# GUARD-FUNCTION-CONSTRAINT-BASED <br> REFINEMENT METHOD TO GENERATE DYNAMIC BEHAVIORS OF WORKFLOW NET WITH TABLE 

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#### Abstract

In order to model complex workflow systems with databases, and detect their data-flow errors such as data inconsistency, we defined Workflow Net with Table model (WFT-net) in our previous work. We used a Petri net to describe control flows and data flows of a workflow system, and labeled some abstract table operation statements on transitions so as to simulate database operations. Meanwhile, we proposed a data refinement method to construct the state reachability graph


[^0]of WFT-nets, and used it to verify some properties. However, this data refinement method has a defect, i.e., it does not consider the constraint relation between guard functions, and its state reachability graph possibly has some pseudo states. In order to overcome these problems, we propose a new data refinement method that considers some constraint relations, which can guarantee the correctness of our state reachability graph. What is more, we develop the related algorithms and tool. We also illustrate the usefulness and effectiveness of our method through some examples.

Keywords: WFT-net, state reachability graph, data refinement, pseudo states, Petri net

Mathematics Subject Classification 2010: 68-Q60

## 1 INTRODUCTION

Due to the complex business logics and a large number of data operations, workflow systems have become increasingly complicated. Thus, it increases the difficulty of verifying the correctness of workflow models. In the design stage of a workflow system, whether the bottom layer that implements specific activity operations, or the upper layer that abstracts its process model into a summary framework, all of their correctness and effectiveness are needed to be guaranteed.

As is well known, the correctness and effectiveness of a process model depends on both control flows and data flows [1]. The control flows record the behavioral profile relations between activities (e.g., strict order relation, exclusiveness relation, and interleaving order relation, etc.) [2]. The data flows reflect the correlation between data items, data operations and guards [3]. If there are unreasonable data operations in the execution of some activities, data-flow errors may occur [4, 5]. In fact, data flows and control flows are unified to detect abnormal data errors, which can strengthen the analysis ability of business process management [6, 7, 8]. A good modeling method contributes to analyzing a workflow system. Petri net, as a good formalization language [9, 10, can greatly describe concurrency and synchronization relations. Currently, it has been widely used in modeling and analyzing of concurrent or distributed systems [11, 12, 13]. In general, the reachability graph of a Petri net is used to detect anomalies $\ddagger$ Especially, the guard-driven reachability graph of a workflow net with data (WFDnet). It can avoid pseudo states and alleviate the state space explosion problem [14, 15].

[^1]Trčka et al. [16, 17, 18 proposed WFD-nets to model workflow systems, and detected their data-flow errors by anti-patterns. Furthermore, data footprint was introduced in [19]. As a directed graph representing data flows, it was abstracted from the state reachability graph of a WFD-net. In order to use a model checker to refine the specifications between states, Smith and Derrick [20] improved state symbols so as to avoid blocking in a process model. Ge et al. [21] and Gardiner and Morgen [22] adopted a task refinement method, which used refinement rules and mutual transformations between predicates to analyse the reachability graph of a Petri net, which can avoid state space explosion. Using action optimization has a good effect on processing causal ambiguity systems [23]. When a complex workflow model deals with massive concurrent data operations (e.g., read, write, delete), it is prone to data-flow errors. In order to improve the accuracy of a workflow model, Sidorova et al. [24] added read/write/delete labelling functions to transitions, and they proposed a new data refinement method to analyze false negative/positive activities in a WFD-net.

Although WFD-nets can describe abstract data operations in business processes [25, 26], the actual workflow systems usually cannot work without background databases. Naturally, some data-flow errors related to table operations or logical defects cannot be reflected in a WFD-net. Given this problem, we proposed Workflow Net with Table (WFT-net) [27]. WFT-net uses a WFD-net to model control flows and data flows of workflow systems, and utilizes data statements related to tables to describe database operations. That is, each transition of a WFT-net is marked by the statement of table operations so as to establish the connection between business logics and databases [27]. In our previous work, a data refinement method was given to generate the state reachability graph of WFT-net, which can describe all possible running information of a workflow system. However, this refinement method has a drawback. That is, when guard functions operate on the same data item, pseudo states may be produced in the state reachability graph (c.f. the motivation example in Section 22, since the refinement method does not consider the constraint relation between guard functions. In order to overcome this problem, a new refinement method is proposed based on guard function constraints in this paper. That is, when different guard functions assign values to the same data item, there is a constraint relationship between them, and their expression of guard function constraints is generated. According to this constraint expression, some states satisfying expression are selected. Furthermore, a guard-driven state reachability graph is constructed.

The rest of this paper is organized as follows. Section 2 presents some basic notations. Section 3 gives an example of motivation. Section 4 formalizes WFTCnet (Workflow Net with Table and Constraints) and its firing rules. Moreover, the principle of data refinement and an algorithm for generating the state reachability graph of a WFTC-net are proposed. Section 5 conducts a case study to illustrate the effectiveness of our method. Section 6 develops our tool and does a group of experiments. Section 7 concludes this paper.

## 2 BASIC NOTATIONS

Definition 1 (Petri net [28, 29, 30, 31]). A net is a triple $N=(P, T, F)$, where $P$ is a finite set of places, $T$ is a finite set of transitions, $F \subseteq(P \times T) \cup(T \times P)$ is a flow relation, and $P \cap T=\emptyset \wedge P \cup T \neq \emptyset$. A marking of a net is a mapping function $M: P \rightarrow \mathbb{N}$, where $\mathbb{N}=\{0,1,2, \ldots\}$ is the set of non-negative integers. In other word, $M(p)$ is the number of tokens in the place $p$. A net $N$ with an initial marking $M_{0}$ is a Petri net and denoted as $P N=\left(N, M_{0}\right)$. Note that our marking is represented by a multi-set. For instance, $M=\left[p_{0}+2 p_{2}\right]$ is a marking, where $M\left(p_{0}\right)=1$ and $M\left(p_{2}\right)=2$. For each node $x \in P \cup T$, its preset is denoted by $\bullet x=\{y \mid y \in S \cup T \wedge(y, x) \in F\}$. Similarly, its postset is denoted by $x^{\bullet}=$ $\{y \mid y \in S \cup T \wedge(x, y) \in F\}$.

Given a Petri net $P N=\left(N, M_{0}\right)$ and one marking $M$, a transition $t$ is enabled at $M$, denoted as $M[t\rangle$, if $\forall p \in{ }^{\bullet} t: M(p) \geq 1$. A new marking $M^{\prime}$ is generated from marking $M$ by firing the transition $t$, which is denoted as $M[t\rangle M^{\prime}$, where for $\forall p \in P$ :

$$
M^{\prime}(p)= \begin{cases}M(p)-1, & \text { if } p \in \bullet t-t^{\bullet} \\ M(p)+1, & \text { if } p \in t^{\bullet}-\bullet t \\ M(p), & \text { otherwise }\end{cases}
$$

Definition 2 (Workflow net [25, 32]). A net $N=(P, T, F)$ is a workflow net (WFnet) if:

1. $N$ has two special places, i.e., one source place start and one sink place end in $P$ such that ${ }^{\bullet}$ start $=\emptyset$ and end $=\emptyset$; and
2. $\forall x \in P \cup T:($ start,$x) \in F^{*}$ and $(x$, end $) \in F^{*}$, where $F^{*}$ is the reflexive-transitive closure of $F$.

Definition 3 (Table). A table $R=\left\{r_{1}, r_{2}, \ldots, r_{n}\right\}$ is a set of finite records. Each record $r_{i}=\left\{d_{1}, d_{2}, \ldots, d_{k}\right\}$ represents the values of $k$ attributes, where $d_{k}$ represents the value of the $k^{\text {th }}$ attribute value [33].

