A MODEL FOR ANALYSIS OF THE IMPACT BETWEEN A TENNIS RACKET AND A BALL

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The purpose of this study is to determine the validity of a vibration model designed to elucidate the impact between tennis rackets and balls. Generally, a vibration model consists of elasticity and viscosity. A rod was added to the existing model to indicate the contact point on the racket face because vibration changes depend on the position of impact. Comparing the results of the model's simulation and the physical experiment that was performed, it was found that the model was appropriate with regards to amplitude and frequency of vibrations. Using such a model, it should be possible to modify the characteristics of rackets. This will be beneficial, not only for racket selection, but also for new design.

KEY WORDS: tennis racket, elasticity and viscosity, collision, vibration, node

INTRODUCTION: Although a tennis player may hit balls at the same swing speed, ball velocities of restitution are different depending on the position of impact. Also, the racket vibrations that occur at the point of collision are different. It is said that this vibration affects the restitution of the ball. Therefore, knowing the racket's vibrations contributes to greater understanding of the uniqueness of each one, for example, the width of the so-called "sweet" area of the face. (Kawazoe, Sakurai & Ichiki, 1999)
In order to clarify the vibration characteristics of rackets, a new model was developed which simulated ball impacts. In general, vibration system models are composed of mass, elasticity and viscosity. However, because a tennis racket consists of shaft, frame and strings, the frequency of vibration changes depended on the impact point on the racket face. Thus, a model composed of only elasticity and viscosity does not determine impact precisely.
Describing the tennis racket as a rod, the vibration is similar to actual vibrations. However, this rod does not manifest the racket's face, shaft or the player's grip. For these reasons, a new model is presented that combines the rod, elasticity and viscosity. Using this model, an attempt is made to clarify the relation between impact point on the face and the vibration of the racket.
Using the index indicated by the new model, players could select a more suitable racket that improves their performance faster. In addition, it will be possible to design rackets to suit various types of players.

METHODS: First, the area of impact on the racket and balls was investigated. Strain gages were put on the racket's shaft and strings as illustrated in Figure1. From the gauges on the shaft, vibrations were picked up, by using an amplifier system. These data were stored in a computer through an A/D converter. Contact times of the ball during impact were measured from the signal of the gauge put on the strings. The kind of the materials used in the manufacture of the racket, was wood and carbon graphite. Collision velocities of the balls were 20 to 30 m/s. The hit point "a" is the area near the tip of the racket face, point "b" is the

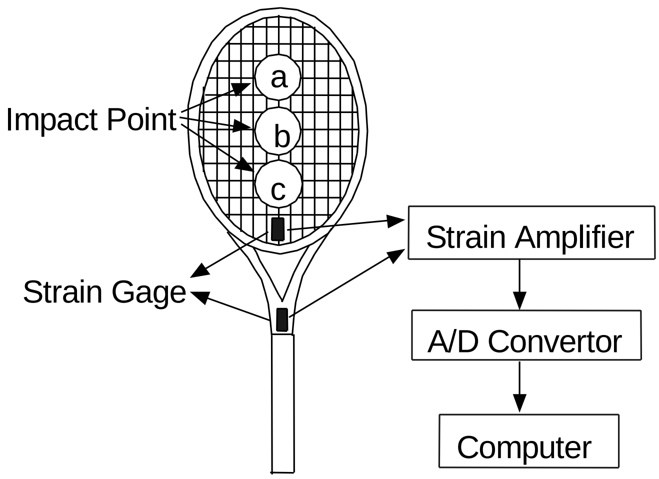


Figure 1 – Diagram of collision of rackets and balls.

