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Affect graphing: leveraging graphical representations in the study of students' affect in mathematics

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Abstract
Affect (e.g., beliefs, attitudes, emotions) plays a crucial role in mathematics learning, but reliance on verbal and written responses (from surveys, interviews, etc.) limits students' expression of their affective states. As a complement to existing methods that rely on verbal reports, we explore how graphing can be used to study affect during mathematical experiences. We analyze three studies that used graphing to represent, stimulate recall, and reflect on affect. In each, students were asked to draw their perception of an affective construct, such as confidence or intensity of emotion, against time. The studies differed in participant populations, target affect, timescales of participant experience, and structural features of the graphs. The affordances of graphing include reduced dependence on verbal data, temporal ordering of participants' recollections, explicit representation of change over time, and the creation of objects (the graph) for discussion. These studies as examples show that well-structured graphing can productively supplement existing methods for studying affect in mathematics education, as a different medium through which students can communicate their experience.

Keywords Affect · Research methods · Confidence · Emotion · Transition-to-proof · Preservice elementary mathematics teachers

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The central role of affect in learning mathematics has become increasingly clear. Historically, affect has encompassed aspects of human experience that involve *feeling* (McLeod, 1988), such as beliefs, attitudes, emotions, motivation, and engagement (Grootenboer & Marshman, 2016; McLeod, 1992; Middleton et al., 2017). Work to understand students' affective experience is important in all school subjects but especially so in mathematics, where students often have negative, even toxic, experiences (Boaler, 2015; Richardson & Suinn, 1972). Success in mathematics is more likely than other subjects to be seen in binary terms ("some can learn; most cannot") and to depend on fixed, genetically determined ability (Boaler, 2015).

The prevalent fixed ability frame makes the recent work in psychology, neuroscience, and mathematics education demonstrating how intelligence and mathematical ability depend on effort and growth (Dweck, 2006; Boaler, 2015) all the more striking and relevant for mathematics educators. Consistent with this view, mathematical thinking often becomes problem solving (Schoenfeld, 1985). When working on mathematical tasks where students do not have go-to solution methods, impasses are inevitable. Depending on prior experience, impasses may either feel normal and familiar or be profoundly upsetting and anxiety-producing. Even among students who have experienced substantial success in mathematics, coping with real problems, impasses, and uncertainty can generate a wide range of affective responses.

A range of methods exists for studying affect, from survey-based quantitative measures (Beswick, 2006; Fennema & Sherman, 1976; Richardson & Suinn, 1972) to qualitative measures using observation (Ingram, 2007; Walter & Hart, 2009), interviews (Hannula, 2002; McDonough & Sullivan, 2014; Op't Eynde et al., 2006), and narrative writing (Di Martino & Zan, 2011; Liljedahl, 2004). These methods assume that students can recognize and/or verbally articulate their feelings, whether in response to prescribed choices (surveys) or open-ended responses to prompts (interviews, journals). But not all aspects of affect are consciously accessible, and students' capacity to communicate their experience in words varies, so methods that rely solely upon verbalization contain inherent limitations. Drawing to represent affect offers an alternative that is less reliant on words. When students graph some aspect of their affect (e.g., Anderson, 2005; Ingram, 2011; McLeod et al., 1990; Smith & Star, 2007), they depict or locate their experience on axes representing time and intensity of the affective construct.

The purpose of this work is to explore what can be learned about affective phenomena via graphing, as a tool for research and teaching in mathematics education. We first review existing methods of studying affect for their strengths and weaknesses and argue that graphing responds to some of these challenges. We then present three studies that used affect graphing (viz., Grant & Levin, 2020; Satyam, 2020; Smith et al., 2017) to illustrate the varied ways in which graphical tools have been used in data collection and analysis. In discussing the affordances of affect graphing across the three cases, we close with guidelines on how to use graphing in alignment with one's research or pedagogical goals, identifying important dimensions to consider in different research and pedagogical contexts. We do not argue for abandoning existing methods based on spoken or written reports but rather to illustrate the productive ways that graphical tools can complement these approaches.

1 Traditional approaches to studying affect in mathematics

Our conceptualization of affect comes from research on mathematical problem-solving: McLeod (1988) characterized affect as having dimensions of magnitude, direction, duration, level of awareness, and level of control. Such dimensions were useful for distinguishing beliefs, attitudes, and emotions as constructs. A more recent perspective is to distinguish between affect that is *trait-like* versus *state-like* (Middleton et al., 2017). Affective traits are seen as long in duration, stable, and resistant to change (e.g., beliefs, attitudes), whereas affective states are cast as short-lived, variable, intense, and rapidly shifting (e.g., emotions). In using this terminology, we do not claim that affective constructs such as beliefs and attitudes are actually traits (and thus fixed). Rather, their long duration and stability suggest a deep-rooted nature similar in structure to a trait, when contrasted with short-lived affect such as emotion. This difference in conceptualization is useful and has methodological consequences: Quantitative methods tend to align with the assumption that affect is trait-like, as qualitative methods are more amenable to the variable and dynamic character of affect as a changing and context-dependent state. We briefly review popular quantitative and qualitative methods used thus far to study student affect, to discuss their affordances and constraints.

1.1 Quantitative methods

Early work on affect in mathematics education largely applied quantitative approaches from psychology. Researchers used large-scale surveys/questionnaires to measure student attitudes and anxiety in regard to mathematics (e.g., Aiken, 1970; Ohlson & Mein, 1977) and correlated them with measures of achievement or engagement (Middleton et al., 2017). Widely used instruments included the *Mathematics Anxiety Rating Scale* (MARS) (Richardson & Suinn, 1972) and *Mathematics Attitude Scales* (Fennema & Sherman, 1976). Subsequently, survey methods have been applied to study beliefs about mathematics learning and teaching (Thompson, 1992).

The use of surveys, and much of quantitative methods in general, assumes affect has a trait-like structure. Surveys are composed of numerous fixed, forced-choice items that ask for Likert-scale or true-false responses. Their use presumes that the individual's affect is stable, in that it can be assessed at a singular point in time through multiple items. Survey structure affords the collection of large data sets with little expense. The format of questions and response choices may reduce students' engagement in communicating their experience. Data analysis of survey results has involved correlations, item analyses, factor analyses, and other statistical techniques to measure latent variables, develop hypotheses about the relationships and interactions between variables, and/or confirm such hypotheses (Middleton et al., 2017).

