Computational Treatment of Transient MHD Slip Flow of an Arrhenius Chemical Reaction in a Convectively Heated Vertical Channel

Muhammed M. Hamza[1*], Muhammad Z. Shehu[2], Halima Usman[1] and Emmanuel Omokhuale[3]

A numerical computational treatment of transient electrically conducting fluid with an Arrhenius chemical reaction in the presence of Navier slip and Newtonian heating is obtained by using implicit finite difference scheme. A transverse magnetic field is applied to the flow direction due to the exothermic nature of the fluid. Numerical computation shows that, higher values of Frank-Kamenetskii parameter (λ) and Biot number (Br) significantly influence the transport phenomenon. Irrespective of smaller or larger time, Magnetic parameter (M) reduces velocity of the fluid as well as wall shear stress.

Keywords: Transient; Magnetohydrodynamic (MHD) flow; Arrhenius chemical reaction; Newtonian heating; Navier slip.

1.   Introduction

At the present time, flow of MHD has a wide area of usage from defense to automotive industry, from medical field to basic science, such as chemistry and biology. Preventing the development of MHD applications is due to a delay in malignant tumor therapy, which results in less bleeding in the case of severe injuries, magnetic reinforcement of some visual and numerous other diagnostic tastes [10]. In addition, MHD flow and heat transfer from fluid along elastic surface have more than one engineering application. Many industries, such as metallurgical operation, thread dragged and refrigerating of continuous strip through an inclusive fluid and refinement of molten metal from non-metallic applications, use these techniques. Many researchers for example [1-10] tried to examine the MHD flow and heat transfer pertaining the stretchable surface under diverse boundary condition.

In the real world, the flow through parallel have numerous applications in aerospace, chemical, civil, environmental, mechanical and bio-mechanical engineering. Many researchers with sufficiently great interest in studying the slip effect in different geometries for different fluid [11-14]. Nandeppanavar et al. [15] carried out a detailed examination of slip flow and heat transfer with non-linear, Navier boundary condition using a stretchable surface. Besides, the study of a magnetic field second order slip and thermal radiation effect on nano fluid flow have been examined by Hakeem et al. [16] using a stretching or shrinking sheet. The boundary layer flow of a nanofluid with the influence of nanomaterial migration and second order slip has been studied by Zhu et al. [17]. Some published work on this topic can be found in reference [18-19]. Exothermic chemical reaction flow has application both in industry and engineering, thus in polymer expulsion, atomic reactor plant, geophysics and energy storage system. In respect to this application, the array solution to the reactive variable viscosity of Couette flow under Arrhenius chemical kinetic in the present of exothermic reaction was examined by Gbadeyan and Hassan [20]. Inherent irreversibility of hydro magnetic third grade reactive Poiseuille flow of a variable viscosity in porous media in convective cooling was studied by Salawu and Fatunmbi [21]. They examined the sensitized, Arrhenius and Bimolecular chemical kinetic of a single step exothermic reaction.

On the other hand, a careful discussion on slip flow outrage has been expressed in particular because of its near-term applicability in mother science and technology. Investigations on free

convection slip flow in a vertical channel with different boundary condition has been studied by many researchers [22-24]. Most recently, Hamza et al. [25] investigated the steady state MHD free convection slip flow of an exothermic fluid in a convectively heated vertical channel. The goal of this study is to extend Hamza et al. [25] work by incorporating time dependent MHD flow of an Arrhenius chemical kinetic in a convectively heated vertical channel. This research is significant because it uses the implicit finite difference scheme to analyze the influence of an Arrhenius chemical reaction in MHD time-dependent free convection flow in a convectively heated vertical channel. This type of results is use in Bioengineering and diverse clinical therapeutic interventions which including drug permeation through human skin, it may also be relevant in paper manufacturing sector as well petroleum industries to ascertain the flow of oil through rock layers.

