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Full Length Research Article

MODELLING QUALITY AS A COOPERATIVE ADVERTISING COORDINATION MECHANISM IN A DECENTRALISED CHANNEL USING GAME THEORY

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ABSTRACT
Considering the scarcity of cooperative advertising models on the interaction between product quality and market variables such as price, advertising effort and subsidy, this paper considers the effect of quality in cooperative advertising in a manufacturer-retailer supply channel in which the channel members engage in a Stackelberg game. The manufacturer is the channel leader, while the retailer is the follower. The research adopts the incorporation of product quality into the traditional cooperative advertising model setting through the multiplicative impact of price, advertising and product quality on demand. It considers two channel structures: an unsubsidised channel structure in which the manufacturer does not provide advertising subsidy to the retailer, and a subsidised channel structure in which the manufacturer provides advertising subsidy for retail advertising. It obtains the prices, the advertising effort, the retailer's payoff and the manufacturer's payoff for both channel structures. The results reveal that for both subsidised and unsubsidised advertising, increase in retail advertising and retailer's payoff resulting from quality improvement is limited due to diminishing marginal returns. Also, quality improvement negatively affects the manufacturer's payoff after a certain quality level. Further, it shows that quality can be substituted for subsidy, and can be used to coordinate the channel.

Keywords: Cooperative advertising, Product quality, Stackelberg game, Supply chain, Subsidy

INTRODUCTION
In advertising literature there is a view that high quality products receive more advertising attention or spending (Nelson, 1974). On the other hand, there is an opposing view to this school of thought, that is, low quality products are given more advertising attention (Comanor & Wilson, 1979). Thus, it is obvious that there exists a positive relationship as well as a negative relationship between quality and advertising (Tellis & Fornell, 1988). This paper considers a special type of advertising known as cooperative advertising. This is an advertising strategy in which the manufacturer pays for a fraction of the advertising cost which the retailer incurs while advertising the manufacturer's product. Cooperative advertising has been modelled with price, demand, advertising effort, subsidy, and scarcely with quality in only a handful works (Zhang et al., 2017; Rabbani & Shahraki, 2021; Yu et al., 2021). These cooperative advertising that considered quality did not compare its effect on other variables in a subsidised channel and an unsubsidised channel. This work centres on these comparisons.

Cooperative advertising models can be categorized into static, dynamic and stochastic models. Berger (1972) is considered to be the first static cooperative advertising mathematical model. This paper was followed by a number of static models (Dant & Berger, 1996; Huang et al., 2002; Xie & Wei, 2009; He et al., 2014; Ezimadu, 2019a). Jorgensen et al. (2000) seems to be one of the first to consider cooperative advertising model in a dynamic setting. This model was followed by other dynamic cooperative advertising models (Jorgensen et al., 2001; Chutani & Sethi, 2012; Ezimadu, 2016; Ezimadu, 2019b). He et al. (2009) may be considered as the first to study cooperative advertising on a stochastic setting. This was extended by Ezimadu & Nwozo (2017). Apart from these extensions which have taken cooperative advertising to unprecedented levels, there are others which considered cooperative advertising with product quality.

De Giovanni (2011) considered a situation where quality and advertising improvements lead to increase in the stock of channel goodwill. In a study of cooperative advertising in relation to product quality Ezimadu and Ogini (2014) modelled cooperative advertising using quality and price. They considered a decentralised channel in which the manufacturer is the Stackelberg leader, and an integrated channel in which the players are involved in a Nash game, and showed that channel integration should be adopted. Another investigation of cooperative advertising in relation to quality was done by He et al. (2014). This work investigated optimal cooperative strategies and coordination in a manufacturer-retailer two-period fashion and textile supply chain. Ezimadu (2017) used the concept of Nash differential game to study the interaction between quality and advertising in a market duopoly.

The present study considers cooperative advertising in a manufacturer-retailer supply chain in a static setting. It extends Ezimadu and Ogini (2014) to consider two decentralised channel structures involving product quality which was incorporated into the classical manufacturer-retailer cooperative advertising model using the multiplicative effect of advertising, price and product quality. The two channel structures are: a situation where the manufacturer does not participate in retail advertising; and a situation where he participates. We will refer to the former as the unsubsidised channel structure, and the latter as the subsidised channel structure. For both channel structures we will consider the effect of product quality on retail advertising, the retailer's payoff and manufacturer's payoff, and the relationship between quality and subsidy.

