A COMPARISON OF TWO FUNCTIONS REPRESENTING VELOCITY OF A HUMAN BODY SUBJECT TO PASSIVE DRAG

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The purpose of this study was to compare the goodness of fit of two functions representing horizontal velocity of a human body subject to passive drag. Hyperbolic and exponential functions were fitted to the horizontal velocity data of three glides following push-off from the wall of five swimmers. Measures of goodness of fit included root mean square errors (RMSE), and the coefficient of determination (R2). The hyperbolic function provided a better fit to the actual values of velocity, provided a closer match to the initial velocity, and predicted better the velocities beyond the fitted interval than the exponential function. It was concluded that for the swimmers and range of glide velocities tested, drag was closer to being proportional to the square of velocity than a linear function of velocity.

KEY WORDS: swimming, glide, passive drag, curve fitting.

INTRODUCTION: Passive glide plays an important role in swimming events specially following starts and turns and in some transitional phases during butterfly and breaststroke (Vorontsov and Rumyantsev, 2000). The average glide speed is highly correlated with the start time (Hay and Guimares, 1983). Mason and Cossor, (2001) found that the most significant aspect of the turns to be the underwater phase. Although the initial push-off velocity is one of the determining factors of glide performance, minimising drag could produce better results than merely increasing effort during push-off, because it does not increase the metabolic cost (Lyttle, et. al. 1998).

Passive drag forces have been measured directly during towing. However, these methods may affect the natural pace and posture of the swimmer. Additionally, these methods do not take into account the effect of unsteady motion and added mass due to the deceleration of a swimmer during actual glide performances. By integrating the equation of motion for a passive swimmer we take advantage of working on kinematic data while eliminating the problems associated with direct methods of drag measurement.

Using passive glide kinematics for finding hydrodynamic drag coefficient has been established by Karpovich and Pestrikov (1939), developed by Klauck and Daniel (1976), and adapted by Takahashi et al. (1982) for the underwater passive glide following static starts from the wall. The 'kinematic method' is usually based on the conventional assumption of quadratic proportionality between drag force and velocity (Amar, 1920, Karpovich, 1933,Counsilman, 1955 and Jiskoot and Clarys, 1975). A quadratic relationship between drag force and velocity yields a hyperbolic velocity-time function for a passively gliding swimmer. Recently, researchers have questioned the validity of the assumption of a quadratic relationship between drag force and velocity (Lyttle, et. al. 1998). Using their towing device and examining different constant speeds and depths the researchers found a linear relationship between velocity and drag force in the tested velocity domain (1.6 to 3.1m/s). A linear relationship between drag force and velocity yields an exponential velocity-time function for a passively gliding swimmer. Sanders and Byatt-Smith (2001) used an exponential function to find a least-squares fit to the velocity-time data of a passively gliding swimmer to predict the time when a swimmer should initiate the kick following the glide in starts and turns. However, while the technique of fitting a function to velocity-time data to predict the most appropriate time to commence kicking is potentially useful, it has yet to be established whether a hyperbolic or exponential function provides a better fit. Further, establishment of whether drag is a linear or quadratic function of velocity may provide insights into the hydrodynamic behaviour of a passively gliding human body in the range of speeds of competitive swimmers during the glide phase of starts and turns. Thus, the purpose of this study was to compare the goodness of fit of exponential (drag force proportional to velocity) and hyperbolic (drag force proportional to the square of velocity) values for horizontal velocity-time data following push-off from the wall.

METHOD: Three female and two male club level competitive swimmers preformed three trials holding a passive glide following a push-off from the wall. The swimmers were at a depth of .5 m and directly above a scale line strained horizontally at a depth of 1m. Balls 5cm in diameter surrounding the cable were spaced exactly 1m apart to allow subsequent calibration. A stationary underwater camera, 12m from the plane of motion of the swimmer and with its axis perpendicular to the swimmer's glide path, recorded motion of the swimmer at 25 fields per second. APAS software was used to digitize the scale line reference markers and hip marker of the swimmer. The hip of the swimmer was regarded as representative of the whole body motion given that the posture during the glide was held constant. Raw data from the hip marker were filtered using a 4th order Butterworth filter with a cutoff frequency of 6Hz. The horizontal coordinate data of the hip were then differentiated to obtain horizontal velocity.

