OPTIMAL STRIDE LENGTH IN RUNNING

Der Chia Lin
Department of Exercise Science
Concordia University
Montreal, Quebec, Canada H4B 1R6

Charles J. Dillman
Biomechanics Laboratory
United States Olympic Committee
Colorado Spring, Colorado 80909

In order to search for the optimal stride length necessary to minimize energy expenditure in running, a comprehensive dynamic analysis needs to be conducted. The procedures of this research are comprised of two main parts, theoretical and experimental.

Seven highly-skilled male runners who attended the American Olympic Development Clinic at the University of Illinois at Champaign-Urbana in August, 1978, acted as subjects for the purpose of experimental study. Based on the experimental data, the stride lengths were adjusted within a reasonable range for each runner. Then, a detailed theoretical analysis was conducted for each assumed stride length. According to the construction of a mathematical model of the lower extremity for running, the equations of motion were derived to depict the kinematic and kinetic properties of the lower extremity. Lagrangian equations were applied to evaluate the resultant effective moments about the three joints of the leg for a full cycle of running. The total mechanical energy expenditure of the total applied moments system was evaluated by the time integral of the instantaneous power. Then for each assumed stride length, the ratio of energy expense to the time duration of a running cycle was obtained. The determinant of the optimal stride length for each runner at a given running velocity was dependent on the assumed stride length which exhibited the minimum power output.

INTRODUCTION

Two components of running combine to directly control running velocity. The velocity at which the athlete runs is the product of two variables: stride length and stride rate. In other words, these two variables are essential factors to the determination of running velocity. But in running, an increase in one parameter may be accompanied by a decrease in the other variable, such that there could be a decrease in running velocity. Therefore, the difficult problem which exists in distance running is how to strike a proper balance between these two variables so that the runner could achieve a more optimal result. In order to search for the optimal stride length necessary to minimize energy expenditure in runnung, a comprehensive dynamic analysis needs to be conducted.

During running, the mechanical system of the body is acted on by such external forces as gravity and ground reactions; and internal forces such as inertia, forces transmitted by the body through the joints of all body segments, and muscular forces which generate the appropriate moments at the articulations. Therefore, the changes in total energies of the body during running is not only the result of mechanical work done by the muscles but also be external forces. In general, mechanical work done with reference to muscular activity can be evaluated in two ways. One measure is the time integral of the instantaneous power, which is the product of the resultant effective moment about a joint and the angular velocity of the limb with respect to that joint. The method was adopted by Beckett and Chang (1968), Gurfinkel et al (1970), and Cappozzo et al (1976). The other measure is the time integral of the product of the force generated by the muscles in causing rotation and its velocity of shortening. The method was used by Chow and Jacobson (1971). In this study, the experimental data with respect to the movement of the lower extremity during running was collected by means of film analysis. In the procedure of film analysis it is not possible to inspect the velocity of muscle shortening. Therefore, in this study the mechanical work done by muscular activities of the lower extremity during running was calculated by the product of the resultant effective moments about the hip, knee, and ankle joints, and the angular velocities of the thigh, shank, and foot segments with respect to their corresponding joint.

A more sophisticated treatment for obtaining the resultant effective moment at a segmented joint under a dynamic equilibrium condition would be the use of D'Alembert's principle or the Lagrangian approach to the formulation of the equations of motion. The former method was adopted by Cappozzo et al (1975), and Zatziorsky and Aleshinsky (1976). The latter one was used by Beckett and Chang (1968), Gurkinkel et al (1970), Jorgensen (1970), Kane (19-73), Chao and Rim (1963), Chow and Jacobson (1971, 1972), and Hatze (1973, 19-76, 1977). Theoretically, the results using D'Alembert's principle or Lagrangian's approach to mechanical analysis is the same for any given mechanical system, it is only the method used to obtain these results that is different. D'Alembert's principle places the emphasis on the forces acting on the system, whereas Lagrangian's method deals with only the kinetic and potential energies which are associated with the system. The procedure of film analysis conducted by this study obtained the kinetic and potential energies for each segment of the lower extremity relatively simply and was more convenient than obtaining the forces acting on each joint. Consequently, the Lagrangian method was the more straightforward approach and was applied to this study.

With regards to dynamical analysis of the segmented human body, it must be remembered that the linear and angular velocities of each segment are measured relative to inertial space, but their components must be written in terms of specific coordinates. Using the body fixed axes and the vector-matrix method, the analysis can then be done in a less tedious and more elegant way. The method was used by Smith and Kane (1967, 1968), Kane and Acher (1969, 1970), Passerello and Huston (1971), Huston and Passerello (1971), Huston et al (19-76), Abdelnour et al (1976) and Hatze (1977); and it was also adopted for this study

The objective of this investigation was to ascertain, through an anylytical and experimental study, the optimal stride length for each selected runner at his given running velocity. It consisted of the construction of a mathematical model for the lower extremity for running; the derivation of the equations for motion; estimation of body segment parameters; cinematographic procedures, film analysis; the evaluation of the resultant effective moments and mechanical work done by the total applied moments system, and the determination of optimal stride length on the basis of minimizing average power expenditure.

MATHEMATICAL MODEL OF THE LOWER EXTREMITY IN RUNNING

A running cycle consists of support phases and recovery phases of both the right and left lower extremities. For the sake of simplifying the mechanical aspects of the problem, some assumptions were employed for a simplified model of the lower extremities in running. These assumptions are delineated as follows:

1. Each limb segment of the lower extremities was assumed to be a rigid body of simple geometric and uniform density.

2. The foot and its toes were assumed to be one link, and hence, no relative motions between them were considered.

3. Each segment was assumed to rotate about a fixed pivot joint.

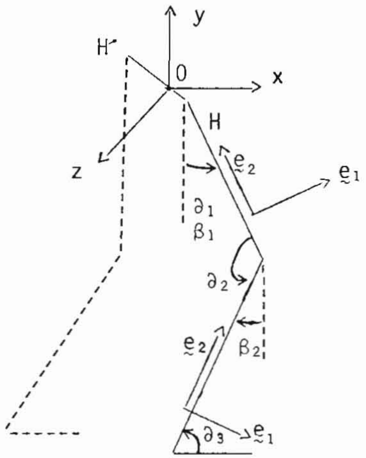
4. Except for the pelvic rotation about the vertical axis, the movement of the pelvis in side-to-side shifting and the rotation about the horizontal axis were neglected.

5. The movement of the lower extremities were assumed to be parallel to the sagittal plane.

6. The movement patterns of both lower extremities were assumed to be the same for all phases and combinations of phases.

7. The movement of the center of gravity of the body was confined to the sagittal plane. In the air-borne phase, the velocity of the body's center of gravity was assumed to be constant in the horizontal direction, and was assumed to have only gravitational acceleration in the vertical direction.

Based on the aforementioned assumptions, the models of a process of the support phase and a process of the recovery phase in running were drawn in Figures 1 and 2. In the figures HH′ is an imaginary line depicting a connection between the right and left hip joint centers. The line HH′ intersects the midsagittal plane of the body at point O. In order to simplify the calculations it was assumed that while running, the relative position of point O is not changed with respect to the mass center of the body. Hence, a relatively fixed coordinate system xyz was set at point O as the origin. If $\underset{\sim}{e}_1$, $\underset{\sim}{e}_2$, $\underset{\sim}{e}_3$ represent a set of mutually orthogonal unit vectors of a moving coordinate system which is either parallel to the centroidal principle axes of each limb segment or corresponds to the pelvic rotation, then the relationship between $\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$ and specific choice of $\underset{\sim}{e}_1$, $\underset{\sim}{e}_2$, $\underset{\sim}{e}_3$ would be made for pelvic rotation and each limb segment.