2.   Formulation of the problem

Consider a time dependent free convection flow of electrically conducting and chemically reacting fluid in a vertical channel with Navier slip condition and Newtonian heating. The flow is induced by the convective heating introduced on the lower surface of the channel wall as well as the reactive property of the fluid. Following Hamza [24] and Hamza et al. [25], the non-dimensional governing equations under the Boussinesq's approximation can be written as:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T'-T_0) - \frac{\sigma B_0^2}{\rho} u' \qquad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 u'}{\partial y'^2} + \frac{QC_0^* A}{\rho C_p} e^{\left(\frac{-E}{RT'}\right)} \qquad (2)$$

The initial and boundary conditions for the present problem are

$$t' \le 0: u'=0, \quad T' \to T'_0, \qquad 0 \le y' \le H$$
$$t' > 0: u' = \gamma^* \frac{\partial u'}{\partial y'}, \quad -k\frac{\partial T'}{\partial y'} = h[T'_0 - T'], \; at \; y'=0 \qquad (3)$$
$$u' = 0, \qquad T' = T'_0, \qquad as \; y' \to H$$

where β is the coefficient of thermal expansion, Q is the heat of reaction, A is the rate constant, E is the activation energy, R is the universal gas constant, ν is the kinematic viscosity, C_0^* is the initial concentration of the reactant species, g is the gravitational force, C_p is the specific heat at

constant pressure, k is the thermal conductivity of the fluid, while ρ is the density of the fluid.

To solve equations (1) and (2), we employ the following dimensionless variables and parameters

$$y = \frac{y'}{H}, \; t = \frac{t'\mu_0}{H^2}, \theta = \frac{E(T'-T_0)}{RT_0^2}, \varepsilon = \frac{RT_0}{E}, U = \frac{u'\mu_0 E}{g\beta H^2 RT_0^2},$$
$$\lambda = \frac{QC_0^* AEH^2}{RT_0^2} e^{\left(\frac{-E}{RT_0}\right)}, \quad M = \frac{\sigma B_0^2 H^2}{\nu\rho}, \qquad \Pr = \frac{\mu_0 \rho C_p}{k}, \qquad (4)$$

Using (4), the equations (1) to (2) take the following form:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial y^2} + \theta - MU \qquad (5)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + \frac{\lambda}{\Pr} e^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} \qquad (6)$$

The initial and boundary conditions in dimensionless form are

$$U - \gamma\frac{\partial U}{\partial y} = 0, \frac{\partial \theta}{\partial y} - Br[\theta - \theta_a] = 0, \; at \; y=0$$
$$U = 0, \qquad \theta = 0, \qquad at \; y=1 \qquad (7)$$

where M, Pr, λ and ε are magnetic parameter, Prandtl number, Frank-Kamenetskii parameter and activation energy parameter respectively. The other non-dimensional quantities are the skin friction (C_f), and the heat transfer rate, (Nu) which are given as:

$$C_f = \left.\frac{\partial U}{\partial y}\right|_{y=0,1}, \; Nu = \left.\frac{\partial \theta}{\partial y}\right|_{y=0,1}$$

where C_f is the skin friction and Nu is the Nusselt number

3.   Numerical Solutions

The set of partial differential equations (5) and (6) with the boundary conditions (7) are solved numerically using implicit finite difference scheme. We used forward difference formulas for all time derivatives and approximate both the second and first derivatives with second order central differences. The equations corresponding to the first derivative and last grid points are modified to incorporate the boundary conditions.

The implicit finite difference equation follows corresponding to equations (5) and (6) are as

$$-r_1 u_{j-1}^{(N+1)} + (1+2r_1)u_j^{(N+1)} - r_1 u_{j+1}^{(N+1)} = r_2 u_{j-1}^{(N)} + (1-2r_2 - M\Delta t)u_j^{(N)} + r_2 u_{j+1}^{(N)} + \Delta t\, \theta_j^N$$

(8)

$$-r_1\theta_{j-1}^{(N+1)} + (\text{Pr}+2r_1)\theta_j^{(N+1)} - r_1\theta_{j+1}^{(N+1)} = r_2\theta_{j-1}^{(N)} + (\text{Pr}-2r_2)\theta_j^{(N)} + r_2\theta_{j+1}^{(N)} + \lambda\Delta t\, \exp\left(\frac{\theta_j^{(N)}}{1+\varepsilon\theta_j^{(N)}}\right)$$

(9)

where $r_1 = \xi\Delta t / \Delta y^2$, $r_2 = (1-\xi)\Delta t / \Delta y^2$ and $0 \le \xi \le 1$. We chose $\xi = 1$ so that we are free from choosing smaller or larger time steps.

