MATHEMATICAL MODELLING OF PERFORMANCE AND UNDERLYING ABILITIES IN SPRINTING

LAMES, M.
University of Mainz
Mainz
FRG

1. Problem

There are two traditional methods of analysing behaviour in sprinting competition: apart from measuring stride-frequency and stride-length, the method of speed curves is common. The latter one is usually the result of time measurements taken on certain intervals on the course. In case of a 100m-dash usually every 10 meters are measured.

In order to get the speed curve the difference between two neighbouring measurements is taken and divided by the distance between them. This procedure produces the mean velocity for every interval.

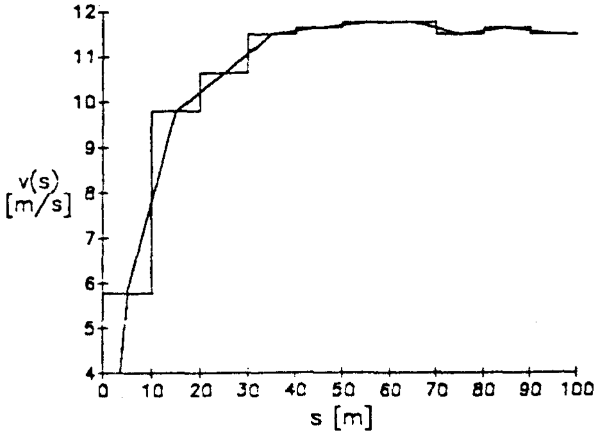

Figure 1. Speed curve drawn as a polygone and step-function

Some critical remarks have to be applied:

- The common represention of speed curves is a polygon which connects the average speed levels at the midpoints of the intervals. This suggests a continuous function and it is a simplification to assume mean speed in the middle of an interval. Actually the procedure supplies a non-continuous step-function (Fig.1).
- Detailed analysis of the results is not satisfactory: if one simply asks for the location of maximum speed the answer can only be the interval with maximum average speed. One cannot even be sure that the actual maximum is located in this interval, because in unlucky cases it might as well be in a neighbouring one.
- Taking differences - in cases of acceleration curves differences of differences - makes the error for velocity measurements systematically larger than the error we have already for a single time measurement. These errors are amplified by the arrangement in a measurement chain, where the random error of one interval becomes the systematic one of the next.

Apart from these more methodological objections one should consider a general aim of analysing performance in competition: to establish a link between behaviour (visible) and its conditions /invisible/, between description and explanation. This desideratum applies to all biomechanical measurements.

In order to generate practically useable information for training it is not always sufficient to stick to mere description of what is going on, but one has to aim at the underlying conditions of this behaviour.

There is broad consensus on underlying abilities in our example, the 100 m - dash: the basic abilities are

reaction time,
ability of acceleration,
sprinting speed and
sprinting endurance.

The operational definitions of these basic abilities appear - exept for reaction time - to be problematic though:

- The ability of acceleration includes the aspects of high acceleration and of acceleration over a long time. It is a complex ability which must not be necessarily one-dimensional. Operational definitions can only aim at one aspect: the initial acceleration represents the maximum amount of acceleration, the distance with positive acceleration or the corresponding time used to reach maximum speed stand for its duration.
- The conventional method measures sprinting speed as the maximum average speed in an interval. So, apart from errors due to the original time measurements, we have a systematic error: maximum average speed underestimates by definition maximum speed. (This holds although - for other error-sources - we usually observe an overestimation of maximum speed (Fig. 3)).
- The operational definition of sprinting endurance as a difference of differences (maximum speed minus final speed) increases the influence of errors from the original measurements.

One reason for the problems cited above is that input data (intermediate times) cannot be transformed into a satisfactory description of behaviour in competition. The resulting step-function is non-continuous and supplies only average speed per interval.

2. Modelling of sprinting behaviour with analytic functions

Facing these problems the idea came up to describe sprinting behaviour with analytic displacement-, speed- and acceleration- functions obtained by (non-linear) regression. The advantages of a regressional approach are obvious:

- Errors in the original measurements are smoothed by the regression function. This holds because one doesn't interpolate but minimizes the Squared Sum of Errors (SSE). Compared with the conventional method we expect a damping of errors instead of an amplification due to the use of differences.
- Speed and acceleration are no longer determinated by using differences but by differentiation of the fitted function.
- Using continuous functions we have speed and acceleration values for any point on the course. Especially the determination of maximum speed location results in a point and not in a 10m-interval. although this point is of course still subjected to errors these are not systematic any more.

2.1. Developement of an appropriate model-function

Since reaction time is an additive parameter it is excluded from the following considerations.

Model-building with regression functions can be performed in two fundamentally distinct ways. Inductive model-building condenses data into a function: "Which function do I know that looks almost the way may data do?" Deductive model-building tries to generate regression functions from asssumptions on the underlying process:" Which function describes the internal functioning of the modelled system?"

