

# Fractal Nature Complex Correction an Inductivity

Vojislav V. Mitic<sup>1,2</sup>, Kouros Khamoushi<sup>3,1</sup>, Cristina Serpe<sup>4</sup>, Branislav M. Randjelovic<sup>1</sup>, Aleksandar Stajcic<sup>5</sup>, Vesna Paunovic<sup>1</sup>, Sanja Aleksic<sup>1</sup>, Branislav Vlahovic<sup>6</sup>

**Abstract** — The microstructures properties predicting are based on their materials characteristics. In ceramic materials, regarding higher miniaturization and integrations, the structure analysis is very important. The main contribution of this research is related to relation between perovskites ceramics electronic properties, especially BaTiO<sub>3</sub> and NZT-ceramics and structural characteristics. We applied advanced analysis, based on fractal nature and introduced complex fractal correction in defining the very important electromagnetic parameter inductivity ( $L$ ). The samples consolidation includes both, powder pressing (cold sintering) and hot sintering. The fractal characterization performs very important role from powders up to the final structures, through which exists structure influence on electro-physical and other ceramic properties. We report the experimental results of BaTiO<sub>3</sub> and NZT-ceramics processing. Also, this is the first time that we apply complex fractal correction (influence of grains and pore surface and as well as particles Brownian motion) on fundamental thermodynamic temperature in Currie-Weiss law. This application includes the influence of Housdorff dimension (DH) from the microstructure images and connect fractal corrections in Currie-Weiss law, for relative dielectric constant  $\epsilon_r$  and magnetic permeability  $\mu_r$ . Through  $\epsilon_r$  and  $\mu_r$ , on this way, for the first time in this scientific area, we present direct relation to inductivity. This complex relation opens quit new advance understanding in structures and parameters in area of ferroelectric-magnetic applications in telecommunications, by introducing the fractals.

**Keywords:** electro-magnetic, fractals, structure, properties, relative dielectric constant, relative magnetic permeability, inductivity.

Vojislav V. Mitic is with the Faculty of Electronic Engineering, University of Nis, A.Medvedeva 14, Nis, Serbia<sup>1</sup> and Institute of Technical Sciences of SASA, K.Mihajla 35, Belgrade, Serbia<sup>2</sup>(Email: [vmitic.d2480@gmail.com](mailto:vmitic.d2480@gmail.com)).

Kouros Khamoushi is with the Faculty of Natural Science, Tampere University, P.O.Box 1001 FI-33014 Tampere, 33720/ Finland<sup>3</sup> and Faculty of Electronic Engineering, University of Nis, A.Medvedeva 14, Nis, Serbia<sup>1</sup> (Email: [kouros.khamoushi@tuni.fi](mailto:kouros.khamoushi@tuni.fi)).

Cristina Serpe, is with ISEL - Instituto Superior de Engenharia de Lisboa CMAFcIO - Centro de Matemática, Aplicações Fundamentais e Investigação Operacional, Faculdade de Ciências da Universidade de Lisboa, Campo Grande, 1749-016 Lisboa, Portugal<sup>4</sup> (Email: [cristinaserpa@hotmail.com](mailto:cristinaserpa@hotmail.com)).

Branislav M. Randjelovic is with the Faculty of Electronic Engineering, University of Nis, A.Medvedeva 14, 18000 Nis, Serbia<sup>1</sup> (Email: [bane@elfak.ni.ac.rs](mailto:bane@elfak.ni.ac.rs)).

Aleksandar Stajcic, is with Center for Microelectronic Technologies, Institute of Chemistry, Technology and Metallurgy, National Institute of the Republic of Serbia, University of Belgrade, Njegoseva 12, 11000 Belgrade, Serbia<sup>5</sup>, (Email: [stajcic@nanosys.ihfm.bg.ac.rs](mailto:stajcic@nanosys.ihfm.bg.ac.rs)).

Vesna Paunovic, is with the Faculty of Electronic Engineering, University of Nis, A.Medvedeva 14, Nis, Serbia<sup>1</sup> (Email: [vesna.paunovic@elfak.ni.ac.rs](mailto:vesna.paunovic@elfak.ni.ac.rs)).