4.   Results and Discussion

Free convection flow of viscous reactive fluid has been considered in a convectively heated vertical channel with Navier slip boundary condition. The numerical computation results are displayed in Figures (1) to (5). The governing parameters controlling the flow systems are: the Prandtl number (Pr), Biot number (Br), Navier slip parameter (γ), activation energy (ε), Frank-Kamenetskii parameter (λ) and non-dimensional time (t). Unless otherwise stated, the following values are assigned to the governing parameters: Pr = 0.71, Br = 0.1, $\lambda = 0.1$, $\gamma = 0.1$, $\varepsilon = 0.01$, M = 1 and $\theta_a = 1$.

Figures 1a and b show the impact of the chemical reaction of Arrhenius kinetic parameter (λ) and non-dimensional time (t) on temperature and velocity distribution respectively. It has been discovered that when the value of λ and t increase, so does the temperature and velocity of the fluid increases. According to Hamza [24], increasing λ and t improves the strength of the chemical reaction and viscous heating source factor in the temperature equation, which improves both temperature and velocity sequentially.

Figures 2a and b present the impact of the local Biot number (Br) and non-dimensional time (t) on temperature and velocity profile sequentially. By increasing the Br and t, Figures 2a and b grows. The presence of Newtonian heating and Navier slip on both temperature and velocity lead to the enhancement of both temperature and velocity at lower plate of the channel. The findings of Hamza [24], and Makinde and Aziz [26] concord with the present results.