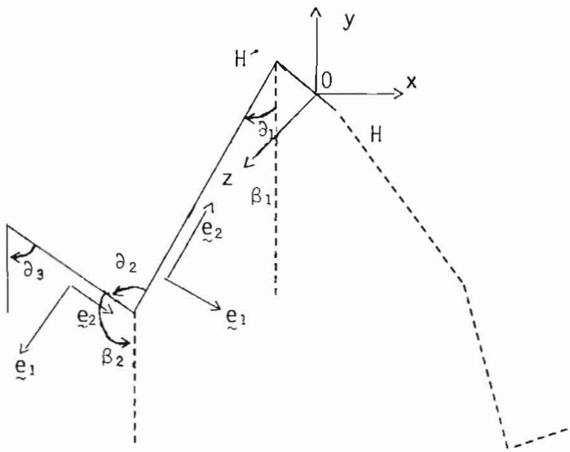


Figure 1:

A model of the support phase.

Figure 2:

A model of the recovery phase

Equation 1 represents that of the relation for pelvic rotation and equation 2 of for each limb segment.

$$(1)\qquad \begin{bmatrix} \underset{\sim}{i} \\ \underset{\sim}{j} \\ \underset{\sim}{k} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \underset{\sim}{e}_1^p \\ \underset{\sim}{e}_2^p \\ \underset{\sim}{e}_3^p \end{bmatrix}$$

$$\begin{bmatrix} \underset{\sim}{i} \\ \underset{\sim}{j} \\ \underset{\sim}{k} \end{bmatrix} = \begin{bmatrix} \cos\beta_1 & -\sin\beta_1 & 0 \\ \sin\beta_1 & \cos\beta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \underset{\sim}{e}_1^t \\ \underset{\sim}{e}_2^t \\ \underset{\sim}{e}_3^t \end{bmatrix}$$

$$(2)\qquad \begin{bmatrix} \underset{\sim}{i} \\ \underset{\sim}{j} \\ \underset{\sim}{k} \end{bmatrix} = \begin{bmatrix} \cos\beta_2 & -\sin\beta_2 & 0 \\ \sin\beta_2 & \cos\beta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \underset{\sim}{e}_1^s \\ \underset{\sim}{e}_2^s \\ \underset{\sim}{e}_3^s \end{bmatrix}$$

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} \cos\beta_3 & -\sin\beta_3 & 0 \\ \sin\beta_3 & \cos\beta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_1^f \\ e_2^f \\ e_3^f \end{bmatrix}$$

Where the angle θ is measured in the horizontal plane and is the angle between the z axis and the unit vector e_3; the angle β is measured in the vertical plane and is the angle between the y axis and the unit vector e_2. (The right hand rule is adopted for the sign convention of the angles.)

In addition, the angle αn ($n = 1, 2, 3$) is measured for the angles between each segment. The relationship between the angles αn and βn, then, could be expressed as indicated below:

(3)
$$\beta_1 = \alpha_1$$
$$\beta_2 = \alpha_1 + \alpha_2 - \pi$$
$$\beta_3 = \alpha_1 + \alpha_2 + \alpha_3$$

THE KINEMATICS OF THE LOWER EXTREMITY

The kinematics of the lower extremity (represented by the left one) for running could be delineated in terms of a series of equations. The angular velocity of the pelvic rotation (ω^p) for running with respect to the fixed coordinate system can be expressed as:

(4-1)
$$\omega^p = \dot{\theta}\, j$$

The angular velocity of the thigh, shank, and foot segment, respectively, in the fixed coordinate system would be expressed as:

(5-1)
$$\omega^t = \omega^p + {}^p\omega^t$$
$$\omega^s = \omega^t + {}^t\omega^s = \omega^p + {}^p\omega^s$$
$$\omega^f = \omega^s + {}^s\omega^f = \omega^p + {}^p\omega^f$$

Where ${}^p\omega^t$, ${}^p\omega^s$, and ${}^p\omega^f$ is the angular velocity of the thigh, shank, and foot segment, respectively, with respect to the pelvis.

In referring to equations (1) and (2), equations (4-1) and (5-1) can be rewritten into a moving coordinate system as illustrated below:

(4-2)
$$\omega^p = \dot{\theta} e_2{}^p$$

(5-2)
$$\omega^t = \dot{\theta}\sin\beta_1 e_1^t + \dot{\theta}\cos\beta_1 e_2^t + \dot{\beta}_1 e_3^t$$
$$\omega^s = \dot{\theta}\sin\beta_2 e_1^s + \dot{\theta}\cos\beta_2 e_2^s + \dot{\beta}_2 e_3^s$$
$$\omega^f = \dot{\theta}\sin\beta_3 e_1^f + \dot{\theta}\cos\beta_3 e_2^f + \dot{\beta}_3 e_3^f$$

When the equations for the rotations of the pelvis and each limb segment were derived, the equations for depicting the rotational kinetic energy of each limb segment was then obtained. It was assumed that the centrolongitudinal axis of each limb segment possesses an axial symmetry, and I_1 was included to represent the transverse moment of inertia about the mass center of the limb segment; I_2 represents the axial moment of inertia. It was then that the rotational kinetic energy of the thigh, shank, and foot segment, respectively, could be expressed as follows:

$$k_\omega^t = \tfrac{1}{2}[I_1^t (\omega_1^t)^2 + I_2^t (\omega_2^t)^2 + I_1^t (\omega_3^t)^2]$$

(6-1) $$k_\omega^s = \tfrac{1}{2}[I_1^s (\omega_1^s)^2 + I_2^s (\omega_2^s)^2 + I_1^s (\omega_3^s)^2]$$

$$k_\omega^f = \tfrac{1}{2}[I_1^f (\omega_1^f)^2 + I_2^f (\omega_2^f)^2 + I_1^f (\omega_3^f)^2]$$

However, based on equation (3) and (5-2), the aforementioned equations can be rewritten as follows:

$$k_\omega^t = \tfrac{1}{2}\,[I_1^t (\dot{\theta}^2 \sin^2 \alpha_1 + \dot{\alpha}_1^2) + I_2^t\, \dot{\theta}^2 \cos^2 \alpha_1]$$

(6-2) $$k_\omega^s = \tfrac{1}{2}\,[I_1^s \{\dot{\theta}^2 \sin^2 (\alpha_1 + \alpha_2) + (\dot{\alpha}_1 + \dot{\alpha}_2)^2\} + I_2^s\, \dot{\theta}^2 \cos^2 (\alpha_1 + \alpha_2)]$$

$$k_\omega^f = \tfrac{1}{2}\,[I_1^f \{\dot{\theta}^2 \sin^2 (\alpha_1 + \alpha_2 + \alpha_3) + (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)^2\} + I_2^f\, \dot{\theta}^2 \cos^2 (\alpha_1 + \alpha_2 + \alpha_3)]$$

