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Load-dependent speed optimization in maritime inventory routing

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ABSTRACT

Maritime inventory routing problems involve determining optimal routes for seagoing vessels between ports while managing the inventory of each port. Normally, such problems are considered with the vessels operating at fixed sailing speeds. However, the speed of vessels can typically be adjusted within an interval, and the actual fuel consumption depends on both the load and the speed of the vessel. The fuel consumption function combines speed and load in a non-linear manner, but can be approximated through linearization. In this work, to evaluate the importance of taking into account that both speeds and load levels influence the fuel costs, the resulting solutions are contrasted with solutions from the case where speeds and travel costs are taken as constants, as well as the case where speed is a decision, but the cost considered to be independent of the load. For either of these cases, load-dependent speed optimization can be added as a post-processing step. Computational experiments show that combining speed and load do have an impact on the selection of routes in maritime inventory routing problems, and that proper modelling of the fuel consumption can reduce sailing costs significantly. On the test instances considered, taking into account speed while ignoring the load leads to cost savings of around 38%. Considering the fuel consumption as a function of speed and load when planning leads to additional cost savings of 28%.

1. Introduction

Maritime transport is important when it comes to trade and development. On specific routes, seaborne trade can compete with rail and road transportation when it comes to accessibility, time, cost, speed, and other constraints. For some types of products, seagoing vessels provide the only viable link between certain regions and continents.

In the past, a strong growth in world trade and development led to the need for higher speeds in shipping. This increase in speed was made possible by technological advances in hull design, hydrodynamic performance of vessels, and engine and propulsion efficiency (Psaraftis and Kontovas, 2013). However, increasing fuel prices, depressed market conditions, and the rising focus on environmental issues has shifted the attention over on the disadvantages of high speed. There is a non-linear dependency between speed and fuel consumption (Psaraftis and Kontovas, 2013). Due to the super-linear dependency between speed and fuel consumption, high vessel speed usually leads to a large increase in the fuel consumption, and hence in the total cost of cargo deliveries, and in emissions of pollutants (Psaraftis and Kontovas, 2013).

In this article, a maritime inventory routing problem (MIRP) with load-dependent speed optimization is introduced and studied. A MIRP is solved to find optimal delivery routes between producers and consumers and at the same time maintain a reasonable inventory level. The objective of the problem is to minimize total costs, which includes transportation and operation costs for each vessel. The transportation costs are to a large extent driven by the fuel consumption, which depends on the sailing speed of a vessel as well as the load of the vessel.

The main objective of this work is to determine how the introduction of load as factor of the fuel consumption influences the routes obtained when solving a MIRP. In standard MIRPs, the travel cost is taken as fixed, implying that speeds are fixed and that the cost is assumed not to depend on the load of the vessels. Some literature has studied routing problems where the travel costs depend on the speed of a vessel, while ignoring the potential influence that load levels have on the fuel consumption (Andersson et al., 2015). Another study considered load-dependent fuel consumption while optimizing the speed of vessels, but for full-load transportation, where the vessel is either totally empty or completely full (Wen et al., 2016). In the MIRP studied here, the fuel consumption depends on both the load and the speed, and both the speed and the load are themselves decisions to be made. To this

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end, we propose a mixed integer programming model to computationally tackle the problem at hand. The model linearizes the fuel consumption by discretizing the speed and load levels, and then allowing other speeds and loads through interpolation.

The rest of this article is organized as follows: Section 2 presents a short literature review. The details of the MIRP considered are given in Section 3. Then, Section 4 introduces a mixed integer programming model for the problem. Next, Section 5 presents a computational evaluation of the model, comparing it with simpler variants either not considering speed as a decision, or ignoring the relationship between load levels and travel costs. Finally, in Section 6 we draw conclusions from this work.

2. Literature review

The problem considered in this article is a MIRP where the fuel consumption, and hence the costs, are minimized by optimizing the speed and load of the vessels. In the following we review some relevant contributions from the literature on MIRPs, then some contributions related to speed optimization, and finally some relevant contributions to modelling fuel consumption. For reviews of the literature on general IRPs, see (Andersson et al., 2010; Coelho et al., 2014).

2.1. Maritime inventory routing

A MIRP is an IRP where the transportation is carried out by a seagoing vessel (Song and Furman, 2013). The literature on MIRPs is extensive, and besides introductions to the area as provided by Christiansen et al. (2009), recent surveys have been provided by Christiansen et al. (2013) and Papageorgiou et al. (2014). Andersson et al. (2010) wrote about industrial aspects of combined inventory management and routing problems and described the industrial practice. In addition, they presented a classification and a comprehensive literature survey of around 90 papers that focused on the IRP and the MIRP.

The MIRP variant studied in this article is similar to variants previously presented in the literature, but with an added speed optimization component. Agra et al. (2015) described a stochastic short sea shipping problem, where the port times and sailing times are considered as stochastic parameters. The company presented is responsible for both distribution between ports and inventory management at the ports. Later, Agra et al. (2016) studied a single product MIRP where the production and consumption rates are constant over the planning horizon. The problem presented involves a heterogenous fleet of vessels and several production and consumption ports with limited storage capacity. As in Agra et al. (2015), the weather conditions are uncertain, and this has an effect on the sailing times. Hence, the travel time between the ports is assumed to be random, following a log–logistic distribution. The authors proposed a two-stage stochastic programming problem with recourse to be able to deal with the random sailing times. Furthermore, they developed a MIP based local search heuristic to be able to solve the problem. Agra et al. (2018) considered a robust optimization model to the same variant of MIRP and presented a decomposition algorithm with several improvement strategies.

De et al. (2017) explored the use of a slow steaming policy within ship routing. They presented a mixed integer non-linear programming model, where a non-linear equation is used to capture sustainability aspects regarding the balance between fuel consumption and vessel speed. Furthermore, they included several time window constraints to enhance the service level at each port, as well as penalty costs associated with vessels arriving to early or finishing too late according to the time windows. The problem was solved heuristically with particle swarm optimization. Recently, Gocmen and Yilmaz (2018) suggested that vessel speeds should be considered in the MIRP, with the goal of reducing the fuel consumption.

2.2. Speed optimization

In traditional ship routing and scheduling problems, the speed of the vessels is fixed and the fuel consumption rate for each vessel is given (Norstad et al., 2011). However, in real life the speed of a vessel can be adjusted within some interval, and the fuel consumption per time unit can be described by a cubic function of speed. Norstad et al. (2011) presented a tramp ship routing and scheduling problem with speed optimization, where the speed of the vessels is represented as a decision variable and a multi-start local search heuristic is applied to solve the problem. To determine speed levels, a specialized method was developed that was later proven to calculate optimal speeds (Hvattum et al., 2013).

It is common to use a sequential approach when planning shipping routes, where routes are first determined as if each vessel sails with a given speed, and then sailing speeds are optimized for the given routes. Andersson et al. (2015) proposed a new modeling approach for integrating speed optimization in the planning of shipping routes and used a rolling horizon heuristic to solve the combined problem. Their work considered a real deployment and routing problem in roll-on roll-off shipping.

Wen et al. (2016) analyzed the simultaneous optimization of routing and sailing speed in full-shipload tramp shipping. The problem consists of different cargos that needs to be transported from load ports directly to discharge ports. There is a heterogenous fleet of vessels, with vessels having different speed ranges and load-dependent fuel consumption. The goal is to determine which cargo to pick up, which route each vessel should follow, and the speed the vessels should have on each leg to maximize the profit. A three-index formulation and a set packing formulation are proposed, and a branch-and-price algorithm is implemented and tested to solve the problem. While Wen et al. (2016) consider a load-dependent fuel consumption function, the ships are either full or empty, meaning that the load is fixed for a given sailing leg.

Norlund and Gribkovskaia (2017) examined how speed optimization strategies perform when accounting for varying weather conditions. They developed a simulation–optimization tool to estimate the fuel consumption of weekly schedules for supply vessels servicing offshore oil and gas installations. The fuel consumption given a fixed speed is higher in rough weather than in calm sea. Speed is optimized using a similar procedure as in (Norstad et al., 2011), and it is shown that speed optimization is still valuable under weather uncertainty.

