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# APPLICATION OF IMAGE ANALYSIS TO THE IDENTIFICATION OF MASS INERTIA MOMENTUM IN ELECTROMECHANICAL SYSTEM WITH CHANGEABLE BACKLASH ZONE 


#### Abstract

This paper presents a new method of identification of inertia moment of reduced masses on a shaft of an induction motor drive being a part of an electromechanical system. The study shows the results of simulations performed on the tested model of a complex electromechanical system during some changes of a backlash zone width. An analysis of wavelet scalograms of the examined signals carried out using a clustering technique was applied in the diagnostic algorithm. The correctness of the earliest fault detection has been verified during monitoring and identification of mass inertia moment for state variables describing physical quantities of a tested complex of the electromechanical system.


## 1. INTRODUCTION

Detection of undesirable changes of states of electromechanical processes takes the form of a sequence of intentional operations executed in assumed time using a set of machines and devices at a determined number of available resources. Faults and other destructive events can be the reasons for these changes of states (Korbicz, Kościelny \& Kowalczuk, 2002).

[^0]In electromechanical systems, such detection is easier due to the possibility of examining the signals simultaneously in the time and frequency, which facilitates the selection of information and, subsequently, correct interpretation of the features that assign the caught signals to different undesirable states of the system indicating the appearance of the fault (Duda, 2007; Zając, 2009).

Recognition of images consists in detection, interpretation, and identification of the correct features in a certain set of the examined object. The process of ascribing the object to a single class is named identification, whereas the ascribing of objects to the respective classes is named classification (Looney, 1997).

There are several scientific studies on the new techniques of detection of faults in dynamic systems using time-frequency methods as well as image analysis. Among these papers, some are very recent (they were published in the last three years) and some are older. The following ones are worth mentioning:

- using the theory of waves in the identification of dynamic systems (Hasiewicz \& Śliwiński, 2005; Wysocki, 2006),
- a comparative analysis of time-frequency methods in diagnostics of electric drives (Sajjad, Zaidi, Zanardelli, Aviyente \& Strangas, 2007),
- analysis of the possibility of application of discrete wavelet transformation to the envelope of stator's current to the early detection of coil short circuits in induction motor indicating a fault (Wolkiewicz \& Kowalski, 2015),
- sing of discrete packet wavelet analysis of signal of stator's current in the detection of faults in windings of stator's induction motor (Mathew, 2017),
- detection of rotor's cracks in induction motor using the algorithm of recognition of binary image created as a result of the transformation of the tested vector of the stator's current (Divdel, Moghaddam \& Alipour, 2016),
- presentation of diagnostics of faults in three-phase induction motor using the method of extraction of features from the thermograms registered for the assumed states of working (Głowacz \& Głowacz, 2017).


## 2. PRESENTATION OF MODEL OF THE INDUCTION MOTOR DRIVE USED IN SIMULATION TESTS OF FAULT IDENTIFICATION

The diagnostic tests have been conducted by means of a working machine in the form of a dynamic mass-absorbing-elastic element being a load to the induction motor. The nominal operating conditions of the induction motor were set, and the model has been placed in a stationary coordinate system related to the stator (model $\alpha, \beta, 0$ ). The gradually increasing backlash zone occurs on the rod as a result of the line's slip on the surface of the drive wheel of the working machine.

Fig. 1 shows a simplified form of a diagram of the tested complex electromechanical system.


Fig. 1. Diagram of the tested complex electromechanical system - the induction motor drive is connected with the dynamic mass-absorbing-elastic element with the clutch

The following parameters of the induction motor were selected for the examination (parameters of its equivalent circuit are expressed in relative units): circuit stator relative resistance $r_{s}=0.059[\Omega]$, circuit rotor relative resistance $r_{w}=0.048[\Omega]$, relative reactance of the dispersion circuit stator $x_{s}=1.92[\Omega]$, relative reactance of the dispersion circuit rotor $x_{w}=1.92[\Omega]$, relative reactance of the dispersed circuit $x_{m}=1.82[\Omega], w=x_{s} \cdot x_{w}-x_{m} \cdot x_{m}=0.374$, mechanical time constant $T_{m}=0.86$ [s]. All simulation tests have been conducted using MATLAB/Simulink environment.

