

# Using inert hot-spots to induce ignition within industrial stockpiles

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## Abstract

The potential for materials undergoing oxidation reactions to spontaneously combust when they are stored in large stockpiles is well known. We consider an application in which such self-heating is desirable and investigate the use of inert hotspots as a means to promote thermal runaway. The size and location of the hotspot are found to have the largest effects on self-heating. Less pronounced are effects due to a periodic ambient temperature. The advection velocity through the stockpile can have large effects.

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## 1 Introduction

In this article we consider the stockpiling of a byproduct from the manufacture of steel. This material undergoes exothermic oxidation reactions which are capable of causing a large temperature increase within the stockpiles. This process is referred to as self-heating of the stockpiles. Excessive self-heating, resulting in thermal runaway, is considered undesirable in coal and compost stockpiles as this results in a loss of material. However, in the storing filter cakes, high temperatures are desirable since the filter cake undergoes a self-sintering process which increases its strength, making it easier for the manufacturer to reuse the material [8]. One mechanism that the manufacturer uses to promote self-heating is to transfer hot material from an already ignited stockpile into a new stockpile. We model this process by treating the hot spot as an inert material with a high temperature and ask, what characteristics of the hot spot ensure spontaneous combustion of the new stockpile?

We approach this problem using Frank–Kamanetskii theory (FK). This uses the non-dimensional equation [2]

$$\frac{\partial \mathbf{u}}{\partial \mathbf{t}} = \frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2} + \delta \exp \left[ \frac{\mathbf{u}}{1 + \epsilon \mathbf{u}} \right], \quad (1)$$

where  $\mathbf{u}$  is the temperature,  $\mathbf{x}$  is the length,  $\mathbf{t}$  is the time,  $\delta$  is the FK parameter and  $\epsilon$  is a another scaled parameter. Bowes [2] describes how these parameters relate to the physical constants. When  $\epsilon = 0$  there is a critical value  $\delta_{\text{cr}}$  of the FK parameter  $\delta$  such that for  $\delta > \delta_{\text{cr}}$  thermal runaway occurs. When

$\delta < \delta_{\text{cr}}$  there exists a stable low temperature solution. However, thermal runaway can still occur depending upon the initial temperature profile [5].

Weber et al. [12] investigated constant, linear, and quadratic initial temperature profiles. Brindley, Griffiths and McIntosh [3] introduced reactant consumption and considered an embedded hotspot. They examined numerically some criteria required to initiate combustion waves. Latter this approach was developed into an analytic method [9]. Both of these papers extended previous work that had been carried out on strongly reactive materials [6, 7], and assumed the embedded hotspots produce a constant power output; this proved challenging to implement in the stockpiles and motivated the current work.

Shah et al. [10] examined hotspots which are more consistent with the type we consider. They examined smoldering behaviour when the reaction rate is controlled by a low oxygen concentration. Their work has since been applied to both numerical and experimental studies into ignition by a heated particle [4, 11].

We build upon previous work by investigating the possibility of inducing ignition using a hot inert material. In doing this we extend the FK equation (1) by allowing for advection through the pile. Our model is

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{\partial^2 \mathbf{u}}{\partial x^2} + \mathbf{v} \frac{\partial \mathbf{u}}{\partial x} + \delta I(x, t) \exp \left[ \frac{\mathbf{u}}{1 + \epsilon \mathbf{u}} \right], \quad (2)$$

on the spatial domain  $[-1, 1]$  with boundary conditions  $\mathbf{u} = \mathbf{u}_o \sin 2\pi t / \omega$ , where  $\mathbf{v}$  is the advection velocity,  $\mathbf{u}_o$  is the amplitude of temperature oscillation and  $\omega$  is the oscillation period. The boundary condition represents the yearly change in ambient temperature around its mean-value  $\mathbf{u} = \mathbf{0}$ . The boundary condition was also used in previous work by the authors [1]. The term  $I(x, t)$  denotes the scaled reactant concentration over the domain; this provides a simple mechanism to include an inert hot spot.

We define the hotspot as the region  $[\mathbf{h}_c - \mathbf{h}_l, \mathbf{h}_c + \mathbf{h}_l]$ ; that is, it is centered at  $x = \mathbf{h}_c$  and has length  $2\mathbf{h}_l$ . To ensure that the hotspot is contained within the

domain we require  $-1 \leq h_c - h_l < h_c + h_l \leq 1$ . The simplest place to add the hotspot is at one end of the stockpile. Subsequently, we predominantly use the condition  $h_c + h_l = 1$  which forces the hotspot to the right edge of the stockpile.