Sanja Aleksic, is with the Faculty of Electronic Engineering, University of Nis, A.Medvedeva 14, Nis, Serbia<sup>1</sup> (Email: [sanja.aleksic@elfak.ni.ac.rs](mailto:sanja.aleksic@elfak.ni.ac.rs)).

Branislav Vlahovic, is with North Carolina Central University (NCCU), Durham, NC 27707, USA<sup>6</sup> (Email: [vlahovic@nccu.edu](mailto:vlahovic@nccu.edu)).

## I. INTRODUCTION

Already known microstructure characterization methods do not provide enough opportunities for material analysis, the structure reconstruction and prediction of the microstructure properties within the miniaturization framework. It is essential to establish adequate mathematical-physical approach, which would lead to set goal within the material consolidation trial synthesis-structure-properties. Unlike all other microstructure analysis tools available so far, which can only passively obtain characterization data, new possibilities are open up for the application methods, based on materials structure fractal nature, which upgrade all these tools, making them available in practice, especially in the microelectronic circuits miniaturization field. The research results related to them in the scientific literature include: material's properties programming, properties prediction, designing, material's programming, and, (the latest) - new simple material's modeling (NSMM) [1].

The problem of all modern and maximally optimized microstructural methods is the question of the flexibility level that can provide the reconstruction and designing the structural units (grains and pores), as well as achieving a higher morphology miniaturization. This is essential for microelectronic components and devices, as well as, electronic circuit integration.

Ferroelectric and related magnetic materials are used in many applications especially for smart materials and multifunctional purposes [2]. Research development has been dedicated to ferroelectric material particularly single-phase crystal materials in the thin film technology. Materials with perovskite structure have a regular atomic order, variation and changes in these repetitive grids like pattern which producing atomic domains disorder in certain direction and materials polarization in some point [3,4].

The diffusion phase transition influence to ferroelectric-magnetic material properties. This mechanism can be described that crystal material, under temperature microstructure influence changes, because of temperature variation and therefore the materials change atomic arrangement and phase transition occur in microscopic scale [5]. This also cause changes in Currie temperature and beginning of polarization and finally generate properties, with materials micro-changes. The phases transition is dominating when solid solution composition becomes homogeneous.

In this scope, the grain and pore analysis based on the solid Euclidean geometric characteristics, what is not enough for this research, almost always involve a microscopic

examination of the different the grain and pore surface geometric features under several levels of magnification.

The fractal dimension of the grains' or pores' space (the Hausdorff dimension - DH), which is the main characteristic of any fractal object A, considerably contributes to the ceramics, physical-chemical properties and other structures is a real number, smaller than the geometric dimension of the minimal space containing A. In the last 20 years, there have been fractals applications wide range in different areas [6–11]. The generalized fractal phenomena were discovered in physical systems [12]. DH is independent on the measure scale for ideal (mathematical) fractals. Natural fractals, like clouds, trees, sea surface, dust in the atmosphere, interstellar dust grains, the solar spectrum and ceramics are almost fractals. These ceramic structures are much closer to ideal fractals than to any Euclidean object and are not self-similar structures as the majority of mathematical fractals (Cantor set, Sierpinski Triangle etc.).

Many fractals (Mandelbrot, Julia sets etc.) also do not confirm self-similarity; this are somehow randomly defined self-similar, which is a kind of a “self-similarity”, with all “natural fractals” characteristic. It was only natural step up the research in the ceramics and similar materials fractal nature, with a special focus on their fractal grainy and porous microstructures [13] and micro-capacity properties [14-16]. By using measured data, from microstructure images, electrical conductivity, dielectric and magnetic permeability could be enlightened by fractals, which is reflecting on inductivity as well.

## II. EXPERIMENTAL

The experimental procedure has been performed on BaTiO<sub>3</sub> and NZT-ceramics perovskite samples.

### 2.1. The BaTiO<sub>3</sub>-ceramic samples preparation

The BaTiO<sub>3</sub>-ceramic samples with different additives were prepared by conventional solid state reaction. As a starting material we used the Murata BaTiO<sub>3</sub> Fluka chemika powders as additive. Further analysis have been conducted by SEM (JEOL JSM-5300), where the microstructure have been performed under different magnifications [17,18] for further fractal analysis. SEM micrographs of BaTiO<sub>3</sub> doped with different additives and sintering temperatures, which includes SEM images, are reported in different research papers [19]. This experimental research was data for fractal analysis corrections within the dielectric properties.