## 3. DESCRIPTION OF EXECUTION OF IDENTIFICATION TESTS OF THE MASS INERTIA MOMENT IN THE ELECTROMECHANICAL SYSTEM CONTAINING FRICTION DESCRIBED BY MEANS POWER EQUATION OF OSTWALD AND DE WAELE

The tests have been carried out in four test groups, with the following four different values of apparent viscosity coefficient $\eta_{k}: 0.0125\left[\mathrm{~Pa} \cdot \mathrm{~s}^{n_{1}}\right], 0.025\left[\mathrm{~Pa} \cdot \mathrm{~s}^{n_{1}}\right]$, $0.0375\left[\mathrm{~Pa} \cdot \mathrm{~s}^{n_{1}}\right]$, and $0.05\left[\mathrm{~Pa} \cdot \mathrm{~s}^{n_{1}}\right]$. Each test group contained seven cases to study with different inertia moment values. The results of simulations for all the physical quantities and for every case of change of inertia moment of reduced masses and connected stiffly with the induction motor drive rotor $J_{1}$ have been written in the matrix $N_{177,2048}$. The elements of the matrix $N_{1}$ have been written for each consistency index $\eta_{k}$. Value of an inertia moment $J_{1}$ have been determined as the percentage in relation to its nominal value $J_{1}$, down at value $\mathrm{A} \%$ and up at value $\mathrm{C} \%$. Formal changes of inertia moment $J_{1}$ have been written in matrix $K_{1}$ in the following order: $K_{1}=\left[\right.$ nominal value of the inertia moment $\left(J_{1}=1.16\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]\right)$,
$\mathrm{A}=2.5 \%\left(J_{1}=1.131\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]\right), \mathrm{A}=5 \%\left(J_{1}=1.102\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]\right), \mathrm{A}=7.5 \%$ $\left(J_{1}=1.073\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]\right), \mathrm{A}=17.5 \%\left(J_{1}=0.957\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]\right), \mathrm{C}=2.5 \%\left(J_{1}=1.189\right.$ $\left.\left.\left[\mathrm{kg} \cdot \mathrm{m}^{2}\right]\right), \mathrm{C}=5 \%\left(J_{1}=1.218\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]\right)\right]$.

In each of the seven cases of changes of inertia moment $J_{l,}$, simulation tests have been conducted for six backlash zone widths. The backlash zone width values have been taken in sequence from the matrix $K_{2}$, in the following order: [ $0.0025,0.00375,0.005,0.0075,0.009,0.01]$. All tests have been executed for the creep index $n_{1}$ value equal 0.93 .

The wavelet type and its order have been selected in such a way that the shape of the basic wavelet approximately would be adequate to the character of the transient course of the tested physical quantity obtained as a result of a simulation for the case of the smallest backlash zone width. Based on the performed tests, the following selections of wavelets have been made for individual physical variables, with decomposition level 10 :
a) linear acceleration of the induction motor drive $a_{1}$-sym5,
b) electromagnetic moment of the induction motor drive $m_{e l}-\mathrm{db} 6$,
c) angular speed of the induction motor drive rotor $\omega_{1}-\operatorname{sym} 5$,
d) linear acceleration of a mass $a_{2}-\mathrm{db} 6$,
e) linear speed of a mass $v_{2}-\operatorname{sym} 5$.

In all the conducted simulations it was assumed that the test of dynamics of the electromechanical system in backlash zone starts precisely when the expression specified in the left part of the following inequality (1) is smaller than its right part:

$$
\begin{equation*}
\left|\alpha_{1}-\alpha_{2}\right|<\frac{K_{2(i)}}{r} ; \quad i=1,2 . .6 \tag{1}
\end{equation*}
$$

where: $r$ - radius of the drive wheel of a working machine [m], $K_{2(i)}-$ value that has been taken sequentially from the $K_{2}$ matrix and corresponding to the given backlash value in mechanical connection,
$i$ - an index number within the $K_{2}$ matrix.
The location change of the angle $\alpha_{1}$ for rotating masses of the induction motor drive [rad] has been calculated using the formula: $\alpha_{1}=\int \omega_{1} \cdot d t$, and the location change of the angle $\alpha_{2}$ of the drive wheel of the working machine [rad], has been calculated using the formula: $\alpha_{2}=\int \omega_{2} \cdot d t$, where $\omega_{1}$ and $\omega_{2}$ stand for angular speeds. The former refers to the angular speed of rotating masses of the induction motor drive $[\mathrm{rad} / \mathrm{s}]$, and the latter to the speed of the drive wheel of the working machine $[\mathrm{rad} / \mathrm{s}]$.

