EFFECT OF JOINT TORQUE ON HORIZONTAL MOTION OF CENTER-OF-MASS DURING SWING ON GYMNASTIC RINGS

Shusuke Imai[1], Taiga Yamasaki[1], Kyoji Yamawaki[2], Xin Xin[1], Shinsaku Izumi[1]

Faculty of Computer Science and Systems Engineering, Okayama Prefectural University, Okayama, Japan[1]

Faculty of Education, Gifu University, Gifu, Japan[2]

To perform a good swing on the rings in gymnastics, it is said to be important to keep the center-of-mass (COM) almost along the vertical line through the cable attachment point on the ring frame. However, the theoretical background of this empirical knowledge is not clear. In this study, we address a question whether a gymnast can always affect the horizontal motion of COM by the input of joint torque during swinging. Using a three-segment model in the sagittal plane, composed of a cables-rings segment, an arms segment, and a head-neck-torso-legs segment (Sprigings et al., 1998), we analyse the relation between the shoulder torque and the COM motion. The results show that there exist a variety of configurations where the shoulder torque cannot affect the horizontal motion of COM.

KEY WORDS: rings event, gymnastics, COM, singular point, three-segment model, shoulder torque

INTRODUCTION: Controling COM is important in a variety of human motions. In this study, we focus on the rings event in men's gymnastics (Sprigings et al., 1998; Brewin et al., 2000; Yeadon & Brewin, 2003). To perform a good swing on the rings from the hanging position, it is said to be important to keep COM almost along the vertical line through the cable attachment point on the ring frame (referred to as the origin). This indicates that the gymnast should control the horizontal position of COM during swinging. However, the theoretical background of this empirical knowledge is not clear. In this study, using a three-segment model of the rings motion (Sprigings et al., 1998) for simplicity, we address a question whether the gymnast can always affect the motion of COM of the whole system by exerting joint torque during swinging.

METHODS: The swing on the rings in the sagittal plane was modeled by the three-segment model (Sprigings et al., 1998) composed of a cables-rings segment, an arms segment, and a head-neck-torso-legs segment as shown in Figure 1. The equations of motion of the model is described as

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = B\tau_3 \tag{1}$$

where $q = [q_1, q_2, q_3]^{\mathrm{T}}$ is the vector of the joint angles, τ_3 is the torque of the shoulder joint, M is the inertia matrix (3×3), C is the vector of Coriolis force and centrifugal force (3×1), G is the vector of gravitational force (3×1), and $B = [0, 0, 1]^{\mathrm{T}}$ is the transformation matrix. $(\cdot)^{\mathrm{T}}$ denotes the transpose of a matrix. The details of the terms are omitted because of the page limitation. The physical parameters of the model were obtained from Sprigings et al. (1998). It is worth noting that a gymnast must control three joints by using only one input torque.
The horizontal position of COM of the whole system is

$$x_g = \gamma_1 \sin q_1 + \gamma_2 \sin(q_1 + q_2) + \gamma_3 \sin(q_1 + q_2 + q_3) \tag{2}$$

where $\gamma_1, \gamma_2, \gamma_3$ are constants determined by the physical parameters. The condition of COM on the vertical line through the origin is described as $x_g = 0$. When $x_g = 0$, q_1 can be determined by q_2 and q_3 as

$$q_1 = \pi + \tan^{-1}\left(-\frac{\gamma_2 \sin q_2 + \gamma_3 \sin(q_2 + q_3)}{\gamma_1 + \gamma_2 \sin q_2 + \gamma_3 \sin(q_2 + q_3)}\right) \tag{3}$$

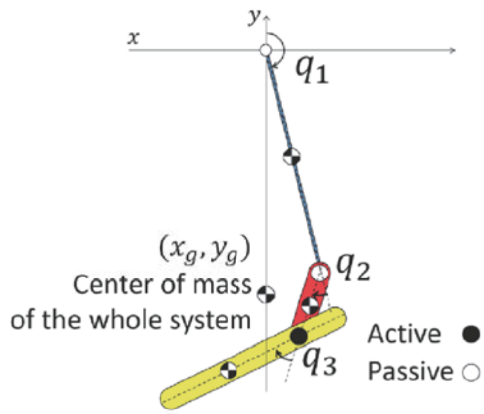

Figure 1: Three-segment model of the swing on the rings in the sagittal plane

To analyse whether the shoulder torque can affect the horizontal motion of COM, we derived the equation of motion with respect to x_g as follows (Khatib, 1990). The horizontal velocity of COM $\dot{x}_g$ is determined by the angular velocity $\dot{q}$ as