More recent work of *experience sampling* has expanded quantitative methods to assess affective states more locally, both in time and specific activity. In studies using experience sampling, students received pagers and reported their engagement whenever their pagers beeped (Leder & Forgasz, 2002; Schiefele & Csikszentmihalyi, 1995; Uekawa et al., 2007). Fundamental to this research is the expectation that students' affect changes over time and activity, so affect is not a stable trait. Experience sampling minimizes the duration between the experience and reporting, but it may also interrupt the experience itself and thus influence the affect in question.

1.2 Qualitative methods

Spurred by long-held concerns about the validity of quantitative data in the study of affect (Batchelor et al., 2019; McLeod, 1988), researchers have explored qualitative methods which allow more for the study of affective states. Such studies typically involve fewer students and are more labor-intensive but have collected richer and more extensive student data. Some common methods are interviews, narrative writing, and direct observation.

Interviews pose a series of questions, here around a central affective focus. In semi-structured interviews, researchers are free to follow up and explore or clarify students' responses. Task-based interviews are useful for engaging students in activities that produce strong affective states and exploring this affect afterward (Goldin, 1997; Maher & Sigley, 2020). Given the challenges of studying emotion, interviews allow researchers to probe as needed about how certain situations made students feel (e.g., Hannula, 2002; Op't Eynde et al., 2006). Like surveys, interviews rely on self-report and a student's understanding of the questions. Unlike surveys, researchers and students have the opportunity to negotiate the meaning of questions. Interviews can however be susceptible to the issue of reactivity (to the interviewer), where students may (knowingly or not) alter their behavior or responses in the presence of another person (Brown, 2015; Roth & Middleton, 2006).

In narrative writing (e.g., journaling), students write about their experience in response to certain prompts (e.g., Hawera, 2004; Liljedahl, 2004; Wilson & Thornton, 2005, 2006). Like other methods, narratives are useful for collecting students' self-reported but here long-form information about their affect. Di Martino and Zan (2011) asked students to write an essay about their relationship with math and analyzed the narratives for emotional dispositions (trait-like), defined through "like/dislike" statements. Written prompts query students about specific moments in the past or present, such as a time they enjoyed (or disliked) mathematics (e.g., O'Keeffe & Paige, 2020) or how they feel after a class lesson—acknowledging the potentially variable and context-dependent nature of affective states. Fine-tuned prompts can further pinpoint affective states of interest.

Direct observation is helpful in understanding the environment and conditions that shape students' affect (Middleton et al., 2017; Walter & Hart, 2009), though it requires the observers' *interpretation* of students' affect. Observations gather information about students' affective states within the environments where they may arise (e.g., in classrooms) and in real time as they occur. Because data are generated by observers, direct observation does not depend on students' self-reports. But because interpreting the meaning of students' actions, postures, and gestures is often difficult, direct observation is primarily used to validate or contextualize data on students' affect gathered from other methods (e.g., surveys and interviews).

In summary, where quantitative measures turn affective experience to numerical data for statistical analyses, qualitative measures produce richer data that can be more fully descriptive but whose analyses are more interpretative. Within qualitative measures, interviews and narratives provide space for students to interpret questions orally and in writing, respectively. While direct observation is less reliant on verbal communication, the lack of self-report means there is no in-built participant check necessarily, for how what was observed compares to that individual student's baseline affect. Qualitative measures still then share basic limitations with surveys: students' awareness of their affect and ability to describe it in words (Schuck & Grootenboer, 2000), whether it be recognizing their affect through given wordings in a survey or finding the words to express their feelings when prompted. As a subset of qualitative measures, we now delve into pictorial including graphical methods, to situate our work.

1.3 Graphical methods

In response to the challenges of verbalization of affective experiences, researchers have explored methods that reduce the dependence on words, asking students to draw maps or pictures to represent their affective states (Gómez-Chacón, 2000; McDermott & Tchoshanov, 2014). Some have asked participants to draw with scales (i.e., graph) to represent changes in affect over time (Anderson, 2005; Ingram, 2011; McLeod et al., 1990; Smith & Star, 2007; Voogt, 2021). The focus on location over time presumes a view of affect as a potentially variable state. The duration of time represented on the x-axis in these studies varied from single problem-solving episodes (McLeod et al., 1990) to years (Anderson, 2005). Anderson (2005) and Ingram (2011) used personal journey graphs to examine teachers' confidence in implementing a reform curriculum and secondary students' feelings about mathematics, respectively. We extend previous usages of graphing of disposition (Smith & Star, 2007) and emotion (McLeod et al., 1990).

To summarize the major approaches to studying affect, prior studies have often framed affect as a trait. Indeed, researchers have historically attended to affective traits for their importance in predicting future behavior as well as for methodological ease. Quantitative methods in particular largely assume that the latent variable to be measured is stable in nature, which aligns with the trait view of affect. But the need to account for affective states (McLeod, 1992) and the variation and dynamics in how they occur have led to increased use of qualitative methods. In particular, experience sampling methods, observation, interviews, and narrative writing have all been used to investigate affective states, but they rely on students' self-awareness, articulation, and verbal report. Observation, by contrast, is direct, but more distant and dependent on observers' interpretative judgment.

In our work, we explore affective states through graphing, as a complementary approach to address the above-mentioned limitations of existing qualitative methods. Affect graphing provides a different channel through which students can communicate their experience: While our graphing task is presented in words, students' primary response—to draw—is non-verbal. We examine emotion and confidence in particular as affective states, even though the latter has often been seen as trait-like. Presenting students with "open" graphical space to render their experience presumes their affect *can* change, so they are free to depict stability or instability over time as they choose. We offer three examples of using graphical methods, along with their study contexts, research goals, and the affective foci that framed their use.

2 What do we mean by "graphing" affect?

We first explain what we mean by graphing and how our use compares to traditional uses of graphs in mathematics as well as in research. Like traditional graphs on a Cartesian plane, our graphs have horizontal and vertical axes: two quantities are coordinated with each other. We refer to portions of the graph as quadrants in the traditional sense. Students draw a graph by drawing a line or marking Xs in the open space provided.