### 2.2. NZT samples preparation

For further researche regarding microvawe and even magnetic properties we analyzed NZT samples - Nd(Zn<sub>0.5</sub> Ti<sub>0.5</sub>) O<sub>3</sub> prepared by conventional solid state reaction, also. It includes X-ray in each temperature from 1250°C to 1500°C. Then there was summation of all X-ray over each other.

To determine the ordering scale, samples were cut and anneald at 1200°C for different period of time in range from 15minutes to 105h. The degree of ordering and the size of order domines were examined by using x-ray diffraction in NZT places Zn<sup>+2</sup> and Ti<sup>+3</sup> located at the B-sites as can be seen in Fig. 1 or NZT ions these ordering leads to an doubling [6] of the lattice parameter of of the original pervoskite cell. and thus can be defined by x-ray diffraction spectrums. The non annealing x-ray diffraction.

Below, a short explanation of the material preparation for the NZT is described. A detail information and complete description are in references [20,21]. The compound was calcined for two hours at 1250 °C temperature and then mixed again for twenty hours in a ball mill zirconia [21,22]. Nd<sub>2</sub>O<sub>3</sub> is substance that willingly attracts water from its environments, through absorption, hence we mixed the powder with distilled water to be saturated and prevent structural changes during the process of dielectric preparation.

TABLE I  
THEORETICAL ESTIMATION BASED ON SIZE CHANGES OF ORDER DOMAIN AND DEGREE OF ORDERING AS A FUNCTIONING AND ANNEALING TEMPERATURE

NZT Material Single Phase	Heat treatment (°C)	Order degree	Ordered domain size
	No Annealing	0.39	12 nm
	17 min at 1100	0.49	22
	35 min at 1100	0.53	32
	60 min at 1100	0.61	77
	3 h at 1100	0.74	95
	5 h at 1100	0.86	102
	7 h at 1100	0.88	102
	10 h at 1100	0.91	102
	26 h at 1100	0.93	102

These prepared powders were mixed with 1wt% of Dispex A40, then powder was kept in 80 °C for twelve hours. The mentioned fine powder was then pressed into the cylinder shape with diameter of 10<sup>-2</sup>m and height of 3·10<sup>-3</sup>m under pressure of 12.75 kg/mm<sup>2</sup>. Samples were sintered at 1400 °C to 1675 in an Al<sub>2</sub>O<sub>3</sub> crucible for six hours in air. To quantify the weight loss; the samples have been measured before and after sintering. Microwave sintered sample dielectric properties were measured using network analyzer to measure permittivity; the post resonator method was used. An invar cavity was used to define  $\tau_f$ . The silver coated cavity was several times larger than size of samples.  $\tau_f$  was measured in a heat area started from -23 to 37°C at a frequency 4 up to 11 GHz. To investigate crystal structure of samples we utilized TEM, SEM, XRD and Differential scanning calorimetry and method for phase evolution as well as Rietveld refinement performed. The phase assemblage and space group of samples checked by the following instruments: Scanning electron microscope (JSM 6300, Joel, Japan) (SEM). Siemens D5000 X-Ray Powder diffractometer. TEM laboratory contains two analytical JEOL instruments JEOL 2010 and HORIBA-Scientific Raman microscope, manufactured by HORIBA Scientific Japan. Differential Scanning Calorimeter (DSC)

and Differential Thermal Analyzer (DTA) Perkin Elmer TGA7 Differential Scanning Calorimeter.

The previously experimental work and Neutron and Raman spectrum of neodymium zinc titanium oxide is differing noticeably (Figure 1 shows a model for NZT structure). These are taken from special direction to be able to investigate existing symmetry elements in atomic layers packing density. For this model pseudocubic parameter equal to  $0.789nm$  was applied [20-23].

### 2.3. Real theoretical experiment

One of the phenomena is applied as Heywang model, based on surface effects and solid states close to the surface electronic structures [28].