Psaraftis and Kontovas (2013) presented a survey and a taxonomy of models in maritime transportation where speed is one of the decision variables. They discussed advantages and disadvantages of reducing the speed of vessels, related to both costs and emissions. Different fuel consumption functions are described, and the authors give examples of how inventory costs can influence the optimal speed. The taxonomy of the different models is based on predefined parameters, such as whether or not the model can find the optimal speed as a function of payload.

Psaraftis and Kontovas (2014) focused on clarifying issues in general speed optimization problems in maritime inventory routing, and then presenting models that optimize the speed of a single vessel for different routing scenarios. Fundamental parameters and considerations were incorporated in the models, such as fuel price, freight rate, inventory cost of the cargo, and the dependency of fuel consumption on payload. The authors considered the difference between solutions that optimize the economic performance and solutions that optimize the environmental performance.

Bialystocki and Konovessis (2016) suggested an approach for constructing an accurate fuel consumption and speed curve. Different factors that can affect the fuel consumption were presented and taken into consideration. An algorithm was introduced and proven to be both simple and accurate when estimating the fuel consumption.

2.3. Fuel consumption models

Andersson et al. (2015) approximated the non-linear fuel consumption function using three speed alternatives and combined these speeds to find a linear overestimation of the consumption. They assumed that the fuel consumption per time unit is approximated by a cubic function, meaning that the consumption per distance unit is quadratic. There is a certain minimum speed for the vessel at which it travels with minimum cost. In addition, a maximum speed can be achieved when there are perfect weather conditions.

Bialystocki and Konovessis (2016) introduced a regression formula with two constant values to represent the non-linear relationship between speed and fuel consumption. The function of the fuel consumption is equal to $0.1727\mu^2 - 0.217\mu$, where μ is the speed of the vessel. This function is independent of load; thus, the load is implicitly taken as a constant. As Fig. 1 shows, the fuel consumption curve is convex. Therefore, the linearization of the curve is an overestimation of the fuel consumption.

When linearizing a fuel consumption function, there can also be an additional overestimation when it comes to the non-linear dependency between time and speed, as pointed out by Andersson et al. (2015). Hence, there is also an overestimation when it comes to the travel time. This is illustrated in Fig. 2. After linearizing the travel time curve, the resulting travel time is higher than the actual time needed for the given speed.

According to Psaraftis and Kontovas (2014), the fuel consumption has a non-linear dependency on both the speed and the load of a vessel. The daily fuel consumption is equal to $k\mu^3(l+A)^{2/3}$, where k is a given constant, μ is the speed of the vessel, l is the payload and A is the lightship weight, that is, the weight of the vessel when it has no load except fuel. The daily sailing costs from this fuel consumption formula is illustrated in Fig. 3, for a single vessel with three different load levels. The part of the formula that is related to speed generates a convex function. However, the part of the formula that depends on load, generates a concave function. A linear approximation of the convex part will give an overestimation of the costs, while a linear approximation of the concave part will give an underestimation of the costs.

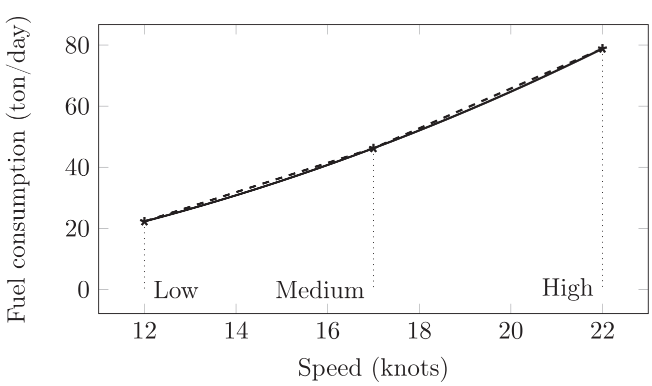

Fig. 1. The non-linear relationship between speed and fuel consumption (Bialystocki and Konovessis, 2016; Andersson et al., 2015). The dashed line is an inner approximation using three break-points.

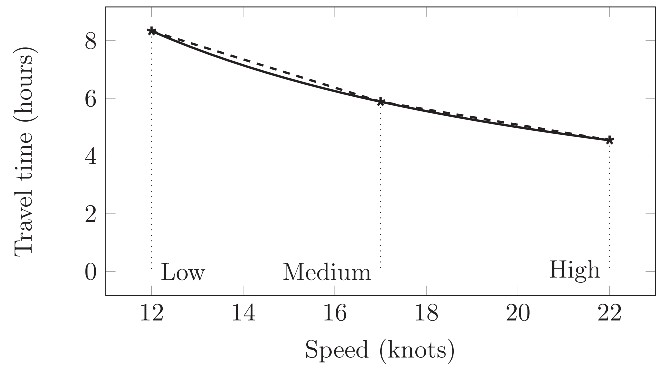

Fig. 2. The non-linear function of travel time as a function of speed (Bialystocki and Konovessis, 2016; Andersson et al., 2015). The dashed line is an inner approximation using three break-points.

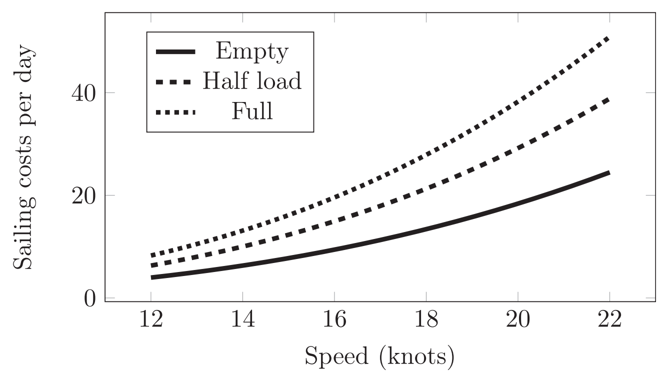

Fig. 3. The relationship between speed, load, and daily sailing costs of a vessel.

3. Problem description

This work examines how load and speed influence the cost function of a maritime inventory routing problem. A geographical region is considered, where a single product is transported between different ports in bulk. The transportation is carried out by a heterogeneous fleet of vessels, with vessels differing in terms of capacity and cost of use. Traveling distances between each pair of ports in the region are given. There are several ports, which are divided into consumption and production ports. The production facilities have fixed production rates, while the consumption facilities have fixed consumption rates, with both production and consumption being continuous over time. Vessels load the product at the production facilities and unload at the consumption facilities at a given rate that depends on the vessel type.

Both the consumption and the production facilities have inventories and hence each port has storage facilities with fixed lower and upper limits, as well as an initial inventory level. The production facilities are not allowed to exceed the maximum storage level, while the consumption ports are not allowed to have shortages. Facilities keep a steady production and consumption, so that the only way to decrease the amount of product at a production facility or to increase the amount of product at a consumption facility is by a vessel loading or unloading the product. Each port can be visited multiple times by different vessels in the planning horizon, depending on the size of its storage and the amount of product to be loaded or unloaded.

Each vessel is given a starting location and executes its route in the best possible way. Throughout the routes, each vessel transports different loads between ports in accordance with the demand and can use different operating speeds during the execution of the routes. A vessel does not need to be fully loaded, nor fully unloaded, upon visiting a port. Thus, routes can involve visiting several production ports or several consumption ports in succession. At the beginning of the planning horizon, each vessel has a given location and may have some product already loaded.

Fig. 4 shows the routes of two vessels. The black ports are production ports, and the white ports are consumption ports. The first vessel, following the solid arrows, goes to port 3 to load cargo before it sails to port 4. There it unloads before sailing to port 5 to fill up again. From port 5 it sails to port 6 to unload and ends its journey. The second vessel, following the dashed arrows, goes to port 1 to load before it sails to port 2 to unload. From there it sails to port 3 to partially fill up and continues to port 5. There it loads additional cargo, before it sails to port 6 to unload and ends its journey.