Our graphs differ from traditional mathematical graphs in important ways. First, we are interested in students' self-perceptions of their own experiences as their lived reality (Abakpa et al., 2017). The graphs produced in our studies are not presented as objective measures of affective experience. They are hand-drawn representations of a student's own perception of their affective response within a recalled experience. A graph of affect produced by biological tools, e.g., heartbeat monitors, would look different (Faust, 1992; Lyons & Beilock, 2012). Second, our affect graphs need not be continuous nor single-valued curves; drawing loops to signify confusion for example is meaningful. Third, heights in our affect graphs lack objective meaning. Because the graphs represent students' own measures of their affect, we cannot compare students in terms of differences in heights on their graphs. Similarly, distances are not necessarily uniform throughout (e.g., an inch on the horizontal axis [time] could represent different durations). Hand-drawn graphs may be tied to certain events and susceptible to memory biases, so time along the horizontal axis may be stretched and shrunk in places.

In summary, the graphs we describe are closer to sketches than traditional graphs. Where comparisons across different graphs are problematic, qualitative and local comparisons within a single graph are more meaningful. We can interpret students' choices to draw their graphs higher or lower relative to their previous points. In our examples, we focus on comparisons within individual students' affect graphs.

3 Affect graphing in three studies

The studies we present took place at three large Midwestern universities and focused on an affective experience of undergraduate students. Each was conducted by different subsets of this article's authors. The basic features of the studies are presented in Table 1.

Study 1 (Smith et al., 2017) investigated the challenges students experienced in a semester-long introduction to proof course. Study 2 (Satyam, 2018, 2020) used the same course setting to focus on students' emotions during proof construction tasks. Study 3 (Grant & Levin, 2020) examined prospective elementary teachers' confidence during a class discussion of a problematic division task in a mathematical content course on numbers and operations.

Studies 1 and 2 shared the same mathematical context but focused on different affect and at different timescales. In contrast, Study 3 illustrated a more pedagogical application of graphing with a different, sometimes non-Cartesian graphing space. For each, we describe (a) the research goals, setting, and data collected, (b) the structure of the graph, and (c) the novel information the graph afforded and then (d) assess the affordances of the specific graphical approach.

Table 1 Features of the three studies

Study	Course	Population	Graph measure	Temporal duration
1	Transition-to-proof	Math majors and minors	Confidence	Course (semester)
2	Transition-to-proof	Math majors and minors	Emotion	Task (15 min)
3	Numbers and operations	Prospective elementary teachers	Confidence	Class (50 min)

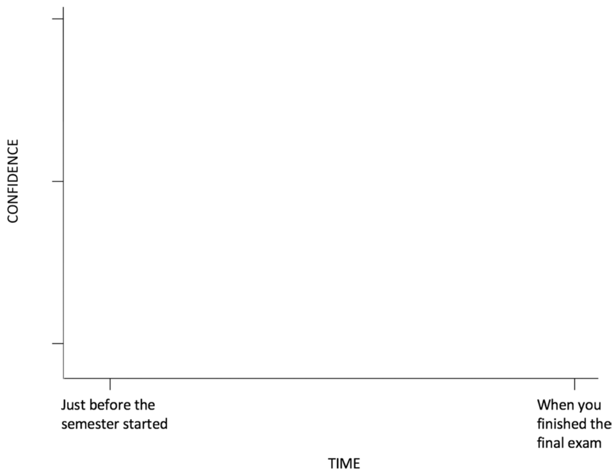
3.1 Characterizing students' transition-to-proof (Study 1)

The *transition-to-proof* project was undertaken to understand the dynamics of students' transition from computation to proof and argument in mathematics (Bae et al., 2018; Smith et al., 2017). Many undergraduates enter their upper-level mathematics coursework with little proof experience and see their ability in and enjoyment of mathematics in terms of speedy and accurate calculation. The project aims to understand the cognitive, affective, and social dimensions of students' adjustment to proof work. One project site was the transition-to-proof course taken by majors and minors between calculus and proof-based courses in analysis and algebra. The course focused on logic, proof methods, and basic set theory, real analysis, and number theory. Data collected included an extensive interview at the end of the course about how students saw the course, how they worked in and out of class, their struggles, and what they learned, as well as student work and course materials. The affect graphing activity was one component of the interview and was given toward the end, after extensive prior discussion occurred.

Our pilot work indicated that an array of cognitive/mathematical, affective, and social factors was involved in students' transition-to-proof. We developed the confidence graph activity to assess one important aspect of students' affective experience: how they felt their confidence changed over their semester-long work. We expected that "confidence" was a linguistically accessible way to assess students' self-efficacy in mathematical work (Bandura, 1997) and that some students would report low points or drops in their confidence because proof work embodied such a deep change in students' sense of doing mathematics.

As Fig. 1 shows, the vertical axis (confidence) provided three marks that interviewers introduced as low, middle, and high confidence. The horizontal axis (time) was minimally structured: The first time point ("just before the semester started") was included because pilot work showed that some students had heard stories about the course prior to the first week. The second point ("when you finished the final exam") was included because we interviewed students shortly after they completed the course. We decided against including other points to avoid imposing structure on students' experience, hoping students themselves would indicate important moments, either to help them construct their graph or to explain features of their graphs (e.g., rising and falling segments,

Fig. 1 The confidence graph presented to students



high and low points). As our procedure, we introduced the activity and described the graph's axes and asked students to graph their confidence. When the graph was complete (students typically needed 3–5 min), we asked students to explain why they drew the graph as they had.

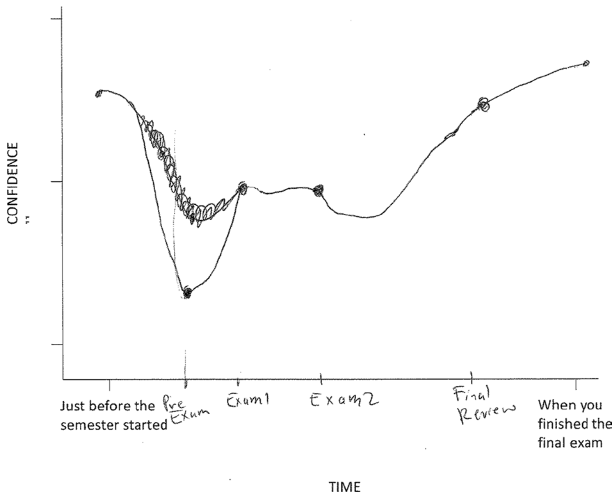
To illustrate how the confidence graph activity contributed to our research goals, we present the case of Kirk, a senior-year male mechanical engineering major pursuing a minor in mathematics. His confidence graph appears in Fig. 2. Kirk received the highest grade possible on the university's grading scale.