After satisfying the condition determined by inequality described by the formula (1) the load moment of the induction motor drive is set to zero in the tested electromechanical system.

In all the conducted simulation tests it was assumed that the tested matrix $N_{1}$ would contain 2000 samples of a signal, chosen in sequence from the moment of appearance of backlash zone in the tested model of the electromechanical system; additionally, 48 samples preceding this moment would be included. Therefore, the total number of samples will amount to the value of $2^{11}=2048$, which enables effectively to carry out a multiresolution analysis.


Fig. 2. Wavelet scalogram registered for test in which connection between the induction motor drive and the mechanical system loading him has been cut off until moment of backlash's taking.

Fig. 2 presents the wavelet scalogram depicting the linear acceleration of the induction motor drive $a_{1}$ with the inertia moment amounting to the value of true mass inertia moment $J_{1}$ decreased by about $7.5 \%$. This simulation test has been conducted with backlash zone width $=0.005$ at the consistency's coefficient $\eta_{k}=0.025$ and the melt flow index $n_{1}=0.93$.

## 4. DESCRIPTION OF THE DIAGNOSTIC ALGORITHM USED IN PROCESSING OF THE WAVELET SCALOGRAMS BY MEANS OF IMAGE ANALYIS METHOD WITH USING OF A CLUSTERING TECHNIQUE

According to the presented diagnostic procedure, the values of inertia moment $J_{1}$ with the assumed backlash zone width values of matrix $X_{3}$ are determined using normalization of values of matrix's elements $X_{2}$.

For all the tested physical quantities, values of elements of matrix $X_{3}$ have been calculated as the transformation of the values $X_{2}$ to the range [ $k_{3}, 2 \cdot k_{3}-h_{3}$ ] according to the formula:

$$
\begin{gather*}
X_{3(e)(a, b)}=\left(\frac{\left(X_{2(e)(a, b)}-h_{2}\right)}{k_{2}-h_{2}}\right) \cdot\left(k_{3}-h_{3}\right)+k_{3} ;  \tag{2}\\
\quad a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in<1,6>
\end{gather*}
$$

where: $X_{2}$ - values of matrix's elements calculated in the test,
$h_{2}$ - the minimal value of elements of matrix $X_{2}$ determined in the test,
$k_{2}$ - the maximal value of elements of matrix $X_{2}$ determined in the test,
$h_{3}$ - the initial value of the range containing normalized values $X_{2}$ in the test,
$k_{3}$ - the final value of the range containing normalised values $X_{2}$ in the test.
Therefore, the values of variables $h_{2}$ and $k$ have been calculated according to the following formulas:

$$
\begin{align*}
& h_{2}=\min \left(X_{2(e)(a, b)}\right) ; a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in\langle 1,6\rangle  \tag{3}\\
& k_{2}=\max \left(X_{2(e)(a, b)}\right) ; a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in\langle 1,6> \tag{4}
\end{align*}
$$

For all the tested physical quantities, values of elements of matrix $X_{2}$ have been calculated as the transformation of pixel colour intensities comprised in matrix $X_{1}$ to the range [ $k_{3}, 2 \cdot k_{3}-h_{3}$ ].

Normalization of values of the elements of the matrix $X_{1}$ used in simulation tests conducted at consistency coefficient $\eta_{k}=0.025$ with the assumed backlash zone width can be described according to the formula:

$$
\begin{equation*}
X_{2(e)(a, b)}=\left(\frac{\left(X_{1(e)(a, b)}-h_{1}\right)}{k_{1}-h_{1}}\right) \cdot\left(k_{3}-h_{3}\right)+k_{3} ; \quad a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in<1,6> \tag{5}
\end{equation*}
$$

where: $X_{1}$ - values of the elements of the matrix recorded in the test conducted with the nominal value of inertia moment $J_{1}$ (in the case of the undamaged drive with the induction motor),
$h_{1}$ - the minimal value of the elements of the matrix $X_{1}$ determined in the test,
$k_{1}$ - the maximal value of the elements of the matrix $X_{1}$ determined in the test,
$h_{3}$ - the initial value of the range containing normalized values $X_{1}$ determined in the test in which one assumed consistency coefficient $\eta_{k}=0.025$ and nominal value of inertia moment $J_{1}$,
$k_{3}$ - the end value of the range containing normalised values $X_{1}$ determined in the test in which one assumed consistency coefficient $\eta_{k}=0.025$ and nominal value of inertia moment $J_{1}$.