$$\dot{x}_g = J(q)\dot{q} \quad (4)$$

where $J(q) = \partial x_g/\partial q$ is the Jacobian matrix (1×3). Taking the derivative of (4) yields,

$$\ddot{x}_g = J(q)\ddot{q} + \dot{J}(q,\dot{q})\dot{q} \quad (5)$$

By substituting (1) into (5), we obtain

$$\bar{M}(q)\ddot{x}_g + \bar{J}^{\mathrm{T}}(q)(C(q,\dot{q}) + G(q)) - \bar{M}(q)\dot{J}(q,\dot{q})\dot{q} = \bar{J}^{\mathrm{T}}(q)B\tau_3 \quad (6)$$

which can be regarded as the equation of motion with respect to x_g, where

$$\begin{aligned} \bar{M}(q) &= \left(J(q)M^{-1}(q)J^{\mathrm{T}}(q)\right)^{-1} \\ \bar{J}^{\mathrm{T}}(q) &= \left(J(q)M^{-1}(q)J^{\mathrm{T}}(q)\right)^{-1}J(q)M^{-1}(q) \end{aligned} \quad (7)$$

The right-hand side of (6) shows the effect of the shoulder torque τ_3 on the motion equation of x_g. We can see that, when $\bar{J}^T(q)B = 0$, the shoulder torque τ_3 cannot affect the horizontal motion equation of COM. Moreover, since the range of q_1 is narrow in rings motion, the condition $\bar{J}^{\mathrm{T}}(q)B = 0$ can be simplified as $J(q)M^{-1}(q)B = 0$. We call the configuration $q = [q_1, q_2, q_3]^{\mathrm{T}}$ that satisfies $J(q)M^{-1}(q)B = 0$ a singular point. We calculated the singular points under $x_g = 0$.

RESULTS & DISCUSSION: Figure 2 shows the singular points calculated numerically by solving $J(q)M^{-1}(q)B = 0$ under $x_g = 0$. Every point (q_2, q_3) in Figure 2 corresponds to the configuration satisfying $x_g = 0$. The other angle q_1 is determined by (3). The points on the solid curves in Figure 2 are the configuration satisfying $J(q)M^{-1}(q)B = 0$, i.e. the singular point, where the shoulder torque cannot affect the horizontal motion of COM. Figure 3 shows the stick figures corresponding to these singular points. The results represent that there exist a variety of singular points in the rings motion. In Figure 2, it can be seen that there is no singular point in the configuration where q_2 is close to zero, and the cables and arms align on a straight line. This means that, when amplitudes of swings are small, the singular point would be avoided by keeping such configurations during swinging. However, when amplitudes of swings are large, for example, the motion ranging from the hanging position ($q_2 = 0$ deg) to the handstand position ($q_2 = 180$ deg), it can be seen that it is unavoidable to intersect with the singular point in Figure 2.