We assume that the scaled concentration of reactants is one outside the hot spot and zero within the hotspot. Thus

$$I(x, t) = I_0(x) = \begin{cases} 0 & x \in [h_c - h_l, h_c + h_l], \\ 1 & x \notin [h_c - h_l, h_c + h_l]. \end{cases}$$

The hotspot is a region of elevated temperature, hence we define the initial condition

$$u_0(x) = \begin{cases} u_h & x \in [h_c - h_l, h_c + h_l], \\ 0 & x \notin [h_c - h_l, h_c + h_l]. \end{cases}$$

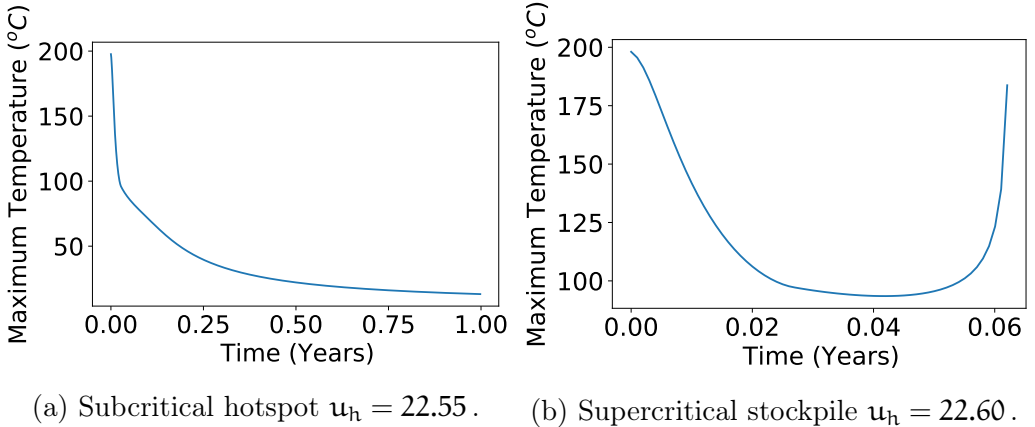
In order to determine whether our stockpiles ignite we need an ignition criterion. As in our previous work we use a fixed temperature ignition criteria [1]. That is, if the stockpile temperature exceeds a defined threshold within the simulation period, then the stockpile is deemed to have ignited. Our threshold temperature is  $u_{ig} = 100$ .

Our main focus is on determining the critical hotspot temperature needed to induce ignition. We consider different hotspot characteristics, such as its size and position, as well as external effects, such as the oscillating boundary condition and the advection velocity through the stockpile.

We also consider the effect that reactant consumption has on the critical hotspot temperature. We do this by introducing a differential equation for the reactant concentration

$$\frac{\partial I}{\partial t} = -\delta_1 I \exp \left[ \frac{u}{1 + \epsilon u} \right], \quad (3)$$

where  $\delta_1$  is analogous to the FK parameter  $\delta$ . For consumption of a solid reactant there is no diffusion or advection.



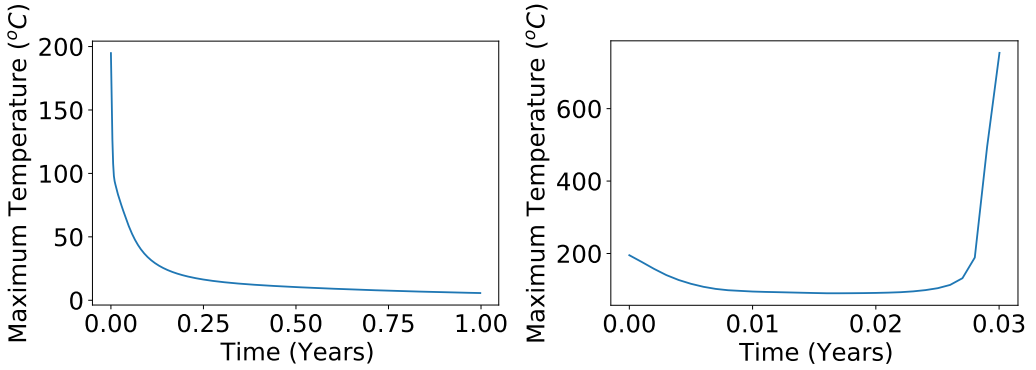
(a) Subcritical hotspot  $u_h = 22.55$ . (b) Supercritical stockpile  $u_h = 22.60$ .