Next, the absolute velocity of the mass center for each limb segment was derived and was used in the calculation for the transitional energy. The velocity of the mass center of the thigh, shank, and foot segment, respectively, is expressed as:

$$\underset{\sim}{v}_t = \underset{\sim}{v} + \underset{\sim}{\omega}^p \times p\, \underset{\sim}{e}_3^p + {}^p\underset{\sim}{\omega}^t \times (-d_1 \underset{\sim}{e}_2^t)$$

(7-1) $$\underset{\sim}{v}_s = \underset{\sim}{v} + \underset{\sim}{\omega}^p \times p\, \underset{\sim}{e}_3^p + {}^p\underset{\sim}{\omega}^t \times (-l_1 \underset{\sim}{e}_2^t) + p\underset{\sim}{\omega}^s \times (-d_2 \underset{\sim}{e}_2^s)$$

$$\underset{\sim}{v}_f = \underset{\sim}{v} + \underset{\sim}{\omega}^p \times p\, \underset{\sim}{e}_3^p + {}^p\underset{\sim}{\omega}^t \times (-l_1 \underset{\sim}{e}_2^t) + p\underset{\sim}{\omega}^s \times (-d_2 \underset{\sim}{e}_2^s) + {}^p\underset{\sim}{\omega}^f \times (-d_3 \underset{\sim}{e}_2^f)$$

Where $\underset{\sim}{v}$ is the velocity vector of the mass center of the body; d_1, d_2, d_3, is the distance from the hip joint center to the mass center of the thigh segment, the knee joint center to the mass center of the shank segment, and the ankle joint center to the mass center of the foot segment, respectively; and l_1, l_2, is the length of the thigh segment from the hip joint center to the knee joint center, and the length of the shank segment from the knee joint center to the ankle joint center.

These equations can then be rewritten as follows:

$$\underset{\sim}{v}_t = (v_x + p\dot{\theta}\cos\theta + d_1\dot{\alpha}_1\cos\alpha_1)\,\underset{\sim}{i} + (v_y + d_1\dot{\alpha}_1\sin\alpha_1)\,\underset{\sim}{j} - p\dot{\theta}\sin\theta\,\underset{\sim}{k}$$

$$\underset{\sim}{v}_s = [v_x + p\dot{\theta}\cos\theta + l_1\dot{\alpha}_1\cos\alpha_1 - d_2(\dot{\alpha}_1 + \dot{\alpha}_2)\cos(\alpha_1 + \alpha_2)]\,\underset{\sim}{i} + [v_y + l_1\dot{\alpha}_1\sin\alpha_1 - d_2(\dot{\alpha}_1 + \dot{\alpha}_2)\sin(\alpha_1 + \alpha_2)]\,\underset{\sim}{j} - p\dot{\theta}\sin\theta\,\underset{\sim}{k}$$

$$\underset{\sim}{v}_f = [v_x + p\dot{\theta}\cos\theta + l_1\dot{\alpha}_1\cos\alpha_1 - l_2(\dot{\alpha}_1 + \dot{\alpha}_2)\cos(\alpha_1 + \alpha_2) + d_3(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)\cos(\alpha_1 + \alpha_2 + \alpha_3)]\,\underset{\sim}{i} + [v_y + l_1\dot{\alpha}_1\sin\alpha_1 - l_2(\dot{\alpha}_1 + \dot{\alpha}_2)\sin(\alpha_1 + \alpha_2) + d_3(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)\sin(\alpha_1 + \alpha_2 + \alpha_3)]\,\underset{\sim}{j} - p\dot{\theta}\sin\theta\,\underset{\sim}{k}$$

(7-2)

Once the equation for the velocity of each limb segment has been derived, then, the equation for the transitional kinetic energy of each limb segment can also be developed using the formula:

(8-1)

$$K_v^t = \tfrac{1}{2} m_t \; v_t^2$$

$$K_v^s = \tfrac{1}{2} m_s \; v_s^2$$

$$K_v^f = \tfrac{1}{2} m_f \; v_f^2$$

Therefore, in accordance with the equation (7-2), the transitional kinetic energy of the thigh, shank, and foot segment, respectively, can be expressed in detail as follows:

$$K_v^t = \tfrac{1}{2} m_t (v_x^2 + v_y^2 + p^2\dot{\theta}^2 + d_1^2\dot{\alpha}_1^2) + m_t (v_x p \dot{\theta} \cos\theta + v_x d_1\dot{\alpha}_1 \cos\alpha_1 + p\, d_1 \dot{\theta} \dot{\alpha}_1 \cos\theta \cos\alpha_1 + v_y d_1\dot{\alpha}_1 \sin\alpha_1)$$

$$K_v^s = \tfrac{1}{2} m_s [v_x^2 + v_y^2 + p^2\dot{\theta}^2 + l_1^2\dot{\alpha}_1^2 + d_2^2 (\dot{\alpha}_1 + \dot{\alpha}_2)^2] + m_s [v_x p \dot{\theta} \cos\theta + v_x l_1\dot{\alpha}_1 \cos\alpha_1 - v_x d_2 (\dot{\alpha}_1 + \dot{\alpha}_2) \cos(\alpha_1 + \alpha_2) + v_y l_1\dot{\alpha}_1 \sin\alpha_1 - v_y d_2 (\dot{\alpha}_1 + \dot{\alpha}_2) \sin(\alpha_1 + \alpha_2) + p\, l_1 \dot{\theta} \dot{\alpha}_1 \cos\theta \cos\alpha_1 - p\, d_2 \dot{\theta} (\dot{\alpha}_1 + \dot{\alpha}_2) \cos\theta \cos(\alpha_1 + \alpha_2) - l_1 d_2 \dot{\alpha}_1 (\dot{\alpha}_1 + \dot{\alpha}_2) \cos\alpha_2]$$

$$K_v^f = \tfrac{1}{2} m_f [v_x^2 + v_y^2 + p^2\dot{\theta}^2 + l_1^2\dot{\alpha}_1^2 + l_2^2 (\dot{\alpha}_1 + \dot{\alpha}_2)^2 + d_3^2 (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)^2] + m_f [v_x p \dot{\theta} \cos\theta + v_x l_1\dot{\alpha}_1 \cos\alpha_1 - v_x l_2 (\dot{\alpha}_1 + \dot{\alpha}_2) \cos(\alpha_1 + \alpha_2) + v_x d_3 (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \cos(\alpha_1 + \alpha_2 + \alpha_3) + v_y l_1\dot{\alpha}_1 \sin\alpha_1 - v_y l_2 (\dot{\alpha}_1 + \dot{\alpha}_2) \sin(\alpha_1 + \alpha_2) + v_y d_3 (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \sin(\alpha_1 + \alpha_2 + \alpha_3)] + m_f [p\, l_1 \dot{\theta} \dot{\alpha}_1 \cos\theta \cos\alpha_1 - l_1 l_2\dot{\alpha}_1 (\dot{\alpha}_1 + \dot{\alpha}_2) \cos\alpha_2 - p\, l_2\dot{\theta} (\dot{\alpha}_1 + \dot{\alpha}_2) \cos\theta \cos(\alpha_1 + \alpha_2) + p\, d_3 \dot{\theta} (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \cos\theta \cos(\alpha_1 + \alpha_2 + \alpha_3) + l_1 d_3\dot{\alpha}_1 (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \cos(\alpha_2 + \alpha_3) - l_2 d_3 (\dot{\alpha}_1 + \dot{\alpha}_2) (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \cos\alpha_3]$$

