A NEW MATHEMATICAL SIMULATION TO STUDY FLIP TURN CHARACTERISTICS IN FRONT CRAWL SWIM

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Swim turns represent an integral factor in determining the final outcome of a swimmer race. The aim of present study was to provide a comprehensive mathematical modelling for achieving kinematic parameters in freestyle flip (pike) turn. In proposed model all attempts have been applied to find out what swimmers should do in order that the turns are accomplished in shorter time. This new mathematical model has been adopted to flip turn but can also be adopted to tuck turn with minor change in calculations. Theoretical considerations suggest that faster upper limbs rotation could lead to a torso pressure gradient, which would induce significant axial flow along the upper limbs toward the torso. Our results demonstrate a better reality of the predicted rotational of body during front crawl swim flip turn. In this new model, we hypothesize that in flip turn the body can be considered and simulated as three thin hinged prisms; upper body, thigh, and shank.

Keywords: Mathematical modelling, flip turn characteristics, front crawl, simulation

INTRODUCTION: Execution of a flip turn requires a swimmer complete a series of complex movements to allow them to change direction. Descriptions of tumble turn technique and performance are found to vary slightly within the literature. Costill et al. (1992) described the process of performing a flip or tumble turn using five separate movement phases. These five turn phases are the approach; the turn; the push-off; the glide; and the pull-out. Maintaining swim speed is considered an important component of the approach to the turn. Despite the importance of turns in the overall performance for competitive swimming, relatively few studies have been carried out. This is probably because there are no simple, accurate and versatile investigatory methods available. Previously, two models considering swimmer; as a bent thin prism, Shahbazi et al. (2005); and as two thin hinged prisms, Shahbazi et al. (2006) were introduced. This study examined another model in which swimmer is considered as three thin prisms of different lengths and hinged together (hip and knee joints), their width equals the swimmer's shoulder to shoulder distance, and their thickness equals the anterior-posterior chest breadth and furthermore, the upper and lower segments are considered equal . This study also examined the reliability of the proposed model during tumbling and the consistency of temporal and kinematic and kinetic aspects of swimmers' turning performance.

MATHEMATICAL MODELLING: In tumbling, the swimmer begins the turn with a straight drop of the head and water rushes over his back aiding the tumbling which has been simulated by two hinged prisms. Then the legs are kept straight until the feet are removed from the water. The bend of the torso is deep and hands reach and pull up. The angles between upper and lower limbs and thigh and shank are α and β respectively. In fact in pike turn, the body assumes only one major bend at the hip throughout most of the turn. The simulated shape is presented in Figure1. In the followings, the swimmer centre of gravity, moment of inertia, and rotational velocity in pike turn are calculated.

Determination of Moment of Inertia: Swimmer rotation is about Z axis and to achieve this we must calculate first upper limb moment of inertia relative to its CM.

Upper body (A): $$I'_{cm} = \frac{1}{12} m' \left(a^2 + b^2 \right) = \frac{M}{24} \left(a^2 + b^2 \right) \quad (1)$$

Thigh (B): $$I'_{cm} = \frac{1}{12} m'' \left(\frac{4a^2}{9} + b^2 \right) = \frac{M}{36} \left(\frac{4a^2}{9} + b^2 \right) \quad (2)$$

Shank (C): $$I'_{cm} = \frac{M}{72}\left(\frac{a^2}{9} + b^2\right) \tag{3}$$

According to parallel axes theorem (Hay, 1985):, the moment of inertia relative to Z axis is:

$$I'_Z = I'_{CM} + \frac{M}{2} Y^2 \tag{4}$$

The moment of inertia of the whole system relative to Z axis is then

$$I_z = \frac{M}{36 \times 9 \times 4}\left(\frac{72a^2}{1} + 12 \times 9b^2\right) + \frac{M}{6}\left(3y_A^{\;2} + 2y_B^{\;2} + y_C^{\;2}\right) \tag{5}$$

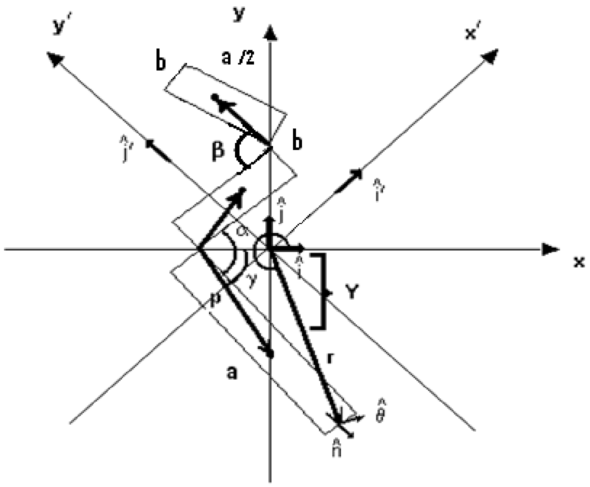

Figure 1- Schematic of swimmer's body in three hinged prisms, X,Y are fixed reference axes while X' Y' are the body references axes in revolution.