For example, a table Student is shown in Figure 1b), which contains two records $r_{1}=\{$ name $1, i d 1$, grad 1$\}$ and $r_{2}=\{$ name $2, i d 2$, grad 2$\}$.

Definition 4 (Workflow Net with Table [27]). A Workflow Net with Table (WFTnet) is a 14-tuples $N=(P, T, F, G, D, R, r d, w t, d t$, sel, ins, del, upd, guard) where

1. $(P, T, F)$ is a WF-net;
2. $G$ is a set of guard functions;
3. $D$ is a finite set of data items;
4. $R=\left\{r_{1}, r_{2}, \ldots, r_{k}\right\}$ is an initial table consisting of $k$ records;
5. $r d: T \rightarrow 2^{D}$ is the labeling function of reading data;
6. $w t: T \rightarrow 2^{D}$ is the labeling function of writing data;
7. $d t: T \rightarrow 2^{D}$ is the labeling function of deleting data;
8. sel : $T \rightarrow 2^{R}$ represents the labeling function of selecting operation in the table $R$;
9. ins : $T \rightarrow 2^{R}$ represents the labeling function of inserting operation in the table $R$;
10. del : $T \rightarrow 2^{R}$ represents the labeling function of deleting operation in the table $R$;
11. upd : $T \rightarrow 2^{R}$ represents the labeling function of updating operation in the table $R$; and
12. guard : $T \rightarrow G_{\Pi}$ is the assigning function of guard functions. $G_{\Pi}$ is a set of guard functions, each of which is a Boolean expression over a set of predicates $\Pi=\left\{\pi_{1}, \pi_{2}, \ldots, \pi_{n}\right\}$, where $\pi_{i}$ is a predicate defined on $D$ or $R$.

In a WFT-net, a guard function is a Boolean expression of some data items especially in tables. It is a formal representation of data conditions related to table operations. $\operatorname{Var}(G)$ represents the variables in the guard function $G$.

Figure 1 is a simple business process of student performance evaluation. Figure 1 a) describes the basic business logics, data operations, database operations, and guard functions assigned to transitions. Figure 1 b) gives an initial table.

1. $D=\{$ name, grad $\}$ is a set of data items, where name represents a student's name and grad means the student's grade, which can be regarded as intermediate variables for a user to operate a database;
2. Student $=\left\{r_{1}, r_{2}\right\}$ is a table, where each record is composed of three attributes, i.e., Sname, Sid and Sgrad, which represent a student's name, id and grade, respectively;
3. $\Pi=\left\{\pi_{1}, \pi_{2}, \pi_{3}\right\}$ is a set of predicates, where $G_{\Pi}=\left\{\pi_{1}, \pi_{2}, \pi_{3}, \neg \pi_{1} \wedge \neg \pi_{2} \wedge \neg \pi_{3}\right\}$ and $\operatorname{Var}\left(\pi_{1}\right)=\operatorname{Var}\left(\pi_{2}\right)=\operatorname{Var}\left(\pi_{3}\right)=\operatorname{Var}\left(\neg \pi_{1} \wedge \neg \pi_{2} \wedge \neg \pi_{3}\right)=\{\operatorname{grad}\}$; and
4. $w t\left(t_{0}\right)=n a m e, \operatorname{sel}\left(t_{0}\right)=n a m e$.

## 3 MOTIVATION

In order to describe all running behaviors of a WFT-net, it is necessary to construct its states and their transition relations. Since the generating states and firing rules of WFT-nets are both related to the data operations in a table, data needs to be further refined. In our previous work, a data refinement algorithm was given in [27]. As shown in Algorithm 1, $B_{d}$ is the value range of data item $d, R_{d}$ is a set of all table data items associated with $d$, and $\Pi_{d}$ is a set of all predicate expressions associated with $d$. For a WFT-net, if a transition $t$ has a write operation on $d$, then $d$ needs to be refined. If $t$ can be fired at a state ${ }^{2}$ then the refinement method can calculate $V_{d}$, i.e., a refinement set of $d$. In Algorithm 1, if $n_{R_{d}}$ and $n_{\Pi_{d}}$ are respectively the cardinality of $R_{d}$ and $\prod_{d}$, we can get its time complexity is $O\left(n_{R_{d}}+n_{\Pi_{d}}\right)$.

[^2]
a)


The data refinement method in Algorithm 1 still has a shortcoming, i.e., it may produce pseudo states in some cases. If a read/write operation on a transition is associated with a data item in $k$ guard functions, then the transition needs to consider all possible assignment values of guard functions, i.e., it will generate $2^{k}$ states. If there are multiple guard functions, it is easy to cause a rapid growth of states, and result in the state space explosion problem. As shown in Figure 1a), grad is written at $t_{1}$, and predicates $\pi_{1}, \pi_{2}$ and $\pi_{3}$ are associated with grad. After firing transition $t_{1}$, it will produce $2^{3}=8$ reachability states (i.e., $C_{2}-C_{9}$ ). All values of the guard functions are described by the truth table in Figure 1 c , Due to the mutual constraint relation between guard functions, the guard function $\pi_{1}$ satisfies the condition, while $\pi_{2}$ and $\pi_{3}$ do not so. Since Algorithm 1 does not consider this constraint relation, pseudo states are generated after firing $t_{1}$. Figure 1d) is the state reachability graph of the WFT-net in Figure 1 a), where a pseudo state is


Figure 1. a) A WFT-net; b) an initial table Student; c) as distribution table of guard function values without constraints; d) a state reachability graph