(8-2)

The total kinetic energy of the lower extremity associated with the support phase or the recovery phase of running can be expressed as:

(9-1) $K = K_\omega^t + K_\omega^s + K_\omega^f + K_v^t + K_v^s + K_v^f$

Where the first three terms on the right hand side of the equation represent the rotational kinetic energies of the thigh, shank, and foot segment; the last three terms respond to the transitional kinetic energies of the thigh, shank, and foot segment, respectively. Therefore, by combining equations (6-2) and (8-2) and by rearranging them and regrouping terms, the total kinetic energy of the lower extremity for running can be constituted as follows:

$$K = \tfrac{1}{2} M_1 (v_x^2 + v_y^2) + \tfrac{1}{2} \dot{\theta}^2 [I_2^t + I_2^s + I_2^f + p^2 M_1 + (I_1^t - I_2^t) \sin^2\alpha + (I_1^s - I_2^s) \sin^2(\alpha_1 + \alpha_2) + (I_1^f - I_2^f) \sin^2(\alpha_1 + \alpha_2 + \alpha_3)] + \tfrac{1}{2} \dot{\alpha}_1^2 \{I_1^t + I_2^s + I_1^f + d_1^2 m_t + (l_1^2 + d_2^2 - 2\, l_1 d_2 \cos\alpha_2)\, m_s + [l_1^2 + l_2^2 + d_3^2 - 2\, l_1 l_2 \cos\alpha_2$$

$$+ 2 l_1 d_3 \cos(\alpha_2 + \alpha_3) - 2 l_2 d_3 \cos\alpha_3] m_f\} + \tfrac{1}{2}\dot{\alpha}_2^2 [I_1^s + I_1^f + d_2^2 m_s + (l_2^2 + d_3^2 - 2 l_2 d_3 \cos\alpha_3) m_f] + \tfrac{1}{2}\dot{\alpha}_3^2 (I_1^f + d_3^2 m_f) + \dot{\theta}\dot{\alpha}_1 \{p \cos\theta \cos\alpha_1 [d_1 m_t + l_1 (m_s + m_f)] - p \cos\theta \cos(\alpha_1 + \alpha_2)(d_2 m_s + l_2 m_f) + p d_3 \cos\theta \cos(\alpha_1 + \alpha_2 + \alpha_3) m_f\} - \dot{\theta}\dot{\alpha}_2 [p \cos\theta \cos(\alpha_1 + \alpha_2)(d_2 m_s + l_2 m_f) - p d_3 \cos\theta \cos(\alpha_1 + \alpha_2 + \alpha_3) m_f] + \dot{\theta}\dot{\alpha}_3 p d_3 \cos\theta \cos(\alpha_1 + \alpha_2 + \alpha_3) m_f + \dot{\alpha}_1\dot{\alpha}_2 \{I_1^s + I_1^f + (d_2^2 - l_1 d_2 \cos\alpha_2) m_s + [l_2^2 + d_3^2 - l_1 l_2 \cos\alpha_2 + l_1 d_3 \cos(\alpha_2 + \alpha_3) - 2 l_2 d_3 \cos\alpha_3] m_f\} + \dot{\alpha}_1\dot{\alpha}_3 \{I_1^f + d_3^2 + l_1 d_3 \cos(\alpha_2 + \alpha_3) - l_2 d_3 \cos\alpha_3 m_f\} + \dot{\alpha}_2\dot{\alpha}_3 [I_1^f + (d_3^2 - l_2 d_3 \cos\alpha_3) m_f] + \dot{\theta} M_1 v_x p \cos\theta + \dot{\alpha}_1 \{(v_x \cos\alpha_1 + v_y \sin\alpha_1)[d_1 m_t + l_1 (m_s + m_f)] - [v_x \cos(\alpha_1 + \alpha_2) + v_y \sin(\alpha_1 + \alpha_2)](d_2 m_s + l_2 m_f) + d_3 [v_x \cos(\alpha_1 + \alpha_2 + \alpha_3) + v_y \sin(\alpha_1 + \alpha_2 + \alpha_3)] m_f\} - \dot{\alpha}_2 \{[v_x \cos(\alpha_1 + \alpha_2) + v_y \sin(\alpha_1 + \alpha_2)](d_2 m_s + l_2 m_f) - d_3 [v_x \cos(\alpha_1 + \alpha_2 + \alpha_3) + v_y \sin(\alpha_1 + \alpha_1 + \alpha_3)] m_f\} + \dot{\alpha}_3 \{d_3 [v_x \cos(\alpha_1 + \alpha_2 + \alpha_3) + v_y \sin(\alpha_1 + \alpha_2 + \alpha_3)] m_f\}$$

(9-2)

Next, considering the potential energy, it was assumed that the position of the pelvic center at point 0 is not changed with respect to the mass center of the whole body, and that the relatively fixed coordinate system xyz was set at point 0 as the origin. Hence, the potential energy of the lower extremity due to gravity only can be written as:

$$V = -g \{m_t d_1 \cos\alpha_1 + m_s [l_1 \cos\alpha_1 - d_2 \cos(\alpha_1 + \alpha_2] m_f [l_1 \cos\alpha_1 - l_2 \cos(\alpha_1 + \alpha_2) + d_3 \cos(\alpha_1 + \alpha_2 + \alpha_3)]\}$$

(10)

Equations of Motion

When the total kinetic energy and the potential energy of the lower extremity associated with the swing and support phase were obtained, the resultant effective moments in the hip, knee, and ankle joints, respectively, could be derived in terms of the Lagrangian equation.

While motion in the recovery phase is considered, the three equations of motion with the variables α_1, α_2, and α_3 could be given by substituting K and V expressions from (9-2) and (10) into the Lagrange's equations.

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{\alpha}_1} - \frac{\partial K}{\partial \alpha_1} + \frac{\partial V}{\partial \alpha_1} = M_H$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{\alpha}_2} - \frac{\partial K}{\partial \alpha_2} + \frac{\partial V}{\partial \alpha_2} = M_N \qquad (11)$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{\alpha}_3} - \frac{\partial K}{\partial \alpha_3} + \frac{\partial V}{\partial \alpha_3} = M_A$$

The recovery phase in running could be divided into two phases. In the recovery phase with non-support the horizontal velocity of the mass center of the whole body is assumed to be constant and its vertical acceleration is assumed due to gravitational acceleration only. In the recovery phase with support by the opposite leg the acceleration of the mass center of the whole body along the x and y axes are, however, represented by a_x and a_y, respectively. Hence, explicitly, the system of equations can be expressed as:

