

A MATHEMATICAL SIMULATION TO STUDY PIKE TURN CHARACTERISTICS IN FRONT CRAWL SWIM

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Swim turns represent an integral factor in determining the final outcome of a swimmer race. The aim of present study was to provide a comprehensive mathematical modelling for achieving kinematic parameters in freestyle flip (pike) turn. In proposed model all attempts have been applied to find out what swimmers should do in order that the turns are accomplished in shorter time. The mathematical model has been adopted to pike turn but can also be adopted to tuck turn with minor change in calculations. Theoretical considerations suggest that faster upper limbs rotation could lead to a torso pressure gradient, which would induce significant axial flow along the upper limbs toward the torso. Our results demonstrate the reality of the predicted rotational of body during front crawl swim pike turn. We hypothesize that in pike turn the body can be considered and simulated as two thin hinged prisms.

KEYWORDS: mathematical modelling, pike turn characteristics, front crawl, simulation

INTRODUCTION: Despite the importance of turns in the overall performance for competitive swimming, relatively few studies have been carried out. This is probably because there are no simple, accurate and versatile investigatory methods available. This study examined a model in which swimmer is considered as two thin prisms of equal length and hinged together (hip joint), their width equals the swimmer's shoulder to shoulder distance, and their thickness equals the anterior-posterior chest breadth and furthermore, the upper and lower segments are considered equal. This study also examined the reliability of the proposed model during tumbling and the consistency of temporal and kinematic aspects of swimmers' turning performance.

Execution of a tumble turn requires a swimmer complete a series of complex movements to allow them to change direction. Descriptions of tumble turn technique and performance are found to vary slightly within the literature. Costill et al. (1992) described the process of performing a flip or tumble turn using five separate movement phases. These five turn phases are the approach; the turn; the push-off; the glide; and the pull-out. Maintaining swim speed is considered an important component of the approach to the turn.

According to Costill et al. (1992), the turn phase incorporates the somersault change of direction movement. To achieve this, the swimmer keeps the opposite arm in the water at the hip when beginning the final arm stroke. Forward rotation of the body is initiated by flexion of the head and a simultaneous small dolphin kick, during the final arm stroke. The legs are drawn to the chest by flexing the hips and knees. This movement causes a decrease in the moment of inertia around the axis of rotation, allowing the swimmer to somersault more easily.

MATHEMATICAL MODELLING: In tumbling, the swimmer begins the turn with a straight drop of the head and water rushes over his back aiding the tumbling which has been simulated by two hinged prisms. Then the legs are kept straight until the feet are removed from the water. The bend of the torso is deep and hands reach and pull up. The angle between upper and lower limbs is (α). In fact in pike turn, the body assumes only one major bend at the hip throughout most of the turn. The simulated shape is presented in Figure1. In the followings the swimmer centre of gravity, moment of inertia, and rotational velocity in pike turn are calculated.

1- *Determination of Moment of Inertia*: Swimmer rotation is about Z axis and to achieve this we must calculate first upper limb moment of inertia relative to its CM

$$I'_{CM} = \frac{M}{24} (a^2 + b^2) \quad (1)$$

According to parallel axes theorem, the moment of inertia relative to Z axis is:

$$I'_{Z} = I'_{CM} + \frac{M}{2} Y^2 = \frac{M}{24} (a^2 + b^2) + \frac{M}{2} Y^2 \quad (2)$$

The moment of inertia of the whole system relative to Z axis is then twice the I'_{Z} and therefore we have

$$I_Z = I_{CM} = 2 I'_{Z} = M \left[\frac{(a^2 + b^2)}{12} + Y^2 \right] \quad (3)$$

2- *Determination of Y^2* : If we define "P" as a line connecting upper segment CM and the mid point of the bottom of the same segment:

$$P = \frac{1}{2} \sqrt{a^2 + b^2} \quad (4)$$

Then the relation between y and P can be defined as

$$Y = P \sin \gamma \quad (5)$$

Where γ is an angle made by P with X axis,

$$\gamma = \alpha/2 + \tan^{-1}(b/a) \quad (6)$$

Therefore:

$$Y^2 = (a^2/4 + b^2/4) \sin^2(\alpha/2 + \tan^{-1}(b/a)) \quad (7)$$

Finally the moment of inertia of the whole system can be established as:

$$I = M \left\{ \frac{(a^2 + b^2)}{12} + \left(\frac{a^2}{4} + \frac{b^2}{4} \right) \sin^2(\alpha/2 + \tan^{-1}(b/a)) \right\} \quad (8)$$