MATERIALS AND METHODS

Model Formulation

The manufacturer transfers the product to the retailer at a certain price P_M, and the retailer in turn sells to the consumer(s) at a price P_R. The manufacturer's production cost is P_C.

The retailer's decision variables are his retail price P_R and advertising expenditure a_R, while the manufacturer's decision variables are his wholesale price P_M, subsidy rate β and product quality Q. The subsidy rate is the percentage of retail advertising expenditure the manufacturer is willing to give to the retailer in support of his local advertising effort.

The demand function is characterized by the retail price P_R, the advertising effort a_R, and the manufacturer's product quality Q. This is given as

$$D(P_R, a_R, Q) = f(P_R)g(a_R)h(Q) \tag{1}$$

We note that $f(P_R)$ reflects the effect of retail price on demand; $g(a_R)$ reflects the effect of advertising on demand; and $h(Q)$ reflects the effect of the manufacturer's product quality on demand.

We observe that the use of multiplicative effect of price and advertising on demand is common in the literature (Thompson & Teng, 1984; Jorgensen & Zaccour, 1999; Yue et al., 2006).

As is common in the literature (Thompson & Teng, 1984; Weng, 1995) we let f to be a linearly decreasing function of the retail price P_R:

$$f(P_R) = 1 - \theta P_R \tag{2}$$

For simplicity, $f(P_R)$ is normalized to 1. P_R is positive, and θ is the price response constant indicating the effect of price on sale.

To ensure that the impact of advertising on sale leads to a saturation point where any additional advertising spending leads to diminishing return, g was given by

$$g(a_R) = \rho a_R^{\frac{1}{2}} \tag{3}$$

which is a concave function of a_R. ρ is the advertising effectiveness parameter. It is an indicator of the impact of advertising on demand. This function was also employed by Xie and Wei (2009).

Generally, it is expected that under normal condition, demand should increase with product quality. Thus we let

$$h(Q) = 1 - \frac{1}{e^Q}. \tag{4}$$

We consider the quality Q on a scale of 1 to 10, with 1 being the least quality, and 10 being the highest or best quality.

We note that the multiplicative effect of f and g will obviously constrain the product $f(P_R)g(a_R)$ to be very low, especially if $f(P_R)$ is very close to 0, which will force $D(P_R, a_R, Q)$ to appear very small even for relatively large $g(a_R)$. This can lead to wrong interpretations. Thus to compensate for such situations we multiply the resulting product by a relatively large constant M, so that

$$f(P_R)h(Q) = M(1 - \theta P_R)\left(1 - \frac{1}{e^Q}\right) \tag{5}$$

Thus from (3) and (5) we have that, the demand function (1) becomes

$$D(P_R, a_R, Q) = M(1 - \theta P_R)\left(\rho a_R^{\frac{1}{2}}\right)\left(1 - \frac{1}{e^Q}\right) \tag{6}$$

The profit of the retailer is

$$R_{pay} = (P_R - P_M)D(P_R, a_R, Q) - (1 - \beta)a_R,$$

while that of the manufacturer is

$$M_{pay} = (P_M - P_C)D(P_R, a_R, Q) - \beta a_R - Q.$$

Model Analysis

The manufacturer who is the Stackelberg leader first discloses his wholesale price P_M, his participation (subsidy) rate β and product quality Q. This is followed by the retailer's decisions on the retail price P_R and advertising effort a_R.

The Retail Price and Advertising Effort

Now, the retailer's problem is

$$\text{Max } R_{pay} = (P_R - P_M)D(P_R, a_R, Q) - (1 - \beta)a_R \quad \text{subject to } P_R,\ a_R > 0. \tag{7}$$

R_{pay} is a concave function of a_R and P_R.

Differentiating R_{pay} partially with respect to P_R we have

$$\frac{\partial R_{pay}}{\partial P_R} = -\theta P_R + \theta P_M + 1 - \theta P_R = 0$$

so that

$$P_R = \frac{1 + \theta P_M}{2\theta}. \tag{8}$$

Also differentiating R_{pay} partially with respect to $a_R$we have

$$\frac{\partial R_{pay}}{\partial a_R} = \frac{\rho(P_R - P_M)(1 - \theta P_R)M\left(1 - \frac{1}{e^Q}\right)}{2a_R^{\frac{1}{2}}} - 1 + \beta = 0,$$

and so

$$a_R = \left(\frac{\rho M(1 - \theta P_R)^2\left(1 - \frac{1}{e^Q}\right)}{8\theta(1 - \beta)}\right)^2. \tag{9}$$