Each port has a given location, demand rate, and an individual visiting cost depending on the vessel. The vessels can operate with different speeds and loads. In addition, the vessels have different sailing costs depending on both speed and load.

The vessels differ in size to better meet the various demands. The size of the vessels is categorized by deadweight tonnage, in other words how much the vessels can transport. In addition, the vessels operate with different speeds, measured in knots. The operating speed of the vessels depends on the size of the vessel, where the larger vessels operate with higher speeds. Furthermore, the vessels have different load rates, which also is dependent on the size of the vessels.

The main objective of the problem is to minimize the total costs, which includes transportation and operating costs for each vessel in the routes they conduct. The fuel consumption, and thus also the sailing cost, depends non-linearly on the speed and load of the vessel. In particular, for a given vessel, the daily fuel consumption follows the relationship discussed by Bialystocki and Konovessis (2016), being equal to $k\mu^3(l+A)^{2/3}$, where k is a constant, μ is the speed, l is the payload and A is the lightship weight.

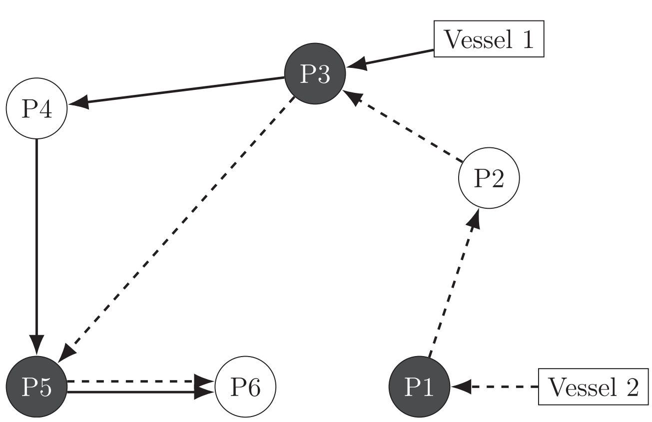

Fig. 4. Example of the structure of a maritime inventory routing problem.

4. Mathematical model

The following mathematical model is based on a model without speed or load optimization provided by Agra et al. (2016). Let V represent a set of vessels and N a set of ports. Each vessel $v \in V$ has its starting point, which can be a point at sea. Each port can have multiple visits during the time horizon. Nodes in the network are indicated by a pair (i,m), where i is a port and m is the visit number. The direct move from node (i,m) to node (j,n), is denoted as (i,m,j,n).

Fig. 5 depicts the origins and destinations of two different vessels and how they move from port to port following the same routes shown in Fig. 1. Vessel 1 starts from origin O_1 and goes to port 3 for the first visit, then sails to port 4 for the first visit, continues to port 5 for the first visit, followed by port 6 for the first visit, and ends up in destination D_1, as the route is completed. The solid line shows this. Vessel 2 starts from origin O_2 and sails to port 1 for the first visit, continues to port 2 for the first visit, continues to port 3 for the second visit, sails further to port 5 for the second visit, followed by port 6 for the second visit, and ends up in destination D_2, as the route is completed. The dashed line shows this.

In the following model vessel routes as well as load and speed of each vessel are decision variables. The model considers the minimization of cost by setting the variables under the model constraints. First we describe the sets, variables and parameters of the model.

Sets

V	:	set of vessels
N	:	set of production and consumption ports
S^A	:	set of possible nodes (i,m)
S^A_v	:	set of nodes that can be visited by vessel v
S^X_v	:	set of all possible moves (i,m,j,n) of vessel v
S^S_v	:	set of breakpoints for the speed of vessel v, with $S^S_v = \{1,2,\dots,U\}$
S^L_v	:	set of breakpoints for the load level of vessel v, with $S^L_v = \{1,2,\dots,R\}$

With these sets we can determine a maximum number of visits for each port (with S^A), prohibit some vessels to visit some ports (with S^A_v) and also prohibit some vessels to perform certain trips (with S^X_v).

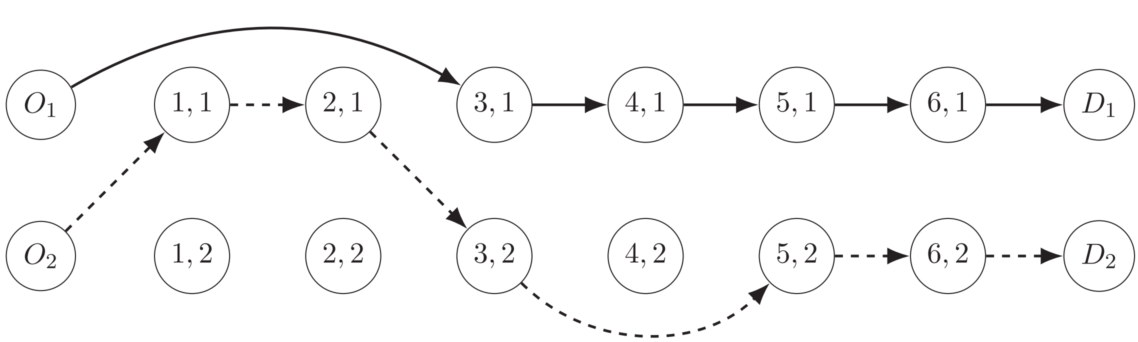

Fig. 5. Example of the resulting network and corresponding routes illustrated in Fig. 4.

Variables

x_{imjnv}	:	1 if vessel $v \in V$ moves directly between nodes (i, m) and (j, n), 0 otherwise, $v \in V, (i, m, j, n) \in S_v^X$
x_{imv}^O	:	1 if vessel v departs from its initial position to node (i, m), 0 otherwise, $v \in V, (i, m) \in S_v^A$
z_{imv}	:	1 if vessel v finishes its route at node (i, m), 0 otherwise, $v \in V, (i, m) \in S_v^A$
z_v^O	:	1 if vessel v is not used in the planning horizon, 0 otherwise, $v \in V$
q_{imv}	:	the amount loaded or unloaded by vessel v at node (i, m), $v \in V, (i, m) \in S_v^A$
f_{imjnv}	:	the amount of product that vessel v transports from node (i, m) to node (j, n), $v \in V, (i, m, j, n) \in S_v^X$
f_{imv}^O	:	the amount of product that vessel v transports from the origin to node (i, m), $v \in V, (i, m) \in S_v^A$
f_{imv}^D	:	the amount of product that vessel v transports from node (i, m) to the destination, $v \in V, (i, m) \in S_v^A$
t_{im}	:	start time of visit number m to port i, $(i, m) \in S_v^A$
o_{imv}	:	1 if vessel v operates in node (i, m), 0 otherwise, $v \in V, (i, m) \in S_v^A$
y_{im}	:	1 if there is a visit (i, m) to port i, 0 otherwise, $i \in N, (i, m) \in S^A$
s_{im}	:	stock levels at ports at the start of visit m to port i, $(i, m) \in S^A$
$g_{imjnvls}$	:	auxiliary variable to determine the speed and load of vessel v when going from (i, m) to (j, n), with s corresponding to a given choice of speed and l of a level of load, $v \in V, s \in S_v^S, l \in S_v^L, (i, m, j, n) \in S_v^X$
g_{imvls}^O	:	auxiliary variable to determine the speed and load of vessel v when going from its origin to (i, m), with s corresponding to a given choice of speed and l of a level of load, $v \in V, s \in S_v^S, l \in S_v^L, (i, m) \in S_v^A$
p_{imjnvl}	:	1 if a level of load in the interval between the two adjacent breakpoints l and $l+1$ is chosen on route (i, m, j, n) of vessel v, 0 otherwise, $v \in V, l \in S_v^L \setminus \{R\}, (i, m, j, n) \in S_v^X$
p_{imvl}^O	:	1 if a level of load in the interval between the two adjacent breakpoints l and $l+1$ is chosen on route from origin to node (i, m) 0 otherwise, $v \in V, l \in S_v^L \setminus \{R\}, (i, m) \in S_v^A$

Variables z_v^O can be used to determine whether or not a vessel is used. Variables $g_{imjnvls}$ are continuous. Constraints (19) ensure that for given values of i, m, j, n, and v, the sum of the variables $g_{imjnvls}$ is 1 if there is a trip from node (i, m) to node (j, n) of vessel v and 0 otherwise. The actual speed and load of the vessel can then be computed as a weighted sum of those values. Variables p_{imjnvl} and p_{imvl}^O are used to enforce the correct interpolation between load levels. To avoid overflow at production ports, it may be necessary to load cargo near the end of the planning horizon. Thus variables f_{imv}^D are needed to allow vessels to be non-empty at the end of the planning horizon. Corresponding variables for the movement of the vessel, or for the travel cost calculation, when moving to the destination node are not required, as the destination node is simply an artificial node, with a travel distance of 0 from the last port visited.