Figures 3a and b are graphed to show the variation of M and t on velocity profiles. The graphical results revealed that a rise in M indicates a decrease in fluid velocity at small time or larger time. As M is increased, a resistive drag force is created that acts in the opposite direction of the transportation, lowering fluid velocity while when t is increased, the opposite development is noticed. Figures 4a and b represent the effect of shear stress against M for different values of t at $y = 0$ and $y = 1$ respectively. At both plates, skin friction decreases as M increase. Also increasing t lead to the substantial increase in the wall shear stress at both plates. Figures 5a and b illustrate the influence of λ on skin friction for different values of t at the plate $y = 0$ and $y = 1$ respectively. Increasing value of λ and t significantly enhance the skin friction at both plates. Table 1 represents the comparison between the present work and the work of Hamza [24]. When M = 0, the result of Hamza [24] is recovered.
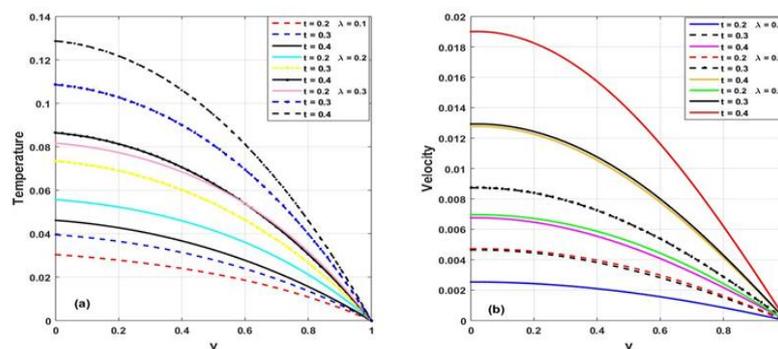
Fig1: Influence of and t on Temperature and Velocity profiles
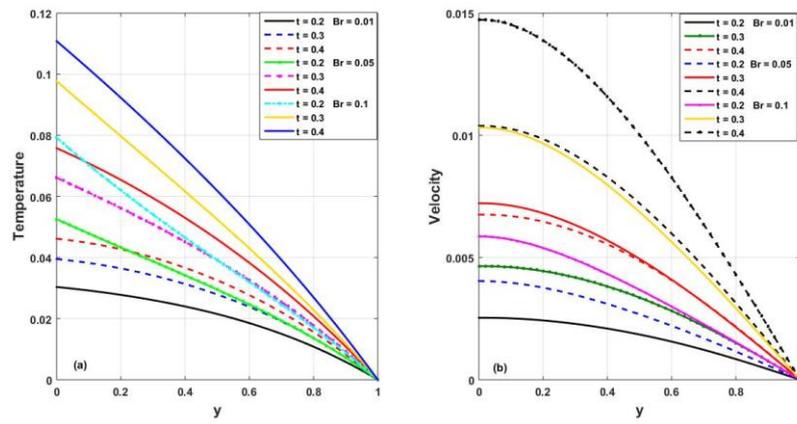
Fig 2: Influence of Br and t on Temperature and Velocity profiles
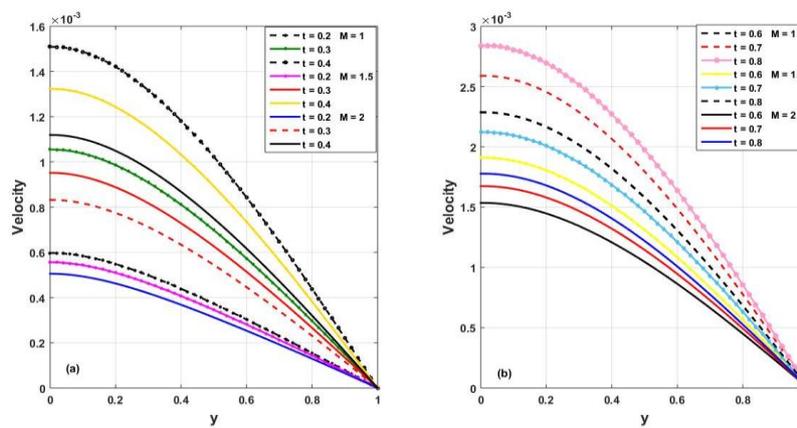
Fig 3: Influence of M and t on Velocity profiles
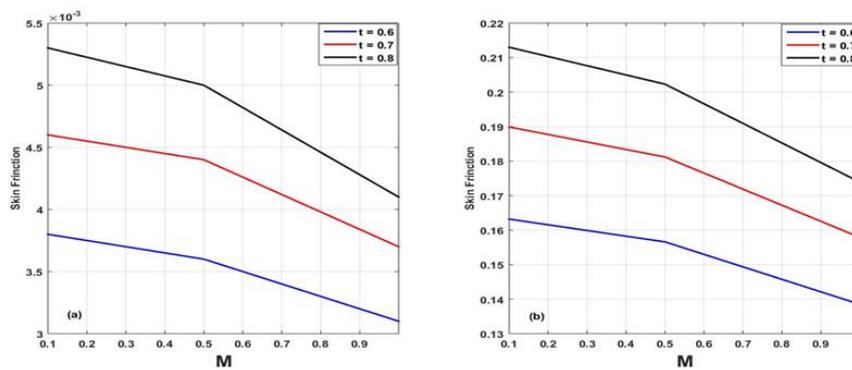
Fig4: Influence of M and t on wall shear stress
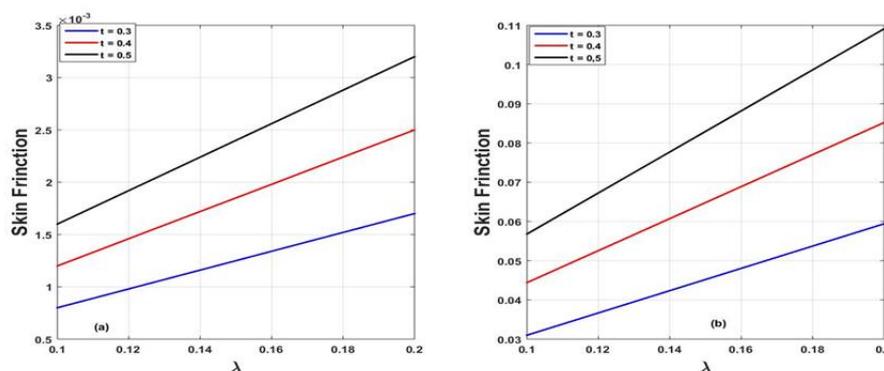
Fig5: Influence of   and t on wall shear stress

