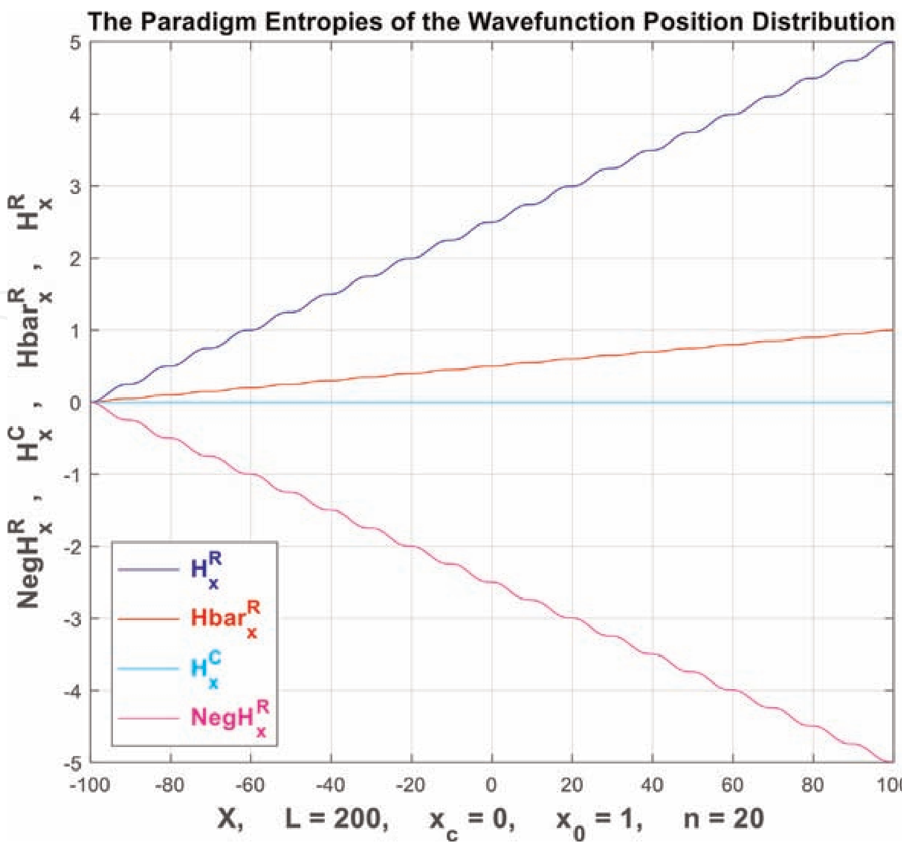

Figure 48.
The graphs of $H_x^R, \overline{H}_x^R, H_x^C, NegH_x^R$ as functions of X for $n = 20$.

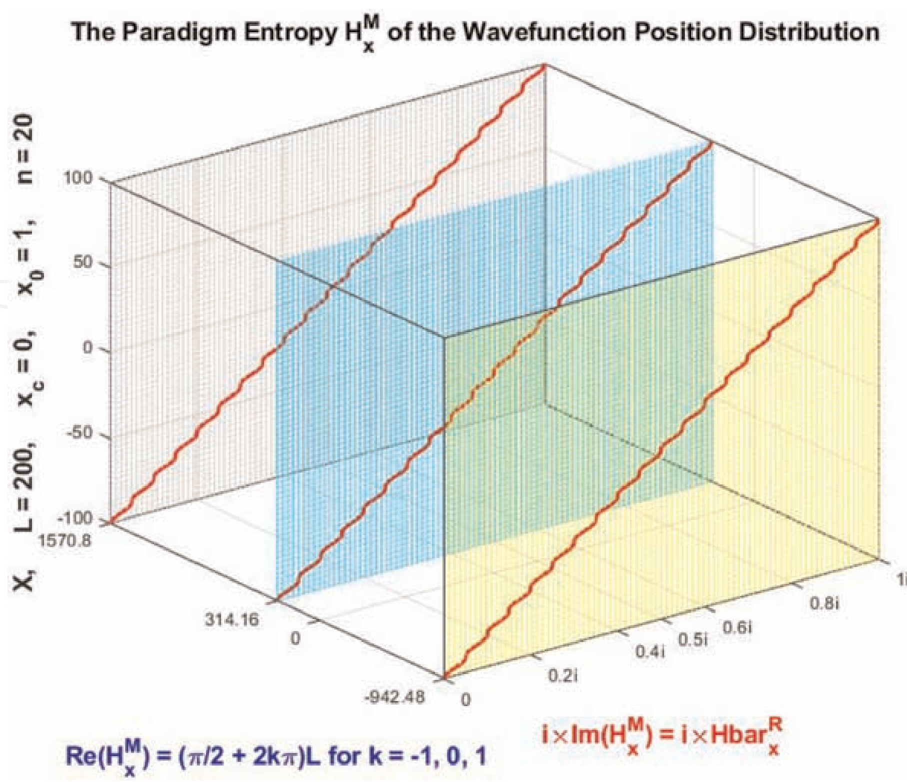

Figure 49.
The graph of $H_x^M = \text{Re}\left(H_x^M\right) + i\text{Im}\left(H_x^M\right)$ in red as functions of X for $n = 20$ and for $k = -1, 0, 1$ in the planes in yellow, in cyan, and in light gray, respectively.