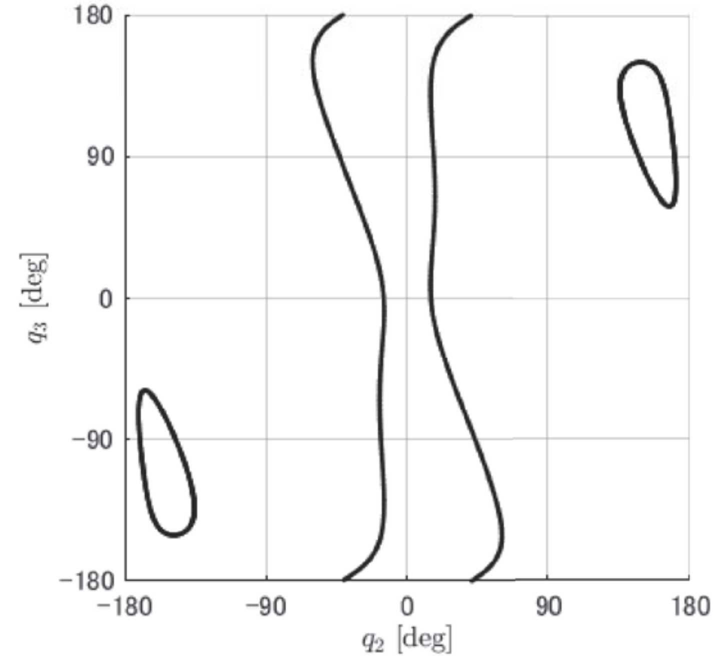

Figure 2: The set of the singular points that satisfies $J(q)M^{-1}(q)B = 0$ under $x_g = 0$.

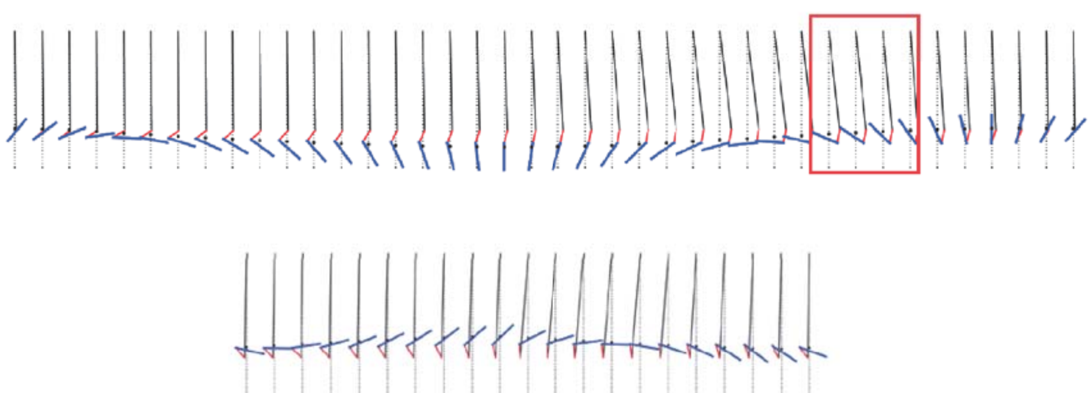

Figure 3: The stick figures corresponding to the singular points shown in Figure 2. In these configurations, the shoulder torque cannot affect the horizontal motion of COM. The configurations enclosed by the red rectangle are similar to the configuration of a gymnast shown in Figure 4.

Figure 4: Swing on the rings by a gymnast. The configuration enclosed by the red rectangle is similar to the configurations shown in Figure 3.

Figure 4 shows the configurations of a gymnast during swinging. It appears that the configurations represented in the right pictures (marked by the red rectangle) are similar to those in Figure 3 (marked by the red rectangle). This would suggest that the gymnast takes the configurations near the singular points during swinging. However, it can be also observed that the gymnast adducts the shoulder and flexes the elbow near the singular points in Figure 4. This implies that the arm motion outside the sagittal plane might be employed to avoid the singular points, in order to control the horizontal motion of COM.

CONCLUSION: The purpose of this study was to address a question whether a gymnast can always affect the horizontal motion of COM by the input of joint torque during swinging on the rings. Using a three-segment model in the sagittal plane, the relation between the shoulder torque and the COM motion was analysed. The results showed that there exist a variety of configurations (singular points) where the shoulder torque cannot affect the horizontal motion of COM. Thus, it is not always easy to keep COM along the vertical line through the origin, as long as the motion is limited to the three-segment model in the sagittal plane. In order to avoid the singular points, it was suggested that the cables and the arms should be aligned almost in a straight line for swings with small amplitude, or the arm motion outside the sagittal plane might be employed near the singular points for swings with large amplitude.

Although this paper used the three-segment model for simplicity and focused on the configurations where COM is along the vertical line through the origin, our approach can be extended to more complex model and any configuration where COM is deviated from the vertical line. Future research should focus on various COM positions and experimental validation with a four-segment model including the hip joint (Sprigings et al., 1998) or a three-dimensional model (Brewin et al., 2000; Yeadon & Brewin, 2003).

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Acknowledgement

This work was partly supported by JSPS KAKENHI Grant Number 16K00356.