Inductive model-building has severe drawbacks. One just can't have the same confidence in an inductive model as in a deductive one, although sometimes complexity of systems or lacking knowledge permit only inductive models (see FUCHS/LAMES 1989).

Trying the deductive approach we assume that the speed curve can be understood as a superposition of two growth processes: acceleration v_A and fatigue v_F. With an additive superposition we arrive at the model-function v as follows:

$$v(t) = v_A(t) + v_F(t) = A \quad (1-e^{-kt}) + F\,(1-e^{lt})$$
$$A,k,l>0, \; F>=0.$$

v is a function of time with 4 parameters and, typical for deductive modeling , these parameters have got interpretations in the original system:

A	=	absolute speed-limit achieved by infinitely long acceleration without fatigue,
k	=	steepness of acceleration process,
F	=	onset of fatigue and
l	=	steepness of fatigue-impact.

Note that one is not dealing with a mechanical model but with a system-oriented one.

In 1951 HENRY and TRAFTON have used a model which is identical to the acceleration component of the introduced one. Their model performed very well in predicting speed-curves of 60y-dashes. Also they found that the parameters A and k were independent.

Practical calculations with our model forced a modification. Having only 11 data points but 4 parameters results in unstable estimates for the parameters. In addition to this, the two parameters of the fatigue-process are only loosely determined by data. The two reasons are that only the last measurements show one significant impact of fatigue and that its overall influence sprinting speed is small compared with the influence of acceleration.

These inductive considerations on lacking quality of data lead to the elimination of parameter l, because the steepness of the fatigue-impact seems even less determinable than its onset. Elimination of a regression parameter means that an appropriate constant value for it is chosen instead of obtaining an estimate by a regression algorithm.

The model-building process is resumed in Figure 2. Several kinematic aspects are involved:

- data consist of intermediate times,
- the model-function is a speed-curve over time,
- usual representations are speed and acceleration curves over the course and
- regression is based on displacement over time (This has the advantage of making use of the raw-data without transformation).

The regression function is obtained by integrating the model function v:

$$s(t) = (A+F)t-A/k(1-e^{-kt}) + F/l(1-e^{lt}).$$

Acceleration is obtained by differentiation:

$$a(t) = Ake^{-kt}-Fle^{lt}.$$

A critical remark has to be made: for a starting runner (t=0) holds s=v=a=0, while the model assumes maximal acceleration at t=0 (see Figure 2). As a consequence one has to admit that the model is not able to describe precisely what is going on on the first few meters. This is not surprising because one can't reasonably expect a description of the building-up of acceleration on the first meters by a model which has just the time for 0 and 10 meters as relevant input. In order to describe this phase more precisely different methods had to be applied.

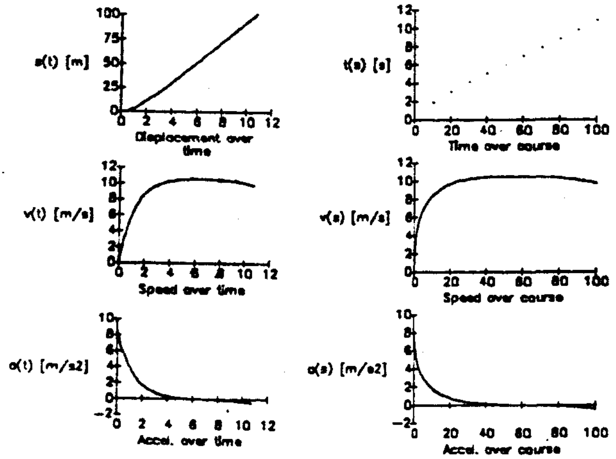

Figure 2: Kinematic aspects of sprinting performance.

2.2. Deriving indicators for basic abilities

With analytic speed- and acceleration functions it is possible to overcome some of the troubles with operational definitions of basic sprinting abilities quoted above.

- The best indicator of <u>reaction time</u> is of course reaction time itself.

- The <u>ability of acceleration</u> is described in its two aspects: amount and duration.

The indicator for the amount is $a_0=a(0)$, that is the initial acceleration.

Usual indicators of duration for acceleration are the time used to reach maximum speed (t_m) and the point on the course for this event $s_m=s(t_m)$. Analysing empirical speed curves reveals that speed is almost constant between 40 and 100 meters though. So, fixing the location of speed maximum is a sort of gambling. For this reason as indicator the time t_{eps} is chosen. At that time acceleration has not yet dropped to zero but to a very small value eps, i.e. eps=0.1 m/s^2. t_{eps} and $s_{eps}=s(t_{eps})$ are by definition smaller than t_m and s_m and we expect them to be much more precise.