Like many other students, Kirk parsed his confidence in the course by exams, including a diagnostic mini-exam ("pre-exam"). He also located a "final review" time point before the final exam. As he later explained, the review period was pivotal for his understanding of content going into the final. As shown in Fig. 2, he corrected his graph around the mini-exam after deciding that his initial two segments did not accurately represent his confidence. Once he made that correction, he described his poor exam performance as "devastating."

Kirk's explanation of the middle portion of his graph was detailed and rich. He explained first that he considered but decided not to drop the course, despite his negative experience on the mini-exam. He judged success was possible if he stepped up his effort: "Okay, I can manage this, it'll have to work." His performance on exams 1 and 2 was considerably better but missing two lectures set him back again (indicated by the dip after exam 2 in Fig. 2). He responded by accessing outside resources that he had not before, spending considerable time in the university's mathematics learning center, which included peer-tutoring. Earlier, Kirk indicated that his time in the center was one key to his success in the course. The graph and his explanation supported and clarified his assessment: Access to the center helped Kirk recover his confidence after set-backs.

Even after this recovery, Kirk drew and explained that the final exam review period was pivotal. His work to "dig deeper" into the content he had missed had a strong positive effect: "Once I understood that, it was where a lot of the older stuff really started to click." When asked if the heights of his first and fifth ("final review") were equivalent, Kirk confirmed their accuracy, because he had recovered his initial confidence. The question, graph, and Kirk's response combined produced a clearer view of the dynamics

Fig. 2 Kirk's confidence graph



of his confidence. The graphical equality of two heights alone would not have explained how Kirk felt about his "recovery."

Kirk's case illustrates the productive contribution of the confidence graph activity in understanding students' experience in the transition-to-proof course. Because the graph named and ordered temporal points across the semester, the graph and subsequent discussion confirmed, expanded on, and clarified Kirk's responses to our interview questions. The graphing activity oriented students to describe their confidence over time that would have been tedious to do with interview questions, requiring multiple inquiries about their confidence at various points in time and how it changed. The graph also generated new content that had not surfaced earlier: Kirk had not described his mini-exam experience as "devastating" beforehand. More generally, the unstructured x-axis invited students to locate important events that they often had not earlier in the interview. The graphing task also forced decisions about magnitude—where to locate heights—where such characterizations were missing or less clear in students' prior verbal descriptions. As Kirk's case shows, decisions about magnitude could be reviewed and revised once magnitudes were explicitly represented. Overall, the confidence graph activity created a more open space for students to represent their experience as they saw fit compared to a series of interview questions. The resulting graph was both a student-constructed representation of their affective experience and a stimulus for richer and more complete verbal descriptions of other issues (e.g., students' use of learning resources).

3.2 Graphing emotion during proof work (Study 2)

This second example illustrates the productiveness of affect graphing in the same course and institution as the previous study but explored a different affective construct—students' emotions while proving (Satyam, 2018, 2020). Task-based interviews were conducted with students; they were given mathematical statements and asked to construct proofs in a period of 15 min. The interviewer observed students and afterwards asked them what emotions they experienced and to draw a graph of their emotions during the task and explain its features.

The emotion graphing task was inspired by McLeod et al.'s (1990) emotion graphs of problem-solving and the confidence graph activity used in Study 1 above (Smith et al., 2017). Though set broadly in the same course context, this study differs from the previous in multiple ways: the affective construct, structural features of the graph, duration of time involved, and that the students' experience was directly observed here. We use the work of one student, Timothy, to illustrate what was learned from observation compared to the construction and discussion of the graph. We first review his proof work and then discuss his emotion graph.

The task was to prove: *If n is an odd natural number, then n^2 - 1 is divisible by 8.* Timothy spent time making sense of the statement and the definitions of an odd integer and odd natural number. He became stuck at a certain step, $4(m^2+m)$; his silence and body language indicated he did not see that expression as divisible by 8. He then had the idea to show $m^2 + m$ is even, but then became stuck again, telling himself, "I know it is going to be even, it's just trying to prove it." He then saw a way using cases: "let m be even and then odd." This would prove that $m^2 + m$ would always be even and thus $4(m^2+m)$ would be divisible by 8.

Observation revealed the following about Timothy's experience. First, it indicated where Timothy had impasses and breakthroughs. There were four: the impasse over $4(m^2+m)$, the breakthrough that m^2+m must be even, the impasse about how to prove m^2+m was even, and the breakthrough on how to do so. Observation identified a count of these significant events and the time elapsed between each event. The observations also provided a general sense of the intensity of each impasse/breakthrough, as indicated by silence, body language, and the length endured. For example, Timothy often fell silent, was still, and did not write or talk to himself when stuck. These observable behaviors gave the interviewer a holistic sense of how the problem affected Timothy emotionally. As a result of observation, both the significant events in time (horizontal axis) and an estimate of the intensity of emotion (vertical axis) were known in advance of his graphing work.

The blank graph given to Timothy is shown in Fig. 3. The horizontal axis was time, and the vertical axis was positive/negative intensity of emotion.

The graph was structured similarly to the previously discussed confidence graph: The horizontal axis had only two temporal locations (when they started and finished working on the problem) because the objective was to learn *what* events influenced and shifted emotions. The vertical axis was minimally structured with only three emotion marks: positive, negative, and "zero" (communicated to students as one's resting state). There were parallels with Study 1; observation here played a similar role as the interview dialogue in Study 1, providing prior information on students' emotions. But there were important differences—the emotion graphing activity sought to clarify students' emotions that remained opaque from observation and had a narrower time frame (from a semester to 15 minutes). Like Study 1, students were also invited to textually annotate their graphs. They first drew their graphs and then talked the interviewer through it.

Timothy's emotion graph for a task (see Fig. 4) provided information that the observation had only suggested: (a) the felt impact of impasses and breakthroughs, (b) comparisons of this felt impact across his work, and (c) the reasons behind his shifts in emotion. We discuss how the graph allowed Timothy to convey a fuller sense of his experience while minimizing the interviewer's influence in the following ways.