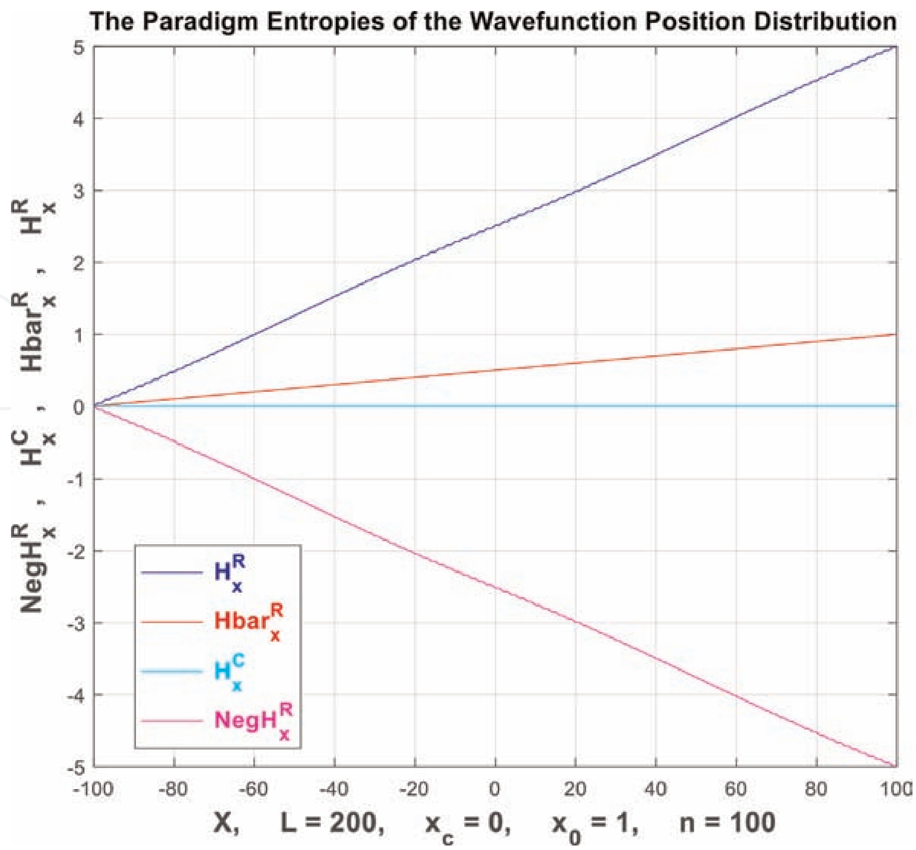

Figure 50.
The graphs of $H_x^R, \overline{H}_x^R, H_x^C, NegH_x^R$ *as functions of* X *for* $n = 100$.

