ROBUST OUTPUT FEEDBACK OF MINIMUM PHASE NON LINEAR SYSTEMS

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ABSTRACT

A robust output feedback controller is synthesized for minimum phase multivariable nonlinear systems based on the differential geometry approach. Using the internal model control structure within the input-output (I/O) linearization framework, the controller is combined with a closed-loop observer to estimate transformed states in the outer-loop. It is shown that the controller-observer combination achieves robust tracking and estimation using simple tuning parameters. The effectiveness of the proposed system is illustrated by a simulation example for control of concentration in a chemical reactor.

Key words: *Nonlinear control, input-output linearization, output feedback, robust control, chemical processes, multivariable systems.*

Many physical systems of engineering importance like, chemical and bio-reactors, robotic manipulators, and polymerization processes are characterized with complex nonlinearities in their models which pose a challenge to control. During the 1980s and 1990s, the nonlinear differential geometric control (Baquette, 1991; Isidori, 1995) has been an area of active research as a promising tool for nonlinear controller design. It involves a coordinate transformation and state-feedback, which transform the nonlinear system to a linear one in the input-output (I/O) sense. The I/O linearization design framework is usually implemented in a two-loop configuration, where in the inner-loop a state feedback controller is used to cancel the nonlinearities and the outer-loop with new states is tuned for off-set free tracking and disturbance rejection.

The I/O linearization app.roach requires accurate mathematical models in order to achieve exact cancellation of nonlinear terms. In many systems however, the physical model may be poorly understood or model parameters such as the heat transfer coefficient, reaction rate constants or damping constants may be inaccurate. This leads to inexact cancellation of non-linearities which may cause ineffective control or even instability. Hence the subject of designing feedback linearizable controllers for nonlinear systems with uncertainty is an important challenge and has gained considerable attention (Calvet and Arkun, 1992; Christofides, 2000; Mahmoud and Khalil, 2002). In order to ensure robust output tracking and preserve closed-loop stability, the I/O framework controller design must explicitly account for uncertainty in a non-conservative manner. Recently, Kolavennu et al. (2000) app.lied the I/O app.roach to design a H_2/H_∞ robust controller in the outer loop for systems with parametric uncertainty. Christofides et al. (1996) designed a robust state feedback controller for nonlinear singularly perturbed system with time-varying variables.

Hitherto, the I/O robust controller design methods require the uncertainty to satisfy some matching conditions (Calvet and Arkun, 1989) and for the case of H_∞ based control frame-work no systematic procedure exists to map perturbations from the nonlinear model to the outer linear plant. Furthermore, the I/O methods assume that the transformed states in the outer plant are available (Wu and Chou, 1999) or can be estimated by open-loop estimators (Hu and Rangaiah, 1999) for open-loop stable systems. In this work, a parameterized controller is proposed in the outer-loop. It employs a simple robustness tuning parameter that arises naturally from perturbations in the nonlinear system. The problem is formulated for fully linearizable multi-input multi-output (MIMO) system which, encompass a wide range of chemical processes and we app.ly the controller-observer combination to an isothermal chemical reactor.

A nonlinear system can be described by a continuous state-space model of the form:

$$\begin{aligned} \dot{x} &= f(x) + \Delta f(x) + g(x)u(t) \\ y &= h(x) \end{aligned} \qquad (1)$$

where $x \in \Re^{n\times 1}$ denotes a state vector, $\Delta f(x) = [\delta_1 f_1 \ \delta_2 f_2 \cdots \ \delta_n f_n]^T \in \Re^{n\times 1}$ is a perturbed function vector, $g = [g_1 \, g_2 \cdots \ g_m] \in \Re^{n\times m}$ and $\delta_i = \Re^1$ are real scalar perturbations. The uncertain vector functions are expressed as $f_j = f_j + \delta_j f_j$ so the Lie algebra (Isidori, 1995) can be written as:

$$L_f h(x) = \sum_{j=1}^{n} \frac{\partial h_i}{\partial x_j}(1+\delta_j)f_j \tag{2}$$

Assumption 1. *For a well posed system, a bound* $\varepsilon < 1$ *exists for a perturbation set* $\Omega(\delta) = \{\delta \in \Re^1 : \| \delta \| \le \varepsilon\}$ *for all time* t *such that* δ's *are real and bounded* $\{\delta_i(t) \in \Omega(\delta)\}$. Assume that a single worst-case perturbation $\delta_i \approx \delta$ exists for all $\{i = 1, \cdots, n\}$ then the Lie algebra can be further simplified to:

$$L_{f+\Delta f} h_i(x) = (1+\delta)\sum_{j=1}^{n} \frac{\partial h_i}{\partial x_j} f_j \tag{3}$$