First, the graph showed the dynamics of Timothy's emotions during his work, however small. Smaller peaks/troughs difficult to discern in the observational phase were clearly

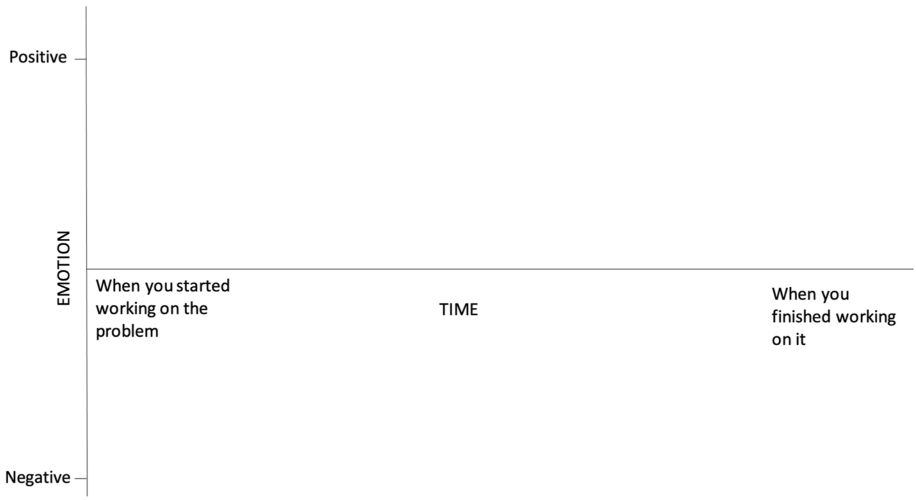


Fig. 3 The emotion graph presented to students

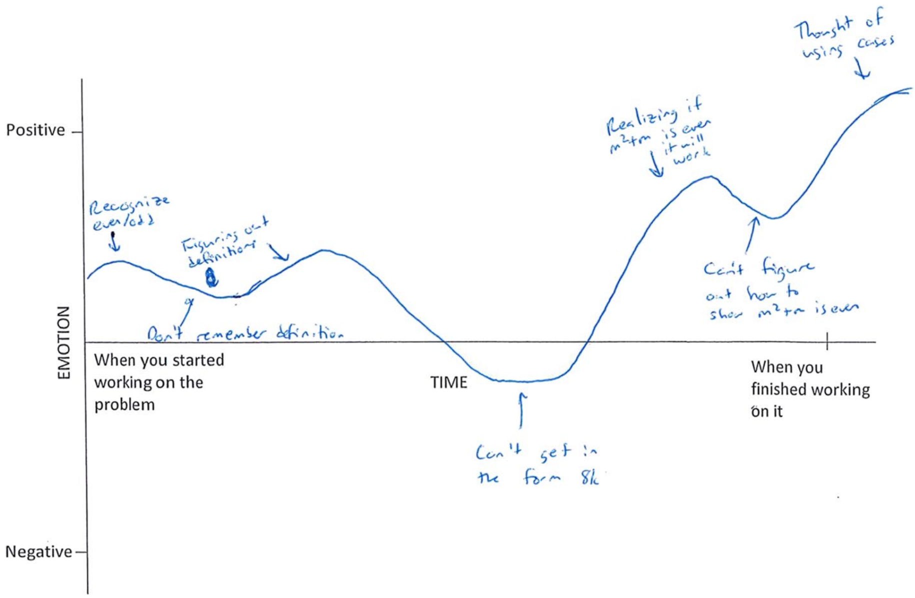


Fig. 4 Timothy's emotion graph for the divisibility task

revealed (e.g., the local peak "Recognizing even/odd" near the beginning). The temporal ordering of the rises/falls in his emotions was also apparent. While the observer could guess at which impasses/breakthroughs were strongest, graphing confirmed and refined these estimates.

Second, the graph supported local comparisons of the emotional impact of events. The vertical scale was crucial. For example, Timothy's emotions at "Don't remember the definition" near the start were less negative than at "Can't get in the form *8k*" near the end. To have these comparisons without the graph, we would have had to (a) interrupt the task or his narrative, potentially breaking the temporality of events or (b) ask a series of highly structured follow-up questions to compare events.

Third, the graph indicated an overall trend of Timothy's graph as positive. Throughout, his emotions after most annotated points were as or more positive than the previous, and his last annotation "thought of using cases" near the end of the graph was the highest point. A macroscopic view of his emotional experience became visible here, which was not clearly evident from observations nor his verbal re-telling.

Lastly, the graphing activity led Timothy to explain his shifts in emotion and leave annotations, clarifying issues that the observation left obscure. For example, for his annotation that he couldn't "remember the definition," he said, "that confused me," revealing the negative emotion was confusion. So while the graph communicated intensity of positive/negative emotions, he (and other students) mentioned specific emotions when talking through their experience, complementing what was observed and directly shown in the graph.

Overall, the graph helped students convey their emotional experience and provided clarity and detail that observations did not. Timothy and other students produced rich accounts of how they felt as they worked, often including multiple peaks and troughs. Had the clarification of emotion depended solely on interview questions, students' narratives would have been filtered through the social dynamics of dialogue (e.g., struggle in verbalizing the experience or worry about talking for too long or cutting out details). To produce the equivalent from only interview questions would require the interviewer to keep track of multiple events and ask corresponding questions comparing events, which would be taxing to the interviewer as well as confusing for a student to respond to. Combined with having control over their drawing, students had a proactive rather than reactive role in communicating their experience. This example combined the strengths of observation (as an outside report) with the strengths of an open self-report, through graphing and subsequent talk, for a more detailed picture of this student's affective experience.

3.3 Graphing confidence during a class discussion (Study 3)

Our third example focuses on understanding the affective experience of a different population of undergraduate students in one class discussion that was designed to surface and provide opportunities for reconciling mathematical doubts (Grant & Levin, 2020). This example shifts attention from solely research goals to research and pedagogy in mathematics teacher education. This study differed from the previous two in its research focus on qualities of a classroom teaching and learning experience, aggregate (rather than an individual's) affect, and content focus.

The study's objective concerned the authors' practice as teacher educators in a number and operations course for prospective elementary teachers (PTs). The authors sought to understand their students' experience in a lesson juxtaposing two contrasting answers to an arithmetic task. They had observed in prior iterations of the lesson that the dynamics of discussions had often influenced students' assessments of their understanding and oriented them to "go along" with their peers' judgment, even when they did not grasp or were uncertain about the reasoning leading to those judgments. The authors began to explore techniques to help PTs continue to engage with challenging tasks in their own terms—with attention to both students' mathematical reasoning and their affect around uncertainty.