Parameters

T	:	number of time units in the planning horizon
H_i	:	minimum number of visits to port $i \in N$
M_i	:	maximum number of visits to port $i \in N$
D_i	:	consumption or demand at port $i \in N$ per unit of time
J_i	:	1 if production facilities are located in port i, and -1 if consumption facilities are located in port $i, i \in N$
P_{iv}	:	port cost at port $i \in N$ for vessel $v \in V$
C_v	:	capacity of vessel $v \in V$
L_v	:	initial load onboard vessel $v \in V$
$\underline{S}_i$	:	lower bound on the inventory level at port $i \in N$
$\bar{S}_i$	:	upper bound on the inventory level at port $i \in N$
S_i^0	:	the initial stock level in port $i \in N$ at the beginning of the planning horizon
A_{im}	:	earliest time for starting visit m to port $i, (i, m) \in S^A$
B_{im}	:	latest time for starting visit m to port $i, (i, m) \in S^A$
K_i	:	minimum time between two consecutive visits to port $i \in N$
$\underline{Q}_i$	:	minimum load/unload quantity in port $i \in N$
U_{im}	:	latest time for finishing visit m to port $i, (i, m) \in S^A$
T_v^Q	:	time for unloading or loading each unit by vessel $v \in V$
T_{ijvs}^{PP}	:	time required by vessel $v \in V$ to sail from port $i \in N$ to port $j \in N$ with speed $s \in S_v^S$
T_{ivs}^{OP}	:	time required by vessel $v \in V$ to sail from its origin to port $i \in N$ with speed $s \in S_v^S$
L_{vl}	:	possible levels of load $l \in S_v^L$ that can be transported on vessel $v \in V$
C_{ijvls}^{PP}	:	sailing cost from port $i \in N$ to port $j \in N$ with vessel $v \in V$ with load $l \in S_v^L$ and with speed $s \in S_v^S$
C_{ivls}^{OP}	:	sailing cost from origin to port $i \in N$ by vessel $v \in V$ with load $l \in S_v^L$, and with speed $s \in S_v^S$

With the sets, variables, and parameters defined the model can be written as follows:

$$\min \sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} \sum_{l \in S_v^L} \sum_{s \in S_v^S} C_{ijvls}^{PP} g_{imjnvls} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} \sum_{l \in S_v^L} \sum_{s \in S_v^S} C_{ivls}^{OP} g_{imvls}^O + \sum_{v \in V} \sum_{(i,m) \in S_v^A} P_{iv} o_{imv} \tag{1}$$

The objective function (1) expresses the minimization of the sum of traveling costs between ports depending on the chosen speed and load, and operational costs in each port. This objective function is optimized over a set of constraints. We present the constraints split in different semantic groups. First we present routing constraints.

$$\sum_{(j,n)\in S_v^A} x_{jnv}^O + z_v^O = 1\, v \in V \quad (2)$$

$$o_{imv} - \sum_{(j,n,i,m)\in S_v^X} x_{jnimv} - x_{imv}^O = 0\, v \in V, (i,m) \in S_v^A \quad (3)$$

$$o_{imv} - \sum_{(i,m,j,n)\in S_v^X} x_{imjnv} - z_{imv} = 0\, v \in V, (i,m) \in S_v^A \quad (4)$$

$$\sum_{v\in V} o_{imv} = y_{im} (i,m) \in S^A \quad (5)$$

$$y_{i(m-1)} - y_{im} \geqslant 0 (i,m) \in S^A : H_i + 1 \leqslant m \leqslant M_i \quad (6)$$

$$y_{im} = 1 (i,m) \in S^A : m \in \{1,\ldots,H_i\} \quad (7)$$

Constraints (2) show that a vessel must either depart from the origin to a port or not be used at all. Constraints (3) define that if a node is visited by vessel v, the vessel must either arrive at the node from the origin or from another node. Constraints (4) ensure that if a vessel is at node i it must either leave to another node or end its route there. Constraints (5) show that a vessel can only visit node (i,m) if there are at least m visits to port i. Constraints (6) guarantee that if a port i is visited m times, then it also has been visited $m-1$ times. Constraints (7) defines the number of mandatory visits for port i. Constraints (8) and (9) are loading and unloading constraints.

$$q_{imv} \leqslant \min\left\{C_v, \bar{S}_i - \underline{S}_i\right\} o_{imv}\, v \in V, (i,m) \in S_v^A \quad (8)$$

$$\underline{Q}_i o_{imv} \leqslant q_{imv}\, v \in V, (i,m) \in S_v^A \quad (9)$$

Constraints (8) ensure the quantity loaded/unloaded cannot exceed the vessel capacity nor the port capacity. Constraints (9) show that if a vessel visits the port, then the amount loaded/unloaded should be at least equal to the minimum quantity. Constraints (10)–(13) concern the product flow.

$$f_{imv}^O = L_v x_{imv}^O\, v \in V, (i,m) \in S_v^A \quad (10)$$

$$f_{jnv}^O + \sum_{(i,m,j,n)\in S_v^X} f_{imjnv} + J_i q_{jnv} = \sum_{(j,n,i,m)\in S_v^X} f_{jnimv} + f_{jnv}^D\, v \in V, (j,n) \in S_v^A \quad (11)$$

$$f_{imjnv} \leqslant C_v x_{imjnv}\, v \in V, (i,m), (j,n) \in S_v^A \quad (12)$$

$$f_{jnv}^D \leqslant C_v z_{jnv}\, v \in V, (j,n) \in S_v^A \quad (13)$$

Constraints (10) define that if a vessel travels from the initial position, then the transported amount is equal to the initial load of the vessel. Constraints (11) guarantees that the amount of incoming product flow plus the amount loaded/unloaded must be equal to the outgoing product flow. Constraints (12) show that the product flow from port to port should be at most equal to the capacity of the vessel. Constraints (13) ensure that the product flow to the destination is at most equal to the capacity of the vessel. Constraints (14)–(18) are time constraints.

$$t_{im} - t_{i(m-1)} - \sum_{v\in V} T_v^Q q_{i(m-1)v} - K_i y_{im} \geqslant 0 (i,m) \in S^A : m > 1 \quad (14)$$

$$t_{im} + \sum_{v\in V} T_v^Q q_{imv} - t_{jn} + \sum_{v\in V}\sum_{l\in S_v^L}\sum_{s\in S_v^S} \max\left\{U_{im} + T_{ijvs}^{PP} - A_{jn}, 0\right\} g_{imjnvls} \leqslant U_{im} - A_{jn} (i,m), (j,n) \in S^A \quad (15)$$

$$\sum_{v\in V}\sum_{l\in S_v^L}\sum_{s\in S_v^S} T_{ivs}^{OP} g_{imvls}^O \leqslant t_{im} (i,m) \in S^A \quad (16)$$

$$t_{im} \geqslant A_{im} (i,m) \in S^A \quad (17)$$

$$t_{im} \leqslant B_{im} (i,m) \in S^A \quad (18)$$

Constraints (14) enforce the minimum time period between two consecutive visits of port i. Constraints (15) relates the start time associated with node (i, m) to the start time associated with node (j, n) when a vessel travels between ports i and j. Constraints (16) show that the travel time for a vessel traveling from origin should not exceed the start time of the visit to the port. Constraints (17)–(18) define time windows for the start and end time of the visits. Constraints (19)–(24) relate the speed and load variables (g), to the routing (x) and flow (f) variables.