The Manufacturer's Participation Rate, Product Quality and Wholesale Price

Now, with the information on the retailer's response, the manufacturer will maximize his profit by deciding on his wholesale price P_M, subsidy rate β and product quality Q. Thus:

$$\text{Max } M_{pay} = \frac{\rho^2(P_R - P_C)(1 - \theta P_M)^3\left(M\left(1 - \frac{1}{e^Q}\right)\right)^2}{16\theta(1 - \beta)} - \frac{\rho^2(1 - \theta P_M)^4\left(M\left(1 - \frac{1}{e^Q}\right)\right)^2}{64\theta^2(1 - \beta)^2} - Q \tag{10}$$

$$\text{subject to } \ P_M > 0,\ \ \beta \in [0,1], Q \in [1,10].$$

so that by Ezimadu and Ogini (2014)

$$\beta = \begin{cases} \dfrac{4\theta(P_M - P_C) - (1 - \theta P_M)}{4\theta(P_M - P_C) + (1 - \theta P_M)}, & 4\theta(P_M - P_C) > (1 - \theta P_M) \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

and $Q = \begin{cases} -\ln\left((4^{-1} - K^{-1})^{\frac{1}{2}} + \frac{1}{2}\right) \\ -\ln\left(-(4^{-1} - K^{-1})^{\frac{1}{2}} + \frac{1}{2}\right) \end{cases}$,

Where

$$K = 2\left[\frac{(P_M - P_C)(1-\theta P_M)^3\rho^2}{16\theta(1-\beta)} - \frac{\beta(1-\theta P_M)^4\rho^2}{64\theta^2(1-\beta)^2}\right]M^2. \qquad (12)$$

Now, we observe that

$$\frac{\partial^2 M_{pay}}{\partial Q^2} = K\left(\frac{1}{e^Q}\right)\left(\frac{2}{e^Q} - 1\right). \qquad (13)$$

Clearly, (13) is positive for $Q < 0.6931471805599$ and negative when $Q > 0.6931471805599$. Thus M_{pay} is maximized when $Q > 0.6931471805599$. This can only be possible for

$$Q = -\ln\left(-(4^{-1} - K^{-1})^{\frac{1}{2}} + \frac{1}{2}\right).$$

Now, maximizing (10) with respect to P_M we have

$$\frac{\partial M_{pay}}{\partial P_M} = \frac{\rho^2\left(M\left(1-\frac{1}{e^Q}\right)\right)^2}{16\theta(1-\beta)}\left(3\theta(P_M - P_C)(1-\theta P_M)^2 + (1-\theta P_M)^3\right) - \frac{\rho^2\left(M\left(1-\frac{1}{e^Q}\right)\right)^2\beta}{64\theta^2(1-\beta)^2}\left(4(1-\theta P_M)^3(-\theta)\right) = 0,$$

implying that

$$P_M = \frac{(1-\beta)(3\theta P_C + 1) - \beta}{4\theta - 5\theta\beta} \qquad (14)$$

so that

$$\beta = \frac{4\theta P_M - 3\theta P_C - 1}{5\theta P_M - 3\theta P_C - 2}. \qquad (15)$$

Equilibrium Characterising Unsubsidised Retail Advertising

If there is no subsidy, then $\beta = 0$, so that (14) becomes

$$P_{M(\beta=0)} = \frac{3\theta P_C + 1}{4\theta}. \qquad (16)$$

From (16) we have that (8) becomes

$$P_{R(\beta=0)} = \frac{5 + 3\theta P_C}{8\theta}. \qquad (17)$$

Now, we have that becomes

$$K_{(\beta=0)} = \left(\frac{2(P_M - P_C)(1-\theta P_M)^3\rho^2}{16\theta}\right)M^2,$$

so that

$$Q_{(\beta=0)} = -\ln\left(-\left(4^{-1} - K^{-1}_{(\beta=0)}\right)^{\frac{1}{2}} + \frac{1}{2}\right). \qquad (18)$$

Thus using (16) and (17) in (9) for $\beta = 0$ we have that

$$a_{R(\beta=0)} = \frac{\rho^2\left(1 - \theta\left(\frac{3\theta P_C + 1}{4\theta}\right)^4\right)\left(M\left(1-\frac{1}{e^Q}\right)\right)^2}{64\theta^2} = \frac{3^4\rho^2(1-\theta P_C)^4\left(M\left(1-\frac{1}{e^Q}\right)\right)^2}{4^4(64)\theta^2}. \qquad (19)$$