Calculations of values of variables $h_{1}$ and $k_{l}$ have been performed using the following formulas:

$$
\begin{align*}
& h_{1}=\min \left(X_{1(e)(a, b))} ; a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in\langle 1,6\rangle\right.  \tag{6}\\
& k_{1}=\max \left(X_{1(e)(a, b)}\right) ; a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in\langle 1,6\rangle \tag{7}
\end{align*}
$$

Values of variables $h_{3}$ and $k_{3}$, which represent the beginning and end of the range to which the values of the elements of matrix $X_{2}$ have been transformed. For all the tested physical quantities, they have been determined using the minimal and maximal value, respectively, according to the formulas:

$$
\begin{align*}
& h_{3}=\min \left(h_{4}, k_{4}\right)  \tag{8}\\
& k_{3}=\max \left(h_{4}, k_{4}\right) \tag{9}
\end{align*}
$$

where: $h_{4}, k_{4}$ - values of variables obtained as a result of calculations conducted using variables calculated for parts of samples recorded in the test.

On the basis of results obtained in the series of tests, it was decided to divide of calculation of values of variables $h_{4}$ and $k_{4}$ into two cases for every simulation test conducted with the assumed backlash zone width:

- the first case concerns linear acceleration on the circuit of the drive wheel of the rotor $a_{1}$ and linear speed of the mass $v_{2}$,
- the second case concerns the electromagnetic moment of the induction motor $m_{e l}$, angular speed of the rotor $\omega_{1}$ and linear acceleration of the mass $a_{2}$.

In the first case, values of variables $h_{4}$ and $k_{4}$ have been calculated according to the following formulas:

$$
\begin{equation*}
h_{4}=\min \left(X_{1(e)(j)}\right)-\max \left(X_{1(e)(\mathrm{j})}\right) ; e \in\langle 1,6\rangle ; j=1,2 \ldots 2048 ; \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
k_{4}=\min \left(X_{1(e)(j)}\right)+\max \left(X_{1(e) \mathrm{j})}\right) ; e \in\langle 1,6>; j=1,2 \ldots 2048 \tag{11}
\end{equation*}
$$

In the second case, calculations of differences between the minimal and maximal values of arithmetic means of appropriate parts of the samples chosen from the matrix $N_{1}$ have been applied. However, values of variables $h_{4}$ and $k_{4}$ have been calculated, respectively, according to the formulas:

$$
\begin{gather*}
h_{4}=\left(m_{e 2}-m_{d 2}\right)+\left(m_{e 2}-m_{d 2}\right)+\left(m_{e 3}-m_{d 3}\right)  \tag{12}\\
k_{d 4}=\left(m_{e 1}-m_{d 1}\right)-\left(m_{e 2}-m_{d 2}\right)-\left(m_{e 3}-m_{d 3}\right)-\left(m_{e 4}-m_{d 4}\right) \tag{13}
\end{gather*}
$$

The minimal values $m_{e 1}, m_{e 2}, m_{e 3}$, and $m_{e 4}$ used to determine the values of variables $h_{4}$ and $k_{4}$ have been calculated by means of the following formulas:

$$
\begin{align*}
& m_{e 1}=\min \left(N_{1(e)(\mathrm{j})}\right) ; e \in<1,6>; j=1,2 \ldots 768 ;  \tag{14}\\
& m_{e 2}=\min \left(N_{1(e) \mathrm{f})}\right) ; e \in\langle 1,6\rangle ; j=1,2 \ldots 1024 ;  \tag{15}\\
& m_{e 3}=\min \left(N_{1(e) \mathrm{j})}\right) ; e \in\langle 1,6\rangle ; j=1,2 \ldots 1280 ;  \tag{16}\\
& m_{e 4}=\min \left(N_{1(e) \mathrm{f})}\right) ; e \in<1,6>; j=1,2 \ldots 1536 ; \tag{17}
\end{align*}
$$

The values of arithmetic means $m_{d 1}, m_{d 2}, m_{d 3}$ and $m_{d 4}$ have been calculated in the following way:

$$
\begin{align*}
& m_{d 1}=\frac{\sum_{j=1}^{768} N_{1(e)(j)}}{768} ; e \in<1,6>  \tag{18}\\
& m_{d 2}=\frac{\sum_{j=1}^{1024} N_{1(e)(j)}}{1024} ; e \in<1,6>  \tag{19}\\
& m_{d 3}=\frac{\sum_{j=1}^{1280} N_{1(e)(j)}}{1280} ; e \in<1,6>  \tag{20}\\
& m_{d 4}=\frac{\sum_{j=1}^{1536} N_{1(e)(j)}}{1536} ; \quad e \in<1,6> \tag{21}
\end{align*}
$$