$$\sum_{l\in S_v^L}\sum_{s\in S_v^S} g_{imjnvls} = x_{imjnv}\, v \in V, (i,m,j,n) \in S_v^X \quad (19)$$

$$\sum_{l\in S_v^L}\sum_{s\in S_v^S} g_{imvls}^O = x_{imv}^O\, v \in V, (i,m) \in S_v^A \quad (20)$$

$$0 \leqslant g_{imjnvls} \leqslant 1\, v \in V, (i,m,j,n) \in S_v^X, s \in S_v^S, l \in S_v^L \quad (21)$$

$$0 \leqslant g_{imvls}^O \leqslant 1\, v \in V, (i,m) \in S_v^A, s \in S_v^S, l \in S_v^L \quad (22)$$

$$\sum_{l\in S_v^L}\sum_{s\in S_v^S} L_{vl} g_{imjnvls} = f_{imjnv}\, v \in V, (i,m,j,n) \in S_v^X \quad (23)$$

$$\sum_{l\in S_v^L}\sum_{s\in S_v^S} L_{vl} g_{imvls}^O = f_{imv}^O\, v \in V, (i,m) \in S_v^A \quad (24)$$

Constraints (19) enforce that speed and a load of a vessel must be set for a travel from node (i,m) to node (j,n) if and only if that travel exists. Constraints (20) are equivalent to constraints (19) but consider the travel from the origin. Constrains (21) and (22) set the upper and lower bounds of variables g. Constrains (23) and (24) compute the load of an arc in terms of the load of the vessel.

The g variables are not binary, and even with constraints (19) many of them can be larger than 0 for the same trip (i,m,j,n,v). To interpolate the cost and time for different load and speed values, at most two variables representing consecutive levels of speed and at most two variables representing consecutive levels of load should be allowed to be larger than 0. Given that the fuel consumption function is convex on the speed of the vessel, the interpolation using the values of g obtains an overestimation of the cost. Then, the selection of the two speed level breakpoints closest to the real speed gives the interpolated value that minimizes the overestimation of the cost. This property is not true when it comes to the load levels due to the concave relation between load level and fuel consumption.

Constraints (25)–(31) deal with selection of breakpoints for the load level of a vessel in a travel between two nodes. They are known in the literature as special ordered set of type 2 (SOS2). Since the fuel consumption function is concave on the load of the vessel, it is necessary to introduce binary variables to ensure that, for a given trip, variables g can only be larger than 0 for at most two consecutive levels of load allowing the linear interpolation between the available breakpoints to function correctly (Williams, 2013).

$$\sum_{l\in S_v^L\setminus\{R\}} p_{imjnvl} = x_{imjnv}\, v \in V, (i,m,j,n) \in S_v^X \quad (25)$$

$$\sum_{s\in S_v^S} g_{imjnv1s} \leqslant p_{imjnv1}\, v \in V, (i,m,j,n) \in S_v^X \quad (26)$$

$$\sum_{s\in S_v^S} g_{imv1s}^O \leqslant p_{imv1}^O\, v \in V, (i,m) \in S_v^A \quad (27)$$

$$\sum_{s\in S_v^S} g_{imjnvls} \leqslant p_{imjnv(l-1)} + p_{imjnvl}\, v \in V, (i,m,j,n) \in S_v^X, l \in S_v^L \setminus \{1,R\} \quad (28)$$

$$\sum_{s\in S_v^S} g_{imvls}^O \leqslant p_{imv(l-1)}^O + p_{imvl}^O\, v \in V, (i,m) \in S_v^A, l \in S_v^L \setminus \{1,R\} \quad (29)$$

$$\sum_{s \in S_v^S} g_{imjnvRs} \leqslant p_{imjnv(R-1)}\, v \in V, (i,m,j,n) \in S_v^X \tag{30}$$

$$\sum_{s \in S_v^S} g^O_{imvRs} \leqslant p^O_{imv(R-1)}\, v \in V, (i,m) \in S_v^A \tag{31}$$

Constraints (25) ensure that if a vessel travels an arc, then a breakpoint delimiting its load level in the arc must be chosen. Constraints (26) show that the value of the speed and the load used by a vessel on route (i, m, j, n) with load between the minimum load and the first level breakpoint can only be larger than 0 if the first interval is chosen. Constraints (27) show the same as constraints (26), but from origin to node (i,m). Constraints (28) ensure that the value of the speed and the load used by a vessel on route (i, m, j, n) can only be larger than 0 if one of the intervals connected to the selected breakpoint is chosen. Constraints (29) ensure the same as constraints (28), but from origin to node (i,m). Constraints (30) guarantees that the value of the speed and the load used by the vessel on route (i,m,j,n) between the last breakpoint and the maximum load can only be larger than 0 if the last interval is chosen. Constraints (31) guarantees the same as constraint (30), but from origin to (i,m). Constraints (32)–(39) are inventory constraints.

$$s_{i1} = S_i^O + J_i D_i t_{i1}\, i \in N \tag{32}$$

$$s_{im} = s_{i(m-1)} - J_i \sum_{v \in V} q_{i(m-1)v} + J_i D_i \left(t_{im} - t_{i(m-1)}\right) (i,m) \in S^A : m > 1 \tag{33}$$

$$s_{im} + \sum_{v \in V} q_{imv} - D_i \sum_{v \in V} T_v^Q q_{imv} \leqslant \overline{S}_i (i,m) \in S^A : J_i = -1 \tag{34}$$

$$s_{im} - \sum_{v \in V} q_{imv} + D_i \sum_{v \in V} T_v^Q q_{imv} \geqslant \underline{S}_i (i,m) \in S^A : J_i = 1 \tag{35}$$

$$s_{iM_i} + \sum_{v \in V} q_{iM_i v} - D_i \left(T - t_{iM_i}\right) \geqslant \underline{S}_i i \in N : J_i = -1 \tag{36}$$

$$s_{iM_i} - \sum_{v \in V} q_{iM_i v} + D_i \left(T - t_{iM_i}\right) \leqslant \overline{S}_i i \in N : J_i = 1 \tag{37}$$

$$s_{im} \geqslant \underline{S}_i (i,m) \in S^A : J_i = -1 \tag{38}$$

$$s_{im} \leqslant \overline{S}_i (i,m) \in S^A : J_i = 1 \tag{39}$$

Constraints (32) set the stock level at the start time of the first visit to a port. Constraint (33) show that the stock level at the start of the m^{th} visit is set by to the stock level at the start of the previous visit, the load or unload operation in the previous visit and the time elapsed between the two visits. Constraints (34)–(35) guarantee that the inventory at each port is within the limit at the end of the visit. Constraints (36)–(37) defines upper and lower bounds on the inventory level until the end of the time horizon for production and consumption ports. Constraints (38)–(39) ensure that the stock level is within their limits at the start of each visit. The rest of the constraints enforce the binary or non-negative nature of the variables.