(i) The recovery phase with non-support

$$M_{A_1} = (I_1^f + d_3^2 m_f)(\ddot{\alpha}_1 + \ddot{\alpha}_2 + \ddot{\alpha}_3) - \dot{\theta}^2 (I_1^f - I_2^f) \sin(\alpha_1 + \alpha_2 + \alpha_3)\cos(\alpha_1 + \alpha_2 + \alpha_3) + (\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta)\, p\, d_3 \cos(\alpha_1 + \alpha_2 + \alpha_3)\, m_f + [\ddot{\alpha}_1 \cos(\alpha_2 + \alpha_3) + \dot{\alpha}_1^2 \sin(\alpha_2 + \alpha_3)]\, l_1 d_3 m_f - [(\ddot{\alpha}_1 + \ddot{\alpha}_2)\cos\alpha_3 + (\dot{\alpha}_1 + \dot{\alpha}_2)^2 \sin\alpha_3]\, l_2 d_3 m_f + 2\, g\, d_3 \sin(\alpha_1 + \alpha_2 + \alpha_3)\, m_f$$

$$M_{N_1} = (I_1^S + d_2^2 m_S + l_2^2 m_f)(\ddot{\alpha}_1 + \ddot{\alpha}_2) - \dot{\theta}^2 (I_1^S - I_2^S)\sin(\alpha_1 + \alpha_2)\cos(\alpha_1 + \alpha_2) - (\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta)\, p\, (d_2 m_S + l_2 m_f)\cos(\alpha_1 + \alpha_2) - (\ddot{\alpha}_1 \cos\alpha_2 + \dot{\alpha}_1^2 \sin\alpha_2)\, l_1 (d_2 m_S + l_2 m_f) - [(\ddot{\alpha}_1 + \ddot{\alpha}_2 + \ddot{\alpha}_3)\cos\alpha_3 - (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)^2 \sin\alpha_3]\, l_2 d_3 m_f - 2\, g \sin(\alpha_1 + \alpha_2)(d_2 m_S + l_2 m_f) + M_{A_1}$$

$$M_{H_1} = [I_1^t + d_1^2 m_t + l_1^2 (m_s + m_f)] \ddot{\alpha}_1 - \dot{\theta}^2 (I_1^t - I_2^t) \sin \alpha_1 \cos \alpha_1 + (\ddot{\theta}^2 \cos \theta - \dot{\theta}^2 \sin \theta) p [d_1 m_t + l_1 (m_s + m_f)] \cos \alpha_1 - [(\ddot{\alpha}_1 + \ddot{\alpha}_2) \cos \alpha_2 - (\dot{\alpha}_1 + \dot{\alpha}_2)^2 \sin \alpha_2] l_1 (d_2 m_s + l_2 m_f) + [(\ddot{\alpha}_1 + \ddot{\alpha}_2 + \ddot{\alpha}_3) \cos (\alpha_2 + \alpha_3) - (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)^2 \sin (\alpha_2 + \alpha_3)] l_1 d_3 m_f + 2 g \sin \alpha_1 [d_1 m_t + l_1 (m_s + m_f)] + M_{N_1}$$

(12)

(ii) The recovery phase with support by opposite leg

$$M_{A_2} = M_{A_1} + [a_x \cos (\alpha_1 + \alpha_2 + \alpha_3) + (a_y - g) \sin (\alpha_1 + \alpha_2 + \alpha_3)] d_3 m_f$$

$$M_{N_2} = M_{N_1} - [a_x \cos (\alpha_1 + \alpha_2) + (a_y - g) \sin (\alpha_1 + \alpha_2)](d_2 m_s + l_2 m_f)$$

$$M_{H_2} = M_{H_1} + [a_x \cos \alpha_1 + (a_y - g) \sin \alpha_1] [d_1 m_f + l_1 (m_s + m_f)]$$

(13)

The resultant effective moments of the lower extremity during the support phase is induced not only by the muscle action about it, but also by the forces of ground reaction or the transmitted forces of ground reaction at the joint of its distal extremity. Such forces could have three mutual orthogonal components. However, the lower extremity is considered to be moving parallel to the sagittal plane; hence, only two of the three components which are parallel to the sagittal plane, are taken into account. As far as the ground reaction forces are concerned, it is assumed to be fixed at the toe; and its effective moment for the ankle joint is then taken into account from the beginning of the thrust phase. However, the effective moment of the ground reaction forces about the hip joint and the knee joint, respectively, is taken into account in the whole period of the support phase. If we let R_x and R_y represent the horizontal and vertical components of the ground reaction forces due to weight bearing and body motion, then it could be appraised as below:

$$R_x = M a_x$$

$$R_x = M (a_y + g)$$

(14)

Therefore, the resultant effective moments about the hip joint, knee joint, and ankle joint, respectively, could be derived as follows:

$$\frac{d}{dt} \frac{\partial K}{\partial \dot{\alpha}_3} - \frac{\partial K}{\partial \alpha_3} + \frac{\partial V}{\partial \alpha_3} = M_A + R_x l_3 \cos (\alpha_1 + \alpha_2 + \alpha_3) + R_y l_3 \sin (\alpha_1 + \alpha_2 + \alpha_3)$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{\alpha}_2} - \frac{\partial K}{\partial \alpha_2} + \frac{\partial V}{\partial \alpha_3} = M_N + R_x\,[l_3 \cos(\alpha_1 + \alpha_2 + \alpha_3) - l_2 \cos(\alpha_1 + \alpha_2)] + R_y\,[l_3 \sin(\alpha_1 + \alpha_2 + \alpha_3) - l_2 \sin(\alpha_1 + \alpha_2)]$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{\alpha}_1} - \frac{\partial K}{\partial \alpha_1} + \frac{\partial V}{\partial \alpha_1} = M_H + R_x\,[l_3 \cos(\alpha_1 + \alpha_2 + \alpha_3) - l_2 \cos(\alpha_1 + \alpha_2) + l_1 \cos\alpha_1] + R_y\,[l_3 \sin(\alpha_1 + \alpha_2 + \alpha_3) - l_2 \sin(\alpha_1 + \alpha_2) + l_1 \sin\alpha_1]$$

(15)

or

$$M_{A_s} = M_{A_2} - M\, l_3\,[a_x \cos(\alpha_1 + \alpha_2 + \alpha_3) + (a_y + g)\sin(\alpha_1 + \alpha_2 + \alpha_3)]$$

$$M_{N_s} = M_{N_2} + M\,\{a_x\,[l_2 \cos(\alpha_1 + \alpha_2) - l_3 \cos(\alpha_1 + \alpha_2 + \alpha_3)] + (a_y + g)\,[l_2 \sin(\alpha_1 + \alpha_2) - l_3 \sin(\alpha_1 + \alpha_2 + \alpha_3)]\}$$

$$M_{H_s} = M_{H_2} - M\,\{a_x\,[l_1 \cos\alpha_1 - l_2 \cos(\alpha_1 + \alpha_2) + l_3 \cos(\alpha_1 + \alpha_2 + \alpha_3)] + (a_y + g)\,[l_1 \sin\alpha_1 - l_2 \sin(\alpha_1 + \alpha_2) + l_3 \sin(\alpha_1 + \alpha_2 + \alpha_3)]\}$$

(16)

Once the equations of the resultant effective moments about the three joints of the lower extremity during both the support phase and the recovery phase have been developed, the total mechanical work done about the hip, knee and ankle joints of the left leg for one cycle of running could be evaluated as below. (However, a comment should be added about the equation. Because the muscular efforts for the activities are not recoverable, the absolute values are used for the integrands in the following equation in order to avoid the cancellation of opposite signs in their summation.)