Values of elements of matrix $N_{1}$ have been determined according to the following order:

$$
N_{1(e)(\mathrm{j})}=\left[M_{(e)(1)}>=M_{(e)(2)}>=\ldots .>=M_{(e)(2048)}\right] ; e \in<1,6>; j=1,2 \ldots 2048 \text { (22) }
$$

where: $M$ - values of the elements of matrix $M$ recorded in the test.
In the presented diagnostic method, determining of values of pattern matrices used in the identification of inertia moment $J_{1}$ was essential. Calculations in the algorithm were conducted according to the clustering method using $k$-harmonic means technique (Frąckiewicz, 2011).

An important element of the diagnostic method was conducting the most profitable initialisation of preliminary values of centres of clusters, membership function and weight function.

The initial values of centres of clusters used in simulations have been calculated by means of the value of variable $k_{3}$ obtained as a result of simulations conducted with consistency coefficient $\eta_{k}=0,025\left[\mathrm{~Pa} \cdot \mathrm{~s}^{n_{1}}\right]$ during which the inertia moment $J_{1}$ was changed at the assumed backlash zone width.

In simulations conducted by means of the described method, values of inertia moments $J_{1}$ have been used and chosen in sequence and according to the order assumed in matrix $K_{1}$.

Calculations of the initial values of centres of clusters $S$ conducted according to the following formula:

$$
\begin{equation*}
S_{(e)(i)}=1-k_{3} ; \quad e \in<1,6>; i=1,2 \ldots 7 \tag{23}
\end{equation*}
$$

where: $k_{3}$ - the value of the variable determined according to the formula (8) by means of values of the matrix elements $X_{1}$ in simulations conducted for all the assumed changes of inertia moment $J_{1}$.

Values of membership function have been written in matrix $M$, whereas the values of weight function have been written in matrix $W$. The initial values in both matrices were set equal to 1 and determined in the following way:

$$
\begin{gather*}
M_{(i)(a, b)}=1 ; \quad a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; i=1,2 \ldots 7  \tag{24}\\
W_{(a, b)}=1 ; \quad a=1,2 \ldots 343 ; \quad b=1,2 \ldots 436 \tag{25}
\end{gather*}
$$

In the next step of the algorithm, the above-described initial values of membership function $M$ and weight function $W$ have been changed using the following formulas (Frąckiewicz, 2011):

$$
\begin{gather*}
W_{(a, b)}=\frac{\sum_{i=1}^{7}\left\|X_{3(e)(a, b)}-S_{(e(i))}\right\|^{-p-2}}{\left(\sum_{i=1}^{7}\left\|X_{3(e)(a, b)}-S_{(e)(i)}\right\|^{-p}\right)^{2}} ;  \tag{26}\\
a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in\langle 1,6\rangle ; p\rangle=2 \\
M_{(i)(a, b)}=\frac{\left\|X_{3(e)(a, b)}-S_{(e(i)}\right\|^{-p-2}}{\sum_{i=1}^{7}\left\|X_{3(e)(a, b)}-S_{(e(i) i}\right\|^{-p-2}} ;  \tag{27}\\
a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in\langle 1,6\rangle ; i=1,2 \ldots 7 ; p\rangle=2
\end{gather*}
$$

where: $p$ - additional optimisation parameter.
In the applied diagnostic method, the "soft" values of the membership function $M$ have been calculated and met the following conditions (Frąckiewicz, 2011):

$$
\begin{gather*}
\left.0<=M_{(i)(a, b)}<=1 ; a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; e \in\langle 1,6\rangle ; i=1,2 \ldots 7 ; p\right\rangle=2  \tag{28}\\
\sum_{i=1}^{7} M_{(i)(a, b)}=1 ; \quad a=1,2 \ldots 343 ; b=1,2 \ldots 436 ; i=1,2 \ldots 7 \tag{29}
\end{gather*}
$$

Changes in the values of the membership function and the weight function have a significant impact on the values of the centres of clusters. It is then followed by determining a degree of membership of every value of the element of matrix $X_{3}$ to the particular clusters.

