Relation between the Branch-to-node Incidence and the Triangular Matrices in Radial Distribution Networks

The power flow solution is a classical problem in electrical engineering that has been studied for more than 60 years [1]. One of the most widely used methods corresponds to the Newton-Raphson approach, which is currently employed for analyzing power systems with meshed configurations and multiple generation sources, i.e., it is typically employed for power systems in high-voltage levels [2]. In the case of medium- and low-voltage levels, the power flow analysis is made with graph-based approaches that consider the radial topology of the network to proposed derivative-free solution methods [3], [4]. Those graph-based approaches reduce the processing times required in the power flow solution because they do not use recursive matrix inversions during the iterative solution process, which is not the case of the derivative-based power flow methods [5].

Two of the most known power flow solution methodologies for radial distribution networks in medium-voltage levels are the backward/forward power flow [4] and the triangular-based power flow approach [3]. Both methods are based on the tree structure that represents radial distribution networks. The main characteristic of both methods is that they are formulated with the usage of two matrices, which are the branch-to-node $(\mathbf{A}_d)$ and the triangular $(\mathbf{T})$ matrices. However, both solution methods are mathematically different. The power flow method based on the triangular matrix is only applicable to strictly radial electrical distribution networks. Still, it is possible to find an analytical relation between both matrices, as demonstrated in this editorial note.

Theorem 1. *The branch-to-node incidence matrix* $\mathbf{A}$ *and the upper-triangular matrix* $\mathbf{T}$ *are related through a negative inverse, including a transposed operation, i.e.,*

$$\mathbf{T} = -\left[\mathbf{A}_d^{\top}\right]^{-1}, \quad (1)$$

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Proof. To demonstrate Equation (1), let us recur to the distribution grid presented in Figure 1. Note that J_b means the current flow through the line b, and I_k corresponds to the net demanded current at node k.

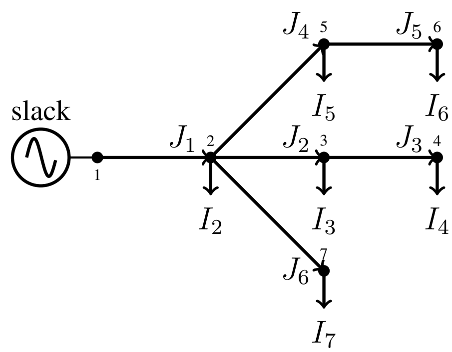

Figure 1: Single-line diagram equivalent for a radial distribution network with 7 nodes

It is worth mentioning that the branch and nodal currents can be related using a matricial operation as presented in Equation (2):

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} \leftrightarrow \mathbb{J} = \mathbf{T}\mathbb{I}, \tag{2}$$

where $\mathbb{J}$ is the vector that contains all the branch currents, and $\mathbb{I}$ contains all the demanded currents, respectively.

Now, if the first Kirchhoff's law is applied to each node of the network, except to the substation node, i.e., node 1, as illustrated in Figure 1, the relation between nodal injected currents and branch currents is reached as presented in (3).

$$\begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} = - \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} \leftrightarrow \mathbb{I} = \mathbf{A}_d^{\top}\mathbb{J} \tag{3}$$

For more details regarding the construction of the branch-to-node incidence matrix refer to [6].

Now to demonstrate that Equation (1) meets, then, if we substitute (3) in (2), then, we have (4):

$$\mathbb{J} = \mathbf{T}\mathbf{A}_d^{\top}\mathbb{J}, \tag{4}$$

which clearly shows that

$$\mathbf{T}\mathbf{A}_d^{\top} = \mathbf{1}_{l\times l}, \tag{5}$$

where l is the number of distribution lines in the distribution grid under analysis, i.e., for Figure 1 $l = 6$. In addition, if the operation in (5) is made for matrices $\mathbf{T}$ and $\mathbf{A}_d^\top$ in (2) and (3), then, the following result is reached.

$$-\begin{bmatrix}1&1&1&1&1&1\\0&1&1&0&0&0\\0&0&1&0&0&0\\0&0&0&1&1&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}-1&1&0&1&0&1\\0&-1&1&0&0&0\\0&0&-1&0&0&0\\0&0&0&-1&1&0\\0&0&0&0&-1&0\\0&0&0&0&0&-1\end{bmatrix}=-\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}, \quad (6)$$

which confirms that

$$\mathbf{T} = -\left[\mathbf{A}_d^\top\right]^{-1},$$

and it completes the proof. □

Remark 1. *The result in Equation (1) is particularly important for power flow studies since it demonstrates that for strictly radial distribution grids, the branch-to-node incidence matrix* $\mathbf{A}_d$ *and the upper-triangular matrix* $\mathbf{T}$ *are related. This is, it will be possible to demonstrate that the upper-triangular power flow method is indeed a particular case of the classical and well-known backward/forward power flow method [7].*

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References

[1] O. D. Montoya, A. Molina-Cabrera, and J. C. Hernández, "A comparative study on power flow methods applied to AC distribution networks with single-phase representation," *Electronics*, vol. 10, no. 21, p. 2573, oct 2021. 1

[2] G. Li, X.-P. Zhang, and X.-F. Wang, "Accelerated newton-raphson power flow," *European Transactions on Electrical Power*, vol. 22, no. 4, pp. 504–517, mar 2011. 1

[3] A. Marini, S. Mortazavi, L. Piegari, and M.-S. Ghazizadeh, "An efficient graph-based power flow algorithm for electrical distribution systems with a comprehensive modeling of distributed generations," *Electric Power Systems Research*, vol. 170, pp. 229–243, may 2019. 1

[4] T. Shen, Y. Li, and J. Xiang, "A graph-based power flow method for balanced distribution systems," *Energies*, vol. 11, no. 3, p. 511, feb 2018. 1

[5] A. Garcés-Ruiz, "Power Flow in Unbalanced Three-Phase PowerDistribution Networks Using Matlab: Theory,analysis, and quasi-dynamic simulation," *Ingeniería*, vol. 27, no. 3, p. e19252, aug 2022. 1

[6] M. C. Herrera-Briñez, O. D. Montoya, L. Alvarado-Barrios, and H. R. Chamorro, "The Equivalence between Successive Approximations and Matricial Load Flow Formulations," *Applied Sciences*, vol. 11, no. 7, p. 2905, mar 2021. 2

[7] S. Ouali and A. Cherkaoui, "An Improved Backward/Forward Sweep Power Flow Method Based on a New Network Information Organization for Radial Distribution Systems," *Journal of Electrical and Computer Engineering*, vol. 2020, pp. 1–11, jan 2020. 3