$$x_{imjnv} \in \{0,1\}\, v \in V, (i,m,j,n) \in S_v^X \tag{40}$$

$$x^O_{imv} \in \{0,1\}\, v \in V, (i,m) \in S_v^A \tag{41}$$

$$o_{imv} \in \{0,1\}\, v \in V, (i,m) \in S_v^A \tag{42}$$

$$z_{imv} \in \{0,1\}\, v \in V, (i,m) \in S_v^A \tag{43}$$

$$z_v^O \in \{0,1\}\, v \in V \tag{44}$$

$$y_{im} \in \{0,1\} (i,m) \in S^A \tag{45}$$

$$p_{imjnvl} \in \{0,1\}\, v \in V, (i,m,j,n) \in S_v^X, l \in S_v^L \setminus \{R\} \tag{46}$$

$$p^O_{imvl} \in \{0,1\}\, v \in V, (i,m) \in S_v^A, l \in S_v^L \setminus \{R\} \tag{47}$$

$$q_{imv} \geqslant 0\, v \in V, (i,m) \in S_v^A \tag{48}$$

$$f_{imjnv} \geqslant 0\, v \in V, (i,m,j,n) \in S_v^X \tag{49}$$

$$f^O_{imv} \geqslant 0\, v \in V, (i,m) \in S_v^A \tag{50}$$

$$f^D_{imv} \geqslant 0\, v \in V, (i,m) \in S_v^A \tag{51}$$

$$s_{im} \geqslant 0 (i,m) \in S^A \tag{52}$$

$$t_{im} \geqslant 0 (i,m) \in S^A \tag{53}$$

A simpler version of this model can be obtained by restricting the available speeds and levels of load of the vessels to discrete sets of values. In this case, no interpolation is needed since a fixed cost may be associated with each combination of vessel, speed, and load. To see a computational comparison between our model and the simpler model the reader is referred to Appendix A.

5. Computational study

This section describes test instances and presents computational results and analysis. The computational tests were run on a computer with 1.90 GHz Intel i5-8350U CPU with 16 GB of RAM under Microsoft Windows 10 Enterprise 64-bit version. The model was coded in AMPL and run in CPLEX 12.9.

5.1. Test instances

The computational study is based on data from seven scenarios, labelled from A to G, from which several instances are obtained. These instances and the corresponding data were obtained from (Agra et al., 2016), being modified from their introduction by Agra et al. (2013). The seven scenarios are based on real data, and vary in the number of ports (from three to six) and ships (from one to five). The instances obtained from each scenario differ in the number of days in the planning horizon (30 or 60 days), initial inventory levels, and demand rates at the ports. To separate the instances from each other, each instance has a specific name based on the characteristics of the data. The name of each instance consists of the scenario letter, the number of ports, the number of ships, the number of days in the planning horizon, and an index number. The names of all the 21 instances used are shown in Fig. 6. The instances are based on short sea shipping, with loading and unloading times being long relative to the sailing times.

Each vessel has operational characteristics which varies from instance to instance. These characteristics are its capacity, its initial load, its possible speeds and the daily sailing costs for different speeds and loads. Three vessel types are considered, with speeds in the ranges $[13.5, 19]$, $[14.4, 20]$, and $[16.2, 21]$. All vessels can perform all trips (set S_v^X has all possible trips). When speed is not optimized, it is assumed that the vessels travel at their maximum speed. In the model with speed optimization, the fuel consumption is linearized, and the breakpoints for the curves are the minimum and maximum speeds, plus a breakpoint selected as 15, 16, or 18, for the three vessel types, respectively.

The instances allow for a heterogeneous fleet of vessels. While instances labelled A only has a single vessel, instances from B to F have two vessels with different capacities and corresponding speed ranges. Instances labelled F contain three different vessels, while instances labelled G have five vessels based on four different capacity levels. In the computational study, the ships have a set of load level breakpoints, with three options for each vessel. The largest load level option equals to the capacity of the vessel, and the smallest load is 0 and represents an empty vessel. The middle breakpoint is set to exactly half of the ship capacity. The vessels are not constrained to select only from the loads and speeds represented by the breakpoints. Rather, the breakpoints are used for the linearization of the objective function and their number and distribution affect the accuracy of the model.

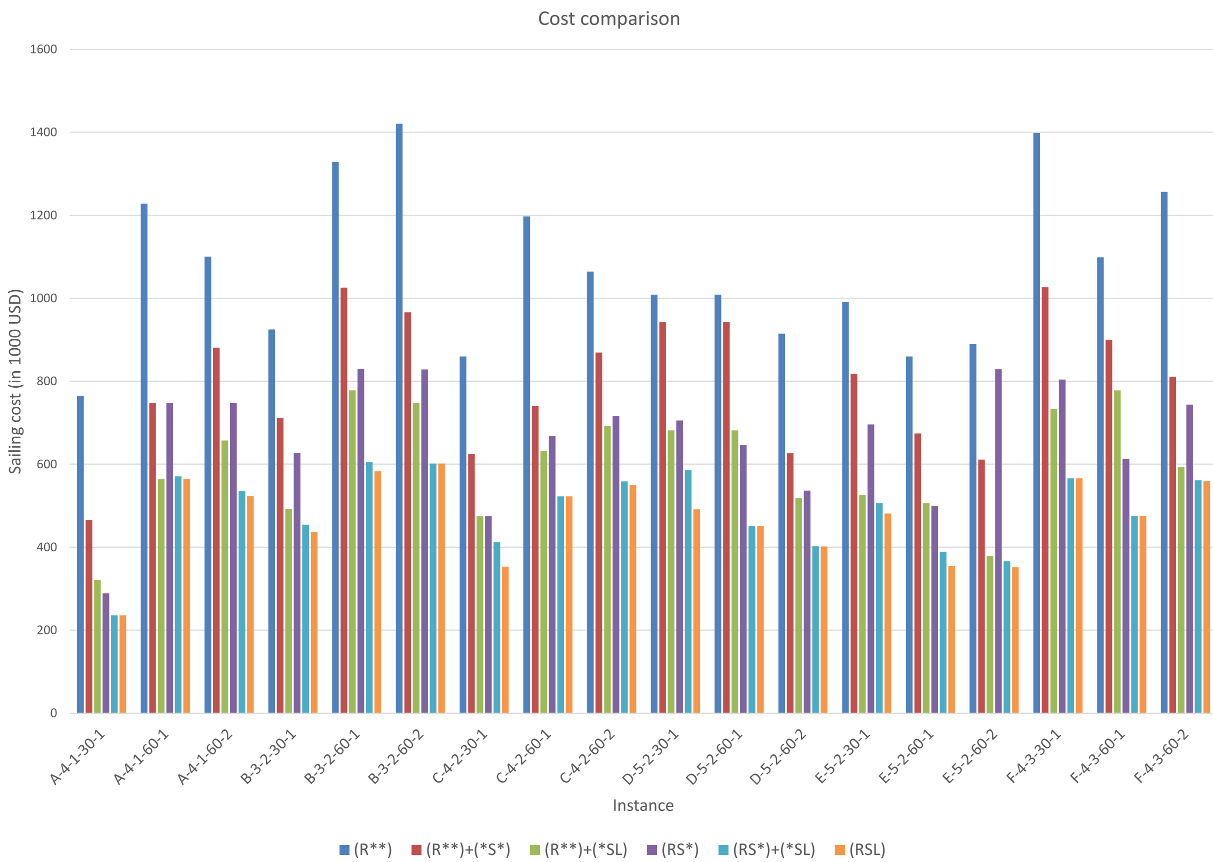

Fig. 6. Cost comparison of the different instances.

5.2. Model variations

To effectively evaluate the influence on solutions from considering load-dependent speed optimization, several variants of the main model are considered. In traditional MIRPs, the speed and the travel cost are considered as fixed. This can be emulated by solving the mathematical model from Section 4 by setting the sailing cost parameter to the fixed travel costs for all load levels, and by introducing only a single sailing speed: $|S_v^S| = 1$. In practice, this reduces to the model described in (Agra et al., 2016). We will refer to this setting as (R**), indicating that routes are optimized without taking into account speed or load. When ignoring speed and load, the cost parameters in the model are calculated assuming that the ship sails at full speed and is full.

Having obtained routes, speed can be optimized as a post-processing step. That is, by fixing all the routing variables, x in the model, the instance becomes much easier to solve and optimal sailing speeds can be determined for the given routes. If this is done without taking into account the load-dependency, it will be referred to as (R**)+(*S*), and if the post-processing is done with the full load-dependent fuel consumption curves, it will be referred to as (R**)+(*SL).