After updating of the values of the membership function $M$ and weight function $W$, new values of centres of clusters $S$ have been determined by means of the following formula (Frąckiewicz, 2011):

$$
\begin{equation*}
S_{(e)(i)}=\frac{\sum_{a=1}^{343}=\frac{\sum_{b=1}^{336}}{3 \times 1}\left(M_{(i)(a, b)} \cdot W_{(e)(a, b)} \cdot X_{3(e)(a, b)}\right)}{\sum_{a=1}^{3435} \sum_{b=1}^{436}\left(M_{(i)(a, b)} \cdot W_{(e)(a, b)}\right)} ; \quad e \in<1,6>; i=1,2 \ldots 7 \tag{30}
\end{equation*}
$$

In the conducted simulation tests, creation of pattern matrices as well as tested matrices have been conducted at the same assumed backlash zone width for all the tested physical quantities.

Values of all the elements of each pattern matrix have been calculated in the series of simulation tests in which the assumed consistency coefficient $\eta_{k}=0.025$.

These values have been written to 7 x 7 matrix $N_{2}$. The tested values have been written into the 1 x 7 matrix $N_{3}$.

Values of the matrix's elements $S$ have been written to the matrix containing pattern values $N_{2}$ and to the tested matrix $N_{3}$. Therefore, matrices $N_{2}$ and $N_{3}$ have been determined according to the following formulas:

$$
\begin{gather*}
\left.N_{2(e)(f, i)}=S_{(e)(f, i)}\right) ; e \in\langle 1,6\rangle ; f=1,2 \ldots 7 ; i=1,2 \ldots 7  \tag{31}\\
N_{3(e)(i)}=S_{(e)(i)} ; e \in\langle 1,6\rangle ; i=1,2 \ldots 7 \tag{32}
\end{gather*}
$$

Determination of the values of the elements of matrix $H$ enables one for correct identification of the value of inertia moment for the calculated wavelet scalogram of the physical quantity at the particular consistency coefficient $\eta_{k}$.

Values of matrix $H$ can be expressed by means of the formula:

$$
\begin{equation*}
H_{(f)}=h_{5} \cdot \sum_{i=1}^{7}\left|N_{2(f, i)}-N_{3(i)}\right| ; \quad f=1,2 \ldots 7 ; i=1,2 \ldots 7 \tag{33}
\end{equation*}
$$

where: $h_{5}$ - the value of the variable used to improve the identification results.
Using of variable's value $h_{5}$ in calculations of matrix $H$ clearly improves the results of the fault identification. The value of this variable causes the increase in the value obtained as a result of the summation of absolute values of differences between the values of the matrix containing pattern values $N_{2}$ and values of the tested matrix $N_{3}$.

The most profitable for identification simulations variable's value $h_{5}$ calculated for tests conducted with all set changes of value of inertia moment $J_{1}$ for assumed backlash zone width and in the group of analysis with the adopted consistency coefficient $\eta_{k}=0.025$.

To calculate the variable's value $h_{5}$ the following formula is used:

$$
\begin{equation*}
h_{5}=\frac{\max \left(S_{(e)(i)}, S_{1(e)(i)}\right)}{\min \left(\left|h_{3}\right|, k_{3}\right)} ; \quad e \in<1,6>; i \in<1,7> \tag{34}
\end{equation*}
$$

Calculations of values of matrix's elements $S_{I}$ were conducted employing the same methodology as for calculations of values of matrix $S$. It can be described by the formula:

$$
\begin{equation*}
S_{1(e)(i)}=1-h_{d 3} ; \quad e \in<1,6>; i=1,2 \ldots 7 \tag{35}
\end{equation*}
$$

where: $h_{3}$ - the value of a variable determined according to the formula (8) by means of values of elements of matrix $X_{1}$ for simulation tests conducted for all the assumed changes of values of inertia moment $J_{1}$.

Index $n r$ (for $n r \in\langle 1,7\rangle$ ) in matrix $H$ determines the column number in the matrix $K_{1}$, which contains the correct inertia moment of reduced masses on the shaft of induction motor drive. The value of index $n r(n r \in\langle 1,7\rangle$ ) for $i=1,2 \ldots 7$ in matrix $H$ has been determined using the following minimum function: $H_{(r r)}=\min \left(H_{(i)}\right)$. Therefore, the number $i$ of a column in the matrix $K_{1}$ refers to the corresponding to it index $n r(i=n r)$.