Assumption 2. *The nonlinear system Eq. (1) is minimum phase and I/O linearizable with well-defined relative degree* r_i*'s that do not change with uncertainty.* This assumption allows us to write I/O equations for the uncertain system as follows:

$$\begin{bmatrix} \dot{\xi}_1^i \\ \dot{\xi}_2^i \\ \vdots \\ \dot{\xi}_r^i \end{bmatrix} = \begin{bmatrix} (1+\delta)\xi_2^i \\ (1+\delta)^2 \xi_3^i \\ \vdots \\ (1+\delta)^r a(\xi,\eta) + (1+\delta)^{r-1} b(\xi,\eta)u \end{bmatrix}$$
$$\begin{aligned} \eta &= q(\xi,\eta) + \Psi(\delta) \\ y_i &= \xi_1^i \end{aligned} \tag{4}$$

where $\{i = 1, \cdots, m\}$, $\xi^i = \left[y, \frac{dy}{dt}, \cdots, \frac{d^{r_i-1}y}{dt^{r_i-1}} \right]$ thus $\xi = \left[\xi_1^1 \cdots \xi_{r_1}^1 \cdots \xi_1^m \cdots \xi_{r_m}^m\right]^T$ is a complete states vector, and $\eta = [\eta_1 \cdots \eta_{n-r}]^T$ represents zero-dynamics. The terms $a = L_f^{r_i} h_i(x)$ and $b = L_g L_f^{r_i-1} h_i(x)$ are elements of the characteristic matrix. If the following nominal transformation is app.lied to cancel out nominal nonlinearities

$$u_i = \frac{1}{b(\xi^i,\eta^i)}\left[v_i - a(\xi^i,\eta^i)\right] \tag{5}$$

the following uncertain MIMO Brunovsky Canonical Form (BCF) (Arkun and Calvet, 1992) is obtained

$$\begin{aligned} \dot{\xi} &= A(\delta)\xi + B(\delta)v + \Delta(\delta) \\ y &= C\xi \end{aligned} \tag{6}$$

where $A(\delta) = \operatorname{diag}\{A_1, \cdots, A_m\}$ and $B(\delta) = [B_1 \cdots B_m]^T$ are uncertain state-space matrices, $C = \operatorname{diag}\{C_1, \cdots, C_m\}$, and $\Delta = [\Delta_1 \cdots \Delta_m]^T$. Equation (6) is a perturbed quasi-linear system because the uncertainties δ's induce state dependent disturbance term $\Delta(\delta,\xi)$ and perturbations on the integrators as well as on the input v. Therefore the individual state-space matrices of the MIMO plant are given for each output as:

$$\begin{aligned} A_i(\delta) &= \begin{bmatrix} 0 & (1+\delta^i) & & & \\ & & (1+\delta^i)^2 & & \\ & & & \ddots & \\ & & & & (1+\delta^i)^{r-1} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \\ B(\delta) &= \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & \delta_g^i \end{bmatrix}^T \\ \Delta(\delta\xi) &= \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & \delta_\varepsilon^i \end{bmatrix}^T \\ C &= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \end{aligned} \tag{7}$$

where $\delta_g^i = (1+\delta^i)^{r-1}$ and $\delta_\varepsilon^i = \delta^i(1+\delta)^{r-1} a(\xi_i, \eta_i)$ are perturbations that arise from variable transformation.

Assumption 3. *Assume that (A,B) a is controllable matrix pair and the state dependent perturbation* $\Delta(\delta,\xi)$ *is bounded in a non-conservative manner as a function of time with an upp.er bound* β *so that* $\Delta(t) = \{\Delta \in \Re^1 : \| \Delta \| \le \beta\}$. Since the nonlinear system is assumed to be minimum phase with stable zero-dynamics, Assumption 3 allows us to bound the state dependent disturbances in Eq. (6) and thus find linear controllers that meet stability and performance objectives for both the I/O quasi-linear sub-system and the original nonlinear plant.