From (7) we have

$$R_{pay(\beta=0)} = \left(\frac{5+3\theta P_C}{8\theta} - \frac{3\theta P_C + 1}{4\theta}\right)\left(1 - \theta\left(\frac{5+3\theta P_C}{8\theta}\right)\right)\rho \times \left(\frac{3^2\rho(1-\theta P_C)^2\left(M\left(1-\frac{1}{e^Q}\right)\right)}{4^2(8)\theta}\right)\left(M\left(1-\frac{1}{e^Q}\right)\right) - \frac{3^4\rho^2(1-\theta P_C)^4\left(M\left(1-\frac{1}{e^Q}\right)\right)^2}{4^4(64)\theta^2}$$

$$= \frac{3^4\rho^2(1-\theta P_C)^4\left(M\left(1-\frac{1}{e^Q}\right)\right)^2}{16384\theta^2}, \qquad (20)$$

where

$$Q = Q_{(\beta=0)} = -\ln\left(-\left(4^{-1} - K^{-1}_{(\beta=0)}\right)^{\frac{1}{2}} + \frac{1}{2}\right).$$

From (15) we have

$$M_{pay(\beta=0)} = \left(\frac{3\theta P_C + 1}{4\theta} - P_C\right)\left(1 - \theta\left(\frac{5+3\theta P_C}{8\theta}\right)\right)\rho\left(\frac{3^2\rho(1-\theta P_C)^2\left(M\left(1-\frac{1}{e^Q}\right)\right)}{4^2(8)\theta}\right) \times \left(M\left(1-\frac{1}{e^Q}\right)\right) - Q$$

$$= \frac{3^4\rho^2(1-\theta P_C)^4\left(M\left(1-\frac{1}{e^Q}\right)\right)^2}{4096\theta^2} - Q, \qquad (21)$$

where $Q = Q_{(\beta=0)}$ as given above.

From the foregoing we have the following result:

Proposition 1 In the unsubsidised channel, the manufacturer's wholesale price $P_{M(\beta=0)}$, product quality $Q_{(\beta=0)}$ and payoff $M_{pay(\beta=0)}$ are given by (16), (18) and (21) respectively, while the retail price $P_{R(\beta=0)}$, advertising effort $a_{R(\beta=0)}$ and payoff $R_{pay(\beta=0)}$ are given by (17), (19) and (20) respectively.

Equilibrium Characterising Subsidised Retail Advertising

From (11) and (15) we have

$$13\theta^2 P_M^2 - (10\theta P_C + 16\theta)P_M + (10\theta P_C + 3) = 0,$$

implying that

$$P_{M(\beta>0)} = \frac{10\theta P_c + 16\theta \pm \sqrt{\left(-(10\theta P_c + 16\theta)\right)^2 + 4(13\theta^2)(10\theta P_c + 3)}}{2(13\theta^2)} \quad (22)$$

$$P_{R(\beta>0)} = \frac{1}{2\theta} + \frac{P_{M(\beta>0)}}{2} \quad (23)$$

where $P_{M(\beta>0)}$ is as given in (22).

Now, from (12) we have

$$K = \frac{2(1-\theta P_M)^3\rho^2}{64\theta^2(1-\beta)^2}\left[(P_M - P_C)4\theta^2 - \left((4\theta+1)\theta P_M - 4\theta^2 P_C - 1\right)\beta\right]M^2. \quad (24)$$

Using (11) in (24) we have

$$K_{(\beta>0)} = \frac{2(1-\theta P_M)^3\rho^2}{64\theta^2\left(1-\frac{4\theta(P_M-P_C)-(1-\theta P_M)}{4\theta(P_M-P_C)+(1-\theta P_M)}\right)^2} \times \left[(P_M-P_C)4\theta^2 - \left((4\theta+1)\theta P_M - 4\theta^2 P_C - 1\right)\frac{4\theta(P_M-P_C)-(1-\theta P_M)}{4\theta(P_M-P_C)+(1-\theta P_M)}\right]M^2$$

$$= \frac{2(1-\theta P_M)^3\rho^2}{64\theta^2}\left(\frac{4\theta(P_M-P_C)-(1-\theta P_M)}{2(1-\theta P_M)}\right)^2 \times \left[4\theta^2(P_M-P_C) - \frac{\left((4\theta+1)\theta P_M - 4\theta^2 P_C - 1\right)\left(4\theta(P_M-P_C)-(1-\theta P_M)\right)}{4\theta(P_M-P_C)+(1-\theta P_M)}\right]M^2.$$