Table 1: Comparison between the present work and the work of Hamza [24]

Y	λ t=0.1	Hamza [24]	Present Work $M \rightarrow 0$	M =1	M = 2
0.1	0.1	0.0272	0.0272	0.0264	0.0241
0.1	0.2	0.0514	0.0514	0.0498	0.0455
0.1	0.3	0.0758	0.0758	0.0735	0.0671
0.1	0.4	0.1004	0.1004	0.0973	0.0890
0.1	0.5	0.1253	0.1253	0.1214	0.1110
0.1	0.6	0.1503	0.1503	0.1457	0.1332
0.1	0.7	0.1756	0.1756	0.1703	0.1557

5.   Conclusion

A numerical approach on transient flow of electrically conducting fluid in a vertical channel is studied along with the impact of pertinent parameters such as MHD, chemical reaction parameter and non-dimensional time. By utilizing implicit finite difference scheme, the non-dimensional governing equations are solved numerically. Some of the major findings are:

(i)   Both chemical reaction parameter and Biot number significantly increases the velocity and temperature of the fluid.
(ii) Magnetic parameter decreases the velocity of the fluid.
(iii) It is noticed that, increasing non-dimensional time greatly influence the transport phenomenon.

Conflict of Interest
The authors declare that there is no conflict of interest.

References

[1]   A.M. Rashad, " Influence of radiation on MHD free convection from a vertical flat plate embedded in porous media with thermophoretic deposition of particles," communications in non-linear science and numerical simulation 13, 2213-2222 (2008).doi:10.1016/j.cnsns.2007.07.002.

[2]   A. Shahzad, R. Ali and M. khan. "On the exact solution of axisymmetric flow and heat transfer over a non-linear radially stretching sheet," Chinese physics letters 29(8),   084705(2012).doi:10.1088/026-307X/29/8/084705.

[3]   T. Hayat, A. Shafiq, A. Alsaedi and M. Awais, "MHD axisymmetric flow of third grade fluid between stretching sheets with heat transfer," computers and fluids. 86, 103-108 (2013).doi:10.1016/j.aej.2015.03.013.

[4]   A. Sokolov, R. Ali and S. Turek. "An AFC stabilized implicit finite element method for partial differential equations in evolving-in-time surface". Journal of Computational and Applied Mathematics, 289,   101-115 (2014).doi:10.1016/j.cam.2015.03.002.

[5]   A.A.Afify,   M.J.   Uddi   and M.Ferdows."Scaling group transformation for MHD boundary layer flow over permeable stretching sheet in presence of slip flow with Newtonian heating effect." Applied Mathematics and Mechanics, 35(11),   1375-1386(2014).doi:10.1007/s10483-014-1873-7.

[6]   M.M. Nejad, K.Javaherdeh and M.Moslemi "MHD mixed convection flow of power low non-newtonian fluids over an isothermal vertical waving plate" Journal of Magnetism and Magnetic Materials. 389, 66-72(2015).doi:10.16/j.jmmm.2015.04.043.

[7]   J.Ahmad, A.Shahzad, M.Khan and R.Ali " A note in convective heat transfer of MHD Jeffrey fluid over a stretching sheet" AIP Advances 5(11), 117117(2015).doi:10.1063/1.4935571.

[8]   R. Ali, A. Shahzad, M. Khan and M. Ayub " Analytic and numerical solutions for axisymmetric flow with partial slip" Engineering with Computers 32(1), 149-154 (2016).doi:10.1007/s00366-015-0405-2.

[9]   T. Hayat, A. Shafiq, A. Alsaedi and S.A. Shahzad. "Unsteady MHD flow over exponentially stretching sheet with slip conditions" Applied Mathematics and Mechanics 37(2), 193-208(2016).doi:10.1007/s10483-016-2024-8.

[10]   M. H. M. Yasin, A. Ishak and I. Pop " MHD heat and mass transfer flow over a permeable stretching/shrinking sheet with radiation effect" Journal of Magnetism and Magnetic Materials 407,235-240(2016).doi:10.1016/j.jmmm.2016.01.0 87.