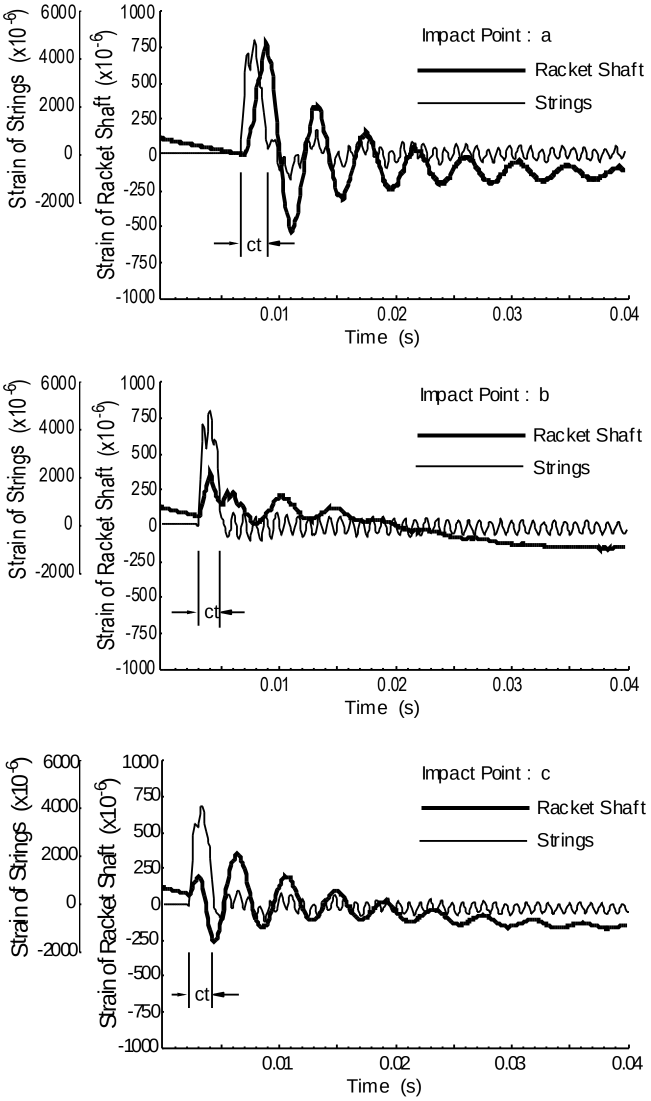


Figure 2 – Strain of racket shaft and strings at impact point a (Top), b (Middle) and c (Bottom).

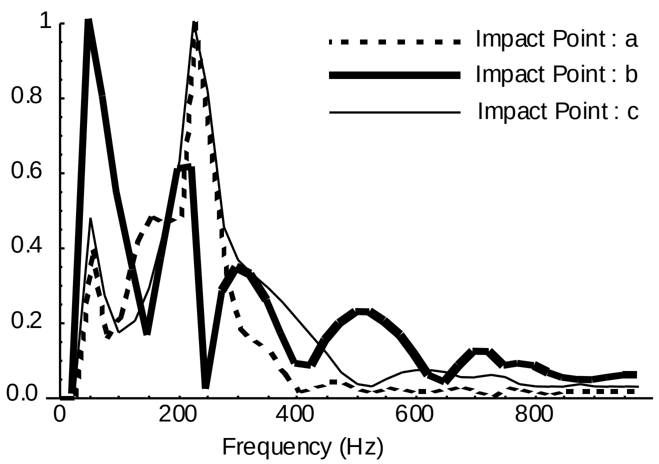


Figure 3 – Layered frequency of racket shaft at 3 impact points.

center of the racket face, and point "c" is the area near to the grip, which are shown in Figure 1.

In Figure 2 the results of hitting the balls at these three points are demonstrated. The bold lines show the strain of the racket shaft, and the thin lines show that of the strings. The contact time is measured from the length of first peak of the string's strain indicated as "ct" in the figure. All were around 1.98ms to 2.31ms. This indicates that the balls' natural frequency range is about 217 to 250Hz. In the top figure, noting the vibration curve of point "a", the first strain of strings and that of racket shaft do not overlap. This indicates there was a phase lag between the tension of the string and the flip of the racket causing the ball's restitution to be low. Moreover, the maximum peak-to-peak strain of the racket shaft is around 1300 $x10^{-6}$, a high value.

In the middle figure, noting the shaft's strain curve of impact point "b", the peak-to-peak value of the strain is reduced to 350 x 10^{-6}. Furthermore, the first peak of the strain overlaps, timed with that of the strings. It is thought that the ball was hit at the node of vibration, and the racket shaft vibrated with the 2^{nd} or 3^{rd} high frequency. In this case, the racket shaft reacted as a rigid body.

In the bottom figure, at impact point "c", the first strain of shaft appears on the opposite side during the ball contact and the peak-to-peak value of the vibration is 600 x 10^{-6}, bigger than the "b".

In Figure 3, racket shafts' layered frequencies calculated by FFT are presented. At the collision of point "a" and "c", the frequency is around 220Hz. But at the impact point "b", the shaft's strain has a higher frequency than the other two points.

Considering the racket as a rod, the physical phenomenon of impact is that of bending vibration. In the case of a rod with both ends unattached, the main vibration of 220Hz is the 1^{st} vibration of all of the impact points. However, it is thought that, only "b" has a 2nd vibration at 300Hz and 3rd at 500Hz. These high frequency vibrations tend to correspond to either ball or string vibrations. It is presumed that this occurrs when the ball is hit at the node of the vibration.