3- *Determination of the Torque applied to the system*: Considering pressure drag force we have:

$$D_P = F_P = \frac{1}{2} \rho_w C_D A_{\perp} V^2 \quad (9)$$

ρ_w is the water density and A_{\perp} is the surface normal to the flow, therefore:

$$A_{\perp} = A(\hat{n} \cdot \hat{\theta}) \quad (10)$$

$\hat{\theta}$ is unit vector in the direction of force and \hat{n} is unit vector of the cross-section and is defined as

$$\begin{aligned} \hat{n} &= \cos \alpha/2 \mathbf{i}' + \sin \alpha/2 \mathbf{j}' \\ &= (\cos \alpha/2 \cos \theta - \sin \alpha/2 \sin \theta) \mathbf{i} + (\cos \alpha/2 \sin \theta + \sin \alpha/2 \cos \theta) \mathbf{j} \end{aligned} \quad (11)$$

And also it is obvious that we can have

$$\hat{\theta} = -\sin \theta \mathbf{i} + \cos \theta \mathbf{j} \quad (12)$$

It is also obvious that

$$\hat{n} \cdot \hat{\theta} = \sin \alpha/2 \quad (13)$$

Therefore we can have for the vertical component of cross section; A_{\perp}

$$A_{\perp} = bc \sin \alpha/2 \quad (14)$$

Where c, is the thickness of the segments. The torque applied can be defined as

$$\vec{\tau} = \vec{r} \times \vec{D} \quad (15)$$

On the other hand, for r we can have

$$r^2 = (a \cos \alpha/2 - b/2 \sin \alpha/2 - P \cos \gamma)^2 + (a \sin \alpha/2 + b \cos \alpha/2)^2 \quad (16)$$

Considering the relation between linear and rotational velocities, we will have

$$\tau = r \cdot \rho_w C_D bc \sin \alpha / 2 \dot{\theta}^2 r^2 / 2 \quad (17)$$

Whereas, we can write for the torque

$$\tau = \frac{r^3 \rho C_D bc \sin \alpha / 2}{2} \dot{\theta}^2 \quad (18)$$

According to Lagrange equation we can finally have for rotational movement of the system knowing on the other hand we have:

$$\tau = I \ddot{\theta} \quad (19)$$

$$\text{Therefore:} \quad \ddot{\theta} = \beta \dot{\theta}^2 \quad (20)$$

$$\text{Where } \beta \text{ is defined as:} \quad \beta = \left(\frac{r^3 \rho C_D bc \sin \alpha / 2}{2I} \right) \quad (21)$$

Integrating (20) yields for the rotational velocity:

$$\dot{\theta} = 1 / (1/\varpi_0 - \beta t) \quad (22)$$

And integrating (22) yields the rotational displacement:

$$\Delta\theta = -\text{Ln} (1 - \varpi_0 \beta t) / \beta \quad (23)$$

RESULTS AND DISCUSSION: Swimming turn techniques have been influenced largely by the subjective opinions of coaches due to the lack of objective evidence. Identification of key kinematic, kinetic, and hydrodynamic variables, and the role of these variables in producing a faster turn in order to reach the wall for push-off. The proposed model would provide a better understanding of the turn phase prior to push-off of the freestyle pike turn. The results of this study can be used in conjunction with the results of other literature on the freestyle turn to optimize total turn performance.

Little has been written regarding turn mechanics prior to wall push-off during freestyle turns. The results of this theoretical study suggest that it may be advantageous for the swimmers to develop as much body bent as possible during this period of turn, in order to reduce the water drag force and to speed up the feet in reaching the wall. A fast body rotation before the turn or simply the body shape of the swimmer might create larger stern waves that would lead to an increase in rotational velocity. A more bent body position throughout turning is like to reduce drag force and speed up the reaching the wall.

To maximize the swimmer's turn rotational velocity, all of the variables in the I , β , and $\dot{\theta}$ equations should be examined together, rather than focussing on each variable individually. These variables are related to the radius of the body bent and the angle made by two segments (upper and lower limbs), while Θ is related to α . The results suggest that the rotational displacement is logarithmic while the resulted velocity is inversely depending on time. As a typical example if the swimmer is of 65Kg of weight, 1.8m of height and at a speed of 2.2 m/s prior to turning then we can get, 15.25 for I_Z and 6.65 for I_{CM} , 0.8m/s for the speed with which the swimmer approaches the wall, 1.97 /s of rotational velocity with which the swimmer rotates while approaching the wall. In Figure 1, the parameters such as Y , I_Z , r , β , and ϖ versus α are depicted and their dependence to each other are revealed

CONCLUSION: A theoretical simulation for pike turn has been developed and presented. The results suggest that it may be advantageous for the swimmers to develop as much bent as possible during the period prior to push-off. The moment inertia of the swimmer depends on the angle made by upper and lower limbs. This model can be regarded as a basic one and the mathematical procedure can be regarded as first step to be developed by other modelling involvers.