For a nominal system, there is no uncertainty hence $\delta_i = 0$, and the desired performance can be specified through tuning parameters via a state feedback law $v_i = -\sum_1^{r_i} \alpha_i \xi_i$ where α_i's are chosen such that the closed-loop polynomial:

$$s^r + \alpha_r s^{r-1} + \cdots + \alpha_2 s + \alpha_1 = 0 \tag{8}$$

is Hurwitz for all $i = \{1, \cdots, m\}$ where s is the Laplace operator. The presence of uncertainty renders the nominal state feedback law ineffective and controller de-tuning is necessary to maintain robust stability and performance. For instance, if the relative degree $r_i = 2$, evaluation of the closed-loop eigen values show that the system is unstable if $\mathrm{Re}\left(\sqrt{\alpha_i^2 - 4\alpha_1\delta_i}\right) > \alpha_2$. Since *a-priori* knowledge of uncertainty magnitudes is seldom available, it is difficult to design a controller that ensures exact cancellation of uncertainties. To overcome this problem, several parameter adaptation schemes and uncertainty estimators have been attempted via the I/O framework as shown in (Jose et al., 1997; Hu and Rangaiah, 1999). In this work the effect of uncertainties is reduced by parameterizing the state feedback controller as outlined in Theorem 1 below.

Theorem 1. *Assume that $\varepsilon = (1+\delta)$ and that the nonlinear system represented by Eq. (1) has a well defined relative degree r, then the following parameterized state feedback law:*

$$v = \frac{1}{\xi^{r-1}}\left[-\alpha_r\xi_r - \frac{\alpha_{r-1}}{\xi^{r-1}}\xi_{r-1} - \frac{\alpha_{r-2}}{\xi^{2r-3}}\xi_{r-2} - \frac{\alpha_{r-3}}{\xi^{3r-6}}\xi_{r-3} - \cdots - \frac{\alpha_1}{\xi^{(r-1)r-p}}\xi_1\right] \quad (9)$$

will cancel uncertainties in the BCF system Eq. (6) and yield a Hurwitz characteristic polynomial for the closed-loop system. The denominator terms can be denoted as given in decreasing order as $\sigma = \left[\varepsilon^{r-1} \;\; \varepsilon^{2r-3} \;\; \varepsilon^{3r-6} \;\; \cdots \;\; \varepsilon^{(r-1)r-p}\right]$, where the second number of the powers raised to ε follow a sequence $\vartheta_{i+1} = \vartheta_i + 1$ whose last element p is obtained from $\vartheta = [1\ 3\ 6\ 10\ 15 \cdots p]$. The proof of this theorem is given in the App.endix.

Remark 1. *If perturbations are zero $\delta = 0$ then $\varepsilon = 1$ and nominal pole-placement will occur. For $\varepsilon \neq (1+\delta)$ partial cancellation of uncertainty will occur but ε can effectively be used as a robustness tuning factor since it affects the closed-loop poles.* The parameterized state-feedback controller $v = K(\varepsilon)\xi$ in Eq. (9) may be difficult to implement because: (i) in case of model mismatch a steady-state off-set will occur; and (ii) the states ξ's consist of the output and its higher derivatives which cannot be measured.

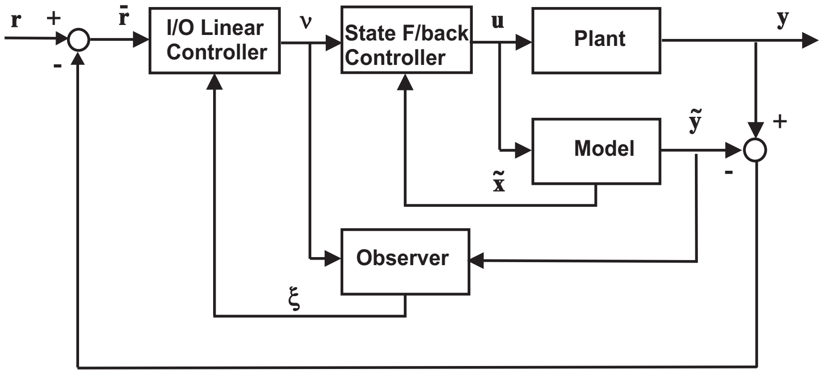

Figure 1: IMC based observer structure

In order to obtain off-set free response and preserve the possibility of using exogenous inputs, the state feedback is modified using output feed-back through the Internal Model Control (IMC) structure as shown in Figure 1 and the controller takes the following form:

$$v = K(\varepsilon(\xi(t) + \Phi(\varepsilon)\bar{r}(t) \quad (10)$$

where $\bar{r} = r - y + \tilde{y}$ and $\Phi = \alpha_1/\sigma_1$. The states are estimated by a dynamic closed-loop observer as outlined in the next section. Substituting Eq. (10) into (6) gives

$$\dot{\xi} = A_{ci}\xi + B\Phi\bar{r} + \Delta(\delta) \quad (11)$$

Where B and $\Delta(\delta)$ are as defined earlier and

$$A_{ci} = \begin{bmatrix} 0 & (1+\delta^i) & & & \\ & & (1+\delta^i)^2 & & \\ & & & \ddots & \\ & & & & (1+\delta^i)^{r-1} \\ -\dfrac{\alpha_1}{\sigma_1} & -\dfrac{\alpha_2}{\sigma_2} & -\dfrac{\alpha_3}{\sigma_3} & \cdots & -\alpha_r \end{bmatrix} \tag{12}$$

