A STUDY OF SELECTED FACTORS AFFECTING TAKEOFF HEIGHT IN THREE-METER SPRINGBOARD DIVING

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The purpose of this study was to find out the connection between some factors in the takeoff process and the takeoff height. A Human-Springboard system, based on a previously developed theory and numerical method, was developed to simulate the takeoff process. In addition, this system could output the takeoff height and other results in response to the input of the control function. Through changing the parameters in a certain form of control function, the relationship between those factors and the takeoff height could be determined. The results of this study could be used as a theoretic base for the coaches and athletes.

KEY WORDS: human-springboard system, numerical computation, emulation

INTRODUCTION: In this study of the three-meter springboard event, the athlete takes off from an elastic board that has been fixed at one end. The movement process could be divided into four phases: running, takeoff, action in the air and entry into the water. In step jumping from one foot and pressing-board jumping, the takeoff action was completed through the pressing and springing process of the board and the conceding and restraining action of the muscles. The greater the distance of the athlete from the board, the more time and space is available to complete the remaining component containing a high-degree of difficulty. Therefore, further research into the factors that influence the height of takeoff action could prove to be very important in the long run. The purpose of this study was to find what kind of influence the parameters have on the takeoff process and to offer guidelines for the coaches and athletes in the training.

METHOD: Jiang et al (1994) built the following system of equations:

$$
\begin{cases}
m_1 \dfrac{d^2 y_1(t)}{dt^2} = m_1 g \oplus N(t) \\
m_2 \dfrac{d^2 y_2(t)}{dt^2} = -N(t) - K y_2(t) \\
y_1(t) - y_2(t) = 2L\cos\theta\,(t) = y(t) \\
y_1(0) = y(0) = y_{10} = y_0 \\
v_0 = v_{10} - v_{20} \\
m_1 g h = \dfrac{1}{2} m_1 v_{10}^2 \oplus \dfrac{1}{2} m_2 v_{20}^2 \\
2L\cos\theta_{\max} = y_{\min} \le y(t) \le y_{\max} = 2L\cos\theta_{\min} \\
v_{\min} \prec y'(t) \prec v_{\max} \\
a_{\min} \prec y''(t) \prec a_{\max}
\end{cases}
$$

In which, $y_1(t)$ was the position of the center of mass of human body and $y_2(t)$ was the position of the board; m_1 was the mass of human body; m_2 was the mass of the board; v_1 was the velocity of the center of mass of human body and v_2 was the velocity of the board. L was the parameter of human body and *K* was a constant that represent the stiffness of the board. The analytic solution was given as follows

$$v_1(t) = -y_0\omega \sin\omega t + v_{10}\cos\omega t + \int_0^t F_1(y(s))\cos\omega(t-s)ds;$$

$$y_2(t) = \frac{v_{20}}{\omega}\sin\omega t - \frac{1}{\omega}\int_0^t \sin\omega(t-s)F_2(y(s))ds.$$

in which

$$\omega = \sqrt{\frac{K}{m_1 + m_2}};$$

$$F_1(s) = \frac{m_2}{m_1 + m_2} y\ (t) + \omega^2 y(t) - \frac{m_1}{m_1 + m_2} g;$$

$$F_2(s) = -\frac{m_1}{m_1 + m_2} y\ (t) - \frac{m_1}{m_1 + m_2} g.$$

From these calculations the height *H* could be determined that the athlete could reach after taking off as follows

$$H = \frac{v_1(T)^2}{2g} + y_2(T).$$

Therefore, the takeoff height *H* could be calculated when the control function $y(t)$ was given.

On the basis of the theory and computational method above that has been established, an simulation system (HBS) is developed in Borland C++, which is used to study the takeoff process.

ANALYSIS METHODS: In the previously developed Human-Springboard system, the control function could be constructed in three ways; (1) To analyze the film recording the action of the athlete; (2) To use the common function, such as sinusoidal function, line or quadric curve; (3) To combine the two ways mentioned above and modify some points of the function with mouse on computer. Here, the second method is used.

From $y(t) = 2L\cos\theta(t)$, $\theta_{\min} \le \theta(t) \le \theta_{\max}$, it can seen that *y* (*t*) is a sinusoidal function of variable period or a cosine function of variable period. To discuss the influence of the parameters v_0, K, h on the system, suppose that *y* (*t*) is a sinusoidal function of fixed period 2T. Let

$$y(t) = y_{\max} - f\sin\frac{\pi}{T}t, \qquad y'(t) = \frac{-f\pi}{T}\cos\frac{\pi}{T}t;$$

$$y(0) = v_0 = \frac{-f\pi}{T}, \qquad f = \frac{-v_0 T}{\pi}.$$

So,

$$y(t) = y_{\max} + \frac{v_0 T}{\pi}\sin\frac{\pi}{T}t.$$

From

$$v_0 = v_{10} - v_{20}, \quad \frac{1}{2}m_1 v_{10}^{\ 2} + \frac{1}{2}m_2 v_{20}^{\ 2} = m_1 gh,$$

in which *h* was the height that proceeding takeoff could reach, to obtain

$$-\sqrt{2gh} \le v_0 \le 0..$$

To get the numerical value of *H*, difference method was used to do differential operation and the complex trapezoidal method was used to integrate.