It is also possible to obtain routes allowing speed optimization and ignoring the load-dependency. This can be done by including breakpoints for speed levels, but calculating the corresponding travel costs based on a fixed load level. This setting will be referred to as (RS*), and an additional setting arises if full load-dependent speed optimization is performed as a post-processing step: (RS*)+(*SL). The full model, where routes and speeds are determined together, while taking into account the load-dependent fuel consumption, is denoted (RSL).

5.3. Solver parameter tuning

To decrease the computational time for solving the models, two CPLEX parameters were tuned using the most challenging model (RSL) and the smaller instances (A to C). The first parameter considered was the branching variable selection technique. Two settings showed the best performance in preliminary tests: Option 0, algorithm decides (CPLEX default) and option 4, based on pseudo reduced costs.

The other parameter tuned was the MIP emphasis, for which three settings showed competitive results in preliminary tests: 0: seeking optimality (CPLEX default), 1: finding feasible solutions and 3: improving best bound. Table 1 shows the CPU time in seconds needed to solve RSL model on CPLEX with different parameter settings. Given the results of the experiments, all further test were performed using the branching variable selection technique based on pseudo reduced costs and the MIP emphasis on seeking optimality.

5.4. Computational time

The emphasis in the computational study is on the effect of planning using load-dependent fuel consumption rates. To this end, a commercial mixed integer programming solver is used to solve the model presented in Section 4 as well as variants of that model. When solving a post-processing step involving fixed routes and the optimization of speed, the model is solved very quickly, with running times in the order of one second or less.

The variants of the model that include routing decisions are potentially time consuming for the instances used. Whereas, (R**) is solved in less than 1000 s for all instances, the full model

Table 1
Solution time (CPU seconds) with different parameters. Columns are identified by (branching variable selection strategy, MIP emphasis). Branching variable selection strategy options are 0: algorithm decides (default) and 4: branch based on pseudo reduced costs. MIP emphasis optoins are 0: seeking optimality (default), 1: finding feasible solutions and 3: improving best bound.

Instance	(0, 0)	(4, 0)	(0, 1)	(4, 1)	(0, 3)	(4, 3)
A-4-1-30-1	0.8	1.1	0.8	1.1	0.8	0.9
A-4-1-60-1	15.2	15.3	11.5	11.6	34.5	34.5
A-4-1-60-2	41.3	10.7	18.0	18.3	25.0	25.3
B-3-2-30-1	283.6	228.6	191.9	175.4	689.3	963.1
B-3-2-60-1	280.9	399.9	556.8	584.7	1498.8	1893.9
B-3-2-60-2	95.1	108.9	148.7	150.0	338.6	598.9
C-4-2-30-1	991.0	471.5	336.8	348.9	1471.7	700.0
C-4-2-60-1	553.7	631.7	1061.1	1078.55	1662.11	2246.53
C-4-2-60-2	10478.0	5285.5	5631.8	5751.8	9753.7	10635.5

(RSL) is difficult to solve for the larger instances. While all instances from scenarios A through F are solved to optimality, with the largest running time being 11,163 s for instance E-5–2-60–2, the instances of scenario G could not be solved to optimality for the full model. The runs of these three instances, G-6–5-30–1, G-6–5-60–1, and G-6–5-60–2, were halted after 40 CPU hours, as continuing the solver after this would lead to issues with computer memory. At this point, the optimality gaps were in the range of 5–17 %, and these instances are therefore not included in the following analyses.

5.5. Sailing costs and savings

In this section, a comparison of sailing costs and savings are presented. The sailing costs are in 1,000 US Dollars. Six methods to produce solutions are considered:

- (R**) where routes are optimized with fixed sailing costs.
- (R**)+(*S*) where (R**) is followed by optimizing speed while ignoring load.
- (R**)+(*SL) where (R**) is followed by load-dependent speed optimization.
- (RS*) where routes and speeds are optimized together while ignoring load.
- (RS*)+(*SL) where (RS*) is followed by load-dependent speed optimization.
- (RSL) where routes are optimized together with load-dependent speed.

The final solutions are evaluated using accurate sailing costs. That is, costs are calculated from the distances, visits, speeds, and loads implied by the solutions, using the formula $k\mu^3(l+A)^{2/3}$ for the sailing costs (Psaraftis and Kontovas, 2014). Fig. 6 shows the results for each of the 18 instances and each of the six solutions obtained.

It is clear that (R**) gives the highest sailing costs in every instance, especially in the three largest instances. However, in instance D-5–2-30–1 and D-5–2-60–1 the sailing costs between (R**) and (R**)+(*S*) are quite similar, which indicates that the optimal speeds found in (R**)+(*S*) are close to the ones which were fixed in (R**). It is (RSL) that provides the best solution in every instance. However, in some of the instances (RS*)+(*SL) can provide the same sailing cost as (RSL), which indicates that the routes given (RS*) are equal to the ones found by (RSL).

Table 2 shows the minimum, average, and maximum cost saving of each solution approach relative to (R**). The lowest saving is from (R**)+(*S*), but even the simple addition of a post-processing step that ignores the load-dependent fuel consumption is able to reduce the costs by more than 25 %. Optimizing routes and speeds together, while ignoring the load-dependency on fuel consumption, provides an even larger cost saving, of 38 % on average. However, the three approaches that involves some step taking into account the load-dependency all lead to greater savings than this. The largest savings is from using (RSL), leading to an average of 56 % lower costs, but even with the fixed routes from (R**), (R**)+(*SL) leads to a 44 % reduction of costs, without increasing the overall computational burden significantly compared to a model with fixed speeds.

Table 2 only includes results for instances solved to optimality. However, given that some models involve approximating non-linear functions, one may ask whether better solutions are missed due to the error in estimation. To this end, we compare the objective function values obtained from the models with the corresponding objective function values when using the non-linear fuel consumption functions. For (R**) there is no estimation error, since the model assumes that fuel consumption does not vary with speed nor load. For the models assuming that fuel consumption depends on speed but not load, such as (RS*) or (R**)+(*S*), the error due to linearization is on average 0.2 %. However, for the models where fuel consumption depends on both speed and load, such as (RSL), the approximation error is on average 0.7 %.

Table 2
Cost savings of different planning strategies, relative to (R**). The rows correspond to different initial model variations, and the columns correspond to different secondary post-processing models.

Initial		Secondary		
		None	(*S*)	(*SL)
(R**)	Min.:	0.0 %	6.6 %	29.2 %
	Avg.:	0.0 %	25.3 %	44.3 %
	Max.:	0.0 %	39.1 %	58.0 %
(RS*)	Min.:	6.9 %		42.0 %
	Avg.:	37.8 %		54.5 %
	Max.:	62.2 %		69.2 %
(RSL)	Min.:	48.4 %		
	Avg.:	56.2 %		
	Max.:	69.2 %		

Table 3
Average speed of vessels for different solution strategies.

	(R**)	(R**)+(*S*)	(R**)+(*SL)	(RS*)	(RS*)+(*SL)	(RSL)
Max. speed	21.0	18.8	18.6	16.3	16.3	16.6
Avg. speed	19.8	16.1	16.0	14.8	14.8	14.8
Min. speed	19.2	14.3	14.3	13.9	14.0	13.9
Avg. reduction	0.0 %	19.1 %	19.2 %	25.6 %	25.5 %	25.4 %

5.6. Speed of vessels

In this section, the average speed in knots of the vessels is compared. The comparison looks at the different solution approaches, and all the speeds used in the instances in total. Each vessel uses only one speed between two ports but can use different speeds for the different visits on the routes. The average speed is simply taken as the average of the speeds in each travel leg, and does not factor in the length or duration of each leg. Table 3 shows the maximum, average, and minimum speed recorded, as well as the average reduction in the average speed compared to (R**). In Table 3, all the average speeds from each of the instances are summarized and divided by the number of instances.