From Theorem 1 the matrix A_{c_i} is Hurwitz hence we can find a symmetric positive definite matrix P that satisfies the following Lyapunov equation

$$A_{c_i}^T P + PA_{c_i} = -I \tag{13}$$

where I is the identity matrix.

Remark 2. *Since it is assumed that the zero-dynamics are stable and the state dependent disturbances* $\Delta(\delta,\xi)$ *are bounded, then if the controller Eq. (11) stabilizes (12), then it stabilizes the nonlinear system.* For closed-loop stability, consider the following Lyapunov function $V_\varepsilon = \xi^T P\xi$ whose time derivative in the direction of ξ is given as

$$\dot{V}_\varepsilon = -\xi^T\xi + 2\Phi\bar{r}B^T P\xi + 2\Delta(\delta)^T P\xi \tag{14}$$

Setting $\varphi = \Phi\bar{r}B^T + \Delta(\delta)$ Eq. (14) can be reduced to:

$$\begin{aligned} \dot{V}_\varepsilon &\le -\frac{3}{4}\|\xi\|^2 - \left[\frac{\|\xi\|^2}{4} - \|2\varphi P\xi\|\right] \\ &\le -\frac{3}{4}\|\xi\|^2 - \left[\left(\frac{\|\xi\|}{4} - \|2\varphi P\xi\|\right)^2 - \|2\varphi P\xi\|^2\right] \\ &\le -\frac{3}{4}\|\xi\|^2 - \|2\varphi P\xi\|^2 \end{aligned} \tag{15}$$

and from Lyapunov stability, the closed-loop system is stable if $\dot{V}_\varepsilon \le 0$ which implies:

$$|\varphi| \le \beta + |\Phi B^T\bar{r}| < \frac{\sqrt{3}\|\xi\|}{4\|P\|} \tag{16}$$

For a nominal plant there are no perturbations $|\Delta(\delta)| = \beta = 0$, hence nominal stability is guaranteed if $|\bar{r}| = r$ is small. For uncertain plant, the parameter ε can be app.lied as a robust tuning factor since it affects the Lyapunov function $\dot{V}_\varepsilon$ through the closed-loop matrix A_{c_i} and Φ. This means that for a fixed set $\{\alpha_1, \cdots, \alpha_r\}$ an optimal ε can be sought that yields desired robustness. The closed-loop performance can be evaluated qualitatively or quantitatively using robust stability measures like the structured singular value (Skogestad and Postlethwaite, 1996) or by the integral squared error (ISE) given as:

$$J(\varepsilon) = \int_0^{t_f} \left[Q|\bar{u}(t)|^2 + R|\bar{y}(t)|^2\right] dt \tag{17}$$

where the bar denotes deviation variables with Q and R as app.ropriate weight functions.

Using the input-output controller design framework, the linearizing state feedback law Eq. (5) requires knowledge of the complete transformed state vector $\left[\xi_1^1 \cdots \xi_{r_m}^m\right]^T$. For most physical systems, the first state $\xi_1^i = y_i$ is usually measurable because typically it represents physical quantities such as temperature, concentration, position or velocity. The remaining states $\xi_2^1 \cdots \xi_{ri}^i$ consist of the first- and high-order derivatives of y which are infeasible to measure so they must be estimated. Due to lack of separation principle for nonlinear systems, many researchers used open-loop observers (Daoutidis and Kravaris, 1992; Hu and Rangaiah, 2001) for estimation of these states. A high-gain observer closed-loop estimator was proposed by Estfandiari and Khalil (1995) and later used by Jose et al. (1997) for simultaneous state and uncertainty estimation. However the observer lacks a systematic procedure to calculate its gain, and is associated with the peaking phenomenon that necessitates imposition of a saturation function on the input. In this work, we explore estimation of the transformed states using a closed-loop observer similar to the one proposed by Ray (1981) but in the context of I/O linearization based on the IMC structure shown in Figure 1.

