A MECHANICAL MODEL FOR LEG STIFFNESS AND AVERAGE AND MAXIMUM FORCE ESTIMATIONS IN SPRINTERS

Morteza Shahbazi-Moghaddam
Physics Department of Tehran University and Biomechanics group of the Faculty of Physical Education, Edinburgh University, UK

In running the leg's complex system of muscle, tendon and ligament has a spring like behaviour, which can be considered as a single non-linear spring. A single spring-mass model, consisting of a single non-linear leg spring and a mass in running has been considered to estimate the leg stiffness and muscle force in running remarkably well. The model has shown that in running, the stiffness of the leg spring is proportional to the squared cosine of the leg angle relative to the axis perpendicular to ground and also to the displacement of CG (centre of gravity) and finally to the squared vertical velocity component. The variation of the leg stiffness with CG displacement and the angle swept by the leg spring, when sprinters alter their supporting leg from braking phase to propulsive phase, at their maximal speed. A 20m-switch pad in conjunction with an electronic interface and a laptop computer has been used for touch down time measurement. The angle swept by sprinter's leg was determined by video filming. The proposed mathematical model enabled us to estimate leg stiffness and muscle force satisfactorily.

KEY WORDS: modelling, leg stiffness, muscle, ground reaction forces.

INTRODUCTION: When sprinters are running, they bounce along the ground using leg spring to alternately store and return elastic energy (Canagna et al., 1964, 1977). Leg muscles, tendon and ligaments can all behave as springs, storing elastic energy when they are stretched and returning it when they recoil (Alexander, 1988). During running, the leg spring does not behave like a single linear spring as was proposed by (Farley and Gonzalez, 1996; Alexander, 1992; Alexander and Veron, 1975; Blickhan, 1989; Blickhan and Full, 1993; Cavagna et al., 1988; Farley et al.. 1991, 1993; He et al., 1992; Ito et al., 1983; McGeer, 1990; McMahon and Cheng, 1990; Thomson and Raibert, 1989). The stiffness of the leg spring varies with speed as experimental evidence showed Farley and Gonzalez, 1996. When sprinters run with increased knee flexion, the stiffness of the leg spring appears to decrease (McMahon et al., 1987). These studies clearly demonstrate that it is possible to change the stiffness of the leg spring during bouncing movements. The aim of the present study was to determine the importance of the variation of the spring stiffness and its dependence upon the angle swept by the leg spring, and finally to estimate the behaviour of the muscle force and acceleration.

METHODS: In kinematic study we have used an appropriate length of white paper in order to get footprints. The footprints on the paper have provided the total stride length (the distance between the toe of support leg and the toe of landing leg), which is the sum of three separate distances; takeoff distance, flight distance and landing distance. During that part of the running stride in which the athlete is not in contact with the ground, the horizontal distance that he travels is determined by the factors that govern the flight of all such projectiles, namely, the speed, angle, and height of release and the air resistance encountered in flight. By far the most important of these is the speed of release, a quantity primarily determined by the ground- reaction forces exerted on the athlete.

The total stride length can be given by the following formulae, Hay, 1993;

$$R=(V_0 \cos\Theta).t_f \quad (1)$$

Where R is the total stride length, V_0 and Θ are CG (Centre of Gravity) release velocity and angle(relative to ground), respectively, t_f is the time of flight which is determined by the counted frames. The precision of t_f measurement depends upon the cares applied in watching the frames of takeoff and landing. Since the CG of the sprinters in each stride is considered as a projectile, then we can also have the following equation Hay,1993;

$$t_{up}=(V0 \sin\Theta)/g \quad (2)$$

Where t_{up} is the time it takes the CG to reach to the peak of flight, and that $t_{up}=t_f/2$.

Dividing (2) by (1), Θ can be calculated, Shahbazi et al. 1998/2000;

$$Tan\Theta\theta=(gtf^2)/(2R) \qquad (3)$$

Whereas;

$$\Theta=ArcTan((gtf^2)/(2R)) \qquad (4)$$

From equations (1) and (4) we can get the CG initial release velocity and its horizontal and vertical components as following;

$$V_0=R/(tf.Cos\Theta) \qquad (5)$$

$$V_{0x}= V0Cos\Theta \qquad (6)$$

$$V_{0y}= V0Sin\Theta \qquad (7)$$

Equation (7) is fundamental for finding mean reaction and muscle forces.

Force estimation at touchdown

Runners experience three kind of forces at touchdown; ground reaction, braking and propulsive. The ground reaction force is applied to oppose gravity during the period of foot-ground contact by sprinters at their top speeds. This force is depending upon the sprinters mass and speed and is significantly greater for faster runners. In fact its average can be given by; (Shahbazi et al.1998; 2000; 2002;)

$$F_{AV}=MV_Y/\Delta t+Mg \qquad (8)$$

Where VY is the vertical component of sprinter's velocity, ?t is the touchdown time.