The highest average speed is from using (R**). This is because the vessels are assumed to operate at maximum speed in (R**), so that there is maximum flexibility in the routing decisions. In (R**)+(*S*) the predetermined routes from (R**) are used, but speed is optimized, and hence the average speed is lower in comparison to (R**). The predetermined routes from (R**) are also used in (R**)+(*SL), but decisions regarding the load of vessels can change together with the speed. As shown in Table 3, the average speed in (R**)+(*SL) is fairly similar to the average speeds provided by (R**)+(*S*). The full model (RSL) which optimizes the routes, speed, and load together, gives a lower average speed compared to the approaches starting with (R**), thus implying that taking into account the load-dependency on fuel consumption also has an effect on the routing decisions.

5.7. Load of vessels

The load of the vessels on each trip is a decision that is closely related to the chosen routes and the chosen speeds. The flow of cargo must meet the demand requirements of the inventories at the ports. Arriving at the same port at different times may lead to the ability to load or unload a different amount of cargo. It is therefore interesting to examine whether the different solutions involve transporting different amounts of product.

The average load of the vessels from the three approaches (R**)+(*SL), (RS*)+(*SL), and (RSL) are compared in the following. That is, we consider only the approaches where load is optimized together with speed in the final solution. The average load for an instance is calculated by adding up the loads of individual sailing legs and dividing it by the total number of sailing legs. Table 4 presents the average of the average load levels of each instance.

When comparing the three selected solution approaches, Table 4 shows that (RS*)+(*SL) has the highest average load level, while (RSL) has the lowest average load level. As (R**)+(*SL) and (RS*)+(*SL) optimizes load on predefined routes, the flexibility is lower when fulfilling the requirement of demand. In (RSL), routes and load (and speed) are optimized at the same time. Thus, adjustments can be made such that the vessels can carry less load and still meet the demand in the ports.

Table 4
Average load of vessels in 1000 tons.

	(R**)+(*SL)	(RS*)+(*SL)	(RSL)
Avg. load on a trip	52.8	55.6	51.4

5.8. Structural analysis of the solution

The structure of the solution can be analyzed by looking at the routing decisions. These will differ depending on whether the routes are decided with a fixed speed (R**), while optimizing speed but ignoring the effect of loads (RS*), or while taking into account load-dependent speed optimization (RSL). By examining the solutions, we find that (R**) and (RS*) leads to different routes for 12 out of 18 instances. This means that the speed of the vessels has an impact on the route structure in 66.7% of the solutions. When comparing the routes of (RLS) to the routes of (R**), we see that the structure changes in 15 out of the 18 instances. Thus, speed and load together have an impact on the route structure in 83.3% of the cases. When looking at the differences in the structure between (RS*) and (RSL), we see that the solutions based on load-dependent speed optimization differs from the optimal routes of (RS*) in 7 out of the 18 instances. This means that load alone has an impact of the routing decision in 38.9% of the instances.

6. Concluding remarks

Maritime transport plays a major role in the global trade business. To gain competitive advantages, costs reductions can be an important factor for the companies that operates within the sector. To minimize the costs, models for MIRP with speed optimization can be used to find the routes and speeds that will generate the lowest costs. Like the speed, the vessels load level do also have an impact on the fuel consumption, and hence the daily sailing costs. This dependency is non-linear, and this paper discusses a model where the fuel consumption curve is linearized in order to find solutions to the MIRP with speed and load optimization.

The computational tests conducted on the different data instances shows that speed optimization alone can generate high savings regarding the total costs. When the effect of load levels on the fuel consumption function are considered, the potential cost savings become even larger. However, even though the results show the importance of taking into account the load when determining the speeds in MIRPs, some limitations of the work remain.

First, the computational time for the largest instances is very high. With the available hardware, the commercial solver was unable to obtain optimal solutions to three of the instances considered. Second, in the computational tests, only three breakpoints are used for both speed and load. When using a relatively small number of breakpoints, the linearization leads to deviations between the approximated costs and the true costs. For fuel consumption functions that depend only on the speed, these deviations are small. However, when involving both speed and load as factors, the deviations are possibly larger. Whether introducing more breakpoints has an effect on the quality of the solutions obtained remains to be seen, but additional breakpoints will enlarge the computational challenges related to solving the full model. Future research may also consider the development of ad hoc exact and heuristic algorithms to efficiently solve the problem at hand.

Table 5
Solution time and cost of the solution found by RSL and RSL[B].

	RSL			RSL[B]		
	Seconds	Solver cost	Real cost	Seconds	Cost	Gap (%)
A-4-1-30-1	0.8	234.252	235.462	268.2	563.632	0.0
A-4-1-60-1	15.2	561.242	563.654	14400.0	–	–
A-4-1-60-2	41.3	520.466	522.919	14400.0	–	–
B-3-2-30-1	283.6	436.242	436.245	1323.8	468.462	0.0
B-3-2-60-1	280.9	578.299	582.698	5679.2	874.095	0.0
B-3-2-60-2	95.1	598.857	601.460	1352.4	Infeasible	–
C-4-2-30-1	991.0	351.090	353.070	3494.5	381.850	0.0
C-4-2-60-1	553.7	521.169	522.524	14400.0	633.162	16.8
C-4-2-60-2	10478.0	547.674	549.286	14400.0	627.145	18.9

CRediT authorship contribution statement

Line Eide: Methodology, Software, Formal analysis, Investigation, Data curation, Writing - original draft, Visualization. **Gro Cesilie Håhjem Årdal:** Methodology, Software, Formal analysis, Investigation, Data curation, Writing - original draft, Visualization. **Nataliia Evsikova:** Methodology, Software, Data curation. **Lars Magnus Hvattum:** Conceptualization, Methodology, Supervision, Writing - review & editing, Project administration. **Sebastián Urrutia:** Methodology, Supervision, Writing - review & editing, Project administration.

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Appendix A. Computational comparison with a plain discretization model

The RSL model proposed in Section 4 uses breakpoints for speeds and loads, and continuous variables to interpolate between the breakpoints. We argue that this modeling approach works better in terms of efficiency and accuracy than a simple discretization of speeds and load levels. The latter approach requires the transformation of speed and load variables into binary variables, which leads to a model that is more challenging to solve.

To show this, we performed a computational experiment with a modified version of model RSL in which variables $g_{imjnvls}$ and g^O_{imvls} are binary. In such model, variables p_{imjnvl} and p^O_{imvl} are not needed since there is no interpolation. Then, variables p_{imjnvl} and p^O_{imvl} and constraints (25)–(31) were removed from the model. We call this model RSL[B]. The breakpoints for this model was taken to be the same as for model (RSL), hence the vessels have only three options of speed and load level for each trip.

Table 5 shows the results for both models in the smaller instances under consideration. The first column shows the instance name. The next three columns show results for our original model: solution time in seconds, cost of the model (interpolated) and real cost of the solution found re-evaluated using the non-linear cost function. All solutions were proved optimal. The last three columns show results for the binary model: solving time in seconds, cost of the model (equal to the real cost since there is no interpolation) and final optimality gap after four hours of CPU time. The results for both models were obtained using CPLEX default parameters.

Model RSL is much easier to solve than RSL[B] due to the lower number of binary variables. In fact, four of the six instances with a time horizon of 60 days were not solved to optimality by the solver in 4 h of CPU time for RSL[B]. Moreover, for two of those instances no feasible solutions were found.

Since speeds and load levels are not limited to just three options in RSL, the real costs of the solutions found are better than the ones found with RSL[B]. Moreover, the interpolation error is small. For RSL[B] to provide solutions of reasonable quality, one should consider many more discretization levels. However, in that case the computational time needed to solve the model would be further increased.

Finally, note that model RSL[B] is automatically infeasible if any vessel has an initial load that does not match any of the discretized load levels available. Added to this, instances can be infeasible even if the initial loads of all vessels match an available breakpoint, as it happens in instance B-3-2-60-2. In this case, the sets of allowed speeds and loads are simply not sufficient to obtain a feasible solution.

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