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Algorithm 1 Generating a refinement set \(V_{d}\) of data item \(d\)
Input: A WFT-net \(N, d, B_{d}, c\);
Output: A refinement set \(V_{d}\);
    if \(R d=\emptyset \wedge \Pi_{d}=\emptyset\) then
    Select an arbitrary \(d_{0}\) from \(B_{d}\), and add \(d_{0}\) into \(V_{d}\);
    else
        if \(R_{d} \neq \emptyset\) then
            for each \(R_{i} \in R_{d}\) do
            Add all stord values of \(d\) in \(R_{i}\) at \(c\) into \(V_{d}\);
            end for
            Select an arbitrary \(d_{0}\) from \(R_{d} \backslash V_{d}\), and add \(d_{0}\) into \(V_{d}\);
        end if
        if \(\Pi_{d} \neq \emptyset\) then
            for each \(\pi_{i} \in \Pi_{d}\) do
                    if \(\pi_{i}\) is defined then
                    if any data item in \(V_{d}\) cannot make \(\pi_{i}\) true then
                        Select a \(d_{i}\) from \(B_{d}\) that make \(\pi_{i}\) true and then add \(d_{i}\) into \(V_{d}\);
                    end if
                    if any data item in \(V_{d}\) cannot make \(\pi_{i}\) be false then
                        Select a \(d_{i}{ }^{\prime}\) from \(B_{d}\) that makes \(\pi_{i}\) false and then add \(d_{i}{ }^{\prime}\) into \(V_{d}\);
                    end if
            end if
        end for
        end if
    end if
    return \(V_{d}\).
```

represented by a dashed box. In order to solve this problem, this paper proposes WFTC-net and a new data refinement method.

## 4 A NEW REFINEMENT METHOD AND REACHABILITY GRAPH GENERATION ALGORITHM

In order to solve the pseudo state problem, WFTC-net is defined and its constraint relations between guard functions are considered. Meanwhile, a new refinement method is proposed to generate an accurate state reachability graph.

### 4.1 Workflow Net with Table and Constraints

By adding constraint relations between guard functions into WFT-net, Workflow Net with Table and Constraints (WFTC-net) is formalized.

Definition 5 (Workflow Net with Table and Constraints). A Workflow Net with

Table and Constraints (WFTC-net) is a 15 -tuples $N=\left(N^{\prime}\right.$, res $)$ where

1. $N^{\prime}$ is a WFT-net; and
2. Given a set of predicates $\Pi=\left\{\pi_{1}, \pi_{2}, \ldots, \pi_{n}\right\}$, its elements are combined by $\wedge$, $\checkmark$ and $\neg$ to form a proposition formula $\omega$. Some formulas are combined by $\vee$ to form a constraint, which is assigned to the values of true $(\mathbf{T})$ or false $(\mathbf{F})$. res is a set of such constraints.

For the example of Figure 1a), there is a set of predicates $\Pi=\left\{\pi_{1}, \pi_{2}, \pi_{n}\right\}$. Since $\pi_{1}, \pi_{2}$ and $\pi_{3}$ are both write operations on the same data item grad, there exist mutual constraint relations between them. Thus, we can define three proposition formulas: $\omega_{1}=\pi_{1} \wedge \neg \pi_{2} \wedge \neg \pi_{3}, \omega_{2}=\neg \pi_{1} \wedge \pi_{2} \wedge \neg \pi_{3}$, $\omega_{3}=\neg \pi_{1} \wedge \neg \pi_{2} \wedge \pi_{3}$, and $\omega_{4}=\neg \pi_{1} \wedge \neg \pi_{2} \wedge \neg \pi_{3}$. At the same time, we can calculate a constraint true $\models$ $\omega_{1} \vee \omega_{2} \vee \omega_{3} \vee \omega_{4}$ and a constraint set res $=\left\{\omega_{1} \vee \omega_{2} \vee \omega_{3} \vee \omega_{4}\right\}$.

Definition 6 (State). Give a WFTC-net $N=\left(N^{\prime}\right.$, res $)$, a four-tuples $c=\left\langle M, \theta_{D}\right.$, $\left.\vartheta_{R}, \sigma\right\rangle$ is called a state of $N$, where

1. $M$ is a marking of $N$;
2. $\theta_{D}: D \rightarrow\{\perp, \top\}$ is the value of the data items in the current state. When a data item is read or written, it indicates that the value of the data item is defined and is represented by the symbol $T$. Otherwise, its value is undefined and is represented by the symbol $\perp$;
3. $\vartheta_{R}: R \rightarrow\{\perp, \top\}$ is the value of the current table, which reflects the situation of records in a table. When a data item is read or written in the table, it means that the value of the data item is defined and is represented by $\top$. Otherwise, its value is undefined and is represented by $\perp$. Each data item of $R$ associated with $N$ is stored in a two-dimensional table, and each tuple information in table $R$ corresponds to a data item $d$;
4. $\sigma: \Pi \rightarrow\{$ true, false, $\perp\}$ represents the assignment state of each predicate. Since each predicate is associated with some data items, when its relevant data items is written to a specific value, it is assigned to true ( $\mathbf{T}$ ) or false ( $\mathbf{F}$ ). Otherwise, its value is still undefined $(\perp)$.

For example, the initial state of the WFT-net in Figure 1 is $c_{0}=\langle$ start, $\{$ name $=$ $\perp, i d=\perp$, grad $=\perp\},\{($ name $1, i d 1$, grad 1$),($ name $2, i d 2$, grad 2$)\},\left\{\pi_{1}=\perp, \pi_{2}=\right.$ $\left.\left.\perp, \pi_{3}=\perp\right\}\right\rangle$. At this time, $t_{0}$ is enabled at $c_{0}$, and the token will move from start to $p_{1}$ after firing $t_{0}$. At the transition $t_{0}$, only the data item name is written, so it is defined, while the data items id, grad are still undefined. Performing a selection operation on the data item name at $t_{0}$ will not change the value of the data items in this table. Therefore, the records $\{($ name $1, i d 1$, grad1 $),(n a m e 2, i d 2, \operatorname{grad} 2)\}$ in the table remain unchanged. Since guard functions are not bound at $t_{0}, \pi_{1}, \pi_{2}$ and $\pi_{3}$ are still undefined. Thus, firing $t_{0}$ generates a new state, i.e., $c_{1}=\langle$ start, $\{$ name $=\top, i d=$ $\perp$, grad $=\perp\},\{($ name $1, i d 1$, grad 1$),($ name $2, i d 2$, grad 2$)\},\left\{\pi_{1}=\perp, \pi_{2}=\perp, \pi_{3}=\right.$ $\perp\}\rangle$.

In order to facilitate the understanding of WFTC-net, the conceptual framework of a WFTC-net is presented in Figure 2 The bottom layer uses a WFD-net to describe the control flows and data flows in a process model, and the upper layer uses a table to represent the database, which realizes the operations of data items in this table. After then a constraint set res is added into a WFT-net, and forms a WFTC-net. By marking some operation statements of tables on transitions, it can reflect some data-flow errors in a workflow system.


Figure 2. WFTC-Net conceptual framework

### 4.2 A Data Refinement Method Based on Guard Function Constraints

Based on guard function constraints, a new data refinement method proposed, as shown in Algorithm 2 . We first use a truth table to enumerate all possible assignments of guard functions. After then, we utilize the constraints between guard functions to construct a set of expressions, and find out reasonable states in the truth table.

For a WFTC-net $N$, if a data item $d$ is written at a transition $t$, then this data item needs to be refined. According to Algorithm2, Rvd is a set of data items refined at the state $c, R$ is a table associated with $N, \operatorname{Sat}$ (guard) is a set of guard functions in $N, \operatorname{Sat}(R)$ is a set of data items in $R, N_{D}$ is a set of data items in $N, N_{R}$ is a set of data items associated with $R$ in $N, \operatorname{guard}(t)$ is a guard function on transition $t$, and res is a set of contraints. Algorithm 2 gives a detailed data refinement method.

First, it chooses a data item $d_{i}$ and initializes Rvd, as shown in step 1. Then Rvd is computed by operating on $d_{i}$ according to different cases, as shown in steps 6-19.

Finally, Rvd and res are computed, as shown in steps 20-22. In Algorithm 2, $n_{N_{d}}$ is the cardinality of $N_{d}$. Then the time complexity of Algorithm 2 is $O\left(n_{N_{d}}\right)$.

Compared with Algorithms 1 and 2 provides a data refinement method under guard function constraints so that our method can avoid generating pseudo states and alleviate the state space explosion problem.

### 4.3 The State Reachability Graph of WFTC-Net

Based on our data refinement method, we give the firing rules of WFTC-net as follows.

Definition 7 (The firing rule of WFTC-net). Let $N=\left(N^{\prime}\right.$, res $)$ be a WFTC-net. $t \in T$ is enabled at one state $c=\left\langle M, \theta_{D}, \vartheta_{R}, \sigma\right\rangle$ (denoted by $c[t\rangle$ ) if and only if:

1. $\forall t \in T: M[t\rangle$;
2. $\forall d \in(r d(t) \cap D): \theta(d)=\mathrm{T} ; \forall d \in(w t(t) \cap D): \theta(d)=\mathrm{T} ; \forall d \in(d t(t) \cap D):$ $\theta(d)=\perp$;
3. $\forall d \in(R \cap \operatorname{del}(t)): \vartheta(d)=\perp ; \forall d \in(R \cap(\operatorname{sel}(t) \cup \operatorname{ins}(t) \cup u p d(t))): \vartheta(d)=\mathrm{T}$; and
4. $\forall d \in \operatorname{var}(G(t)): \theta(d)=\mathrm{T} \cap \vartheta(d)=\top$ and $\sigma(G(t))=$ true.

After firing the transition $t$, a transition $t$ is enabled at state $c$. A new state $c^{\prime}=\left\langle M^{\prime}, \theta_{D}^{\prime}, \vartheta_{R}^{\prime}, \sigma^{\prime}\right\rangle$ is generated, which is denoted as $c[t\rangle c^{\prime}$ such as:

1. $M[t\rangle M^{\prime}$;
2. $\forall d \in d t(t): \theta^{\prime}(d)=\perp ; \forall d \in(w t(t) \bigcup r d(t)): \theta^{\prime}(d)=\mathrm{T} ; \forall d^{\prime} \in R v d: \theta^{\prime}(d)=d^{\prime} ;$
3. $\forall d \in(w t(t) \backslash d t(t)): \theta^{\prime}(d)=\mathrm{T} ; \forall d \in D \backslash(d t(t) \cup w t(t)): \theta^{\prime}(d)=\theta(d)$;
4. $\forall R^{\prime} \in \operatorname{ins}(t), \forall \operatorname{ins}\left(R^{\prime}\right) \cap R \neq \emptyset: \vartheta^{\prime}\left(R^{\prime}\right)=\vartheta\left(R^{\prime}\right) ; \forall R^{\prime} \in \operatorname{ins}(t), \forall \operatorname{ins}\left(R^{\prime}\right) \cap R=\emptyset$ : $\vartheta^{\prime}\left(R^{\prime}\right)=\vartheta\left(R^{\prime}\right) \cup \operatorname{ins}\left(R^{\prime}\right)$;
5. $\forall R^{\prime} \in \operatorname{del}(t), \forall \operatorname{del}\left(R^{\prime}\right) \subset R: \vartheta^{\prime}\left(R^{\prime}\right)=\vartheta\left(R^{\prime}\right) \backslash \operatorname{del}\left(R^{\prime}\right) ; \forall R^{\prime} \in \operatorname{del}(t), \forall \operatorname{del}\left(R^{\prime}\right) \not \subset$ $R: \vartheta^{\prime}\left(R^{\prime}\right)=\vartheta\left(R^{\prime}\right)$;
6. $\forall R^{\prime} \in \operatorname{upd}(t), \operatorname{upd}\left(R^{\prime}\right) \subset R^{\prime}: \vartheta^{\prime}\left(R^{\prime}\right)=\vartheta\left(R^{\prime}\right) \backslash \operatorname{upd}\left(R^{\prime}\right) \cup \operatorname{upd}\left(R^{\prime}\right)^{\prime} ;$
7. $\forall R^{\prime} \in \operatorname{sel}(t), \forall \operatorname{sel}\left(R^{\prime}\right) \subset R: \vartheta^{\prime}\left(R^{\prime}\right)=\top ; \forall R^{\prime} \in \operatorname{sel}(t), \forall \operatorname{sel}\left(R^{\prime}\right) \not \subset R: \vartheta^{\prime}\left(R^{\prime}\right)=\perp$;
8. $\exists g \in G, d \in(w t(t) \cup r d(t)) \cap(u p d(t) \cup i n s(t) \cup \operatorname{sel}(t)): \sigma^{\prime}(g)=$ true;
9. $\exists g \in G, d \in(d t(t) \cap \operatorname{del}(t)): \sigma^{\prime}(g)=$ false;
10. $\forall g \notin G, d \in\left(\theta_{D} \cup \vartheta_{D}\right): \sigma^{\prime}(g)=\perp$; and
11. $\forall g \in G, d \in \operatorname{var}(r e s): \sigma(g)=$ true.

According to Definition 11, a constraint true $\models\left(\pi_{1} \wedge \neg \pi_{2} \wedge \neg \pi_{3}\right) \vee\left(\neg \pi_{1} \wedge \pi_{2} \wedge\right.$ $\left.\neg \pi_{3}\right) \vee\left(\neg \pi_{1} \wedge \neg \pi_{2} \wedge \pi_{3}\right) \vee\left(\neg \pi_{1} \wedge \neg \pi_{2} \wedge \neg \pi_{3}\right)$ is obtained from Figure 1 a), and its truth table is shown in Figure 3a). The result shows that there are only four reasonable assignments for the three guard functions.

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Algorithm 2 A data refinement method for WFTC-net
Input: A WFTC-net \(N, R=\left\{D_{1}, D_{2}, \ldots, D_{n}\right\}, \operatorname{Sat}(\) guard \()\), \(\operatorname{Sat}(R)\);
Output: Data refinement set Rvd, constraint set res;
    Select data items \(d_{i}\), and initialize \(R v d \leftarrow \perp\);
    if \(d_{i} \in \operatorname{Sat}(\) guard \() \wedge d_{i} \in \operatorname{Sat}(R)\) then
        \(\operatorname{guard}(t) \leftarrow \mathrm{T}\);
        for \(N_{d}\left(d_{i}\right) \in N_{R}\) do
            \(R v d \leftarrow d_{i} ;\)
        end for
    end if
    if \(d_{i} \in \operatorname{Sat}(\) guard \(\left.)\right) \wedge d_{i} \notin \operatorname{Sat}(R)\) then
        \(\operatorname{guard}(t) \leftarrow \perp\);
        if \(d_{i} \in N_{d}\) then
            \(R v d \leftarrow d_{i} ;\)
        else
            \(R v d \leftarrow R v d+d_{i} ;\)
        end if
    end if
    if \(d_{i} \notin \operatorname{Sat}(\) guard \() \wedge d_{i} \in \operatorname{Sat}(R)\) then
        \(\operatorname{guard}(t) \leftarrow \operatorname{guard}\left({ }^{\bullet} t\right)\);
        for \(d_{i} \in N_{R}(j)\) do
            \(R v d \leftarrow d_{i} ;\)
        end for
    end if
    if \(d_{i} \notin \operatorname{Sat}(\) guard \() \wedge d_{i} \notin \operatorname{Sat}(R)\) then
        \(\operatorname{guard}(t) \leftarrow \operatorname{guard}\left({ }^{\bullet} t\right)\)
        if \(d_{i} \in N_{D}\) then
            \(R v d \leftarrow d_{i} ;\)
        else
            \(R v d \leftarrow R v d+d_{i} ;\)
        end if
    end if
    if \(d_{i}=\) null then
        \(\operatorname{guard}(t) \leftarrow \operatorname{guard}(\bullet t), \operatorname{Rvd} \leftarrow R v d ;\)
    end if
    if \(d_{i} \in \operatorname{Sat}\left(\right.\) guard \(\left._{i}\right)\) then
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    end if
    return res, Rvd.
```