Therefore

$$Q_{(\beta>0)} = -\ln\left(-\left(4^{-1} - K_{(\beta>0)}^{-1}\right)^{\frac{1}{2}} + \frac{1}{2}\right). \quad (25)$$

Using (11) in (9) we have

$$a_{R(\beta>0)} = \frac{\rho^2(1-\theta P_M)^4\left(M\left(1-\frac{1}{e^Q}\right)\right)^2}{64a_R^2}\left(\frac{4\theta(P_M-P_C)-(1-\theta P_M)}{2(1-\theta P_M)}\right)^2 = \frac{\rho^2(1-\theta P_M)^2\left(M\left(1-\frac{1}{e^Q}\right)\right)^2\left(4\theta(P_M-P_C)-(1-\theta P_M)\right)^2}{256\theta^2}, \quad (26)$$

where $P_M = P_{M(\beta>0)}$ and $Q = Q_{(\beta>0)}$ as given above.
Using (11) in (7) we have

$$R_{pay(\beta>0)} = \left(\frac{1-\theta P_M}{2\theta}\right)\left(\frac{1+\theta P_M}{2}\right)\left(M\left(1-\frac{1}{e^Q}\right)\right) \times \rho\sqrt{\frac{\rho^2(1-\theta P_M)^2\left(M\left(1-\frac{1}{e^Q}\right)\right)^2\left(4\theta(P_M-P_C)-(1-\theta P_M)\right)^2}{256\theta^2}} - \frac{2(1-\theta P_M)^3\left(M\left(1-\frac{1}{e^Q}\right)\right)^2\rho^2\left(4\theta(P_M-P_C)-(1-\theta P_M)\right)}{256\theta^2}, \quad (27)$$

where $P_M = P_{M(\beta>0)}$ and $Q = Q_{(\beta>0)}$ are as given above.
Also using (11) in (10) we have

$$M_{pay(\beta>0)} = \frac{(P_M-P_C)(1+\theta P_M)(1-\theta P_M)\left(M\left(1-\frac{1}{e^Q}\right)\right)^2\rho^2\left(4\theta(P_M-P_C)-(1-\theta P_M)\right)}{32\theta} - \frac{\left(4\theta(P_M-P_C)-(1-\theta P_M)\right)^2(1-\theta P_M)^2\left(M\left(1-\frac{1}{e^Q}\right)\right)^2\rho^2}{256\theta^2} - Q_{(\beta>0)}, \quad (28)$$

where $P_M = P_{M(\beta>0)}$ and $Q = Q_{(\beta>0)}$ are as stated above. Thus we have the following result:

Proposition 2 In the subsidised channel, the manufacturer's wholesale price $P_{M(\beta>0)}$, product quality $Q_{(\beta>0)}$ and payoff $M_{pay(\beta>0)}$ are given by (22), (25) and (28) respectively, while the retail price $P_{R(\beta>0)}$, advertising effort $a_{R(\beta>0)}$ and payoff $R_{pay(\beta>0)}$ are given by (23), (26) and (27) respectively.

RESULTS AND DISCUSSION

Parameter Values

We recall that $\rho \in [0,1]$ being the advertising effectiveness parameter. Thus we let $\rho = 0.2$. θ is the price response constant indicating the effect of price on demand. We let $\theta = 0.02$. We take the manufacturer's production cost to be $P_C = 20$. M is a correction constant which should have a large value. Thus we let $M = 50$.