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Determination of y_A, y_B, and y_C: If we define two angles as

$$\hat{S} = (\alpha_0 + \alpha + \gamma_0) - \gamma$$
$$\hat{P} = (\beta_0 + \beta + \alpha_0) - \hat{S} \tag{6}$$

In which α_0, β_0, γ_0 are given by followings:

$$Sin\,\alpha_0 = \frac{b}{\sqrt{b^2 + \frac{4a^2}{9}}}, Sin\,\gamma_0 = \frac{b}{\sqrt{a^2 + b^2}} \quad Sin\beta_0 = \frac{b}{\sqrt{b^2 + \frac{a^2}{9}}} \tag{7}$$

Then we can have the followings:

$$\begin{cases} y_A = \dfrac{\sqrt{a^2 + b^2}}{2} Sin\,\gamma \\ y_B = \dfrac{\sqrt{b^2 + \frac{4a^2}{9}}}{2} Sin\left[(\alpha_0 + \alpha + \gamma_0) - \gamma\right] \\ y_c = \dfrac{\sqrt{b^2 + \frac{a^2}{9}}}{2} \sin\left[(\beta_0 + \beta + \alpha_0) - \hat{S}\right] + \sqrt{b^2 + \frac{4a^2}{9}} Sin\,\hat{S} \end{cases} \tag{8}$$

Inserting these data in moment of inertia relations already obtained, we will finally have:

Upper body (A): $$I'_z = \frac{M}{24}\left(a^2 + b^2\right)\left[1 + 2Sin^2\gamma\right] \tag{9}$$

Thigh (B):
$$I'_z=\frac{M}{36}\left(\frac{4a^2}{9}+b^2\right)+\frac{M}{3}\left[\frac{\frac{4a^2}{9}+b^2}{4}+\frac{a^2+b^2}{4}Cos^2\gamma-\right. \tag{10}$$
$$\left.\frac{\sqrt{a^2+b^2}}{2}\sqrt{b^2+\frac{4a^2}{9}}Cos\,\gamma\,Cos\left(Sin^{-1}\frac{b}{\sqrt{\frac{4a^2}{9}+b^2}}+Sin^{-1}\frac{b}{\sqrt{a^2+b^2}}+\alpha-\gamma\right)\right]$$

Shank (C):
$$I'_z=\frac{M}{72}\left(\frac{a^2}{9}+b^2\right)+\frac{M}{6}\left[\frac{\frac{17a^2}{9}+5b^2}{4}+\frac{4a^2+b^2}{4}Cos\left(\beta_0+\beta+\alpha_0-2\hat{S}\right)+\frac{a^2+b^2}{4}Cos^2\gamma\right. \tag{11}$$
$$\left.-\sqrt{b^2+\frac{4a^2}{9}}\sqrt{a^2+b^2}\left(Cos\,\hat{S}\,Cos\lambda+\frac{1}{2}Cos\,\gamma\,Cos\left(\beta_0+\beta+\alpha_0-\hat{S}\right)\right)\right]$$

The force due to water pressure is: $D=\frac{1}{2}\rho_{water}C_C A_\perp V^2$, the vertical plane of swimmer body is; $A_\perp=A(\hat{n}.\hat{\theta})$, where $\hat{\theta}$ is the force direction and $\hat{n}$ is the unit vector representing the surface, on the other hand we can have; $A_\perp=bc\,Sin\,\alpha/2$, therefore we can have for $\hat{\theta}$ and $\hat{n}$ the followings:

$$\hat{n}=\left(Cos\,\alpha/2\,Cos\,\theta-Sin\,\alpha/2\,Sin\,\theta\right)i+\left(Cos\,\alpha/2\,Sin\,\theta-Sin\,\alpha/2\,Cos\,\theta\right)j \tag{12}$$

And also; $\hat{\theta}=-Sin\,\theta_i+Cos\,\theta_j, and\hat{n}.\hat{\theta}=Sin\,\alpha/2$, then; $D=\frac{1}{2}\rho C_D bc\,Sin\,\alpha/2\,\dot{\theta}^2r^2$ (13)