$$W = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \{ |M_H|\,|\dot{\alpha}_1| + |M_N|\,|\dot{\alpha}_2| + |M_A|\,|\dot{\alpha}_3| \}\, dt$$

(17)

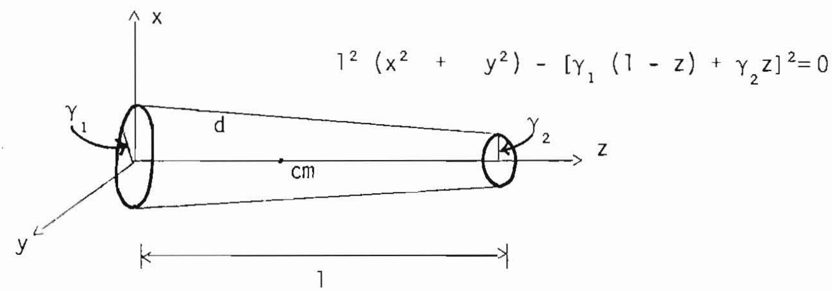
Where the eight components of the time interval are describing a sequence of the following phases: the restraint phase, the thrust phase, the thigh and shank backward swing while the body is in the first air-borne phase, the thigh forward swing and shank backward swing while the body is still in the first air-borne phase, the thigh forward swing and shank backward swing while the opposite leg is in the support phase, the thigh and shank forward swing while the opposite leg is still in the support phase, the thigh and shank forward swing while the body is in the second air-borne phase,and the leg decsent phase.

RESULTS OF EXPERIMENTAL AND ANALYTICAL STUDIES

This experiment included the filming of seven highly-skilled male runners who participated in the U.S. Olympic Committee, Track and Field National Governing Board - Olympic Development Mini-Clinic Branch. Each runner was asked to run at his own competitive pace on a rubberized asphalt track at Memorial Stadium on the campus of the University of Illinois at Urbana-Champaign on August 2nd. and 3rd., 1978. A Locam 16mm high speed camera, which was loaded with Tri-X Reversal film ASA 200, was used for the filming. Film analysis was carried out by using a model M-16C projection head of the Vanguard Motion Analyzer and an enlarged projected screen. In two dimensional plane of film the kinematics angular parameters of the hip, knee, and ankle joint, respectively, were determined by means of coordinates of their joint centers and segmental length. The Kinematics angular parameters of the pelvis relative to a frontal plane (θ, $\dot{\theta}$ and $\ddot{\theta}$), however, were estimated in terms of the coordinates of midpoint of the imaginary line which connected the right and left hip joint centers, the coordinates of the hip joint center, and half of the pelvic width. The movements of mass center of the whole body were obtained from the information of body segment parameters and the coordinates of thirteen points of body landmarks. These thirteen points of body landmarks included: the left tragion, the left and right shoulder, elbow, wrist, hip, knee and ankle joint centers, respectively.

Each runner's body weight, as well as segmental lengths of the left leg and their upper and lower circumferences were measured just before the filming. The mass of each segment was estimated on the basis of the results of Dempster's (1955) and Clauser's (1966) studies. Each limb segment was simulated as frustum of a right circular cone which has the graph of an equation of the form (see Figure 3).

Figure 3: A Model of Limb Segment



The value of the moment of inertia and the distance from joint to the segmental center of the mass were, then, calculated by means of the following formulas:

$$m\,d = \rho \iiint_R z\,dx\,dy\,dz$$

$$I_2 = \rho \iiint_R (x^2 + y^2)\,dx\,dy\,dz \qquad (18)$$

$$I_2 = \rho \iiint_R (y^2 + z^2)\,dx\,dy\,dz - md^2$$

Where R is a bounded closed region described by inequalities such as the following:

$$0 \le z \le 1 \qquad -\frac{1}{l}[\gamma_1(1-z)+\gamma_2 z] \le y \le \frac{1}{l}[\gamma_1(1-z)+\gamma_2 z]$$

$$-\frac{1}{l}\sqrt{[\gamma_1(1-z)+\gamma_2 z]^2 - l^2 y^2} \le x \le \frac{1}{l}\sqrt{[\gamma_1(1-z)+\gamma_2 z]^2 - l^2 y^2}$$

Table 1 listed these physical parameters for each subject. The values were treated as constants and were used for necessary computation throughout this study.

TABLE 1: The Physical Parameters for Each Subject

Physical Parameters	Subject						
	#1	#2	#3	#4	#5	#6	#7
I_1^t (Kg-m²)	.1388	.1375	.1013	.1038	.1269	.1089	.1072
I_2^t (Kg-m²)	.0208	.0204	.0158	.0152	.0196	.0150	.0174
I_1^s (Kg-m²)	.0630	.0589	.0457	.0452	.0600	.0468	.0443
I_2^s (Kg-m²)	.0042	.0040	.0034	.0033	.0038	.0031	.0033
I_1^f (Kg-m²)	.0115	.0109	.0090	.0090	.0109	.0092	.0088
I_2^f (Kg-m²)	.0008	.0007	.0007	.0007	.0007	.0006	.0007
d_1 (m)	.2094	.2113	.1962	.2013	.2053	.2033	.2005
d_2 (m)	.1982	.1936	.1828	.1838	.2000	.1905	.1818
d_3 (m)	.1373	.1373	.1330	.1330	.1330	.1201	.1201
p (m)	.1700	.1675	.1600	.1600	.1650	.1550	.1650
l_1 (m)	.4702	.4760	.4350	.4425	.4644	.4589	.4355
l_2 (m)	.4752	.4740	.4456	.4437	.4851	.4515	.4219
l_3 (m)	.3200	.3200	.3100	.3100	.3100	.2800	.2800
m_t (Kg)	7.1850	6.9605	6.0624	6.0175	6.7419	5.9553	6.3374
m_s (Kg)	3.4111	3.3045	2.8781	2.8568	3.2007	2.8273	3.0087
m_f (Kg)	1.0886	1.0546	.9185	.9117	1.0215	.9023	.9602
M (Kg)	72.5760	70.3080	61.2360	60.7824	68.1000	60.1550	64.0140

On the basis of film analysis, some kinematic data which contained the necessary factors for the analytical study were obtained for each subject. These results are illustrated in Table II.