The braking force occurs at the moment of landing, and at knee's flexion, while the propulsive force occurs at knee extension and when runner prepares himself/herself for another stride and flying. Practically all the force platforms results are showing that the brake and propulsive phases are about 40% and 60% of total touchdown time, respectively, Mero & Komi, 1994; Kyrolainen et al., 1999; Nummela, 1993; Coh,
et al., 1998; Weyand et al., 2000. In the present study, 42% and 58% for braking and propulsive phases have been chosen.

Force estimation at braking

We have considered each leg as a spring, as was already considered by, McMahon and Cheng, 1990; Luhtanen and Komi, 1980; Jacobs et al., 1996. At braking phase knee is in flexion and the quadriceps is in eccentric situation, all is as if the simulated spring is compressed. The spring like energy of the leg is equal to the sum of kinetic energy of sprinter at the moment of landing and the potential energy of CG (centre of gravity) Figure (2);

$$(½)KX^2=Mgh_B+(½)MV^2_Y \qquad (9)$$

where X is the maximum compression, K, the leg stiffness, hB, the CG down displacement and VY, is the velocity vertical component. X and h_B are related by;

$$X_B=h_BSec\alpha \qquad (10)$$

Where a is the angle which shank makes with the perpendicular axis.

Inserting (10) into (9) and solving for K, we get;

$$K=M(V^2Y/h^2_B+2g/h_B)Cos^2\ \alpha \qquad (11)$$

As can be seen, the stiffness of the leg is not only dependent directly to the mass and quadratic vertical velocity component but also inversely to the CG displacement and Cosa, quadratically .The variations of K relative to h and a are shown in Figures (a and b). The braking force can then be given as;

$$F_B=K_B\ X_B \qquad (12)$$

Inserting (10) and (11) into (12), braking force for sprinters can finally formulated as;

$$F_B=M((V^2Y/h_B+2g)Cos\alpha \qquad (13)$$

The variations of FB relative to h and a are shown in Figures (c and d).

Braking acceleration a_B

The braking acceleration can easily be found by dividing FB by the mass of sprinters that means;

$$a_B=(V^2Y/h_B+2g)Cos\alpha \qquad (14)$$

The variation of braking acceleration is similar to braking force.

Estimation of propulsive force

In this phase the knee is in extension and the CG is in higher position relative to stance position and the angle of shank is also varied (greater than in brake phase). In fact in this case the energy of the spring like leg, is transferred to the gravitational potential energy of CG and its kinetic energy, that means, we can write exactly the same equation as (9) but with different height and compression;

$$(½)KX^2{}_P=Mgh_P+(½)MV^2{}_Y \quad (15)$$

The X_P and h_P are related to each other by following relationship;

$$X_P=h_P Sec ß \quad (16)$$

Inserting (16) into (15) and solving for K, we get a similar equation as (11);

$$K_P=M(V^2{}_Y/h^2{}_P+2g/h_P)Cos^2 ß \quad (17)$$

The variations of the stiffness of leg in propulsive phase is similar to the variations of K in braking phase. The propulsive force can be given as;

$$F_P=K_P X_P \quad (18)$$

Inserting (16) and (17) in (18) we'll get for F_P;

$$F_P=M((V^2{}_Y/h_P+2g)Cos ß \quad (19)$$

The propulsive acceleration a_P can be given as below;

$$a_P= (V^2{}_Y/h_P+2g)Cos ß \quad (20)$$

RESULTS AND DISCUSSION: As can be seen from Table 2, the GRF value is about 2.55 times the subject's body weight. Comparing with the results obtained by Farley et al., which for vertical GRF component 2.3 times body weight has been reported, there is a remarkable agreement. In fact the difference between 2.55 from the present study and 2.3 from Farley study is due to the fact that the GRF is normally considered to be the resultant of the vertical and the two horizontal forces. Therefore this difference is the resultant of the two horizontal forces. In present study the total GRF is given, which referring to the results obtained by forceplate, vertical and two horizontal forces can be deduced. The variations of leg's stiffness and muscles relative to CG displacement and the foot angle relative to the axis perpendicular to the ground are presented in Figures 1 (a, b, c, and d). The force curves show that for small displacement of CG, and also small angle of foot enormous muscle force should be provided (specially in braking). Numerical results, mean ± SD, for five sprinters are indicated in Tables 1 and 2. In Figure 2, leg's muscles, tendons, and ligaments have been considered as a spring in both braking and propulsive phases.

Table 1 Stride length and time characteristics, Mean ± SD.