Heywang model, based on surface effects of electronic structures, was applied in the theoretical experiment. The specific surface electronic states formation influences the interruption of the periodic grid structures, allocated in the restricted zone, and formation of the non-degenerated zone, as the basic zone with the surface acceptor or donor types of states at the grain boundaries.

The idea of the theoretical experiment is to examine the particles (such as electrons and oxygen vacancies) motion, in the grain boundaries and between the grains, in the frame of Brownian motion fractal nature similarities. Particles on the one side of the boundary collide with particles on the other side of the boundary, influencing changes in their trajectories and stochastic motion. The goal is to determine and predict particles trajectories by applying fractal analysis, in order to transform chaotic Brownian motion to an orderly motion. In all of these processes, working temperature should be considered, due to its dependance on ceramic material fractality.

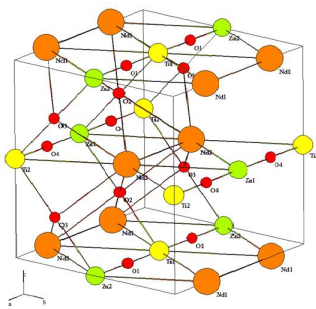


Fig.1. Structure model, based on monoclinic P2<sub>1</sub>/n crystal structure, showing the atomic positions in NZT.

Microstructural properties depend on grains and pores, as fractal objects. Thus, if we want to design a material with desired properties, we have to define grains and pores structure, by applying fractal analysis, and when we reconstruct some shape once, we can repeat it every time by the same procedure. A very important thing is that the features of the consolidated material are represented through

intergranular and interporal relations, which are basically super-micro-intergranular fractal capacitors, which are parts of intergranular micro RLC impedances.

In order to apply fractal nature analysis, SEM images of the sample have to be prepared, because we need to translate the grain contour into a mathematical analytical form, which was introduced by Barnsley, as a fractal interpolation method.

Topological dimension,  $D_T$ , is related to Euclidean geometry objects, whilst fractal dimension,  $DH_f$ , is a characteristic of fractal objects, influencing the grains' and pores' surfaces and ferroelectric properties in BaTiO<sub>3</sub> NZT and other perovskite ceramics. Mathematically, the difference between the topological and fractal dimension is:  $DH_f - D_T = DH_f - 2 > 0$  and the same method is applied throughout the whole experiment. To enable application of different mathematical methods, certain parameters are established [19]:  $\alpha_S$ , normalized surface fractality ( $0 < \alpha_S < 1$ ),  $\alpha_P$ , fractal parameter that transforms fractality of porosity ( $0 < \alpha_P < 1$ ), and  $\alpha_M$ , dynamic correction factor characterized by the Brownian 3D motion ( $0 < \alpha_M < 1$ ):

$$\alpha_S = 1 - [DH_f(S_S) - D_T(S_S)],$$

$$\alpha_P = 1 - [DH_f(S_P) - D_T(S_P)],$$

$$\alpha_M = 1 - [DH_f(L_M) - D_T(L_M)].$$

In our previous research [28] we already established that based on the assumption that the measured temperature  $T$  of sintering BaTiO<sub>3</sub>-ceramics must be influenced by these three fractality factors, a unique fractal corrective factor  $\alpha$  was introduced, which enables to obtain real value of the sample temperature  $T_f$  by correcting the measured temperature  $T$ :

$$T_f = \alpha T.$$

Based on the Curie-Weiss Law

$$\varepsilon_r(T) = \frac{C_c}{T - T_C},$$

we have [28] expression for dielectric constant

$$\varepsilon_r(T_f, \alpha) = \varepsilon_r(T_f, \alpha_S, \alpha_P, \alpha_M) = \frac{C_c}{\left( \frac{3T_f}{\alpha_S + \alpha_P + \dim \sum_{t=0}^n X(t) Bi(t) - 1} \right) - T_C}$$

Based on those formulas we can clearly establish relation between microstructure and electronic characteristics, if we have the material relative dielectric constant as a fractal constants function.