Algorithm 3 is developed to generate all reachable states from $c_{0}$. In this algorithm, res and Rvd are the results, where operation $\left(\theta_{D}\right)$ represents operations (i.e., read, write, delete) on data in $N$, operation $\left(\vartheta_{R}\right)$ represents operations on data items in $R$, and $a d d(c)$ represents adding a new state. In this algorithm, steps 5-29 generate a new state $c^{\prime}$. Steps 31-34 skip one transition and look for another new enabled transition since the generated state is repeated. In Algorithm 3, the time complexity is $O(1)$.


Figure 3. a) A distribution table of the guard function values with constraints; b) A state reachability graph of WFTC-net

According to the firing rule of WFTC-net, we propose an algorithm for generating its state reachability graph. Based on the depth-first idea, as shown in Algorithm 4.

According to Algorithm 4, the state reachability graph of Figure 1 a) is generated as shown in Figure 3b). When the data item grad is written at $t_{1}$, since there is a constraint true $\models\left(\pi_{1} \wedge \neg \pi_{2} \wedge \neg \pi_{3}\right) \vee\left(\neg \pi_{1} \wedge \pi_{2} \wedge \neg \pi_{3}\right) \vee\left(\neg \pi_{1} \wedge \neg \pi_{2} \wedge \pi_{3}\right) \vee\left(\neg \pi_{1} \wedge\right.$ $\left.\neg \pi_{2} \wedge \neg \pi_{3}\right)$, some assignments of $\pi_{1}, \pi_{2}$ and $\pi_{3}$ such as $\{(0,1,1),(1,0,1),(1,1,1)\}$ in Figure 1c) do not exist. Therefore, such pseudo states need to be removed. In fact, the state reachability graph generated by Algorithm 4 is more in line with the actual demand. In Algorithm 4, if $n_{t}$ and $n_{c}^{\prime}$ are respectively the cardinality of $t$ and $c^{\prime}$, its time complexity is $O\left(n_{t}+n_{c}^{\prime}\right)$.