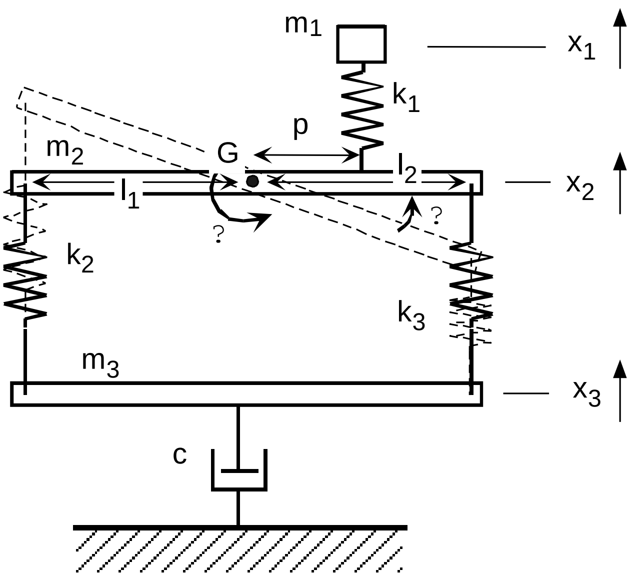


Figure 4 – Model of racket and ball.

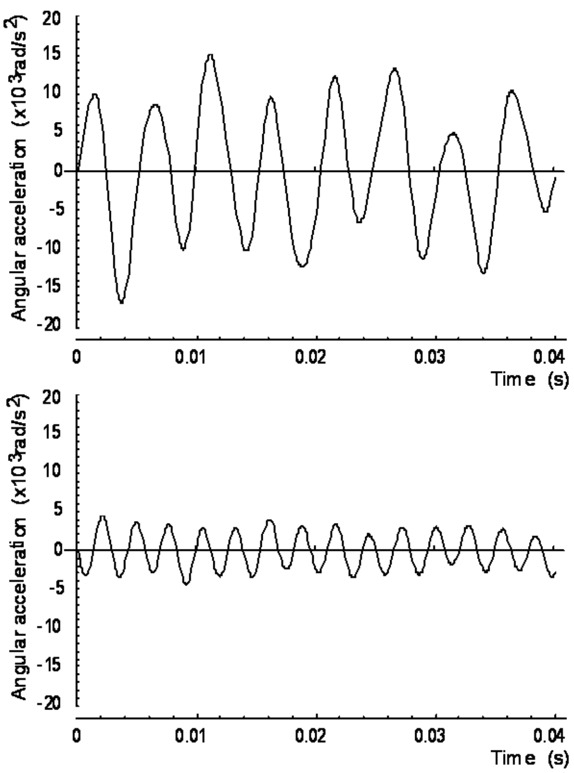
Referring to these vibrations and frequencies, a model was made as illustrated in Figure4. Considering m_1 to be the ball and the string mass, m_2, m_3 are the partial mass of the racket shaft. The elasticity is denoted by k_1 , referring to the elasticity of the strings and the ball. This is followed by k_2, k_3 ,showing strong and weak elasticity of the shaft. C is assumed to be the viscosity of the hand grasping the racket. G is the center of gravity of m_2, and l_1, l_2 show the distance from k_2, k_3's joints to G. P indicates the distance from the impact point of the balls to G. Changing P, it is possible to simulate a collision at various points of impact. q shows the angle of shaft, and regard this as the vibration of the racket. x_1, x_2 and x_3 indicate the displacements of the m_1, m_2 and m_3. This model's equations of motion are as follows:

$$m_1 \frac{d^2 x_1}{dt^2} = -k_1(x_1 - x_2 - p\theta) \quad (1)$$

$$m_2 \frac{d^2 x_2}{dt^2} = k_1(x_1 - x_2 - p\theta) - k_2(x_2 - x_3 - l_1\theta) - k_3(x_2 - x_3 + l_2\theta) \quad (2)$$

$$I \frac{d^2 \theta}{dt^2} = -k_1(x_1 - x_2 - p\theta)p + k_2(x_2 - x_3 - l_1\theta)l_1 - k_3(x_2 - x_3 + l_2\theta)l_2 \quad (3)$$

$$m_3 \frac{d^2 x_3}{dt^2} = -c\frac{dx_3}{dt} + k_2(x_2 - x_3 - l_1\theta) + k_3(x_2 - x_3 + l_2\theta) \quad (4)$$



RESULTS AND DISCUSSION: Solving the equations, a model of collision was simulated between the racket and the ball. Each parameter is shown as follows:

m_1:0.05 [kg] mass of a ball and string
m_2: 0.12 [kg] partial mass of the racket
m_3: 0.24 [kg] partial mass of the racket
I: 0.04 [kg m^2] moment inertia of the racket
k_1: 80000 [N/m] elasticity of string
k_2: 280000 [N/m] elasticity of the racket
k_3: 60000 [N/m] elasticity of the racket
c: 20.0 [N/m/s] viscosity of hand at the grip (Maeda, 1988)
l_1: 0.2 [m] half length of the racket
l_2: 0.2 [m] half length of the racket
p: -0.2 ~ 0.2 [m] length of impact points to the C.G.
Initial condition dx_1/dt: 30[m/s] ball velocity
All other initial conditions are 0.