[11]   R. Ellahi " Exact solutions of flows of an oldroyd 8-constant fluid with nonlinear slip conditions". Naturforsch.doi: 65a, 1081-1086(2010). doi:10.1515/zna-20210-1211.

[12]   R. Ellahi, T. Hayat and F.M. Mahomed " Generalized Couette flow of a third-grade fluid with slip, the exact solutions " 2.Naturforsch 65a, 1071-1076(2010). doi:10.1515/znz-2010-1209.

[13]   A. Hussain, S.T. Mohyud-din and T.A. Cheema. " Analytical and numerical approaches to squeezing flow and heat transfer between two parallel disks with velocity slip and temperature jump" Chinese Physics Letter. 29, 114705 (2012).doi:10.1088/0256-307X/29/11/114705.

[14]   N.S. Akbar and S.Nadeem. "Thermal and Velocity slip effects on the peristaltic flow of a six constant jeffrey's fluid model" International Journal of heat Mass transfer. 55, 3964-3970(2012).10.1016/j.ijheatmasstransfer. 2012.03.026.

[15]   M. M. Nandeppanavav, K. Vajravelu, M.S. Abel, M.N. Siddalingappa."Second order slip flow and heat transfer over a stretching sheet with non-linear Navier boundary condition" International journal of thermal science. 58, 143-150(2012).doi:10.1016/j.ijthermalsci.2012. 02.019.

[16]   A. K. A. Hakeem, N. V. Ganesh and B. Ganga. "Magnetic field effect in second order slip flow of nanofluid over a stretching/shrinking sheet with thermal radiation effect" Journal of magnetism and magnetic material.381, 243-257(2015).doi:10.1016/j.jmmm.2014.12.0 10.

[17]   J. Zhu, D. Yang, L. Zheng and X.Zhang."Magnetic field effect in secod order velocity slip and nano particles migration in flow of Buongiorno nanofluid" Applied mathematics letters. 52,183-191(2016).doi:10.1016/j.aml.2015.09.003.

[18]   F. Mabood, A. Shafiq, T. Hayat and S. Abelman."Radiation effects in stagnation point flow with meting heat transfer and second order slip" Results in Physics. 7, 31-42(2017).doi:10.1016/j.rinp.2016.11.051.

[19]   N. Najib, N. Bachok, N. Arifin and F. M. Ali."Stability analysis of stagnation point flow in a nanofluid over a stretching /Shrinking sheet with second order slip, Soret and Dufour effects: A revised model. Applied Science. 8(4), 642(2018).doi:10.3390/app8040642.

[20]   J. A. Gbadeyan and A. R. Hassan. "Multiplicity of solutions for a reactive variable viscous coquette flow under Arrhenius Kinetic" Mathematical Theory and Modeling. 2(9), 39-49(2012).

[21]   S.O. Salawu and E.O.Fatunmbi."Inherent Irreversibility of hydromagnetic third grade reactive poiseuille flow of a variable viscosity in porous media with convective cooling" Journal of the Serbian Society for Computational Mechanics. 11(1), 46-58(2017).doi:1024874/jsscm.2017.11.01. 05.

[22]   Pal D, Talukdar B. Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Commun Nonlinear Sci Numer Simulat 2010;15:1813–30. 15.doi:10.1016/j.cnsns.2009.07.011.

[23]   Makinde O D. Computational modelling of MHD unsteady flow and heat transfer toward a flat plate with Navier slip and Newtonian heating. Braz J Chem Eng 2012;29:159–66. Doi:10.1590/S0104-66322012000100017.

[24] Hamza M M. Free Convection slip flow of an exothermic fluid in a convectively heated vertical channel. Ain Shams Engi journal 2018;9:1313-23.doi:10.1016/j.asej.2016.08.011.

[25] M. M. Hamza, M .Z .Shehu and B. H. Tambuwal. "Steady State MHD free Convection Slip flow of an exothermic fluid in a convectively heated vertical channel" Saudi J.Eng.Technol. 6(10): 364-370(2021).doi:10.36348/sjet.2021.v06i10.006.

[26] Makinde O.D and Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with convective boundary condition. Int. J. Thermal Sci. 50: 1326-1332 (2011).doi:10.1016/j.ijthermalsci.2011.02.019.