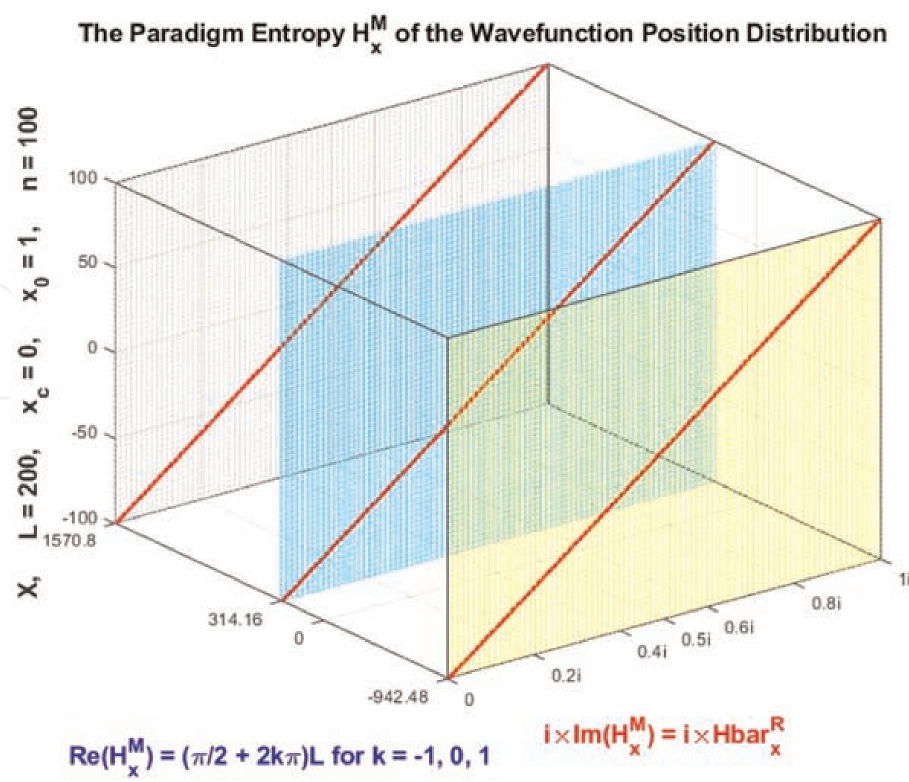

Figure 51.
The graph of $H_x^M = \text{Re}\left(H_x^M\right) + i\text{Im}\left(H_x^M\right)$ *in red as functions of* X *for* $n = 100$ *and for* $k = -1,0,1$ *in the planes in yellow, in cyan, and in light gray, respectively.*

and finished their task on the probabilistic phenomenon. During the execution of the nondeterministic phenomenon and experiment we also have: $0.5 \leq DOK < 1$, $-0.5 \leq Chf < 0$, and $0 < MChf \leq 0.5$. We can see that during this entire process we have incessantly and continually $Pc^2 = DOK - Chf = DOK + MChf = 1 = Pc$, that means that the simulation which behaved randomly and stochastically in the real set and universe $\mathcal{R}$ is now certain and deterministic in the complex probability set and universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$, and this after adding to the random experiment executed in the real universe $\mathcal{R}$ the contributions of the imaginary set and universe $\mathcal{M}$ and hence after eliminating and subtracting the chaotic factor from the degree of our knowledge. Furthermore, the real, imaginary, complex, and deterministic probabilities and that correspond to each value of the momentum random variable P have been determined in the three probabilities sets and universes which are $\mathcal{R}$, $\mathcal{M}$, and $\mathcal{C}$ by P_r, P_m, Z and Pc respectively. Consequently, at each value of P, the novel quantum mechanics and CPP parameters P_r, P_m, P_m/i, DOK, Chf, $MChf$, Pc, and Z are surely and perfectly predicted in the complex probabilities set and universe $\mathcal{C}$ with Pc maintained equal to one permanently and repeatedly.

In addition, referring to all these obtained graphs and executed simulations throughout the whole research work, we are able to quantify and visualize both the system chaos and stochastic effects and influences (expressed and materialized by Chf and $MChf$) and the certain knowledge (expressed and materialized by DOK and Pc) of the new paradigm. This is without any doubt very fruitful, wonderful, and fascinating and proves and reveals once again the advantages of extending A. N. Kolmogorov's five axioms of probability and hence the novelty and benefits of my inventive and original model in the fields of prognostics, applied mathematics, and quantum mechanics that can be called verily: "The Complex Probability Paradigm".

As prospective research, we aim to develop the novel prognostic paradigm conceived and implement it in a large set of nondeterministic phenomena in quantum mechanics.

Author details

Abdo Abou Jaoudé
Department of Mathematics and Statistics, Faculty of Natural and Applied Sciences, Notre Dame University-Louaize, Lebanon

*Address all correspondence to: abdoaj@idm.net.lb

References

[1] Wikipedia, the free encyclopedia, Quantum Mechanics. Available from: https://en.wikipedia.org/

[2] Wikipedia, the free encyclopedia, Particle in a Box. Available from: https://en.wikipedia.org/

[3] Abou Jaoude A, El-Tawil K, Kadry S. Prediction in complex dimension using Kolmogorov's set of axioms. Journal of Mathematics and Statistics, Science Publications. 2010;**6**(2):116-124

[4] Abou Jaoude A. The complex statistics paradigm and the law of large numbers. Journal of Mathematics and Statistics, Science Publications. 2013;**9**(4):289-304

[5] Abou Jaoude A. The theory of complex probability and the first order reliability method. Journal of Mathematics and Statistics, Science Publications. 2013;**9**(4):310-324

[6] Abou Jaoude A. Complex probability theory and prognostic. Journal of Mathematics and Statistics, Science Publications. 2014;**10**(1):1-24

[7] Abou Jaoude A. The complex probability paradigm and analytic linear prognostic for vehicle suspension systems. American Journal of Engineering and Applied Sciences, Science Publications. 2015;**8**(1):147-175

[8] Abou Jaoude A. The paradigm of complex probability and the Brownian motion. Systems Science and Control Engineering, Taylor and Francis Publishers. 2015;**3**(1):478-503