Effect of Product Quality on the Advertising Effort, Product Demand and Payoff

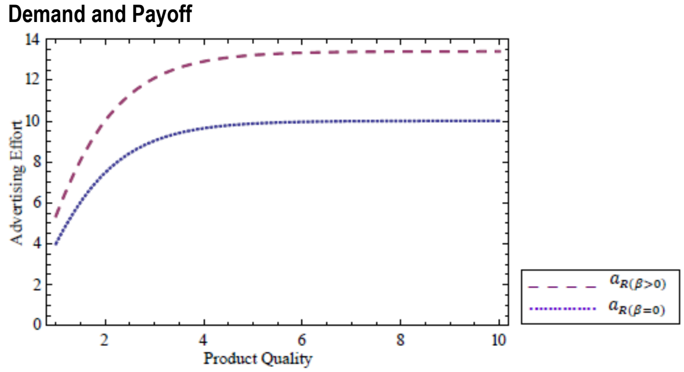

Figure 1: Effect of product quality on retail advertising

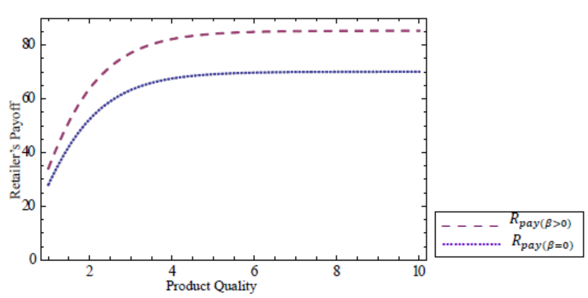

Figure 2: Effect of product quality on the retailer's payoff

From Figure 1 we observe that increase in quality leads to increase in retail advertising for both when retail advertising is subsidised and when it is not subsidised. However, with the provision of

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subsidy, the retailer is motivated towards increasing advertising effort with increase (improvement) in quality. This is reflected in the payoff as can be seen in Figure 2. Thus for all quality levels, as subsidised advertising effort is larger than unsubsidised advertising effort, we have that the retail payoff is larger with the provision of subsidy.
Further we observe from Figure 1 that the retail effort increases with quality, and then stabilizes after a certain quality level value. Thus beyond a certain quality level improvement, the retail effort is no longer necessary. At this point and beyond, the product has the tendency of aiding its sales without increase in the retail advertising effort.
Also, we observe that continuous increase in quality does not lead to an equivalent increase in payoff as can be seen in Figure 2. This is because of diminishing return from quality. In short, it is obvious that after a certain quality level, the marginal return is zero. Thus, such a quality level should not be exceeded! Further, we note that advertising effort and retailer's payoff are better with subsidy

Effect of Product Quality on the Manufacturer's Payoff

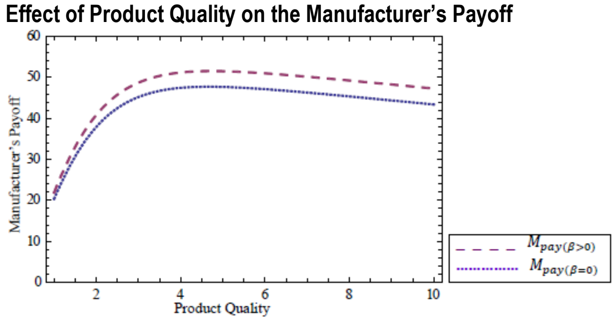

Figure 3: Effect of product quality on product demand

Considering Figure 3 we observe an increase in manufacturer's payoff for both subsidised and unsubsidised advertising as quality level increases for lower quality levels. This eventually plunges. This implies that the manufacturer's product quality improvement results in negative effect on the payoff. In short, it leads to diminishing returns. Thus the improvement should be done with caution! It is advisable for him to focus on his optimal quality to avoid this scenario. Further, it is clear that for all quality levels, the manufacturer's payoff is larger with subsidy.

The Effect of Price on Quality

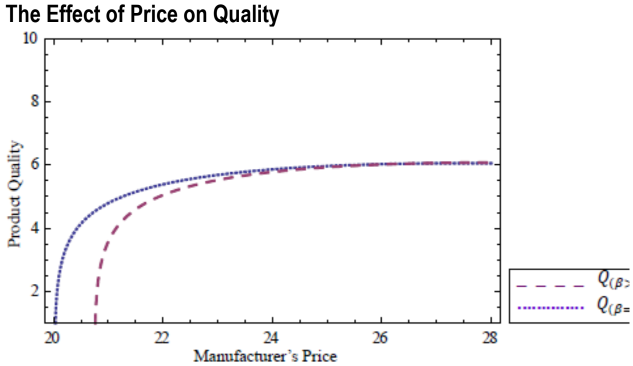

Figure 4: Effect of the manufacturer's price on product quality

From Figure 4 we observe that as the manufacturer's price increases, the quality level also increases for both subsidised and unsubsidised retail advertising. Thus, increase in price is a motivation for product quality improvement. In other words, increase in price serves as a compensation for increase in quality spending. However, it is important to note that for manufacturer's prices not above the optimal value, the quality level is higher in the absence of subsidy (and lower with subsidy). That is, a high quality level is needed whenever the manufacturer does not provide subsidy. With subsidy, the retailer is motivated towards advertising the product. But in the absence of subsidy, the retailer will not engage much in advertising, thus leading to low patronage and profit. Thus to compensate for this, the manufacturer will have to increase the product quality. This shows that the manufacturer has the option of switching between quality and subsidy.