- <u>Sprinting speed</u> is indicated as usual by maximum speed: $v_m=v(t_m)$.

- The quotient q_V of final speed by maximum speed is taken as indicator of <u>sprinting endurance</u>: $q_V = 100\ v(t_{100}) / v(t_m)$ [%].

3. Results of a pilot study

3.1. Robustness of the suggested method

Earlier the sensitivity of the conventional differences-method to errors in measurements was criticized and a higher robustness of the regression-method was postulated. A chance for testing these assumptions are the remarkable differences between interval times reported for the 100m-final at Rome 1987. LETZELTER (1989) pointed out that interval times reported immediately after the event deviated from those published by the official biomechanical commission some months later. Obviously the first measurement suffers much more from errors than the last one which used high-frequency techniques.

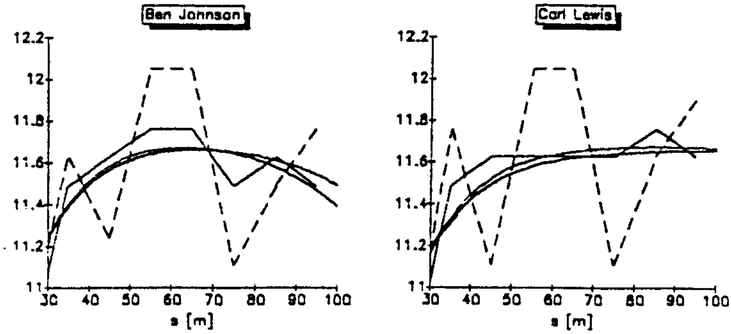

Figure 3: Speed curves from interval times reported immediately after the final at Rome 1987 (dashed) and reported by the biomechanical commission (solid). The curved lines are the results of the regression method for each set of data.

In Figure 3 in addition to the step-functions the regression curves are drawn. One sees that they are not only remarkably smoother but in particular that the two curves lead almost to identical results. This is a convincing indication of robustness because we know that one set of raw data suffers a lot from errors. Even the largest deviation between the two curves at the end of Ben Johnson's dash is smaller than 0.1m/s.

3.2 Results on sprinting abilities

The introduced method supplies estimates for the parameters mentioned above, which are only a selection of possible variables. With this data as input, ideally based on a large number of cases, very sophisticated analyses of sprinting behaviour and underlying abilities are possible. Such analyses are inappropriate though to the data base of this pilot study: the 16 100m-finalists of Rome 1987. It's aim is a methodological one. But even with 2x8 cases descriptive and correlative results seem to be very interesting.

TABLE 1
Descriptive statistics of 100m-finals at Rome 1987.

variable	abbr.	m/f	mean	st. dev.	min	max
total time	t_{100} (s)	m f	10.14 11.07	0.183 0.094	9.83 10.90	10.34 11.19
reaction time	t_r (s)	m f	0.18 0.19	0.043 0.030	0.109 0.142	0.232 0.241
initial acceleration	a_0 [m/s²]	m f	9.34 8.85	0.558 0.377	9.15 8.33	10.98 9.50
time for acceleration	t_m (s)	m f	8.08 6.24	1.507 0.281	6.43 5.88	10.00 6.77
length of acceleration	s_m (m)	m f	78.86 53.07	17.44 2.73	59.82 49.57	100.00 58.17
time for acc. to 0.1m/s2	t_{eps} (s)	m f	5.21 5.05	0.288 0.194	4.65 4.76	5.53 5.33
length of acc. to 0.1m/s2	s_{eps} (m)	m f	46.35 40.76	3.32 1.80	40.26 38.25	49.36 43.47
maximum speed	v_m (m/s)	m f	11.32 10.37	0.274 0.111	10.97 10.25	11.66 10.55
quotient for endurance	q_v (%)	m f	99.28 92.04	0.67 2.06	98.41 88.91	100.00 95.39

The descriptive statistics of 9 variables are given in Table 1. From the methodological point of view it is important to note that t_{eps} and s_{eps} are obviously better estimates than t_m and s_m. For men s_m has a range of 40 meters while the range of s_{eps} is less than 10 meters. The same tendency can be observed for women, but, as all women have a decrease in sprinting speed, their maximum speed can be more precisely determined, whereas men do not noticeably reduce their speed (min q_V=98.41%).

TABLE 2
Intercorrelations of variables describing performance on 100m (upper half: men, lower half: women: levels for significance: 0.71 (5%) and 0.83 (1%)).

	t_{100}	t_r	a_0	t_{eps}	s_{eps}	v_m	q_v
t_{100}	1	.41	.28	-.57	-.74	-.97	-.11
t_r	.67	1	.42	-.34	-.35	-.34	-.47
a_0	.16	-.29	1	-.93	-.82	-.46	-.22
t_{eps}	-.32	.22	-.97	1	.97	.73	.36
s_{eps}	-.46	.12	-.93	.99	1	.88	.34
v_m	-.83	-.27	-.62	.71	.81	1	.11
q_v	-.18	.18	-.34	.51	.49	.16	1

Table 2 shows the intercorrelations of variables for men and women. Two aspects are of particular interest: the determination of the complex criterion of performance (t_{100}) and the intercorrelations of basic abilities.