The centre of mass components can be obtained considering three parts (Hay 1985):

$$x_{cm}=\frac{1}{4}\times\left[\frac{3}{2}\sqrt{a^2+\frac{b^2}{4}}Cos\left(\alpha_0+\alpha+\gamma_0-\gamma\right)-\sqrt{a^2+b^2}\,Cos\,\gamma-\frac{\sqrt{a^2+\frac{b^2}{4}}}{2}Cos\left(\beta_0+\beta-\alpha-\gamma_0+\gamma\right)\right]$$

$$y_{cm}=\frac{1}{4}\times\left[\sqrt{a^2+b^2}\,Sin\,\gamma+\frac{3}{2}\sqrt{a^2+\frac{b^2}{4}}\,Sin\left(\alpha_0+\alpha+\gamma_0-\gamma\right)+\frac{1}{2}\sqrt{a^2+\frac{b^2}{4}}\,Sin\left(\beta_0+\beta-\alpha-\gamma_0+\gamma\right)\right]$$

Moment to the swimmer body: The moment applying to swimmer body can then obtained as:

$$N=\vec{r}\times\frac{1}{2}\rho_W C_D bcSin\frac{\alpha}{2}\dot{\theta}^2r^2=\left(\frac{r^3\rho_w C_D bcSin\alpha/2}{2}\right)\dot{\theta}^2=\left(\frac{\rho_w C_D bcSin\alpha/2\,\dot{\theta}^2}{2}\right)\left[\left(y_{cm}-y\right)^2+\left(x_{cm}-x\right)^2\right]^{3/2}$$

(15)

As can be seen, the moment is quadratically proportional to swimmer rotational velocity and linear displacement of his/her CG in which two angles; α and β are involved. On the other hand we can write:

$N=\mathrm{I}\,\ddot{\theta}$ (16)

Therefore: $\ddot{\theta}=\mathrm{K}\dot{\theta}^2$ (17)

Where β is defined as: $\mathrm{K}=\left(\frac{r^3\rho C_D bc\sin\alpha/2}{2I}\right)$ (18)

RESULTS AND DICUSSION:

Little has been written regarding turn mechanics prior to wall push-off during freestyle turns. The present model would provide a better understanding of the turn phase prior to push-off of the freestyle pike turn because two hinges were considered in modelling, which is very similar to the reality and a complementary to previous model, Shahbazi et al. (2006). Identification of key kinematic, kinetic, and hydrodynamic variables and the role of these variables in producing a faster turn, in order to reach the wall for push-off were what we tried to improve the turn modelling. The results of this study can be used in conjunction with the results of other literature on the freestyle turn to optimize total turn performance.
Our findings suggest that it may be advantageous for the swimmers to develop as much body bent as possible during this period of turn, in order to reduce the water drag force and to speed up the feet in reaching the wall. A fast body rotation before the turn or simply the body shape of the swimmer might create larger stern waves that would lead to an increase in rotational velocity. A more bent body position throughout turning is like to reduce drag force and speed up the reaching the wall**.** The variation of modelling parameters are depicted on Fig.1.

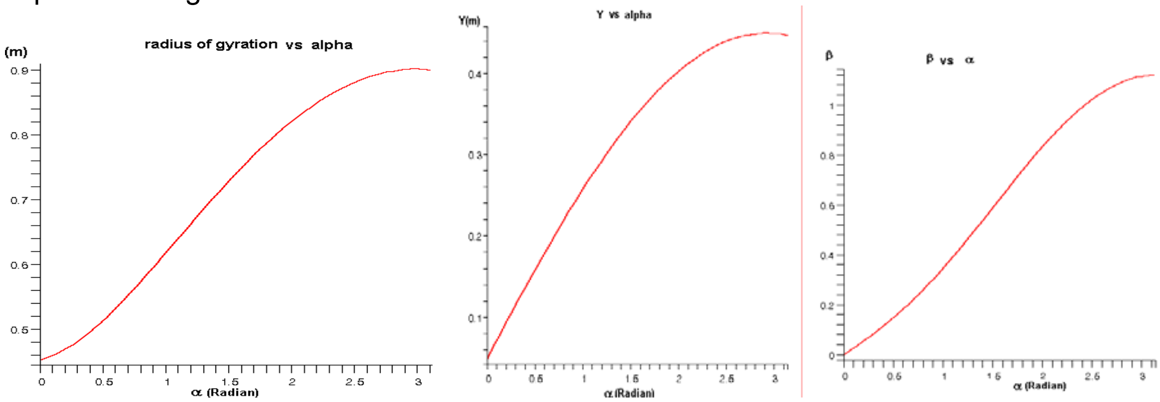

Figure 1- The three segments hinged model through which the variation of moment of inertia and other parameters involved in the model versus α are depicted.

CONCLUSION: A theoretical simulation for pike turn with two hinges has been developed and presented. The results suggest that it may be advantageous for the swimmers to develop as much bent as possible during the period prior to push-off. The moment of inertia of the swimmer depends on all three angles made by upper and lower limbs (α) and thigh and shank (β). This model can be regarded as a complementary to previous model and the mathematical procedure can be regarded as new step to be developed by other modelling involvers.