**Figure 1:** Comparison of the thermal evolution in two stockpiles with slightly different hotspot temperatures. The simulations do not include fuel consumption. Parameter values:  $h_c = 0.9$ ,  $h_l = 0.1$ ,  $\delta = 0.45$ ,  $\epsilon = 0.027$ ,  $\nu = 0$ .

## 2 Results

We first consider whether our model is sensitive to our ignition criterion. Our results, not shown here, show that the critical hotspot temperature is practically independent of our choice of ignition temperature  $u_{ig}$ . One exception is when the ignition temperature is low. To understand the independence of the critical hotspot temperature, it is useful to compare the maximum temperature within two stockpiles with slightly different hotspot temperatures.

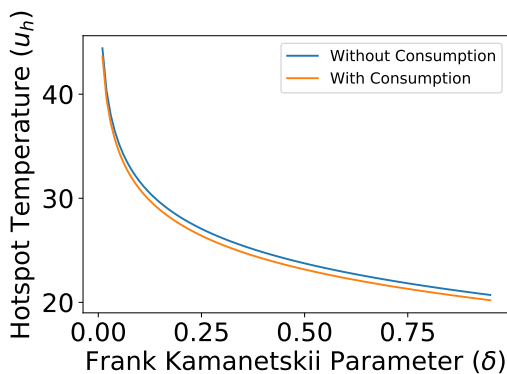
In our simulations we use the parameter values from Berry et al. [1]. Figure 1 compares the behaviour of two stockpiles with slightly different hotspot temperatures where we do not consider the reactant consumption. In Figure 1a the scaled temperature of the hotspot is  $u_h = 22.5$  whereas in Figure 1b it is slightly higher,  $u_h = 22.6^\circ\text{C}$ , corresponding to a difference of approximately  $0.5^\circ\text{C}$ . In the former, the stockpile gradually cools throughout the

(a) Subcritical hotspot  $u_h = 23.5$ .(b) Supercritical hotspot  $u_h = 23.55$ .

**Figure 2:** Comparison of the thermal evolution in two two stockpiles with slightly different hotspot temperatures. The simulation includes fuel consumption. Parameter values:  $h_c = 0.9$ ,  $h_l = 0.1$ ,  $\delta = 0.45$ ,  $\epsilon = 0.027$ ,  $\nu = 0$ .

integration period whereas in the latter, the stockpile also cools but then begins to undergo significant self-heating with the simulation terminated once the maximum temperature exceeded the predefined threshold  $u_{ig} = 100$ . This figure shows clear distinction between stockpiles that are subcritical and those that are supercritical. The sharp rise in temperature that is observed in the supercritical stockpile continues after the displayed integration period. This rapid heating will cause the stockpile temperature to exceed any reasonable, predefined ignition criteria for our application. As a result, further analysis of this model we regard as not sensitive to our choice of ignition temperature. This difference is also observed when we include reactant consumption, see Figure 2, where we use the same difference in hotspot temperature.

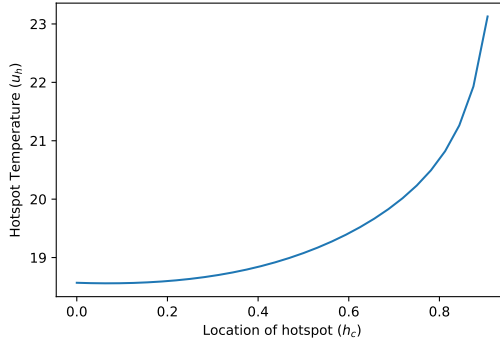
We now investigate how the critical value of the hotspot temperature changes as a function of the FK parameter  $\delta$ . Figure 3 shows that as the FK parameter increases the critical hotspot temperature decreases. This is unsurprising, as increasing the value for the FK parameter increases the amount of heat



**Figure 3:** The effect the FK parameter has on the critical hotspot temperature. Parameter values:  $h_c = 0.9$ ,  $h_l = 0.1$ ,  $\delta = 0.45$ ,  $\epsilon = 0.027$ ,  $\nu = 0$ .

released by the reaction. This figure also indicates that the inclusion of reactant consumption increases the hotspot temperature but the trend line is maintained. For the remainder of our analysis we ignore the consumption of reactant. Although consumption changes the critical hotspot temperature we find that the relationships between the parameters remains the same. The consumption of material has a larger effect on the long term temperature evolution and a small effect on questions relating to criticality.