Regarding the microwave and even magnetic materials we applied another fractal method based on regression. This method application, based on real data fractal analysis include regression method defined by the mathematical formulations given in [24-26]. This consists of obtaining the equations system coefficients that best fit the data. This system is based on

$$\varphi\left(\frac{x+j}{p}\right) = a_j \varphi(x) + b_j x + c_j.$$

where  $x \in [0, 1]$ ,  $0 \leq j \leq p-1$ , and  $a_j, b_j, c_j$  are the parameters (real numbers) to estimate, with  $0 < |a_j| < 1$ . The default domain  $[0, 1]$ . The solution of this system is a function  $\varphi: [0, 1] \rightarrow \mathbb{R}$  and is called a fractal function [27]. In fact, it is proved in [25] that such functions have mathematical fractal structure, meaning that the plot of its graph is a fractal curve. Theoretical mathematical properties and explicit solutions are provided in papers [24,25]. Parameters  $a_j$  are the fractal coefficients and  $b_j$  are the directional coefficients. Bigger fractal coefficients mean strong fractal oscillations. Parameter  $p$  is the fractal period and  $L$  is the fractal level of a curve, defined by the system. This means that the first fractal level in the entire domain will be replicated in each sub-interval (there are  $p$  sub-intervals), composing the second fractal level.

The fractal regression method consists of estimating the parameters  $a_j, b_j, c_j$  such that they fit the real data. For doing this, we consider the form of explicit solution of the problem, that depends on the  $p$ -expansion of numbers in  $[0, 1]$ . For  $L=2$ , this solution is

$$\varphi(0) = \frac{c_0}{1-a_0}, \quad \varphi\left(\frac{\xi}{p}\right) = a_{\xi_1} \frac{c_0}{1-a_0} + c_{\xi_1}, \quad \xi_1 \neq 0$$

$$\varphi\left(\frac{\xi_1}{p} + \frac{\xi_2}{p^2}\right) = a_{\xi_2} \left( a_{\xi_1} \frac{c_0}{1-a_0} + c_{\xi_1} \right) + b_{\xi_2} \frac{\xi_2}{p} + c_{\xi_2}, \quad \xi_1, \xi_2 \neq 0$$

For obtaining the best coefficients, the theoretical approach computes the SSR - sum of square residuals in between the formal definition and the real values. Then it uses the partial derivatives of this SSR and equals to zero, for minimizing this error. The best solution of the problem is given when

$$\frac{\partial SSR}{\partial a_j} = 0, \quad \frac{\partial SSR}{\partial b_j} = 0, \quad \frac{\partial SSR}{\partial c_j} = 0,$$

for all  $j=0, 1, 2, \dots, p-1$ . This is a problem with  $3p$  parameters to estimate where the equations to solve are nonlinear.

The mathematical analytical solution of this partial derivative system is not possible to compute, and a numerical approach is needed. With the software for numerical computation of the solution, called Fractal Real Finder, we worked samples and obtained estimated curves and estimates of Hausdorff dimension. Fractal Real Finder is a software that is available online<sup>1</sup>. With input of real data, running the program it gives an output with a fractal curve as modelled above. With the estimated fractal curves, we may upper estimate the Hausdorff dimension [25].

**Proposition.** *The Hausdorff dimension  $DH$  of the graph of the function  $\varphi$  solution of the above system is upper bounded by the solution of:*

$$\sum_{j=0}^{p-1} \beta_j^D = 1,$$

<sup>1</sup> Support material as well as links for the software Fractal Real Finder are available in [https://www.researchgate.net/profile/Cristina\\_Serpa](https://www.researchgate.net/profile/Cristina_Serpa)

where  $\beta_j = \max\left\{\frac{1}{p}, |a_j|\right\}, 0 \leq j \leq p-1$ .

The coefficients with fractal relevance are those  $a_j$  such that  $|a_j| > 1/p$ . The Hausdorff dimension is an indicator of the chaotic/irregularity behaviour of data. The classical dimension is integer: 1 for lines and curves, 2 for 2D objects, 3 for solid 3D objects. There are structures that have characteristics in between two integer dimensions. In that case, we may estimate a non-integer dimension. The fundamental theoretical mathematical non-integer dimension is the so called Hausdorff dimension, sometimes referred as fractal dimension. The box dimension is a simplified indicator that provides estimates for the real Hausdorff dimension of real data.