[9] Abou Jaoude A. The paradigm of complex probability and Chebyshev's inequality. Systems Science and Control Engineering, Taylor and Francis Publishers. 2016;**4**(1):99-137

[10] Abou Jaoude A. The paradigm of complex probability and analytic nonlinear prognostic for vehicle suspension systems. Systems Science and Control Engineering, Taylor and Francis Publishers. 2016;**4**(1):99-137

[11] Abou Jaoude A. The paradigm of complex probability and analytic linear prognostic for unburied petrochemical pipelines. Systems Science and Control Engineering, Taylor and Francis Publishers. 2017;**5**(1):178-214

[12] Abou Jaoude A. The paradigm of complex probability and Claude Shannon's information theory. Systems Science and Control Engineering, Taylor and Francis Publishers. 2017;**5**(1):380-425

[13] Abou Jaoude A. The paradigm of complex probability and analytic nonlinear prognostic for unburied petrochemical pipelines. Systems Science and Control Engineering, Taylor and Francis Publishers. 2017;**5**(1):495-534

[14] Abou Jaoude A. The paradigm of complex probability and Ludwig Boltzmann's entropy. Systems Science and Control Engineering, Taylor and Francis Publishers. 2018;**6**(1):108-149

[15] Abou Jaoude A. The paradigm of complex probability and Monte Carlo methods. Systems Science and Control Engineering, Taylor and Francis Publishers. 2019;**7**(1):407-451

[16] Abou Jaoude A. Analytic prognostic in the linear damage case applied to buried petrochemical pipelines and the complex probability paradigm. Fault

Detection, Diagnosis and Prognosis, IntechOpen. 2020;**1**(5):65-103. DOI: 10.5772/intechopen.90157

[17] Abou Jaoude A. The Monte Carlo techniques and the complex probability paradigm. Forecasting in Mathematics - Recent Advances, New Perspectives and Applications, IntechOpen. 2020;**1**(1): 1-29. DOI: 10.5772/intechopen.93048

[18] Abou Jaoude A. The Paradigm of Complex Probability and Prognostic Using FORM. London Journal of Research in Science: Natural and Formal (LJRS), London Journals Press. 2020; **20**(4):1-65. DOI: 10.17472/LJRS

[19] Abou Jaoude A. The paradigm of complex probability and the central limit theorem. London Journal of Research in Science: Natural and Formal (LJRS), London Journals Press. 2020;**20**(5):1-57. DOI: 10.17472/LJRS

[20] Abou Jaoude A. The Paradigm of Complex Probability and Thomas Bayes' Theorem. The Monte Carlo Methods - Recent Advances, New Perspectives and Applications. IntechOpen. 2021. DOI: 10.5772/intechopen.98340

[21] Abou Jaoude A. The paradigm of complex probability and Isaac Newton's classical mechanics: On the foundation of statistical physics. The Monte Carlo Methods - Recent Advances, New Perspectives and Applications. IntechOpen. 2021. DOI: 10.5772/intechopen.98341

[22] Abou Jaoude A. The Computer Simulation of Monté Carlo Methods and Random Phenomena. United Kingdom: Cambridge Scholars Publishing; 2019

[23] Abou Jaoude A. The Analysis of Selected Algorithms for the Stochastic Paradigm. United Kingdom: Cambridge Scholars Publishing; 2019

[24] Abou Jaoude A. The Analysis of Selected Algorithms for the Statistical Paradigm. Vol. 1. The Republic of Moldova: Generis Publishing; 2021

[25] Abou Jaoude A. The Analysis of Selected Algorithms for the Statistical Paradigm. Vol. 2. The Republic of Moldova: Generis Publishing; 2021

[26] Abou Jaoude A. Forecasting in Mathematics - Recent Advances, New Perspectives and Applications. London. United Kingdom: IntechOpen; 2021

[27] Abou Jaoude A. The Monte Carlo Methods - Recent Advances, New Perspectives and Applications. London. United Kingdom: IntechOpen; 2022

[28] Abou Jaoude A. Ph.D. Thesis in Applied Mathematics: Numerical Methods and Algorithms for Applied Mathematicians. Spain: Bircham International University; 2004. Available from: http:// www.bircham.edu

[29] Abou Jaoude A. Ph.D. Thesis in Computer Science: Computer Simulation of Monté Carlo Methods and Random Phenomena. Spain: Bircham International University; 2005. Available from: http:// www.bircham.edu

[30] Abou Jaoude A. Ph.D. Thesis in Applied Statistics and Probability: Analysis and Algorithms for the Statistical and Stochastic Paradigm. Spain: Bircham International University; 2007. Available from: http://www.bircha m.edu