Assumption 4. *Assume that the nonlinear system Eq. (1) there is a diffeomorphism* $(\eta,\xi) = \mathrm{T}(\eta,\xi)$ *given by:*

$$\xi = \begin{bmatrix} \xi_1^i \\ \xi_2^i \\ \vdots \\ \xi_{r_i}^i \end{bmatrix} = \begin{bmatrix} h_i(x) \\ L_f h_i(x) \\ \vdots \\ L_f^{r_i-1} h_i(x) \end{bmatrix} \quad (18)$$

If these equations are linearly independent, then each sub-system $i = \{1, \cdots, m\}$ *has full rank* $r = \sum r_i$ *where* $r \le n$ *and the nonlinear system is observable.* The diffeomorphism gives rise to the normal form Eq. (4). If we define a new state vector z which is related to ξ by

$$z(t) = \Gamma(t)\xi(t) \quad (19)$$

then the observability full rank condition in Assumption 3 allows us to express the original state vector ξ in terms of the new state vector z and y as:

$$\xi = \begin{bmatrix} \Gamma \\ C \end{bmatrix}^{-1} = \begin{bmatrix} z \\ y \end{bmatrix} = \Omega_1 z + \Omega_2 y \quad (20)$$

The matrices Ω_1 and Ω_2 are partitions of the $r \times r$ square inverse. Since the output *y* is measurable, to get the full state vector now requires only the knowledge of *z* which can be obtained from Eq. (19) as

$$\dot{z} = \dot{\Gamma}\xi + \Gamma\dot{\xi} \quad (21)$$

Substituting $\dot{\xi}$ from nominal form of the Brunowsky Canonical Form Eq. (6) and using the following identities:

$$\begin{bmatrix} \Gamma \\ C \end{bmatrix} [\Omega_1 \ \Omega_2] = \begin{bmatrix} \Gamma\Omega_1 & \Gamma\Omega_2 \\ C\Omega_1 & C\Omega_2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (22)$$

$$\Omega_1\Gamma + \Omega_2 C = I$$

obtained from left and right matrix multiplication in Eq. (20) we can simplify Eq. (21) to:

$$z = [\Gamma A\Omega_1 - \Gamma\dot{\Omega}_1]z + [\Gamma A\Omega_2 - \Gamma\dot{\Omega}_2]y + \Gamma Bv \quad (23)$$

If all elements of the state transformation matrix Γ are assumed to be real constants $\{\gamma_{ij}\} \in \Re^1$, then derivatives $\dot{\Omega}_{1,2} = 0$ and the full-order state estimator can be constructed from Eqs. (19) and (23) as follows:

$$\begin{aligned} \dot{\hat{z}} &= \Gamma A\Omega_1\hat{z} + \Gamma A\Omega_2 y + \Gamma Bv \\ \hat{\xi} &= \Omega_1\hat{z} + \Omega_2 y \end{aligned} \quad (24)$$

where $\hat{z}$ and $\hat{\xi}$ refers to estimated state vectors. The advantage of this observer is that the elements of Γ can be selected to yield desired error convergence rates through pole-placement.

The proposed controller and state estimator have been app.lied to control concentration of an isothermal continuous stirred tank chemical reactor (Kolavennu et al., 2000) with liquid phase multi-component reaction system $A \leftrightarrow B \rightarrow C$. The dynamic equations of the system take the following form:

$$\begin{aligned} \dot{x}_1 &= -k_1x_1 + \frac{F}{V}(C_{A_f} - x_1) + k_2x_3^2 \\ \dot{x}_2 &= -\frac{F}{V}x_2 + k_3x_3^2 \\ \dot{x}_3 &= k_1x_1 - \frac{F}{V}x_3 - (k_2 + k_3)x_3^2 + u \end{aligned} \quad (25)$$

Where $x_1 = C_A/C_{AF}$, $x_2 = C_B/C_{AF}$ and $x_3 = C_C/C_{AF}$ are dimensionless concentrations, and $u = N_{BF}/FC_{AF}$ is the dimensionless manipulated variable. The objective is to keep the concentration x_2 at a desired set-point by manipulating N_{BF} - the molar feed rate of x_3. The process variables and other parameters are given in Table 1 where the subscript s refers to steady-state.