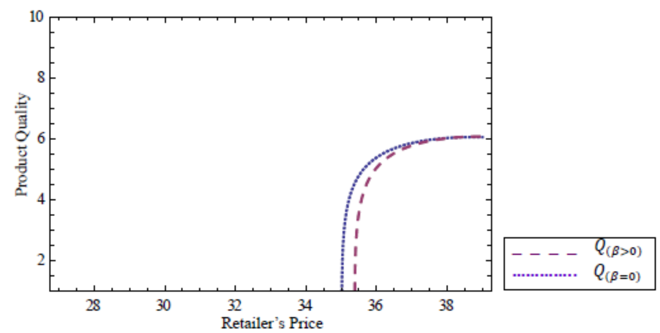

Figure 5: Effect of the retail price on product quality

In Figure 5 we observe a similar trend discussed in Figure 4 above. This stems from the direct linear relationship between the retailer's price and the manufacturer's price as given in (8) where an increase in the manufacturer's price signals an increase in the retail. As the manufacturer's price increases, the quality level also increases as shown in Figure 4 for both subsidised and unsubsidised retail advertising, thus leading to increase in quality in both situations as the retail price increases. Further, the provision of subsidy by the manufacturer which implies lower quality invariably implies lower quality for all corresponding retail prices.

Switching between Manufacturer's Subsidy and Product Quality

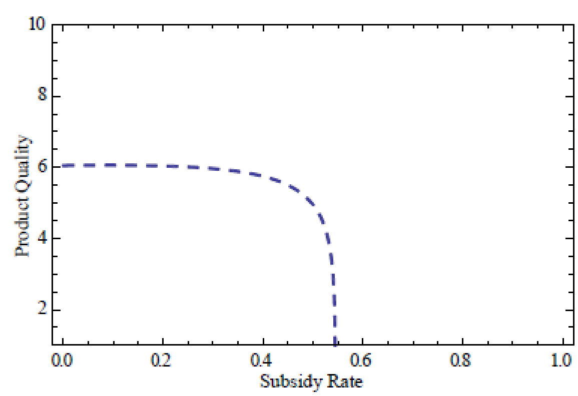

Figure 6: Effect of subsidy on product quality

Figure 6 shows that quality decreases with subsidy. This is in consonance with the observation on Figure 4 where we noted that the manufacturer can switch between improving product quality to the consumers and advertising subsidy to the retailer. Both Figure 4 and Figure 6 show that an increase in one implies a decrease in the other, and vice versa, affirming that this quality-subsidy switch is possible. The choice of which approach to adopt depends on the manufacturer's assessment of the situation. We note that naturally, the use quality may be far more appealing to the manufacturer especially if there is any uncertainty regarding the retailer's utilization of subsidy if provided. Thus, just like retail subsidy, the manufacturer can use product quality to coordinate a supply channel.

Conclusion

This work considered cooperative advertising involving a manufacturer and a retailer in a channel. Both channel members are assumed to be involved in a static Stackelberg game in which the manufacturer is the Stackelberg leader, while the retailer is the follower. The work considered two channel structures and obtained the optimal advertising effort, retail price, wholesale price, the retailer and the manufacturer's payoffs for both channel structures. It observed that increase in price is a motivation for quality improvement. Further, quality improvement leads to diminishing marginal returns on retail advertising effort and retailer's payoff which eventually becomes zero with higher quality levels. Worst still, quality improvement adversely affects the manufacturer's payoff. This suggests that the manufacturer should adopt his optimal quality level. In addition, the work shows that quality improvement can be used as substitute for retail advertising subsidy, thus extending the cooperative advertising literature.
This work considered the effect of product quality in cooperative advertising. An extension can factor in other marketing concerns. This can lead to a better insight on cooperative advertising. The work used a multiplicative model to consider the effect of quality on advertising, subsidy, retailer's payoff, and manufacturer's payoff. Exploring other model-types can lead to different results. This can lead to better understanding, and eventually extend the cooperative advertising literature.

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