The instructors used affect graphing to examine changes in students' confidence within a particular discussion orchestration pattern, *diverge then converge*, that juxtaposes two answers, asks students to provide justifications leading to both (the correct and incorrect) answers, and then determines the correct pattern of reasoning. The instructors had two primary hypotheses: first is that juxtaposing contrasting answers would perturb PTs' thinking and foster continued engagement as they worked to resolve the perturbation and second is that engaging in the discussion would support PTs in resolving the perturbation. To evaluate their approach to orchestrating discussion, they wanted to assess how much students felt settled/perturbed as they contrasted two answers and patterns of reasoning. "Confidence" here was seen as an accessible way of measuring how resolved or uncertain students felt about the competing answers, arguably a different, if related meaning of "confidence" from Study 1.

The task was to solve $189 \div 11$ using $220 \div 11 = 20$ as a first step, assuming a sharing meaning of division. The choice of starter was purposeful in that $220 \div 11$ requires students to grapple with what the overestimate means in terms of sharing division (fair sharing 31 more objects among 11 people than in the original problem). The overestimate provides the opportunity for students to clarify their understanding of remainder. Many students stop after they remove 22 objects (2 each from 11 groups) and conclude that the remainder must be 9, because they cannot take one more item from each group without going below the original dividend. However, they have lost track that the items they are removing to get back to the original dividend of 189 were not "real" items—they were only introduced as a calculational device.

As expected, the PTs worked individually or in small groups and found two answers—18 remainder 9 and 17 remainder 2 (the correct answer). A sensible chain of reasoning leads to each answer. 220 ÷ 11 can be represented as 11 groups of 20. Because 200 is 31 more than 189 (the dividend), we take away 2 items from each group (or 22 in total). There would then be 11 groups of 18, with 31-2*11 = 9 items not in any group. This leads to 18 R.9. Alternatively (and correctly), another path is to continue the process of removing items evenly from each of the 11 groups, taking it one step further: 220–22 = 198 (now 11 groups of 18), which is 9 more than the dividend of 189. Taking away 1 more item from 9 of the 11 groups leads to 9 groups of 17 and leaves 2 groups of 18. The extra items in the 2 groups are interpreted as 17 R.2.

Immediately after the discussion had concluded, we asked PTs to rate their confidence at seven points during the lesson using five indicators from low to high (as shown in Fig. 5). This brief delay in making retrospective judgments of confidence was judged necessary to preserve the PTs' focus on the mathematics rather than interrupting the discussion.

The authors adapted the approach taken in Smith et al. (2017), restructuring the x-axis so that each column represented a known point in the discussion, to capture PTs' reflections across the discussion in a uniform way. The students' produced graphs ranged from those with X marks in each column to continuous graphs connecting successive points. As with the previous two studies, the authors did not take levels of confidence to be directly comparable across students. Instead, they were interested in seeing where and how many drops in confidence the PTs would report.

Figure 6 presents two confidence graphs that illustrate a dip in confidence when presented with the possibility that 17 R.2 was a correct answer to the task; some students connected the points of their graph using lines while others did not.

As in Studies 1 and 2, the authors collected additional data to inform their interpretation of PTs' experience of the discussion. In written form, on the back of the graph, they asked PTs first to choose one high confidence point and one low confidence point and describe what was happening at each point. Second, the PTs rated their confidence at the end of the activity, specifically focusing on whether aspects of the task or discussion remained unclear. Next, they were asked to find and explain a chain of reasoning given pictorially that produced the incorrect answer. Lastly, they were asked to fix what was incorrect in the given chain of reasoning and use their correction to help someone understand why it was faulty. This four-element written follow-up served as a formative assessment of how well

My Confidence Level during the Activity

HIGH							
MEDIUM							
LOW							
	MY work on 189 ÷ 11 = ?	MY work considering 18 R.9	MY work considering 17 R.2	Presentation for 18 R.9	Presentation for 17 R. 2	Discussion of Validity	End of Activity

Fig. 5 Confidence graph presented to the class

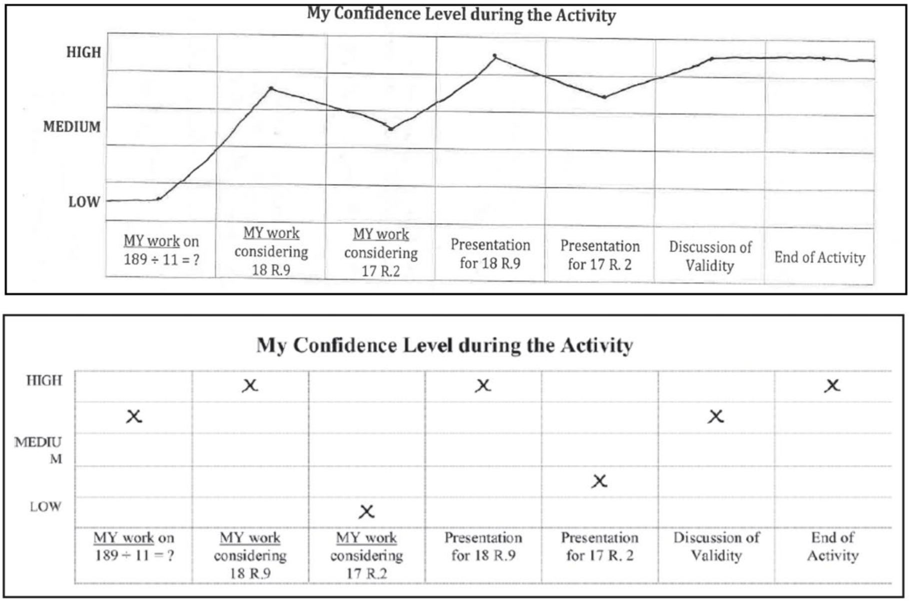


Fig. 6 Two prospective teachers' confidence graphs: The top graph is discrete, using X marks, and the bottom graph is continuous, where points are connected by lines.

the lesson achieved the instructors' conceptual goals. Almost universally, PTs were able to adequately explain how to fix the strategy leading to the incorrect answer, and only three PTs (of 22) expressed any concerns about their understanding at the end of the lesson.