## 5 CASE STUDY

This section shows the effetiveness of our method through a case study of private car application for access control in a community. In order to simplify this process, some irrelevant operations are omitted. We first use a WFT-net to model this process, as shown in Figure 4 A user inputs an account $i d$ to $\log$ in the system $\left(t_{0}\right)$. If $\mathrm{s} / \mathrm{he}$ has never registered in this system, $\mathrm{s} /$ he needs to re-apply for registration $\left(t_{2}\right)$. $\mathrm{S} / \mathrm{he}$

```
Algorithm 3 Generate state set of a WFTC-net
Input: \(R\), WFTC-net \(N\), res, Rvd;
Output: sat( \(\left.c^{\prime}\right)\);
    Transition \(t \leftarrow N U L L\), State \(c \leftarrow N U L L\), Hashtable \(h \leftarrow N U L L\);
    The initial state \(c_{0}=\langle\) start \(, \perp, \top, \perp\rangle=\left\langle m, \theta_{D}, \omega_{R}, \sigma\right\rangle ;\)
    if \(t_{i} \in T \wedge m\left[t_{i}\right\rangle \wedge \operatorname{guard}\left(t_{i}\right)=\sigma\left(\pi_{i}\right)\) or \(t_{i} \in T \wedge m\left[t_{i}\right\rangle \wedge \operatorname{guard}\left(t_{i}\right)=\) null then
        \(t \leftarrow t_{i} ;\)
    else
        \(i \leftarrow i+1 ;\)
        if \(t_{i} \in t \wedge m\left[t_{i}\right\rangle m^{\prime}, a d d\left(m^{\prime}\right)!=h\) then
            \(c\left[t_{i}\right\rangle c^{\prime}\); // There is no repeat state
            if \(\forall d \in \operatorname{operation}\left(\theta_{D}\right), \forall d \in R v d\) then
                \(\theta^{\prime}(d) \leftarrow d ;\)
            else
            \(\forall d \in \operatorname{operation}\left(\theta_{D}\right): \theta^{\prime}(d) \leftarrow \perp ;\)
        end if
        if \(\forall R^{\prime} \in \operatorname{operation}\left(\vartheta_{R}\right), \forall \vartheta_{R}[d] \in R v d\) then
            \(\vartheta^{\prime}\left(R^{\prime}\right) \leftarrow \vartheta(R) ;\)
        else
            \(\forall R^{\prime} \notin \operatorname{operation}\left(\vartheta_{R}\right), \vartheta^{\prime}\left(R^{\prime}\right) \leftarrow \perp ;\)
        end if
        if \(\forall \operatorname{guard}(t) \in \operatorname{operation}(\Pi), \operatorname{guard}(t) \in \operatorname{Rvd}\) then
            \(\sigma^{\prime} \leftarrow\) true;
        else
            \(\sigma^{\prime} \leftarrow\) false
        end if
        if guard \(_{1} \in\) res, \(\ldots\), guard \(_{i} \in\) res \(^{\text {then }}\)
            guard \(_{j} \in\) true,\(\left(\right.\) res - guard \(\left._{j}\right) \leftarrow\) false;
        end if
        \(c^{\prime} \leftarrow\left\langle m^{\prime}, \theta^{\prime}(d), \vartheta^{\prime}\left(R^{\prime}\right), \sigma^{\prime}\right\rangle\), then add \(c^{\prime}\) into \(\operatorname{Sat}\left(c^{\prime}\right)\);
        else
            if \(t_{i} \in t \wedge c\left[t_{i}\right\rangle c_{m}, \operatorname{add}\left(c_{m}\right)=h\) then
                \(t_{i} \leftarrow t_{i}+1 ; / /\) There are repeat state
        end if
        end if
    end if
    return \(\operatorname{Sat}\left(c^{\prime}\right)\).
```

```
Algorithm 4 Generate the state reachability graph of a WFTC-net
Input: WFTC-net \(N\), Sat ( \(c^{\prime}\) );
Output: \(R G(N)\);
    Take \(c_{0}\) as the root node of \(\mathrm{RG}(\mathrm{N})\) and mark it as \(\boldsymbol{n e w}\);;
    while there is a node marked new do
        Make the node as \(c\);
    end while
    if there is a directed path from \(c_{0}\) to \(c\) and the marking of a node is \(c\) then
        Change the marking of \(c\) to old, and return to step 2;
    end if
    if \(\forall t \in T: \neg c[t\rangle\) then
        Change the marking of \(c\) to endpoint, and return to step 2 ;
    end if
    for \(\forall t \in T: c[t\rangle\) do
        Calculate Sat( \(c^{\prime}\) ) according to Algorithm 3;
    end for
    if \(\forall t \in T: c[t\rangle\) then
        Calculate \(c^{\prime}\) in \(c[t\rangle c^{\prime}\) according to Definition 11,
    end if
    for \(\forall c^{\prime} \in \operatorname{Sat}\left(c^{\prime}\right)\) do
        if \(c^{\prime}\) already exists in the directed path from \(c_{0}\) then
            Draw a directed arc from \(c\) to \(c^{\prime}\), and mark the side of the arc as \(t\);
        else
            Generate a node \(c^{\prime}\) and mark it as new in RG(N); draw a directed arc from
            \(c\) to \(c^{\prime}\) and mark the side of the arc as \(t\); erase the new label of node \(c\), and
            return to Step 2.
        end if
    end for
```

first enters the registration interface to access this system $\left(t_{5}\right)$ and then submits the materials information related to the driver license $\left(t_{7}\right)$. If the review is passed, $\mathrm{s} / \mathrm{he}$ can continue to modify or update other information $\left(t_{10}\right)$. Otherwise, it is necessary to resubmit the information again $\left(t_{8}\right)$. If $\mathrm{s} /$ he registers successfully, $\mathrm{s} /$ he becomes a registered user. This user can choose to exit this system $\left(t_{3}\right)$ or modify his/her personal information $\left(t_{4}\right)$. If this user needs to modify the license plate number information, $\mathrm{s} / \mathrm{he}$ needs to enter his/her name first, and then update the license plate number information. At the same time, $\mathrm{s} / \mathrm{he}$ needs to submit the relevant copy materials and upload them to the system $\left(t_{11}\right)$. After all the copy materials (copy) are submitted and approved ( $t_{12}$ ), the permission can be obtained. Finally, this user can exit and the process ends $\left(t_{13}\right)$.