Table 1: Process parameters for the chemical reactor

x_{2s}	=	3.775	[-]	θ	=	0.8	[-]
x_{1s}	=	3.665	[-]	c_2	=	2.9	[s^{-1}]
x_{3s}	=	0.869	[-]	k_1	=	1	[s^{-1}]
u_s	=	5.0	[-]	$\hat{k}_2$	=	3	[s^{-1}]
C_{Af}	=	3.3	[mol/m^3]	k_3	=	5	[s^{-1}]
V	=	3.0	[$m^{3]}$	F	=	3	[m^3/s]

The subscript *s* in Table 1 refers to steady-state parameters, and the parameters k_i's are uncertain chemical reaction rate constants with k_2 given as $k_2 = \hat{k}_2 + c_2\theta$ such that $|\theta|<1$. Equation (24) can be written in the form of Eq. (1) and smooth vectors f(x) and g(x) can be obtained to compute the following I/O linearization

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= L_f^2h(x) + L_gL_fh(x)\,u \\ y &= \xi_1 \end{aligned} \quad (26)$$

The concentration x_2 is the measured output and the system has a relative degree r = 2 because $L_gh(x) = 0$ but $L_gL_fh(x) = 2k_3x_3 \neq 0$. According to Theorem 1 the following IMC based parameterized controller is obtained: $v = \frac{1}{\varepsilon^2} - \alpha_2\xi_2 - \frac{\alpha_1}{\varepsilon}\xi_1 + \frac{\alpha_1}{\varepsilon}\bar{r}$. Figure 2 shows the effect of the tuning parameter on the nominal plant response when x_2 is controlled to steady-state x_{2s} =3.775 from an initial state vector x_0 = [1 8.33 1].

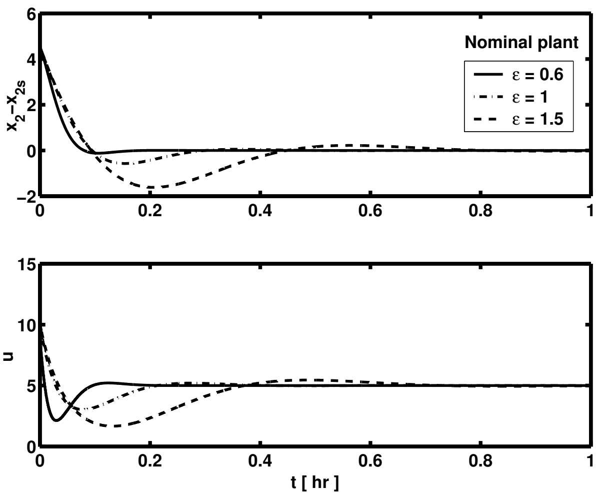

Figure 2: Response for control of x_2 to steady-state x_{2s} with $\alpha_1 = 1$,