In practical applications we are most interested in the parameters we can control, primarily the characteristics of the hotspot. We now investigate what effect changing the centre of the hotspot  $h_c$  has on the critical hotspot temperature. Figure 4 shows that as the center of the hotspot moves towards the center of the stockpile, the critical temperature decreases. This is expected, as moving the hotspot away from the boundary decreases the amount of its initial heat content which is lost to the surroundings. This analysis could be paired with a cost function for the effort it would take to place a hotspot in the centre of an already built stockpile, to determine whether this is worthwhile. A new stockpile could be build around a hotspot and a centred hotspot is recommended for such a pile.

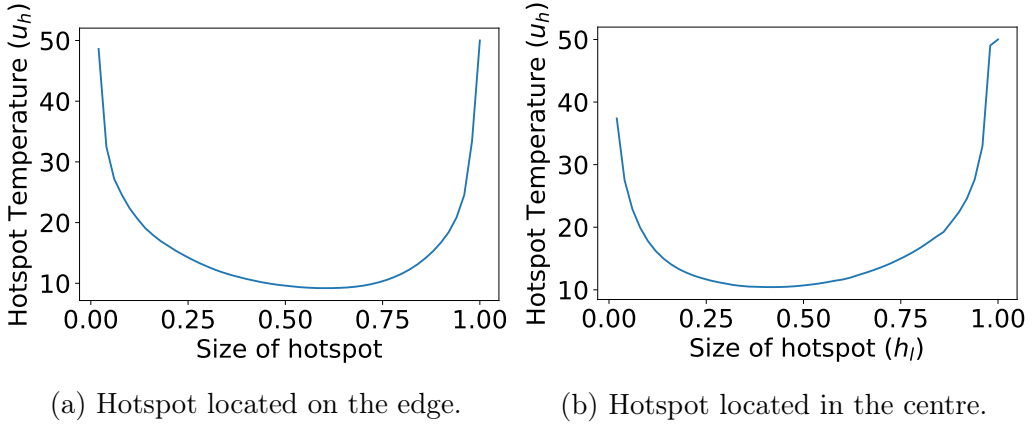


**Figure 4:** The effect that hotspot location has on the critical hotspot temperature. Parameter values:  $h_1 = 0.1$ ,  $\delta = 0.45$ ,  $\epsilon = 0.027$ ,  $\nu = 0$ .

We now investigate the effect of changing the width of the hotspot  $h_1$ . Recall that the parameter  $h_1$  is the proportion of the stockpile that the hotspot takes up, that is, when  $h_1 = 0.1$  the hotspot is 10% of the stockpile's length. Figure 5 shows the two limiting cases when the hotspot is placed either on the edge or in the center of the stockpile. In both cases, as the size of the hotspot initially increases, the critical hotspot temperature decreases. However, once the hotspot size increases past a certain size, the critical temperature increases. It is important to note that this is a feature of the inert hotspot. This large hotspot condition could be of interest in a scenario in which one stockpile has reacted to completion, in which case a significant amount of material is available to add to the end of a subcritical stockpile. This result assumes that the stockpile is maintained at a fixed length. If we fix the length of the reactive material and increase the hotspot size, then this increases the length of the stockpile increasing the FK parameter  $\delta$ . From Figure 3 we know that this decreases the critical hotspot temperature.

We are also interested in how external effects such as variation in the ambient temperature and advection through the stockpile affect the critical hotspot temperature. The stockpiles are porous and stored in an outdoor setting for periods of many months. Consequently, variation in the the ambient