The initial state of the WFT-net in Figure 4 is $c_{0}=\langle$ start, $\{i d=\perp, l p n=$ $\perp$, copy $=\perp\},\{(i d 1, l p n 1$, copy 1$),(i d 2, \operatorname{lpn} 2$, copy 2$)\},\left\{\pi_{1}=\perp, \pi_{2}=\perp, \pi_{3}=\perp, \pi_{4}=\right.$ $\left.\left.\perp, \pi_{5}=\perp, \pi_{6}=\perp\right\}\right\rangle$. The transition $t_{0}$ is enabled at $c_{0}$, and the token will move

a)
User

| Id | Lpn | Copy |
| :---: | :---: | :---: |
| id1 | lpn 1 | copy1 |
| id2 | lpn2 | copy 2 |

b)

Figure 4. WFT-net for vehicle management system
from $p_{0}$ to $p_{1}$ after firing $t_{0}$. The data item $i d$ is defined, but $l p n$ and copy are still undefined, so only $i d$ is written at $t_{0}$. Guard functions $\pi_{1}, \pi_{2}$ are both about this write operation. According to Algorithm 1, all assignments of $\pi_{1}$ and $\pi_{2}$ will be included in a truth table after this write operation is performed at $t_{0}$, as shown in Figure 5a). Firing $t_{0}$ generates four new states, i.e., $c_{1}=\left\langle p_{1},\left\{i d_{3}, l p n=\perp\right.\right.$, copy $=$ $\perp\},\{(i d 1$, lpn 1, copy 1$),(i d 2, l p n 2$, copy 2$\left.),(i d 3, \perp, \perp)\},\left\{\pi_{1}, \pi_{2}, \perp, \perp, \perp, \perp\right\}\right\rangle, c_{2}=\left\langle p_{1}\right.$, $\{i d, l p n=\perp$, copy $=\perp\},\{(i d 1, l p n 1$, copy 1$),(i d 2, l p n 2$, copy 2$)\},\left\{\neg \pi_{1}, \pi_{2}, \perp, \perp, \perp\right.$,
$\perp\}\rangle, c_{3}=\left\langle p_{1},\left\{i d_{3}, l p n=\perp\right.\right.$, copy $\left.=\perp\right\},\{(i d 1, l p n 1$, copy 1$),(i d 2, l p n 2$, copy 2$),(i d 3$, $\left.\perp, \perp)\},\left\{\pi_{1}, \neg \pi_{2}, \perp, \perp, \perp, \perp\right\}\right\rangle$, and $c_{4}=\left\langle p_{1},\{i d\right.$, lpn $=\perp$, copy $=\perp\},\{(i d 1$, lpn 1 , copy1), (id2, lpn 2 , copy 2$\left.)\},\left\{\neg \pi_{1}, \neg \pi_{2}, \perp, \perp, \perp, \perp\right\}\right\rangle$. Similarly, all assignments of $\pi_{3}$ and $\pi_{4}$, and $\pi_{5}$ and $\pi_{6}$ are included in their corresponding truth tables, as shown in Figures 5b) and 5c), respectively. Figure 5d) shows data information in an updatad table, and $i d 3$ is an updatad data item. Figure 5 e) shows a state reachability graph of the WFT-net in Figure 4 , and pseudo states are represented by a dotted line. The state information in the state reachability graph is shown in Table 1. where $\theta_{D}$ is the value of $\{i d, l p n$, copy $\}, \vartheta_{R}$ is the records of table $U s e r$, and $\sigma$ is one assignment of guard functions $\left\{\pi_{1}, \pi_{2}, \pi_{3}, \pi_{4}, \pi_{5}, \pi_{6}\right\}$ ( $T=$ true, $F=$ false $)$.

| $C$ | $m$ | $\theta_{D}$ | $\vartheta_{R}$ | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: |
| $C_{0}$ | $p_{0}$ | $\{\perp, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=\perp, \pi_{2}=\perp, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{1}$ | $p_{1}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{2}$ | $p_{1}$ | $\{i d, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{3}$ | $p_{1}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{4}$ | $p_{1}$ | $\{i d, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{5}$ | $p_{3}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, p 4=\perp, p 5=\perp, p 6= \\ & \perp \end{aligned}$ |
| $C_{6}$ | $p_{10}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp \pi_{4}=\perp, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{7}$ | $p_{5}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{8}$ | $p_{7}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \\ & \pi_{6}=F \end{aligned}$ |
| $C_{9}$ | $p_{7}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F \\ & \pi_{6}=T \end{aligned}$ |
| $C_{10}$ | $p_{7}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=F \end{aligned}$ |
| $C_{11}$ | $p_{7}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{12}$ | $p_{6}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{13}$ | $p_{8}$ | \{id3, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{14}$ | $p_{9}$ | \{id3, lpn, copy \} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{15}$ | $p_{10}$ | \{id3, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F \\ & \pi_{6}=T \end{aligned}$ |
| $C_{16}$ | $p_{6}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{17}$ | $p_{8}$ | \{id3, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{18}$ | $p_{9}$ | \{id3, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{19}$ | $p_{10}$ | \{id3, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{20}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{21}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{22}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=F, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |


| $C_{23}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{array}{\|l} \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ \text { copy } 2)\} \end{array}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=F, \pi_{4}=F, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| $C_{24}$ | $p_{10}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=F, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{25}$ | $p_{10}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{26}$ | $p_{4}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{27}$ | $p_{6}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{28}$ | $p_{8}$ | \{id, lpn, copy \} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{29}$ | $p_{9}$ | \{id, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{30}$ | $p_{10}$ | \{id, lpn, copy\} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{31}$ | $p_{4}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{32}$ | $p_{6}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{33}$ | $p_{8}$ | \{id, lpn, copy \} | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{34}$ | $p_{9}$ | \{id, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| ${ }^{\text {C35 }}$ | $p_{10}$ | \{id, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| ${ }^{\text {C36 }}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{37}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=F, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{38}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{39}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=F, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{40}$ | $p_{10}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{41}$ | $p_{4}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{42}$ | $p_{6}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{43}$ | $p_{8}$ | \{id, lpn, copy \} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{44}$ | $p_{9}$ | \{id, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{45}$ | $p_{10}$ | \{id, lpn, copy \} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{46}$ | $p_{10}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=F, \pi_{5}=\perp \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{47}$ | $p_{4}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{48}$ | $p_{6}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{49}$ | $p_{8}$ | \{id, lpn, copy \} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{50}$ | $p_{9}$ | \{id, lpn, copy\} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| ${ }^{5} 51$ | $p_{10}$ | \{id, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| $C_{52}$ | $p_{3}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{53}$ | $p_{10}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(\text { id } 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |


| $C_{54}$ | $p_{5}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \\ & \pi_{6}=\perp \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| $C_{55}$ | $p_{7}$ | $\{i d 3,1 p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=F \end{aligned}$ |
| $C_{56}$ | $p_{7}$ | $\{i d 3, \mathrm{lpn}, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F \\ & \pi_{6}=F \end{aligned}$ |
| $C_{57}$ | $p_{7}$ | $\{i d 3,1 p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F \\ & \pi_{6}=T \end{aligned}$ |
| $C_{58}$ | $p_{6}$ | $\{i d 3,1 p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F \\ & \pi_{6}=T \end{aligned}$ |
| $C_{59}$ | $p_{8}$ | $\{i d 3$, lpn,$\perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lp} n 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F \\ & \pi_{6}=T \end{aligned}$ |
| $C_{60}$ | $p_{9}$ | $\{i d 3,1 p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lp} n 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{61}$ | $p_{10}$ | $\{i d 3$, lpn,$\perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F \\ & \pi_{6}=T \end{aligned}$ |
| $C_{62}$ | $p_{7}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lp} n 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{63}$ | $p_{6}$ | $\{i d 3$, lpn,$\perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{64}$ | $p_{8}$ | \{id3, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{65}$ | $p_{9}$ | \{id3, lpn, copy \} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |
| $C_{66}$ | $p_{10}$ | \{id3, lpn, copy $\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \\ & \pi_{6}=T \end{aligned}$ |