## 5. RESULTS OF SIMULATIONS OF THE DIAGNOSTIC ALGORITHM APPLIED FOR IDENTIFICATION OF INERTIA MOMENT OF REDUCED MASSES ON SHAFT OF INDUCTION MOTOR DRIVE IN ELECTROMECHANICAL SYSTEM

In the tables presented below, in the column named Test Parameters, backlash zone widths have been placed assumed in the process of identification of the value of inertia moment of reduced masses connected stiffly with the motor's rotor $J_{l}$. Besides, in the following tables, one bolded the correct results that were obtained finally in the process of identification of fault and that are the final results of calculations of matrix $H$.

For all tested physical quantities, pattern matrices $N_{1}$ have been created in a series of tests in which the assumed value of consistency coefficient $\eta_{k}$ was equal to $0.025\left[\mathrm{~Pa} \cdot \mathrm{~s}^{n_{1}}\right]$. Pattern matrices $N_{1}$ have been created for analyses in which an additional optimisation parameter $p$ has been adopted corresponding to the assumed value of this variable. This value has been placed in the tables presented below.

Tab. 1. Exemplified results of tests in matrix $\boldsymbol{H}$ for linear acceleration of the induction motor drive $a_{1}$

| Test parameters | Results | Test parameters | Results |
| :---: | :---: | :---: | :---: |
| inertia moment | 9.5409 | inertia moment | 9.4615 |
| $J_{1}=0.957$, | 8.4485 | $J_{1}=0.957$, | 7.3730 |
| backlash zone $=0.00375$, | 7.3105 | backlash zone $=0.00375$, | 5.9764 |
| $\eta_{k}=0.0375$, | 6.0374 | $\eta_{k}=0.0375$, | $\mathbf{0 . 0 6 7 5}$ |
| optimisation parameter | $\mathbf{0 . 0 6 7 1}$ | optimisation parameter | 10.4579 |
| $p=2$ | 10.5406 | $p=3$ | 11.3810 |
| 1.4658 |  | 0.0150 |  |
| inertia moment | 0.0115 | inertia moment | $\mathbf{0 . 0 0 0 2}$ |
| $J_{1}=1.131$, | $\mathbf{0 . 0 0 0 1}$ | $J_{1}=1.131$, | 0.0166 |
| backlash zone $=0.01$, | 0.0128 | backlash zone $=0.01$, | 0.0357 |
| $\eta_{k}=0.0375$, | 0.0276 | $\eta_{k}=0.0375$, | 0.1333 |
| optimisation parameter | 0.1080 | optimisation parameter | 0.0276 |
| $p=3$ | 0.0212 | $p=9$ | 0.0380 |

Tab. 2. Exemplified results of tests in matrix $\boldsymbol{H}$ for electromagnetic moment of the induction motor drive $\boldsymbol{m}_{e l}$

| Test parameters | Results | Test parameters | Results |
| :---: | :---: | :---: | :---: |
| inertia moment | 0.7347 | inertia moment | 1.0650 |
| $J_{1}=1.102$, | 0.3730 | $J_{1}=1.102$, | 0.5381 |
| backlash zone $=0.0075$, | $\mathbf{0 . 0 0 1 2}$ | 0.4137 | backlash zone $=0.0075$, |
| $\eta_{k}=0.0125$, | $\eta_{k}=0.0125$, | 0.5904 |  |
| optimisation parameter | 2.4923 | optimisation parameter | 3.4786 |
| $p=2$ | 1.0490 | 1.5264 |  |
|  | 1.3497 |  | 1.9718 |
|  | 0.4008 | inertia moment | 0.3424 |
| inertia moment | 0.8187 | $J_{1}=1.189$, | 0.7000 |
| $J_{1}=1.189$, | 1.3102 | backlash zone $=0.01$, | 1.1203 |
| backlash zone $=0.01$, | 1.7962 | $\eta_{k}=0.05$, | 1.5364 |
| $\eta_{k}=0.05$, | 4.2535 | optimisation parameter | 3.6411 |
| optimisation parameter | $\mathbf{0 . 0 0 5 0}$ | $p=9$ | $\mathbf{0 . 0 0 4 6}$ |
| $p=3$ | 0.3993 |  | 0.3415 |