TABLE II: Kinematic Parameters for Each Subject

Kinematic Parameters	Subjects						
	#1	#2	#3	#4	#5	#6	#7
Horizontal Velocity (m/sec)	6.9780	8.2890	6.9580	5.9540	7.6550	6.0770	6.5080
Stride Length (m)	2.0450	2.4260	2.0360	1.8880	2.4270	2.0500	1.8250
Air-Borne Phase (sec)	.1423	.1789	.1463	.1423	.1626	.1789	.1220
Support Phase (sec)	.1504	.1138	.1463	.1748	.1545	.1585	.1585
Percentage of Air-Borne (%)	48.6200	61.1200	50.0000	44.8800	51.2800	53.0200	43.4900
Percentage of Support (%)	51.3800	38.8800	50.0000	55.1200	48.7200	46.9800	56.5100
Events (m)	1500	800	1500	*3000	800	*3000	*3000
Best Performance	3'40"7	1'45"2	3'40"	8'30"1	1'51"2	8'31"2	8'28"8
Average Velocity (m/sec)	6.7970	7.6000	6.8180	5.8810	7.1940	5.8690	5.8960

* 3000 m is Steeple Chase

Chow and Jacobson (1971) carried out the method in their human locomotion study and concluded that, in the normal range of activity, the sum total of mechanical energy expenditure by the muscle-activity system is proportional to the integral of the square of the net muscular moment. In accordance with the conclusion, the angular parameters of each limb segment of the leg relates to its joint with minimum expense of the energy were obtained by applying variational methods. In other words, let:

$$J = \int_{t_0}^{t} f\,(\alpha_1 \dot{\alpha}_1 t)\,dt$$

Where $f\,(\alpha_1 \dot{\alpha}_1 t)$ is a function to provide a profile of the behavior of rotation of a limb segment about its joint. The Euler-Langrange differential equation $\frac{\alpha f}{\alpha \alpha} - \frac{d}{dt}\frac{\alpha f}{\alpha \dot{\alpha}} = 0$ is applied to minimize the performance measure. The method of Langrange multipliers is, then, utilized to determine the solution. Once the behaviors of the angular parameters for each limb segment in a full cycle of running were determined, they were substituted into the equations (12), (13) and (16). The physical parameters shown in Table I and the kinematic data shown in Table II were also substituted into those three equations to carry out the resultant effective moments applied at the hip joint, knee joint, and ankle joint, respectively, for each subject in the period of a full cycle of running. The mechanical work done by the total applied moments system at the lower extremity in the period of a full cycle of running was, then, calculated in terms of applying the equation (17). Results relating to the amounts of mechanical energy expenditure in a full cycle of running for each subject were listed in Table III.

TABLE III: Mechanical Energy Expenditure in a Running Cycle for Each Subject

Subject	Horizontal Velocity (m/sec)	Stride Length (m)	Mechanical Work (joules)	Period of Running Cycle (sec)	Power Expenditure (watts)
#1	6.9870	2.0450	882.9799	.5854	1508.4060
#2	8.2890	2.4260	1376.5730	.5854	2351.6929
#3	6.9580	2.0360	840.7721	.5852	1436.6631
#4	5.9540	1.8880	588.9545	.6342	928.6640
#5	7.6550	2.4270	1097.0860	.6341	1730.1593
#6	6.0770	2.0500	656.9621	.6747	973.7460
#7	6.5080	1.8250	639.9881	.5608	1141.1075

In order to determine the optimal stride length for each subject at his particular running velocity, a range of possible stride lengths was assumed for each one on the grounds of his experimental results. The assumed stride lengths were varied by 42 cm between the shortest assumed stride length and the longest assumed stride length. Computations of the mechanical work done by the total applied moments system at the lower extremity were made for each assumed stride length which was varied by 1 cm throughout the whole range. The kinematic parameters of each assumed stride length were changed on the basis of the hypothesis that: 1) The periods of the support phase and the air-borne phase in a running cycle were such that they were proportional to those of the experimental data. 2) The ratio of the stride length to the stride time was always equal, that is, the average horizontal velocity was equal to its raw data. 3) The angular coordinates of the pelvic rotation at the point where the leg takes-off and where the opposite leg takes-off were such that they were proportional to the raw data. 4) The angular coorinates of the hip joint at the point where the leg takes-off, touches-down, and where its extremity of backward swing (the maximum extension) and forward swing (the maximum flexion) were also such that they were proportional to those of the raw data. 5) The angular coorinates of the knee joint at the point where the maximum flexion during the recovery phase and the support phase were such that they were inversely proportional to those of the raw data.

Corresponding to each assumed stride length, while the changes of the kinematic data were determined, the governing equations of the angular parameters for each joint of the leg were constructed by means of the method of Langrange multipliers. The mechanical work done by the total applied moments system at left leg were computed. The ratio of the mechanical work done to the time duration of its corresponding running cycle was determined for each assumed stride length. The optimal stride length for each subject was then ascertained on the basis of the minimum ratio of the mechanical energy expenditure. Typical examples of the results pertaining to the variations of the mechanical system at the leg and the ratio of mechanical energy corresponding to each assumed stride length for each subject were plotted graphically for subject #2 and #3 in Figures 4 and 5.

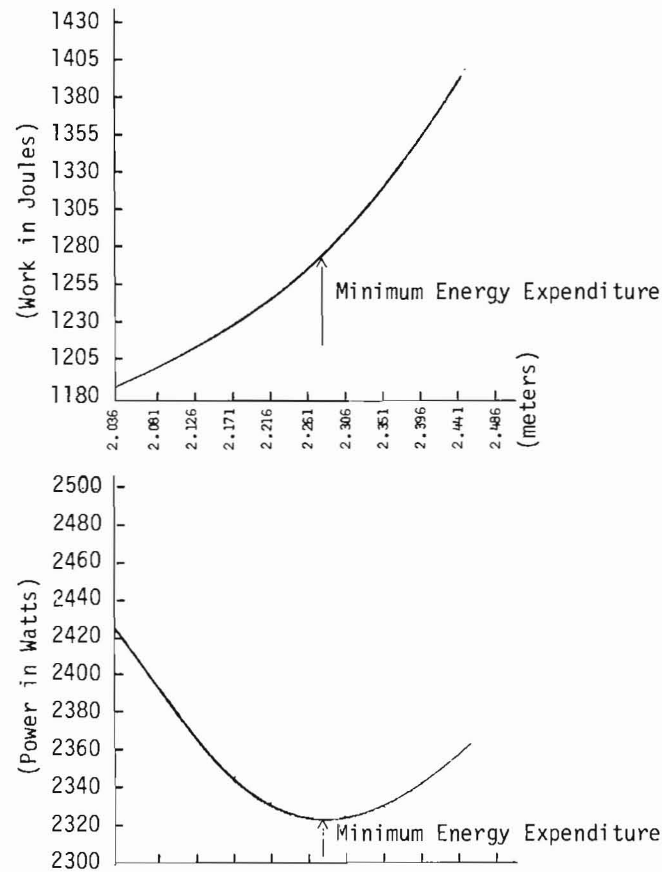


Figure 4: The Variations of the Amount of Mechanical Work and Energy Expenditure in Different Stride Length for Subject #2 with V = 8.289 m/sec.