Table 1. Concrete states information in the state reachability graph
The pseudo states cannot exist in actual process model, as shown in Figure 5e). According to Algorithm 3, when $i d$ is written at $t_{0}$, there is a constraint relation between guard functions $\pi_{1}$ and $\pi_{2}$. Given two proposition formulas $\omega_{1}=\left(\neg \pi_{1} \wedge \pi_{2}\right)$ and $\omega_{2}=\left(\pi_{1} \wedge \neg \pi_{2}\right)$, the constraint true $\models \omega_{1} \vee \omega_{2}$ is calculated by Definition 2, and its truth table is shown in Figure 6 a). Then only transitions that satisfy true can be enabled. When $t_{0}$ is enabled at $c_{0}$, Algorithm 1 generates four states $c_{1}, c_{2}, c_{3}$ and $c_{4}$. In fact, guard functions in $c_{1}$ and $c_{4}$ do not satisfy the constraint true. Similarly, two constraints true $\models\left(\neg \pi_{3} \wedge \pi_{4}\right) \vee\left(\pi_{3} \wedge \neg \pi_{4}\right)$ and true $\models\left(\neg \pi_{5} \wedge \pi_{6}\right) \vee\left(\pi_{5} \wedge \neg \pi_{6}\right)$ are contructed and their truth tables are shown in Figure 6 b ) and 6 c ), respectively. Then the constraint set res $=\left\{\left(\neg \pi_{1} \wedge \pi_{2}\right) \vee\left(\pi_{1} \wedge \neg \pi_{2}\right),\left(\neg \pi_{3} \wedge \pi_{4}\right) \vee\left(\pi_{3} \wedge \neg \pi_{4}\right),\left(\neg \pi_{5} \wedge\right.\right.$ $\left.\left.\pi_{6}\right) \vee\left(\pi_{5} \wedge \neg \pi_{6}\right)\right\}$ is constructed. Figure 6 d$)$ shows the state reachability graph without pseudo states of the WFTC-net in Figure 4 generated by Algorithm 4 . The state information is recorded in Table 2. The reachability graph without pseudo states is more in line with actual requirements when characterizing the dynamic behavior of this system.

## 6 TOOL AND EXPERIMENTS

Based on our algorithms, we develop a tool to generate the state reachability graph of a WFC-net, which is written in C++ programming language. After inputting a WFTC-net (.txt file) and a table (.txt file), our tool can read them, and generate state reachability graphs. Figure 7 a) describes an abstract file information of the WFT-net in Figure 7 , where 7 b ) shows a constraint set res, 7 c ) represents an initial

| $\pi_{1}$ | $\pi_{2}$ |
| :---: | :---: |
| 0 | 0 |
| 1 | 0 |
| 0 | 1 |
| 1 | 1 |

a)

b)

| $\pi_{5}$ | $\pi_{6}$ |
| :---: | :---: |
| 0 | 0 |
| 1 | 0 |
| 0 | 1 |
| 1 | 1 |

c)
User

| Id | Lpn | Copy |
| :---: | :---: | :---: |
| Id1 | Lpn1 | Copy1 |
| Id 2 | Lpn 2 | Copy2 |
| Id 3 | $\perp$ | $\perp$ |

d)

e)

Figure 5. a), b) and c) are the truth tables of the three sets of guard functions without constraints; d) An updatad table User; e) A state reachability graph of the WFT-net
table user, 7 d ) is a state reachability graph of WFT-net, and 7 e ) shows a state reachability graph of WFTC-net.

The results show that the WFT-net in Figure 4 produces a total of 91 states and 146 state arcs, while WFTC-net produces a total of 63 states and 82 state arcs. The main reason is that WFT-net does not consider the constraint relation between guard functions and generates pseudo states while our method overcomes this problem. In order to further show the effectiveness of our tool, we do the experiments on 10 different examples, and their results are shown in Table 33 All

| C | $m$ | $\theta_{D}$ | $\vartheta_{R}$ | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: |
| $C_{0}$ | $p_{0}$ | $\{\perp, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=\perp, \pi_{2}=\perp, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{1}$ | $p_{1}$ | $\{i d, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{2}$ | $p_{1}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{3}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=F, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{4}$ | $p_{2}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{5}$ | $p_{10}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=T, \pi_{4}=F, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{6}$ | $p_{4}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{7}$ | $p_{6}$ | $\{i d, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{8}$ | $p_{8}$ | \{id, lpn, copy\} | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{9}$ | $p_{9}$ | \{id, lpn, copy\} | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{1} 0$ | $p_{10}$ | \{id, lpn, copy\} | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=F, \pi_{2}=T, \pi_{3}=F, \pi_{4}=T, \pi_{5}=\perp, \pi_{6}= \\ & \perp \end{aligned}$ |
| $C_{11}$ | $p_{3}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \pi_{6}=$ $\perp$ |
| $C_{12}$ | $p_{10}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \pi_{6}=$ $\perp$ |
| $C_{13}$ | $p_{5}$ | $\{i d 3, \perp, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=\perp, \pi_{6}=$ $\perp$ |
| $C_{14}$ | $p_{7}$ | $\{i d 3$, lpn, $\perp$ \} | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=T, \pi_{6}= \\ & F \end{aligned}$ |
| $C_{15}$ | $p_{7}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \pi_{6}= \\ & T \end{aligned}$ |
| $C_{16}$ | $p_{6}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \pi_{6}= \\ & T \end{aligned}$ |
| $C_{17}$ | $p_{8}$ | $\{i d 3, l p n, \perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \pi_{6}= \\ & T \end{aligned}$ |
| $C_{18}$ | $p_{9}$ | $\{i d 3$, lpn,$\perp\}$ | $\begin{aligned} & \{(i d 1, \text { lpn } 1, \text { copy } 1),(i d 2, \text { lpn } 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \pi_{6}= \\ & T \end{aligned}$ |
| $C_{19}$ | $p_{10}$ | $\{i d 3$, lpn,$\perp\}$ | $\begin{aligned} & \{(i d 1, \operatorname{lpn} 1, \text { copy } 1),(i d 2, \operatorname{lpn} 2, \\ & \text { copy } 2),(i d 3, \perp, \perp)\} \end{aligned}$ | $\begin{aligned} & \pi_{1}=T, \pi_{2}=F, \pi_{3}=\perp, \pi_{4}=\perp, \pi_{5}=F, \pi_{6}= \\ & T \end{aligned}$ |

Table 2. Concrete states information in the state reachability graph

| Model | WFT-NET |  |  |  |  |  |  |  | WFTC-RG |  |  | WFT-RG |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. of Guards | No. of Data Operations |  |  | No. of Table Operations |  |  |  | No. of States | No. of Arcs | Time | No. of States | No. of Arcs | Time | No. of Pseudo States |
|  |  | wt | rd | dt | sel | ins | del | upd |  |  |  |  |  |  |  |
| 1 | 2 | 3 | 8 | 1 | 6 | 1 | 1 | 2 | 63 | 82 | 8.348 | 91 | 146 | 9.704 | 28 |
| 2 | 4 | 3 | 8 | 1 | 6 | 1 | 1 | 2 | 66 | 80 | 10.791 | 91 | 140 | 16.732 | 25 |
| 3 | 6 | 3 | 8 | 1 | 7 | 1 | 1 | 1 | 39 | 43 | 8.918 | 86 | 145 | 12.786 | 47 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11 | 14 | 7.46 | 11 | 14 | 8.448 | 0 |
| 5 | 0 | 2 | 7 | 1 | 6 | 1 | 1 | 2 | 66 | 79 | 8.58 | 128 | 155 | 11.49 | 62 |
| 6 | 4 | 3 | 8 | 1 | 7 | 1 | 1 | 2 | 30 | 36 | 8.818 | 178 | 304 | 9.399 | 148 |
| 7 | 6 | 3 | 8 | 0 | 5 | 2 | 1 | 3 | 29 | 34 | 34.866 | 126 | 210 | 38.164 | 97 |
| 8 | 2 | 3 | 9 | 1 | 7 | 1 | 1 | 3 | 34 | 38 | 8.906 | 92 | 118 | 9.683 | 58 |
| 9 | 3 | 2 | 10 | 0 | 6 | 1 | 1 | 2 | 41 | 51 | 8.152 | 99 | 163 | 9.774 | 58 |
| 10 | 4 | 2 | 11 | 0 | 7 | 2 | 1 | 2 | 50 | 56 | 8.688 | 72 | 83 | 9.414 | 16 |

Table 3. The test results

| $\pi_{1}$ | $\pi_{2}$ |
| :---: | :---: |
| 1 | 0 |
| 0 | 1 |

a)

b)

c)

d)

Figure 6. a), b) and c) are the truth tables of the three constraints; d) A state reachability graph of the WFTC-net
the experiments are done on the Intel Core $\operatorname{I5}-8500 \mathrm{CPU}(3.00 \mathrm{GHz})$ and 8.0 GB memory.