Based on Newton's Second Law of Motion the equation of motion of the swimmer can be written as:

$$P(t) - D(t) = M.\frac{dv}{dt}$$

Where P(t) is the propulsive force, D(t) is the resistive force, v(t) is the velocity of the swimmer and M represents the total mass of the swimmer plus the added mass of water.
Given that propulsive force is zero during a passive glide, we have:

$$P(t) = 0 \qquad \text{then:} \qquad -D(t) = M.\frac{dv}{dt}$$

Quadratic Relation:
On the basis of a traditional quadratic relation between drag and velocity we have:

$$D(t) = c_2.v^2 \qquad \text{then:} \qquad -c_2.v^2 = M.\frac{dv}{dt}$$

Where c_2 is the drag coefficient in Kg/m. Solving this differential equation and taking into account the initial velocity condition (V_m: maximum velocity at release), we have:

$$\int -c_2.v^2 = \int M.\frac{dv}{dt} \qquad \text{and} \qquad v(0) = V_m$$

Then we have the velocity as:

$$v(t) = \frac{1}{\frac{c_2}{M}.t + \frac{1}{V_m}}$$

Where c_2 / M is the integrated drag coefficient (C_2) in m^{-1}.

Linear Relation:
On the basis of a linear relation between drag and velocity we have:

$$D(t) = c_1.v \qquad \text{then:} \qquad -c_1.v = M.\frac{dv}{dt}$$

c_1 is the drag coefficient in Kg/sec. Solving this differential equation and taking in to account the initial velocity condition(V_m : maximum velocity at release),we have:

$$\int -c_1.v = \int M.\frac{dv}{dt} \qquad \text{and} \qquad v(0) = V_m$$

Then we have the velocity as:

$$v(t) = V_m.Exp(-\frac{c_1}{M}.t)$$

Where c_1 / M is the integrated drag coefficient (C_1) in Hz.

MATLAB® curve-fitting toolbox was used to fit the actual velocity-time data with the parametric hyperbolic and exponential functions using a least squares fit.

RESULTS: Table1 shows the average values for each subject, maximum velocity estimated (Vm), integrated drag coefficient (C), coefficient of determination (R2) and sum of the Root Mean Squared of Errors (RMSE) for the exponential and hyperbolic least-squares fits. Mean R^2 of all subjects was 0.03 greater for the hyperbolic function than for the exponential function. Mean RMSEs were 0.089 for the hyperbolic function and 0.112 for the exponential function.

Table 1 Comparison of Different Fit Parameters for Exponential and Hyperbolic Fits.

Fit	Exponential Fit				Hyperbolic Fit			
Subject	V_m(m/s)	C_1 (Hz)	R_	RMSE	V_m(m/s)	C_2 (m^{-1})	R_	RMSE
CAR	2.321	0.381	0.907	0.116	2.523	0.252	0.930	0.101
CHR	2.391	0.464	0.948	0.094	2.548	0.292	0.966	0.073
STE	2.248	0.401	0.949	0.073	2.348	0.246	0.971	0.053
JOA	2.532	0.382	0.853	0.151	2.372	0.267	0.901	0.121
KEL	2.309	0.371	0.910	0.127	2.496	0.250	0.949	0.096
Average	2.360	0.400	0.913	0.112	2.458	0.261	0.943	0.089

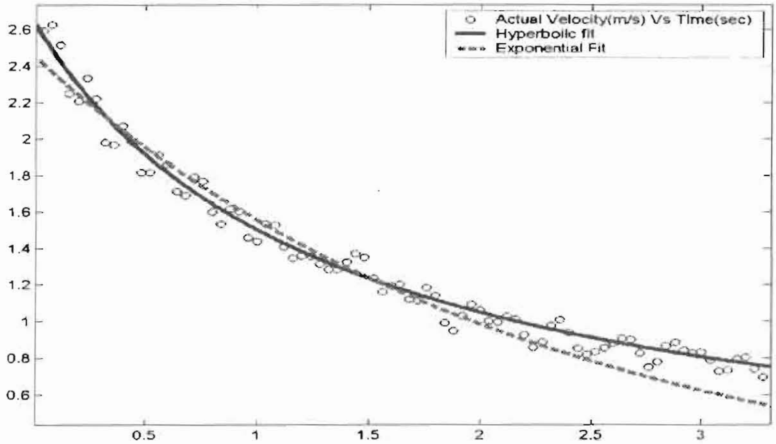

Figure 1: Actual values and fitted function for one of the trials for subject CHR.

Figure 1 is an example of actual velocity values and fitted horizontal velocity values plotted against time.

CONCLUSION: The mean R^2 and mean RMSEs of all subjects indicated that the fit was better for the hyperbolic function than the exponential function. From the graphs it was evident that the hyperbolic function provided a better estimate of the initial velocity (Vm). Also, although the differences between the values for R^2 were not large, the graphs indicated that the hyperbolic

fit could predict the actual velocity values beyond the range of fitted values.
These results supported the conventional assumption of quadratic proportionality between drag force and velocity.

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