Two examples of simulated results are

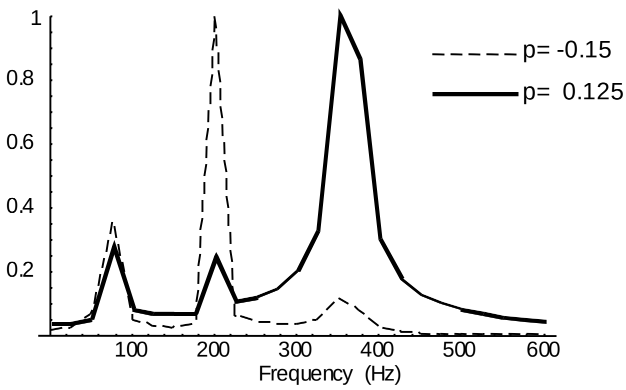


Figure 6 – Result of frequencies simulation at two impact points.

demonstrated in Figure 5. The top figure shows the angular acceleration of q at the impact points p=-0.15. The peak-to-peak value is around $30x10^3$, and it is larger than that of the bottom figure which was simulated at the impact point p=0.125. This result is similar to the characteristics found when amplitude of a racket's vibration changes due to the position of impact. These two vibrations' frequencies are shown in Figure 6. There are three peaks at 73Hz, 200Hz and 350 Hz. The frequency of simulation at p=-0.15 shows the highest peak at 200 Hz, but that of p=0.125 shows the highest peak at 350Hz. In this case the position of p=0.125 is the "sweet" area of this model. Comparing the results of the experiment in Figure3, the peaks of frequencies are similar to each other in that there are two or more peaks and the high frequency vibration appears at the "sweet" area. Judging from this, it is thought that the model shows the collision of the racket and ball properly.

Next, the model was simulated by changing the value of elasticity of k_2 or k_1. In Figure 7, the result of shifting the p from –0.2 to 0.2 is demonstrated. The altitude component indicated by "p" shows the hit point of the ball, and the vertical component indicates the angular acceleration of q, which is the vibration of the racket. Around these angular acceleration markers, 's,' 't' and 'u', the "p" point is supposed to be the "sweet" spot of the racket. When changing k_2 $28x10^4$ to $14x10^4$[N/m], the minimum values of angular acceleration are not so different, but the "sweet" spot marker 's' shifts to the left marker 't', close to the center of gravity. When changing k1 $8x10^4$ to $4x10^4$, string tension is reduced, and the marker 's' shifts downward closer to the marker 'u'. It is thought that the "sweet" spot vibrates less. Therefore, by changing the parameters and simulating the model, it will be possible to find the best characteristics of a racket.

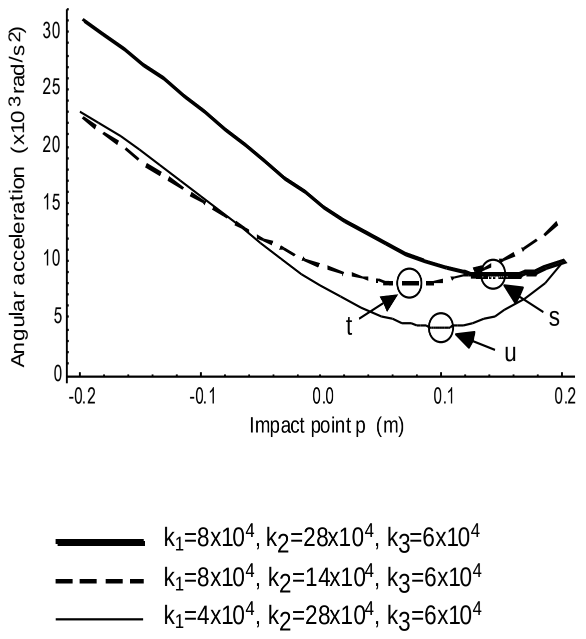


Figure 7 – First peak-to-peak angular acceleration in the vibration shifting impact point p on the rod in the model of Figure 4.

CONCLUSION: For this study, a tennis racket model was designed, consisting of elasticity, viscosity and rod that made it possible to change the position of the hit point. Further, the impact of the racket and the balls was simulated. The computer model showed the same characteristics of vibrations as that of actual play. For example, the amplitudes of the vibration were reduced at the "sweet" area of the racket face, and the high frequencies also appeared there. From these results it was felt that this model properly simulated the actual impact of a racket and a ball. Using this model and changing the parameters, people will be able to regulate the characteristics of a racket from the viewpoint of mechanical vibration. This will be useful when a player selects rackets or in the designing of new rackets.

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