Maximum speed accounts almost singulary for the total 100m-time. The correlation is higher for men (r=-0.97) than for women (r=-0.83) but the men's sample has a broader range (0.51s versus 0.29s for women). Figure 4 shows the impressive correlation.

Maximum speed itself is correlated with duration and length of acceleration and to a smaller degree with initial acceleration. Reaction time and sprinting endurance seem to be of minor importance for performance in the two samples.

A very astonishing result is the marked but negative correlation between initial amount and duration of acceleration. Although these findings are consistent with a one-dimensional concept of the ability of acceleration, the two aspects seem to be antithetic: one can either have a large initial acceleration or a long acceleration. An explanation of this finding could be selective adaption of strenght abilities to contact time on the ground which decreases considerably.

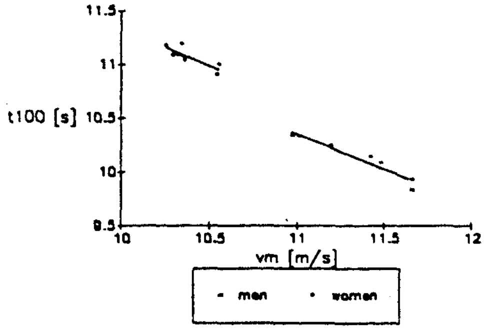

Figure 4: Scatter diagram for maximum speed t_m and total time t_{100} for the finals at Rome 1987.

4. Summary and discussion

a) Method

The regression method proved to be superior to the differences method. The advantages are:

- smoothing of raw data versus amplification of errors,
- continuous curves versus non-continuous step-functions,
- values for any point on course versus values only for intervals.

One reason for the excellent fit is that 100m-dashes are run with maximum acceleration and no tactical manipulation of sprinting speed occurs. Exceptions could be the last meters of eliminating heats when qualification is sure. But even in this case only variables quantifying sprinting endurance would be affected. This objection implies though that the model-function is not suitable for events longer than 200 meters, because in those events running speed is very much determined by tactical considerations.

The applied model is obtained by deduction and describes the additive superposition of an acceleration-process and a fatigue-process. Practical calculations impose two restrictions: one fatigue-parameter must be held constant and the model is not able to describe precisely the building-up of speed and acceleration on the first meters.

The main sciencetific advantage is that the method allows for calculation of parameters which can be interpreted as precise indicators of basic sprinting abilities.

b) Practical results

A first result is that values for maximum speed reported by the differences method have to be doubted. Because of the arrangement of measurements in a chain it is very likely that at least one interval shows values that they are too high. The error-struck measurement published immediately after Johnson's victory at Rome 1987 reported a maximum speed of 12.05m/s, the more precise biomechanical commission 11.76m/s. The regression method results in a maximum speed of 11.66m/s.

The length and duration of positive acceleration is a question of practical interest. It can now be answered by pointing out a certain point on the course. A better indicator for this aspect of the ability of acceleration is the length and duration of positive acceleration greater than an almost negligible threshold (suggestion: $eps=0.1m/s^2$).

Concrete results of the pilot study on the two finals at Rome 1987 are:

- with the exceptions of reaction time and duration of acceleration men are significantly superior to women in all variables,
- extreme groups differ demonstrably in duration and length of acceleration and especially in maximum speed,
- maximum speed is clearly the most important ability accounting for overall performace and
- the ability of acceleration seems to show a conflict between initial acceleration and its duration.

The special impact of the introduced method is that the gap between description and explanation, between performance in competition and underlying abilities is closed. The abilities can now be tested under optimal conditions: during competition.

If further investigation confirms its excellent suitability and technical progress makes data more available, the introduced method could become a routine-procedure of future training in sprint.

REFERENCES:

FUCHS, P. and LAMES, M.: Mathematische Modellierung und Leistungsprognosen in der Leichtathletik (Mathematical modeling and predicting performance in track and field-events). In: Sportwissenschaft 19(1989)4, 420-435.

HENRY, F.M. and TRAFTON, I.V.:The velocity curve of sprint running. In: The Research Quarterly (1951) 22.

LETZELTER, M.: Wettkampfverhalten und Sprinttraining: Zur Diagnose des Tempoverhaltens von Weltklassesprintern (Performance in competition and training of sprint: Diagnosis of speed behaviour of word-class athletes). In: Die Lehre der Leichtathletik 28(1989)1, 50-67.