Sub. Mass	Tot.SL(m)	Eff.SL(m)	Tot.TD(ms)	Brak.Pha.	Prop.Pha.	Vert.velo.
73.4± 2.7 Kg	2.16 ± 0.21	1.28± 0.14	1.09±0.04	39.2±2.8 ms	61 4±2.3 ms	0.62±0.03 m/s

Table 2 CG displacement, acceleration, reaction, braking and propulsive forces.

CG Br. dis.	CG Pro.dis	CG Accel.	CG Accel.	Reac.force	Brak.force	Prop.force
0.13±0.02 m	0.17±0.02 m	-13.2±1.4 ms^{-2}	+9.3±2.1ms^{-2}	1875±34 N	1649±58 N	1530±42 N

CONCLUSION: The proposed model seems to be simpler, comprehensive, easy to use and yields reasonable dynamic results, comparable to the results achieved by other researchers engaging force-platform. The aim of present study was to offer a practical simple technique, which does not engage expensive apparatus for estimation of dynamic parameters, especially when comparison between athletes is to be considered.

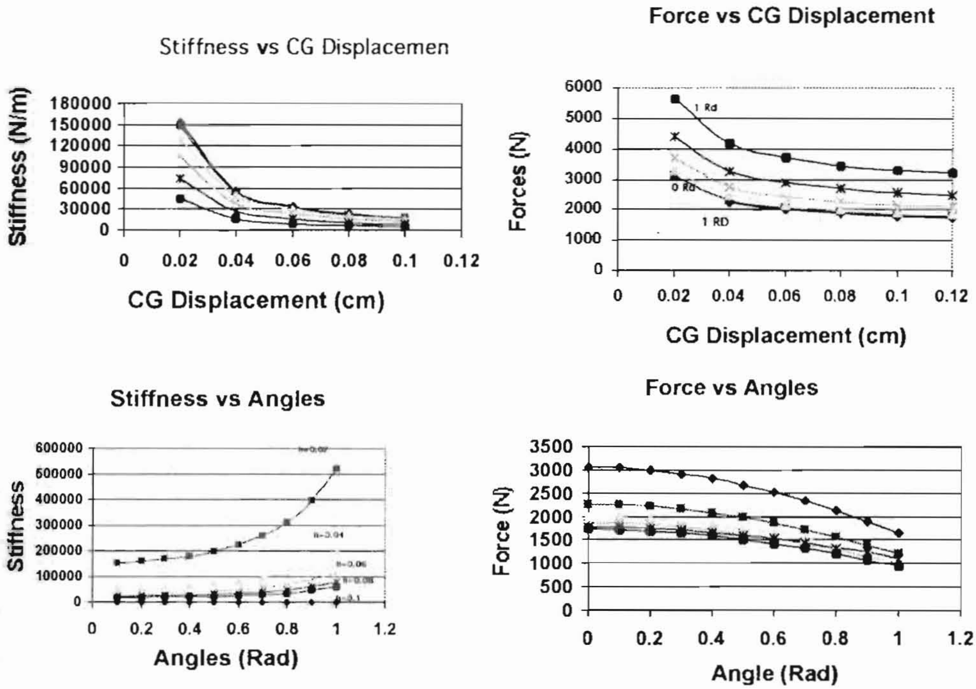

Figure 1: The variations of stiffness (a&b) and muscle force (c&d) versus CG displacement and angle of foot relative to axis perpendicular to the ground are presented.

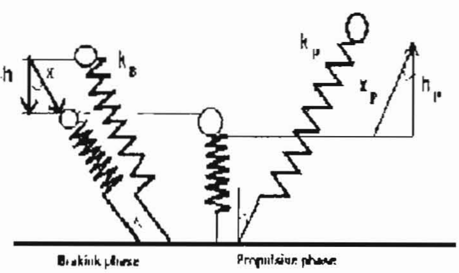

Figure 2: The leg's muscles, tendons, and ligaments have been considered as a spring. In braking phase the spring is compressed and the CG is shifted downward while at propulsive phase CG is upward shifted and the spring is decompressed. X and h dependence can be seen from the triangles presented in the Figure.

REFERENCES:

Alexander, R. McN. (1988). Elastic Mechanisms in Animal Movement. Cambridge University Press, Cambridge.

Alexander, R. McN. (1992). A model of bipedal locomotion on compliant legs. Phil. Trans. Roy. Soc. B338, 189-198.

Alexander, R. McN.and Vernon, A. (1975). Mechanics of hopping by kangaroos (Macropodidae). J. Zool. Lond. 177, 2650303.

Acknowledgement

The author would like to thank Professor Ross Sanders Chair of Sport Science and Leisure Study of Edinburgh University for the all facilities he provided and the Tehran University Research Council for support provided.