The authors found alignment between the PTs' written responses and their confidence graph ratings (e.g., "I thought my work for 18 R.9 was correct so I didn't feel confident that the answer could be 17 R.2."). Cautious about interpreting this ordinal scale as representing equal intervals, the authors computed "mean" confidence levels for each phase indicated on the x-axis. This supported class-level inferences: The class was less confident when asked to consider the correct, but counter-intuitive, answer (mean confidence of 2.6 on a 5-point scale), but by the end of the discussion, the class was highly confident (4.9 on a 5-point scale). They also analyzed the graphs for downward turns in confidence. Nearly three-quarters of the PTs indicated at least one downward turn, supporting the claim that PTs experienced some perturbation during the lesson. Twelve of the 22 PTs drew a downward turn in confidence when asked to consider the correct yet counter-intuitive answer. The downward turn analysis clearly distinguished students whose graphs showed upward and downward change in confidence across the discussion from those who reported either high or low confidence throughout the lesson.

The graph served as a way for PTs to communicate and reflect on the affective aspects of uncertainty. The affect graph became an object to refer to, granting them permission to talk about lost confidence as well as establishing a common set of lesson checkpoints. The written reflections helped with interpreting the confidence graph, but the confidence graph also prompted the written reflection in turn. Collecting the PTs' reflections in graphical and written form captured data that would have been difficult to produce from observation and videotape records. Both of these methods typically generate impressions that at least some of the PTs were deeply engaged in discussions, but the confidence graph activity gave researchers (and teachers) a tool for measuring where the entire class was in their understanding and how understanding shifted throughout the discussion. The graphs and written reflection provided a more complete view of the group as a sum of individuals for pedagogical purposes.

4 Discussion

This paper has argued for and illustrated the use of graphical methods as a data collection tool in studies of students' affective experience in mathematics, as in Smith and Star (2007) and Satyam et al. (2018). While other studies have used similar techniques (Anderson, 2005; Ingram, 2007), we have focused on affective states in particular and productive insights that can be generated from graphical methods. This work speaks to how verbal reports, written or oral, are emphasized in mathematics education data collection and analysis. Researchers may too easily assume that individuals' words (spoken or written, intentional or not) are accurate records, relatively complete, and interpretable. While words are essential in human communication, they are not our only resource; non-verbal tools are powerful stimuli and referents for describing affective states that are not easily articulated. Graphing allows for self-expression of one's experience but with more structure and coordination of constructs. A unity of both verbal and non-verbal tools may provide new insights, so we maintain that graphing is not a panacea but rather a complement to existing popular approaches for studying affect.

We have presented three studies that used graphing to explore some aspect of affect in students' mathematical experiences in different contexts—from confidence in a semester-long mathematics course, to emotions during a single instance of proof construction, to confidence in a whole class discussion of competing answers and lines of reasoning. As the three examples share commonalities, we summarize the main affordances of graphing in each study before synthesizing across them. The core argument is two-fold: (1) Graphing provides a fundamentally different way from verbal description for students to render their affective experience during mathematical work and (2) the activity of graphing and the resulting object of a graph support students' further clarification and deepening of their verbal accounts.

In the analysis of students' experience in a transition-to-proof course (Study 1), the confidence graphs served as data—where students represented their confidence and its change over time—and stimuli, drawing out richer descriptions and corroborating their earlier verbal responses. In the analysis of emotion in proving (Study 2), where observation revealed important events and indicators of students' emotion, the graphs provided more and deeper insights—via direct descriptions of those emotions, qualitative comparisons of highs and lows, and richer pictures of the affective course of students' solution attempts. In both cases (Studies 1 and 2), the graphs provided insights into students' affect that would have been difficult to assemble using interview questions or observations alone. In the study of confidence during a class discussion (Study 3), a differently structured confidence graph revealed each student's confidence in their thinking at pivotal moments of a class discussion and how their confidence shifted. Pedagogically, the set of graphs provided a view of the entire class's experiences, supporting instructors' self-evaluation of their effectiveness in orchestrating the discussion. The collection of graphs and responses was a more complete and reliable record of the discussion than what is typically available to instructors at the end of lessons—their impressions based on the participation of the most vocal students.

Taken together, these examples show how graphs can be useful in the study of affect for different settings and different timescales and to understand different aspects of student affective experience in mathematics (e.g., Lee et al., 2021). The graphing activities in all three studies contributed to understanding changes in students' affect over some mathematical experience—working through a challenging course, developing a proof, and deciding between two plausible answers. The act of graphing facilitated students' reconstruction of temporally ordered events and changes in affect across them. This mental process involved identifying and comparing the magnitudes of the affect at different time points. Where the reconstructive nature of the graphing activity and ensuing discussion may trouble some readers (compared, for example, to experience sampling (Schiefele & Csikszentmihalyi, 1995)), the graphs have the valuable affordance of being inspectable—and therefore adjustable—by their creators, as shown by Kirk's work in Study 1. Relative to interviews and fixed-item surveys, the graphs provided students a more open, but still ordered, space in which to represent their affect.

The structure of the affect graphs varied across the studies, reflecting different purposes for examining student affect. The x-axes of the blank graphs given to students in Studies 1 and 2 indicated time only at the beginning and end of the target experience, inviting students to add significant events, characterize their affect at those points, and so indicate change over time. These three added features (events, affect magnitudes, and changes) then became foci for explanation and discussion. Study 3 intentionally pursued the opposite approach, listing key events in the class discussion on the x-axis and asking all students to rate their confidence at just those points. The resulting class set of graphs were uniform in structure and comparable. These two approaches to structuring the graph highlight the need for researchers to be clear about their inquiry goals and the graph structure that best supports them.

Asking students to construct affect graphs can provide important insights into their experience in mathematics (Voogt, 2021), but we do not argue for their stand-alone use. In all three studies, graphing was used with other data to support the analysis of the graphs themselves and deepen the resulting portraits of the students' (or the class's) affective experience. In Studies 1 and 2, researchers asked students about specific figural aspects on their graphs, such as locations of points and trends across time, to ensure the graphs represented what the students intended. In Study 1, students' responses earlier in the interview provided context for the graphical focus on confidence. In Study 2, observation was helpful for understanding and examining the students' emotion graphs. In Study 3, researchers' follow-up questions allowed students to describe what was happening at the highest and lowest confidence in their graphs, adding substantially to what they could learn from ordinal ratings alone. Affect graphing is a powerful tool for rendering experience, when bolstered by students' clarifications.