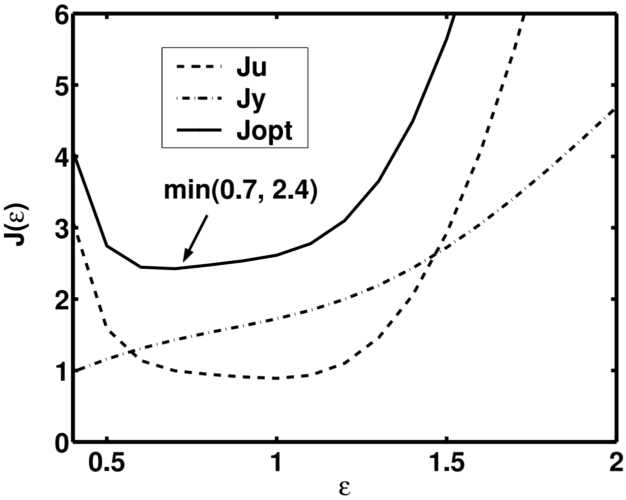

Figure 3: ISE performance objective versus tuning parameter ε

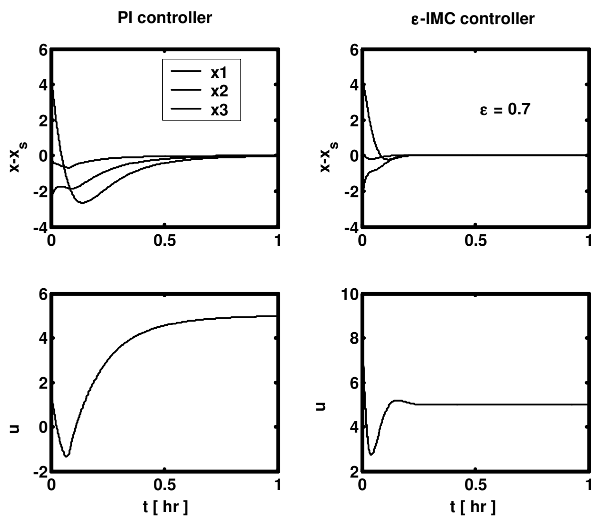

Figure 4: Performance comparison between the PI and parameterized IMC Controller

A small ε gives good output response but is associated with aggressive manipulated variable moves which in a real process may cause undue wear of actuators. On the other hand, a bigger ε yields moderate moves but result in sluggish closed-loop performance. Therefore optimal tuning for robustness should combine qualitative measures based on user's knowledge of the process, and quantitative performance measures like the ISE described earlier. As shown in Fig. 3 for weighting functions Q = 1500 and R = 1000 in Eq. (17), a minimum value of the ISE objective function is obtained at ε = 0.7. As seen in Fig. 4 for this optimal value, the parameterized controller gives superior performance compared to a PI controller in the outer-loop designed using state-feedback linearizing procedure outlined by Daoutidis and Christofides (1995). The PI controller was designed using $\alpha_0 = 1$, $\alpha_1 = 0.5$, $\alpha_2= 0.1$ whereas the controller gain K_c = 0.1 and the integral time τ_I = 0.49 were obtained using IMC tuning rules (Morari and Zafiriou, 1989) based on the second-order plant $y(s) = v(s)/(\alpha_{2S}{}^2+\alpha_1 s+\alpha_0)$ in the outer-loop. The initial move of the parameterized controller is computed based on the feedback error $\bar{r}$ and system states x thus no controller initialization is required. The PI controller however must be initialized from arbitrary values which may cause performance degradation or instability.

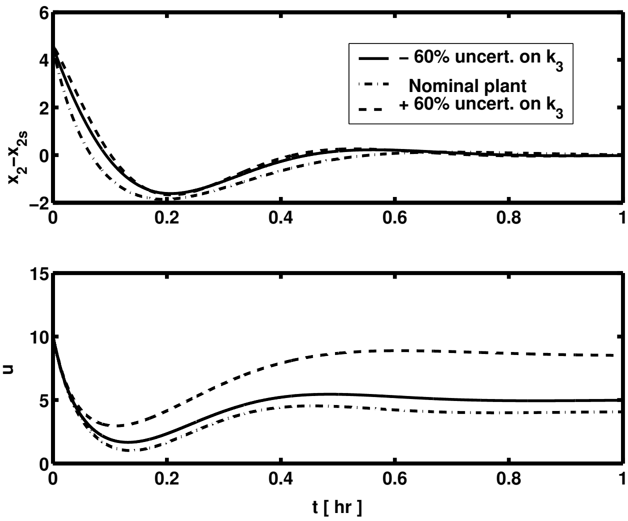

Figure 5: Closed-loop response for different plant uncertainty in k_3 for a constant ε

Figure 5 shows robustness of the parameterized controller against model-mismatch in the reaction constant k_3 when a tuning constant ε = 1.5 was used. Similar simulation runs showed that even for non-optimal tuning parameters the controller could bring the output to the desired steady-state operating point when significant uncertainty is present in the reaction rate constants.

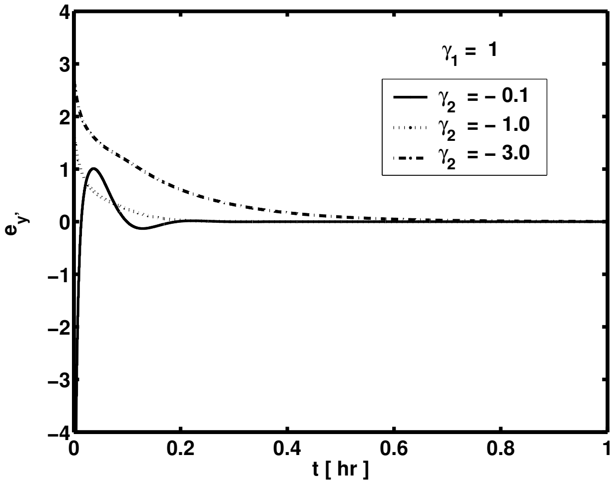

Figure 6: Error between actual and estimated states $e_{y'} = \hat{\xi}_2 - \xi_2$ for different observer parameters

The state ξ_2 represents time derivative of the output ξ = dy/dt which usually cannot be measured. If measurement noise is low it can be estimated from time series data using $\xi_k = (y_k - y_{k-1})/T_s$ where T_s is the sampling time. In this work, the complete state vector is estimated using the observer described in Section 4. The estimated state variable $\hat{\xi}_1$ showed negligible difference from the actual state $\xi_1 = y$.

Figure 6 shows the error profiles between the actual and estimated state ξ_2 for an observer tuning matrix Γ whose elements are assumed constant $\Gamma = [\gamma_1 \;\; \gamma_2]$. As seen in Fig. 6, the elements of Γ can be adjusted to alter the speed of convergence of the error and hence improve the closed-loop speed of response and performance.