Tab. 3. Exemplified results of tests in matrix $H$ for angular speed of the rotor of the induction motor drive $\omega_{1}$

| Test parameters | Results | Test parameters | Results |
| :---: | :---: | :---: | :---: |
| inertia moment | 0.8648 |  | 0.8659 |
| $J_{1}=1.218$, | 1.3365 | inertia moment | 1.3383 |
| backlash zone $=0.005$, | 1.8056 | $J_{1}=1.218$, | 1.8080 |
| $\eta_{k}=0.0125$, | 2.3249 | backlash zone $=0.005$, | 2.3280 |
| optimisation parameter | 4.8432 | $\eta_{k}=0.0125$, | 4.8496 |
| $p=2$ | 0.4357 | optimisation parameter | 0.4363 |
|  | $\mathbf{0 . 0 6 0 4}$ | $p=3$ | $\mathbf{0 . 0 6 0 4}$ |
| inertia moment | 0.9500 |  | 0.9391 |
| $J_{1}=1.218$, | 1.6000 | inertia moment | 1.5815 |
| backlash zone $=0.0025$, | 2.2706 | $J_{l}=1.218$, | 2.2445 |
| $\eta_{k}=0.05$, | 2.9889 | backlash zone $=0.0025$, | 2.9548 |
| optimisation parameter | 6.4174 | $\eta_{k}=0.05$, | 6.3460 |
| $p=3$ | 0.3959 | optimisation parameter | 0.3913 |
|  | $\mathbf{0 . 1 7 6 2}$ | $p=6$ | $\mathbf{0 . 1 7 4 1}$ |

According to the results of presented simulation tests, obtaining of correct results of identification of fault for all the tested physical quantities is possible if optimisation parameter $p$ is equal or bigger than 2 .

Tab. 4. Exemplified results of tests in matrix $\boldsymbol{H}$ for linear speed of the mass $\boldsymbol{v}_{\mathbf{2}}$

| Test parameters | Results | Test parameters | Results |
| :---: | :---: | :---: | :---: |
| inertia moment | 1.7737 | inertia moment | 1.8340 |
| $J_{1}=1.218$, | 2.7766 | $J_{1}=1.218$, | 2.8699 |
| backlash zone $=0.01$, | 3.8581 | backlash zone $=0.01$, | 3.9862 |
| $\eta_{k}=0.0125$, | 5.0250 | 5.1898 |  |
| optimisation parameter | 10.7233 | $\eta_{k}=0.0125$, | 11.0535 |
| $p=2$ | 0.8414 | optimisation parameter | 0.8704 |
| $\mathbf{0 . 0 2 6 3}$ | $p=3$ | $\mathbf{0 . 0 2 7 2}$ |  |
| inertia moment | 3.3018 | inertia moment | 1.8634 |
| $J_{1}=1.073$, | 2.2717 | $J_{1}=1.073$, | 1.2977 |
| backlash zone $=0.009$, | 1.1624 | backlash zone $=0.009$, | 0.6724 |
| $\eta_{k}=0.0375$, | $\mathbf{0 . 0 3 3 0}$ | $\eta_{k}=0.0375$, | $\mathbf{0 . 0 1 9 5}$ |
| optimisation parameter | 5.8461 | 4.2600 | optimisation parameter |
| $p=3$ | $p=6$ | 2.6275 |  |
|  | 5.1521 | $p=$ | 2.8415 |

One should pay attention to the fact that if the value of additional optimisation parameter $p$ exceeds 3 , it triggers noticeable differences in results of fault's identification. Such conclusions can be drawn from the minimal values of matrix $H$ obtained for the calculated wavelet scalograms. It can be noticed in all the presented tables.

## 6. CONCLUSIONS

The presented system of damage detection - based on image analysis enabling for the determination of centres of clusters by means of using the changes of value of membership function and weight function - found its application in the identification of the value of the inertia moment of rotating masses connected stiffly with the induction motor's rotor. The identification process has been conducted in the assumed time ranges.

The use of image analysis greatly increases the efficiency of analysis of nonstationary signals. This method, used in a diagnostic system, can effectively reduce the consequences of appearing of faults because it can be successfully applied already in the initial phase of the development of a fault. The conducted simulation confirmed the functioning of the proposed methodology of fault detection.

On the basis of the conducted research, it can be stated that the efficiency of detection process and fault identification can be greatly enhanced through ensuring appropriate ranges of changes of parameters and through the transformation the colour intensity of pixels recorded at wavelet scalogram in a system containing non-zero backlash zone to adequate ranges.

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