#### 2.4. Fractal data reconstruction

From the following image Fig. 2, we selected a circle part and zoomed in it to a new image. We perform a fractal reconstruction in both inner and outer curves around the grains' clusters. This reconstruction solution is important for completing the analysis related to microstructure grains' clusters morphology.

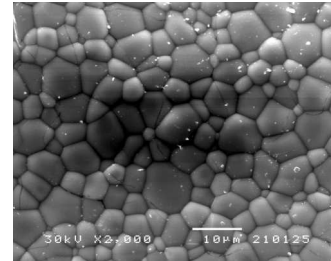


Fig. 2. SEM image of NZT sample under magnification 2000. We introduced red points in two different border lines, the inner one and the outer one.

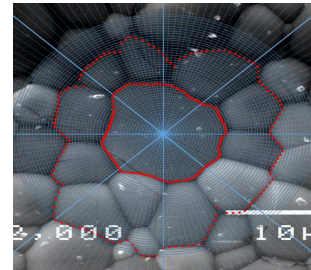


Fig. 3. Selected morphology of different grains' clusters which are circled as inner and outer one (notified by curves of red colour points).

This is also important from the point of view that we have in the dielectric and as well as magnetic materials some related to core-shell structures which are not here the research target, but all of these could be interesting in future exploring. This is plotted in the Fig 4.

By variables change, from polar to Cartesian variables, we run the Fractal Real Finder software for the sequence of radiuses and obtain the estimated fractal model with coefficients, as in Table II. The sequence of points is the set of radiuses corresponding to  $p^l = 12^2 = 144$  angles around the image.

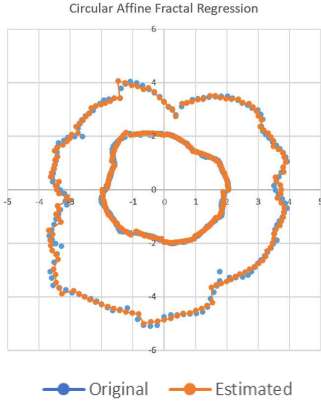


Fig. 4 The successful reconstruction curve around grains' cluster selected at SEM images (Figure 2).

We observed that the estimated fractal curves are so close to the original curves generated by points, so it is hardly to see the difference; it is almost imperceptible at sight.

TABLE II  
ESTIMATED COEFFICIENTS FOR THE INNER FRACTAL CURVE RADIUS

	$a_j$	$b_j$	$c_j$
0	0.029	-0.057	1.992
1	-0.003	-0.365	2.035
2	0.014	0.355	1.737
3	0.004	0.28	2.039
4	0.005	-0.511	2.452
5	-0.006	-0.011	1.839
6	0.051	0.038	1.866
7	-0.037	0.01	2.058
8	0.034	0.047	1.728
9	-0.007	0.113	1.975
10	0.02	-0.042	1.956

This fractal reconstruction reveals no fractal coefficients (those bigger than  $1/12=0.08(3)$ ) and, in consequence, the corresponding estimated Hausdorff dimension is 1. The coefficients of the outer fractal estimated curve are in the Table III.

TABLE III  
ESTIMATED COEFFICIENTS FOR THE OUTER FRACTAL CURVE RADIUS

	$a_j$	$b_j$	$c_j$
0	0.14	1.498	2.38
1	0.101	-0.182	3.421
2	0.146	-0.017	3.068
3	-0.338	0.331	4.598
4	-0.194	1.464	4.528
5	-0.01	-0.025	4.788
6	-0.051	-0.533	5.191
7	-0.038	0.224	4.051
8	-0.079	-0.102	4.31
9	-0.215	0.599	4.372
10	-0.136	0.485	4.316

The relevant fractal coefficients are  $a_0 = 0.139859$ ,  $a_1 = 0.101052$ ,  $a_2 = 0.145667$ ,  $a_3 = -0.337943$ ,  $a_4 = -0.193711$ ,  $a_9 = -0.215169$ ,  $a_{10} = -0.135588$ ,  $a_{11} = 0.102129$ , (those bigger than  $1/p=0.08(3)$ ). The corresponding estimated Hausdorff dimension for the curve at Figure 4, is 1.29565.