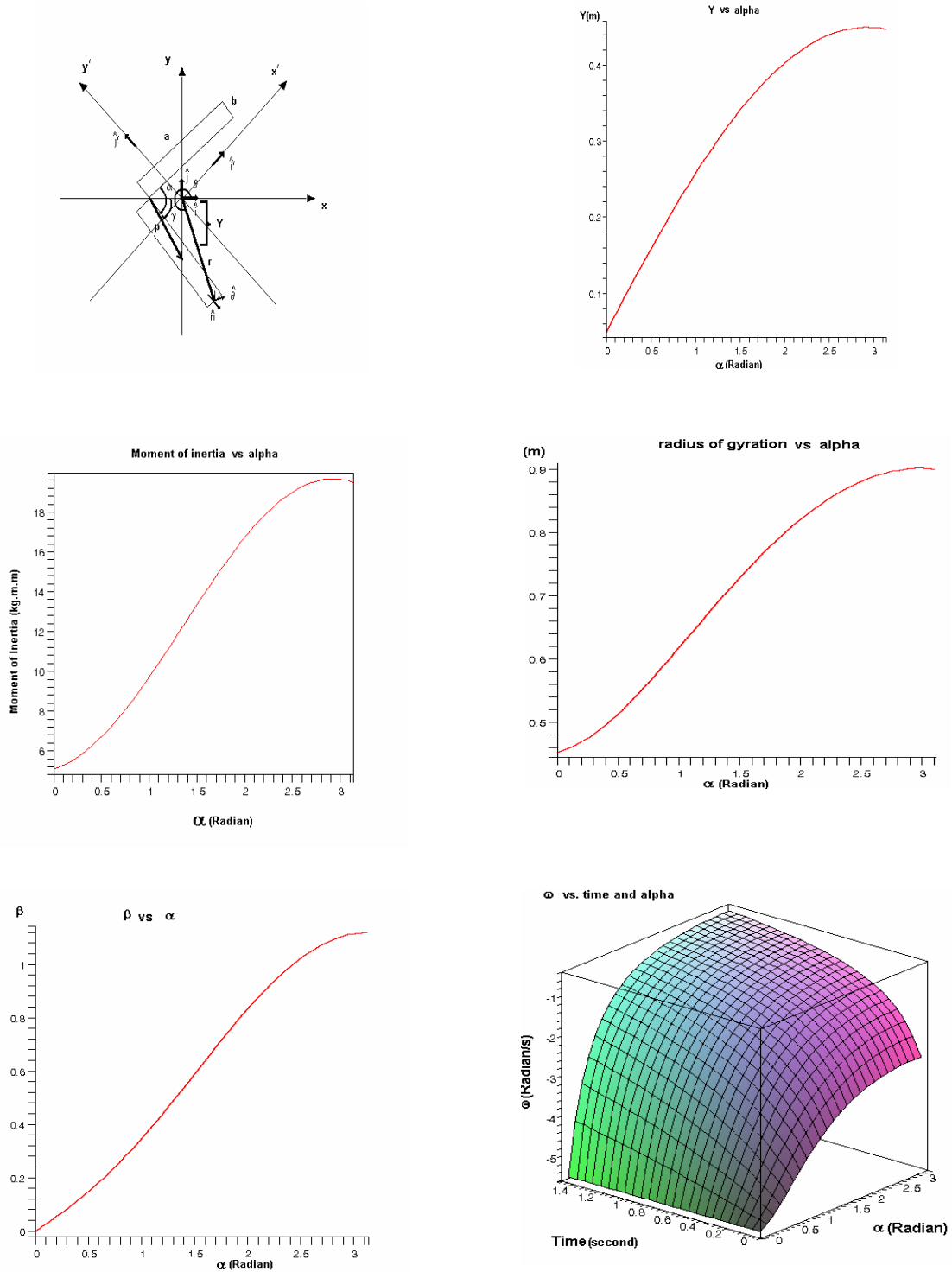


Figure 1- The two segment hinged model through which the variation of moment of inertia and other parameters involved in the model versus α are depicted.

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