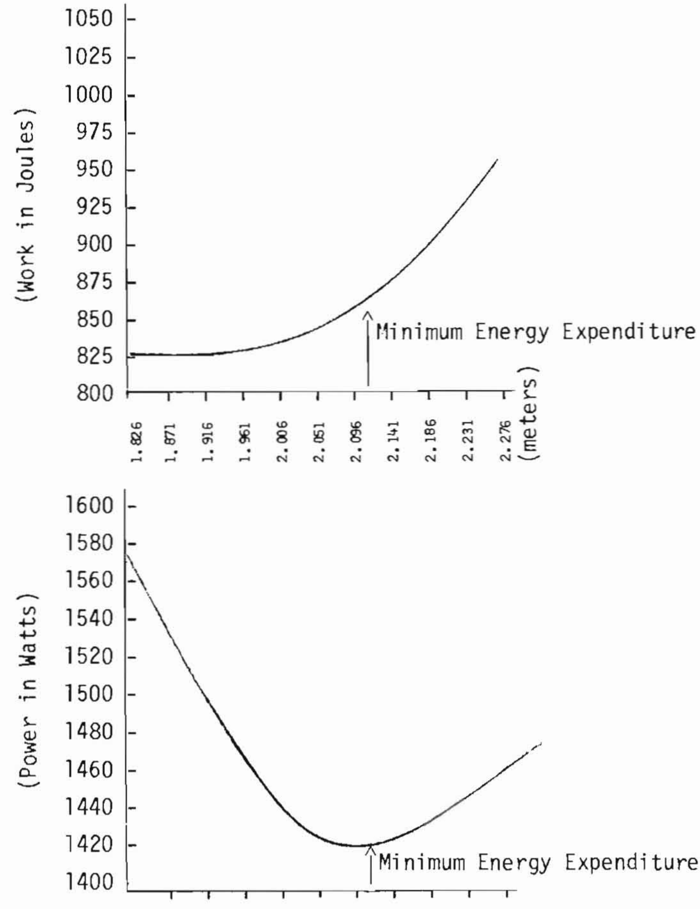


Figure 5: The Variations of the Amount of Mechanical Work and Energy Expenditur in Different Stride Lengths for Subject #3 with V = 6.958 m/sec.

The numerical results of the optimal stride length for each subject were then illustrated in Table IV.

TABLE IV: The Experimental Stride Length and the Optimal Stride Length for Each Subject

Subject	Average Velocity (m/sec)	Experimental Stride Length (m)	Optimal Stride Length (m)	Length of Thigh and Shank (m)
#1	6.9870	2.0450	2.1500	0.9454
#2	8.2890	2.4260	2.2850	0.9500
#3	6.9580	2.0360	2.1050	0.8806
#4	5.9540	1.8880	2.0700	0.8862
#5	7.6550	2.4270	2.2950	0.9495
#6	6.0770	2.0500	2.0250	0.9104
#7	6.5080	1.8250	1.8500	0.8574

DISCUSSION

The mechanical work done by the total applied moments system reveal that in general, shortening the length of the stride would result in lessening the cost of energy for each subject at a given running velocity. Increasing the length of stride would increase energy expense in a somewhat non-linear relationship. However, while checking the variations in the ratio of mechanical energy expenditure to the time interval of a running cycle against its corresponding assumed stride length, it is apparent that the minimum ratio does not necessarily occur at the shortest assumed stride length. On the contrary, the ratios of energy expenditure for shorter assumed stride lengths were mostly higher than those for longer assumed stride lengths among most of the subjects. These results indicate that by shortening the length of the stride thereby increasing the rate of stride inorder to run at a given velocity is always more inefficient than slightly elongating the length of stride. This finding lends support to the concept of Dillman's description (1975) that the majority of present research findings indicate that the "better or more skilled" runners tend to have a greater length of stride than the "poor or less skilled" runners at a given velocity.

The prime objective of this study was to analyze the experimental data and evaluate the mechanical properties of the lower expremity when subjected to run a full cycle of motion. Then, the optimal stride length for each individual runner at a particular running velocity was ascertained by means of theoretical procedures. Four out of seven runners in this study needed to elongate their stride length slightly. While, on the contrary, the other three subjects required a shortening of their stride length to achieve their optimal performance.

REFERENCES

Abdelnour, T.A.; Passerello, C.E. and Houston, R.L. "An Analytical Analysis of Walking", Office of Naval Research Technical Report, 1976.

Beckett, R. and Chang, K. "An Evaluation of the Kinematics of Gait by Minimum Energy", Journal of Biomechanics, Vol: 1, pp 147-159, 1968.

Cappozzo, A.; Leo, T. and Pedotti, A. "A General Computing Method for the Analysis of Human Locomotion", Journal of Biomechanics, Vol: 8, pp 307-320, 1975.

Cappozzo, A.; Figura, F. and Marchetti, M. "The Interplay of Muscular and External Forces in Human Ambulation", Journal of Biomechanics, Vol: 9, pp 35-42, 1976.

Chao, E.Y. and Rim, K. "Application of Optimization Principles in Determining the Applied Moments in Human Leg Joints During Gait", Journal of Biomechanics, Vol: 6, pp 497-510, 1973.

Chow, C.K. and Jacobson, D.H. "Studies on Human Locomotion Via Optimal Programming", Mathematical Bioscience, 10, pp 239-306, 1971.

Chow, C.K. and Jacobson, D.H. "Further Studies of Human Locomotion; Postural Stability and Control", Mathematical Bioscience, 15, pp 93-108, 1972.

Clauser, C.E., McConville, J.T. and Young, J.W. "Weight, Volume, and Centers of Mass of Segments of the Human Body", AMRL Technical Report, pp 69-70, Wright-Patterson Air Force Base, Ohio, 1969.

Dempster, W.T. "Space Requirements of the Seated Operator", WADC Technical Report, pp 55-159, Wright-Patterson Air Force Base, Ohio, 1955.

Dillman, C.J. "Kinematic Analysis of Running", Exercise and Sport Science Review, Vol: 3, pp 193-218, 1975.

Gurfinkel, V.S.; Fomin, S.V., and Shtilkind, T.I. "Determination of the Joint Moments on Locomotion", Biophysics, 16(2), pp 404-407, 1970.

Hatze, H. "Optimization of Human Motions", Biomechanics III, pp 138-142, 1973.

Hatze, H. "Biomechanical Aspects of a Successful Motion Optimization", Biomechanics V-B, pp 5-12, 1976.

Hatze, H. "A Complete Set of Control Equations for the Human Musculo-Skeletal System", Journal of Biomechanics, Vol: 10, pp 799-805, 1977.

Huston, R.L. and Passerello, C.E. "On the Dynamics of a Human Body Model", Journal of Biomechanics, Vol: 4, pp 369-378, 1971.

Huston, R.L.; Passerello, C.E.; Hessel, R.E. and Harlow, M.W. "On Human Body Dynamics", Annals of Biomedical Engineering, 4, pp 25-43, 1976.

Jorgensen, T. Jr. "On the Dynamics of the Swing of a Golf Club", American J. of Physics, 38(5), pp 644-651, 1970.

Kane, T.R. and Scher, M.P. "A Dynamical Explanation of the Falling Cat Phenomenon", Int. J. Solid Structures, Vol: 5, pp 663-670, 1969.

Kane, T.R. and Scher, M.P. "Human Self-Rotation by Means of Limb Movements", J. of Biomechanics, Vol: 3, pp 39-49, 1970.

Kane, T.R. "Inertia Parameters Required for the Dynamic Analysis of Human Motions", Mechanics and Sports, The American Society of Mechanical Engineering, pp 1-8, 1973.

Passerello, C.E. and Huston, R.L. "Human Attitude Control", J. of Biomechanics, Vol: 4, pp 95-102, 1971.

Smith, P.G. and Kane, T.R. "The Reorientation of a Human Body in Free Fall", Technical Report, 171, Stanford University, California, 1967.

Smith, P.G. and Kane, T.R. "On the Dynamics of the Human Body in Free Fall", J. of Applied Mechanics, pp 167-168, March 1968.

Zatziorsky, V.M. and Aleshinsky, S.Y. "Simulation of Human Locomotion in Space", Biomechanics V-B, pp 387-394, 1976