### III. RESULT AND DISCUSSION

The main point of our research paper is introducing the complex fractal correction on inductivity  $L$  through applied fractal correction for  $\epsilon_r$  and  $\mu_r$ , using

$$L = \frac{\lambda}{\sqrt{\epsilon_r \mu_r}}.$$

The main goal in our research is to implement the complex fractal correction in Curie-Weiss law for relative dielectric constant  $\epsilon_r$  and magnetic permeability  $\mu_r$  as well. Furthermore, at first, relative dielectric constant fractalization, as already published result, what we already discussed in section 2.3. Regarding the complex fractal correction applied on Curie-Weiss law for  $\epsilon_r$  we will perform some physical-mathematical approach from our previous research [21,31]. Based on the assumption that the measured temperature  $T$  of sintering NZT-ceramics must be influenced by these three fractality factors, united in one fractal corrective factor  $\alpha$ , in order to obtain real value of the sample temperature  $T_f$  by correcting the measured temperature  $T$ :

$$T_f = \alpha T.$$

$\alpha$  depends on the parameters  $\alpha_s$ ,  $\alpha_p$  and  $\alpha_M$ , in the following way [28]:

$$\alpha = u \alpha_s + v \alpha_p + w \alpha_M,$$

where  $u, v, w > 0$  are real coefficients satisfying condition  $u + v + w = 1$ . The Curie-Weiss Law for microwave and magnetic properties materials, expressed by:

$$\mu_r(T) = \frac{C_c}{T - T_c},$$

with mention temperature dependence  $T_f = \alpha T$ , magnetic permeability can be written as:

$$\mu_r(T_f, \alpha) = \frac{C_c}{T - T_c} = \frac{C_c}{(T_f/\alpha) - T_c} = \frac{\alpha}{T_f} \frac{C_c}{1 - \alpha(T_c/T_f)}.$$

Using similar approach as in [28],  $u = v = w = 1/3$ , it becomes:

$$\mu_r(T_f, \alpha_s, \alpha_p, \alpha_M) = \frac{C_c}{\left( \frac{3T_f}{\alpha_s + \alpha_p + \dim \sum_{i=0}^n X(t_i) Bi(t) - 1} \right) - T_c}$$

and inductivity is

$$L = \frac{\lambda}{\sqrt{\epsilon_r \mu_r}} = \frac{\lambda}{\sqrt{\frac{C_c}{\left( \frac{3T_f}{\alpha_s + \alpha_p + \dim \sum_{i=0}^n X(t_i) Bi(t) - 1} \right) - T_c}}}$$

$$= \frac{\lambda}{K \cdot C_c} (3T_f - K \cdot T)$$

where we denote

$$K = \alpha_s + \alpha_p + \dim \sum_{i=0}^n X(t_i) Bi(t) - 1$$

On this way, we definitely for the first time, within the complex relation of material sciences and telecommunication applications, involve the physical mathematical approach and analysis, finalized with the electromagnetic inductivity with complex fractal correction. This is very important that we practically have one very complete overview relation between the electromagnetic parameters and very fine and precise microstructure characterization introduced by fractal nature analysis.

#### IV. OUTLOOK

In our future research there is a plan to continue to clarify the microwave and magnetic NZT ceramics materials properties.

#### V. CONCLUSIONS

In our research we continue to develop the complex relations between microstructures characteristics and ferroelectric-magnetic, microwave and other electromagnetic field properties, with consequent fractal nature analysis in ceramics material sciences. We originally applied our previous results based on relative dielectric constant ( $\epsilon_r$ ), with BaTiO<sub>3</sub> ceramics data, where is introduced the complex fractal correction. After this, regarding the microwave and other possible magnetic properties, we extended our results on Curie-Weiss law for magnetic permeability ( $\mu_r$ ), what was based on real fractal characterization with different fractal characterization methods and have got the experimental-theoretical Housdorff dimension (DH) results, on NZT ceramics. Complete data for NZT is based on real experimental samples consolidation. This DH value could be applied in further research papers within some mathematical calculations. This reported results, related to complex function inductivity  $L$ , connected with fractal nature microstructure analysis parameters, open very important advance perspective for microwave telecommunication miniaturization applications.

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