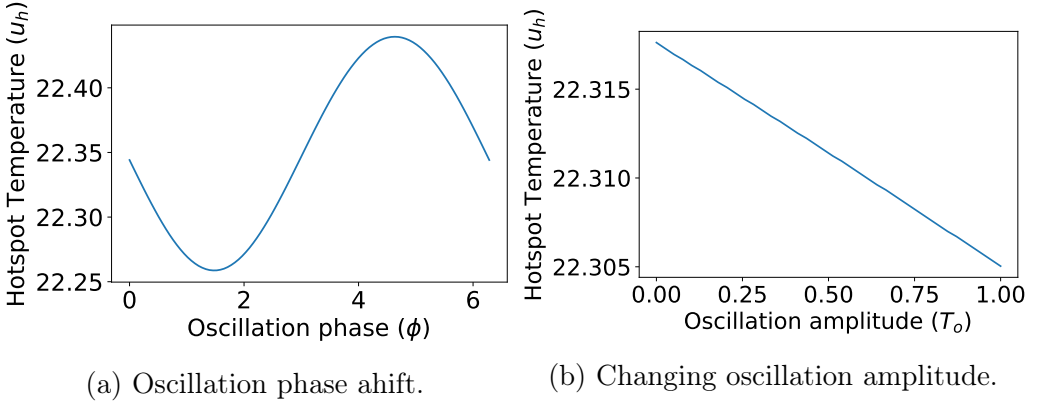




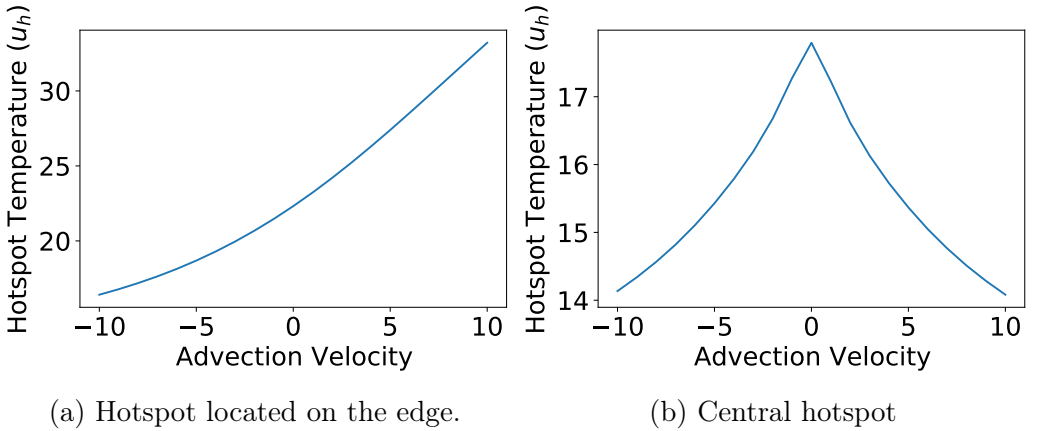
**Figure 5:** The effect that the size of a hotspot has on the critical hotspot temperature for a centrally located hotspots and edge hotspots. Parameter Value:  $\delta = 0.45$ .

conditions are considerable. Figure 6 shows that the effect of the periodic boundary condition is negligible. We attribute this to the results displayed in Figure 1b where the stockpile ignites in a short period of time, thus limiting the possible effects of the boundary condition. The effects might be greater for marginally subcritical stockpiles.

Advection currents through the stockpile have a more significant effect on the critical hotspot temperature than periodic boundary conditions, as displayed in Figure 7. We observe that the greatest benefit is when advection moves energy from the hotspot towards the centre of the stockpile. When the hotspot is located at the centre of the stockpile any advection decreases the critical hotspot temperature, as heat is transported away from the inert hotspot into the reactive stockpile. We find that the advection term has the potential for large effects. However, it is challenging to determine a realistic advection term; in reality the effect on the hotspot temperature may only be small.



**Figure 6:** The effect of the seasonal temperature oscillation and the oscillation phase have on the critical hotspot temperature. Parameter values:  $h_c = 0.9$ ,  $h_l = 0.1$ ,  $\delta = 0.45$ .



**Figure 7:** The effect of the advection on the critical hotspot temperature. Parameter values:  $h_c = 0.9$ ,  $h_l = 0.1$ ,  $\delta = 0.45$ .

### 3 Conclusion

We have investigated the use of a hotspot to kick-start self-heating within a stockpile. In particular we have explored some of the relationships between

the characteristics of the hotspot; that is, its size and location, and the environment of the hotspot temperature required for thermal runaway. We have displayed some crucial relationships to promote ignition that can be applied to specific stockpiles once the underlying parameters are known. Centrally located hotspots require less heat, that is, have a lower critical temperature, to ensure ignition. This is attributed to a decrease in energy loss at the boundary. Under the right conditions, hotspots are an effective tool in promoting ignition. They can also be used to reduce the time to ignition for stockpiles that are supercritical when  $\delta > \delta_{cr}$ . This latter application has the potential to increase the recycling of the by-products in the steel-making process.

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