RESULTS AND DISCUSSION:

1. The influence of v_0 on the takeoff height *H*

Let $m_1 = 72.00000$, $m = 45.0$, $K = 6543.31179$, $h = 0.67000$, $y_{\max} = 1.20000$, $y_{\min} = 0.49325$, $T = 0.51299$, and so obtain $-3.62381 \le v_0 \le 0$. Input different values of v_0 into the system, and the output data are shown in Table 1.

Table 1 The Influence of v_0 on the Takeoff Height

v_0	Y_{1out}	v_{1out}	Y_{2out}	v_{2out}	H	E_f
-3.62	0.7388	6.9063	-0.4493	3.2853	1.9842	0.5637
-3.5	0.7638	6.7239	-0.4246	3.2264	1.8820	0.5700
-3.0	0.8645	5.9863	-0.3255	2.9875	1.5028	0.5986
-2.5	0.9640	5.2471	-0.2275	2.7469	1.1772	0.6309
-1.5	1.1601	3.7640	-0.0344	2.2612	0.6884	0.2954
-0.5	1.3524	2.2751	0.1548	1.7695	0.4189	0.0000

From the results, the effect of v_0 can be seen clearly:

1) The takeoff height *H* has direct relation with $-v_0$ because the greater the value of $-v_0$, the higher the takeoff height. That is to say, the action of pressing board quickly as soon as human body reaches the board is helpful for increasing the takeoff height. But it can be seen from table 1 that in some cases *H* is larger while the utilization ratio of energy E_f is smaller, that is, the remaining vibration of board is larger.

2) Theoretically $-v_0$ can reach the maximum when $v_{20} = 0$, which is impossible in practice.

So practically, the result is $v_{20} < 0$.

3) The above analysis shows that the takeoff height *H* could be improved if a proper initial velocity was taken v_0.

2. The influence of *K* on the takeoff height *H*

Let $v_0 = -3.50$ while other parameters remain the same and the output data are shown in Table 2.

Table 2 The Influence of *K* on the Takeoff Height *H*

K	Y_{1out}	v_{1out}	Y_{2out}	v_{2out}	H	E_f
6543	0.7638	6.7239	-0.4246	3.2264	1.8820	0.5700
6500	0.7572	6.7141	-0.4312	3.2167	1.8688	0.5620

6000	0.6757	6.5595	-0.5127	3.0631	1.6825	0.4601
5500	0.5833	6.3201	-0.6052	2.8247	1.4327	0.3389
5000	0.4788	5.9821	-0.7096	2.4878	1.1162	0.1959
4500	0.3613	5.5304	-0.8271	2.0371	0.7334	0.0278

The results show a very different effect can be obtained when the stiffness of the board *K* is different while $y(t)$ is same. The smaller the value of *k* is, the lower the center of mass can reach after taking off.

3.The influence of *h* on the takeoff height *H*

By using input of different value to *h* into the system while $v_0 = -3.50$ and other parameters remains same, the results are shown in Table 3.

Table 3 The Influence of *h* on the Takeoff Height *H*

h	Y_{1out}	v_{1out}	Y_{2out}	v_{2out}	H	E_f
0.00	1.0727	0.9474	-0.1263	0.9404	-0.0805	0.0000
0.25	0.8687	4.5873	-0.3236	2.3728	0.7500	0.4864
0.50	0.7842	6.0951	-0.4053	2.9661	1.4900	0.5445
1.00	0.8512	6.8595	-0.3371	3.3620	2.0635	0.6346
1.50	0.9576	7.0246	-0.2308	3.5271	2.2867	0.6978
2.00	1.0463	7.1623	-0.1420	3.6648	2.4752	0.7095

From Table 3, it can be seen that the value of *H* and E_f is increasing with *h*, which is practically right.

CONCLUSION: From the discussion of the data, the effect that the parameters v_0, K, h have on the takeoff height can be seen. If an athlete wanted to jump higher from the board, he should press the board as quickly as possible when he reaches the board, choosing the hardest board within his ability, and in the preceding jump, should jump as high as possible.

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