4.1 Issues for researcher application

Our goal is to encourage researchers to consider how graphical methods can contribute useful data about affect during mathematical work and experiences that would be difficult to collect via other methods. In that spirit, we highlight a set of issues that researchers should consider in exploring and planning the use of affect graphing. We focus on the features of the blank graphs and how the structuring of graphical space should align with researchers' purposes in studying affect.

1. *The two constructs to coordinate*. The graphs in our studies indicated time along the x-axis and an affective construct along the y-axis.
2. *The timescale of the experience, when magnitude and change over time are focal*. Researchers should consider what is a meaningful and accessible start and end time for the experience and whether blank space before or after is relevant. For example, Study 1's blank graph provided space and then a tick mark along the x-axis labeled the "start of the semester," where students could represent their confidence going into the course.
3. *The graph's quadrants*. The structuring of the graphical space into quadrants should follow from the nature of the target affect. If the affect can have negative valence (e.g., negative emotion), then two quadrants (assuming time along the x-axis) are appropriate. If negative valence does not make sense (e.g., low confidence rather than negative), one quadrant is sensible. Though the three graphs examined here included only one or two quadrants, "negative time" may be appropriate for exploring retrospective or anticipated affect. In this case, two or four quadrants would be appropriate.
4. *The structure of the axes in terms of specific locations.* The number of tick-marks on the graph's axes and their labels largely influence the graphs students produce. Locating a point on the x-axis demands a corresponding vertical location along the y-axis. Researchers should therefore think about how much to structure the graphical space, according to their goals.
   a. *More or fewer locations* along the axes influence the graphs produced and lead to follow-up questions. On the x-axis, more locations call students to indicate how they felt at particular points. Fewer locations open up space for students to identify events significant for them. On the y-axis, the number of locations (levels) reflects the researcher's judgment about what levels of affect students can sensibly distinguish. If the researcher's focus concerns shifts in affect, fewer locations may be appropriate, as this structure emphasizes relative (over actual) heights over time.
   b. Locations on the axes may be *labeled*, or not, with descriptive levels (e.g., high, medium, low; negative, zero, positive on the y-axis). When particular points in time and/or ways of characterizing affect are of interest and meaningful, providing those descriptions and clarifying them with students is appropriate. Alternatively, the absence of descriptions encourages students to characterize levels of affect (on the y-axis) or important events (on the x-axis) in their own meaningful ways.
5. *Structuring the space for continuous or discrete graphing.* Beyond the axes, researchers should consider how else they may want to structure the graphical space, to guide the graphing. For example, in Study 3, the highly structured x- and y-axes provided boxes for students to mark with an X. This discrete structure was consistent with the study's purposes. In contrast, the more open space in Studies 1 and 2 encouraged students to draw continuous graphs that were consistent with the researchers' focus on change—rises and falls and the experience that caused them.

In summary, the numerous decisions that researchers make in designing their graphs for representing affect lead to multiple levers of control over the character of the resulting data—both graphical and verbal. These decisions of how to structure affect graphs should flow from and be consistent with one's research purpose(s). The design of blank graphs shapes—not determines—the graphs that participants draw. Those graphs are in turn only steps into the nature of their affective experience. Follow-up questions, whether written or spoken, are important for attending to specificities of students' graphs (e.g., comparing heights and slopes at different points or on intervals), verifying their graphs as accurate portrayals of their experience and for triangulation with other data. Our experience has been that the act of drawing helps students render their experience, to make sense of and reflect on said experience which may help them more easily discuss it.

4.2 Issues for classroom application

While we primarily propose affect graphing for research purposes, we also believe graphing activities can be productive in mathematics classrooms. Graphing, when suitably structured, can be used across grade levels (despite our exclusive focus on college students here) to provide valuable space for students to represent their experience in mathematics. With modification, even young children as early as third grade can draw meaningful graphs in one- or two-dimensional spaces (Blanton et al., 2015). Affect graphing activities in particular can signal to students that how one feels when doing mathematics is of legitimate importance in school mathematics.

First, affect graphing can provide useful formative assessment of students' current states in the classroom. Teachers may ask students to graph some aspect of their affect, such as confidence or confusion, during or after a class activity or discussion. As seen in Study 3, affect graphing allowed the authors to record any perturbances in students' confidence during a classroom activity in order to evaluate their approach to orchestrating the class discussion using data. Teachers often rely on their holistic judgment of how an activity or discussion went—an atmosphere of sustained engagement with infrequent lulls or whether students are engaging with each other. This judgment can be influenced by a few expressive students. But with individual reports from all students, it is possible to tell how every student in the class has experienced a discussion, especially for quieter students who may not share their experience vocally. Looking across a set of graphs provides a complete view. Indeed, interactive digital tools (e.g., starter screens in Desmos) that ask students to graph as an emotional check-in are already being used in the context of distance learning. Thus, affect graphing is one approach to making visible aspects of students' classroom experience that would otherwise remain below the surface.

Second, affect graphing allows teachers to test hypotheses they may hold about the affective contour of a planned discussion (as in Study 3). The resultant graphs can support or counter their hypotheses and thus allows for teacher experimentation, grounded in student reports. Affect graphs can also be used as a more exploratory tool (as in Studies 1 and 2). Teachers may provide a blank graph leaving the x-axis relatively unstructured to discover what events of an experience were most salient to students. Ultimately, just as for researchers, teachers may structure graphs to learn how students are feeling at certain times or what events led to their highs, lows, and shifts in feeling.

Lastly, affect graphing promotes students' meta-cognitive awareness, as in their self-monitoring of past learning experiences. Through the act of graphing, students may identify for themselves particular coursework, mathematical topics, and/or learning activities that led to changes in their affect. For students in upper elementary onward, graphing supports reflection on challenges experienced and learning strategies that have been effective for them. Asking students to draw and think about changes in their affect particularly over a long period of time (such as a semester) encourages them to reflect on the entirety of their experience. Doing so may help students notice patterns in the resilience of their affective states (confidence, enjoyment, engagement, etc.) that were positive for their past mathematical development and that these reflections may positively inform their future.

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Conflict of interest The authors declare no competing interests.

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