A parameterized controller that uses a simple robustness tuning factor was developed for multivariable minimum phase I/O linearizable nonlinear system. The tuning factor arises naturally by mapp.ing real-scalar perturbations from nonlinear model to I/O linearized model, and its optimal value can be determined using the ISE criteria for a given level of uncertainty. A full-order state observer was constructed for fully observable nonlinear system and combined with the controller to achieve output feedback using the IMC structure. The effectiveness of the controller-observer combinations was illustrated for control of concentration in an isothermal chemical reactor. Compared to a PI controller, the parameterized controller yielded robust output tracking and regulation even for arbitrary initialization. Using simple tuning parameters, the observer showed robustness against model uncertainty with rapid error convergence hence improving the overall closed-loop performance.

English Symbols

A	=	State space matrix
B	=	state space matrix
C	=	output matrix
F	=	flow rate
H	=	output scalar field
k	=	reaction rate constant
K	=	controller gain
M	=	number of output points
n	=	system order
P	=	positive-definite matrix
Q	=	weighting matrix
Re( )	=	Real part of ( )
R	=	weighting matrix
$\Re^{n\times m}$	=	$n\times m$ real number matrix
r	=	set-point
t	=	time
u	=	manipulated variable
v	=	external input
V	=	volume
x	=	state variable vector
y	=	output variable vector

Greek Symbols

α	=	tuning parameter
η	=	zero-dynamic states
Γ	=	observer tuning parameter
Δ, δ	=	perturbation
Ω	=	observer tuning parameter
ξ	=	transformed state vector
φ	=	parameter
τ	=	time constant
Φ	=	parameter
ς	=	tuning parameter

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Appendix

Proof of Theorem 1. The proof is by inspection. For $v_i = -\sum_1^{r_i} \alpha_j \xi_j^i$, the closed-loop characteristic polynomial of the uncertain Brunovsky Canonical Form Eq. (6}) for different *relative degrees* is given by:

$$\begin{aligned} p_{r_i}(\delta) &= |\lambda I - A(\delta)| \\ &= \lambda^{r_i} + \alpha_{r_i} \lambda^{r_i - 1} \alpha_{r_i - 1} \lambda^{r_i - 2} \xi^{r_i - 1} + \cdots + \alpha_{r_i - 1} \lambda^{r_i - 2} \xi^{r_i - 1} \end{aligned} \tag{27}$$

where $\varepsilon = 1 + \delta$ as assumed. λ represents eigen-value and I is the identity matrix. The powers raised to the ε 's associated with each α_i are tabulated below for $r_i = \{1, \cdots, 7\}$

	α_1	A_2	α_3	α_4	α_5	α_6	α_7
$r_i = 2$	ε	1					
$r_i = 3$	ε^3	E^2	1				
$r_i = 4$	ε^6	E^5	ε^3	1			
$r_i = 5$	ε^{10}	E^9	ε^7	ε^4	1		
$r_i = 6$	ε^{15}	E^{14}	ε^{12}	ε^9	ε^5	1	
$r_i = 7$	ε^{21}	E^{20}	ε^{18}	ε^{15}	ε^{11}	ε^6	1

Since closed-loop stability depends on the eigen-values in Eq. (27) which in turn depend on perturbations δ embedded in ε we can eliminate them in the closed-loop characteristic polynomial using the following feedback law

$$\begin{aligned} v_i &= K(\varepsilon)\xi \\ &= \frac{1}{\varepsilon^{r-1}}\begin{bmatrix}-\alpha_1 & -\alpha_2 & \cdots & -\alpha_r\end{bmatrix}\Delta_\varepsilon \xi \end{aligned} \quad (28)$$

where $\xi = \begin{bmatrix}\xi_1^i & \cdots & \xi_{ri}^i\end{bmatrix}^T$ and Δ_ε is a diagonal matrix that consist of α_i's coefficients for a particular r_i tabulated above.

$$\Delta_\varepsilon = \begin{bmatrix} \frac{1}{\varepsilon^{(r_i-1)r_i-p}} & & & & \\ & \ddots & & & \\ & & \frac{1}{\varepsilon^{2r_i-3}} & & \\ & & & \frac{1}{\varepsilon^{r_i-1}} & \\ & & & & 1 \end{bmatrix} \quad (29)$$

The control law Eq. (28) performs a similarity transformation of Eq. (6) into a Hurwitz matrix Eq. (12) whose characteristic polynomial does not depend on δ's any more but on the tuning parameters α_i's.