The results in Table 3 show that our method can effetively reduce pseudo states when producing the state reachability graph of a WFT-net. When multiple guard functions are operating on the same data item, WFT-net lacks consideration of the constraint relation between guard functions. Thus, it is easy to generate pseudo states in its state reachability graph. WFTC-net considers this constraint relation, and thus pseudo states can be avoided. Naturally, the operating behaviors of the system can be described more accurately. Obviously, the state reachability graph of WFTC-net (WFTC-RG) spends less time in comparison with the corresponding WFT-RG.

Additionally, in order to study the influence of user numbers on the state reachability graph of a WFT-net, we choose the models of group 1 and group 10 in Table 3 to do the experiments. The results are shown in Table 4, where $x / y$ means the model of $x$ users and group $y$.

As shown in the experimental results in Table 4, when the user information in the table increases gradually, the number of states generated in the state reachability graph of WFT-net and WFTC-net also increases. With the increase of data items in the table, the operations on the data in the table will also increase. Thus, the state will gradually increase in the process of generating the state reachability graph. Obviously, WFTC-RG spends less time in comparison with the corresponding WFTRG, which further illustrates the superiority of our method.

|  |  |
| :---: | :---: |
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|  |  |
|  |  |

a）

| 1 | gpnr | goperator | gformer | glatter |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 1 | pi1 | 0 | 0 |
| 3 | $*$ |  |  |  |
| 4 | 1 | pi1 | 0 | 0 |
| 5 | 2 | not | 0 | 1 |
| 6 | $*$ |  |  |  |
| 7 | 1 | pi2 | 0 | 0 |
| 8 | $*$ |  |  |  |
| 9 | 1 | pi2 | 0 | 0 |
| 10 | 2 | not | 0 | 1 |
| 11 | $*$ |  |  |  |
| 12 | 1 | pi3 | 0 | 0 |
| 13 | $*$ |  |  |  |
| 14 | 1 | pi3 | 0 | 0 |
| 15 | 2 | not | 0 | 1 |
| 16 | $@$ |  |  |  |
| 17 | cpnr | coperator | cformer | clatter |
| 18 | 1 | g1 | 0 | 0 |
| 19 | 2 | g2 | 0 | 0 |
| 20 | 3 | not | 0 | 1 |
| 21 | 4 | not | 0 | 2 |
| 22 | 5 | and | 3 | 2 |
| 23 | 6 | and | 1 | 4 |
| 24 | 7 | or | 5 | 6 |
| 25 | $*$ |  |  |  |
| 26 | 1 | g3 | 0 | 0 |
| 27 | 2 | g4 | 0 | 0 |
| 28 | 3 | not | 0 | 1 |
| 29 | 4 | not | 0 | 2 |
| 30 | 5 | and | 3 | 2 |
| 31 | 6 | and | 1 | 4 |
| 32 | 7 | or | 5 | 6 |
| 33 | $*$ |  |  |  |
| 34 | 1 | g5 | 0 | 0 |
| 35 | 2 | g6 | 0 | 0 |
| 36 | 3 | not | 0 | 1 |
| 37 | 4 | not | 0 | 2 |
| 38 | 5 | and | 3 | 2 |
| 39 | 6 | and | 1 | 4 |
| 40 | 7 | or | 5 | 6 |
| 41 | $@$ |  |  |  |
|  |  |  |  |  |

b）

| 1 | User | Id | Lpn | Copy |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 0 | id1 | lpn1 | copy1 |
| 3 | 1 | id2 | lpn2 | copy2 |
| 4 | $@$ |  |  |  |

c）

d）

e）

Figure 7．a）A WFT－net；b）A constraint set res；c）An initial table user；d）A state reachability graph of WFT－net；e）A state reachability graph of WFTC－net

| User/ <br> Model | WFT-NET |  |  |  |  |  |  |  |  | WFTC-RG |  |  | WFT-RG |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. of Guards | No. of Data Operations |  |  | No. of Table Operations |  |  |  |  | No. of States | No. of Arcs | Time | No. of States | No. of Arcs | Time | Pseud <br> eudo States |
|  |  | wt | rd | dt | sel | ins | del | 1 | upd |  |  |  |  |  |  |  |
| 1/1 | 2 |  |  | 1 |  | 6 | 1 |  | 2 | 43 | 52 | 9.037 | 55 | 77 | 12.843 | 34 |
| 2/1 |  |  | 3 |  |  |  |  |  | 63 | 82 | 8.348 | 91 | 146 | 9.704 | 28 |  |
| 3/1 |  |  |  |  |  |  |  |  | 79 | 122 | 9.192 | 123 | 255 | 10.79 | 44 |  |
| 4/1 |  |  |  |  |  |  |  |  | 100 | 173 | 9.295 | 160 | 403 | 10.476 | 60 |  |
| 5/1 |  |  |  |  |  |  |  |  | 120 | 231 | 9.504 | 196 | 582 | 16.533 | 76 |  |
| 6/1 |  |  |  |  |  |  |  |  | 140 | 297 | 11.509 | 232 | 793 | 15.136 | 92 |  |
| 1/10 | 4 | 2 | 11 | 0 | 7 | 2 | 1 |  |  | 2 | 32 | 34 | 9.021 | 45 | 50 | 11.172 | 13 |
| 2/10 |  |  |  |  |  |  |  |  | 50 |  | 56 | 8.688 | 72 | 83 | 9.414 | 16 |
| 3/10 |  |  |  |  |  |  |  |  | 58 |  | 65 | 8.493 | 89 | 101 | 9.919 | 31 |
| 4/10 |  |  |  |  |  |  |  |  | 75 |  | 84 | 9.665 | 115 | 130 | 10.311 | 40 |
| $5 / 10$ |  |  |  |  |  |  |  |  | 92 |  | 103 | 9.211 | 141 | 159 | 9.486 | 49 |
| 6/10 |  |  |  |  |  |  |  |  | 109 |  | 122 | 10.149 | 167 | 188 | 13.945 | 58 |

Table 4. The test results

To further illustrate that our constraints also apply to WFD-net, we do a set of experiments in order to compare WFD-RG and WFDC-RG in terms of state space and construction time. The following benchmarks are used:

- B1 is an example with two threads accessing two shared variables, then it produces concurrency bugs 34.
- B2 is a classic algorithm for solving the mutual exclusion problem in concurrent systems [35].
- B3 is a concurrent program, where multiple concurrent threads manipulate a shared hash table [36].
- B4 is a system-level modeling language that offers a wide range of features to describe concurrent systems at different levels of abstraction [37].
- B5 is a tutorial program to detect and fix data races [38].
- B6 is a sequence of instructions where any branch is at the end and there are shared variables access [39].
- B7 is a simple producer-consumer example. It is a set of interconnected modules communicating through channels using transactions, events and shared variables [40].
- B8 is a test driver for a simplified version of a Bluetooth driver [41].

For each benchmark, we first use WFD-net to model it, and use our tool to obtain their WFD-RG and WFDC-RG, respectively. Each benchmark tested 10 times, and the result of running time is their averages. Table 5 is the results of our experiments. It shows the number of states, the number of arcs, and the construction time of WFD-RG and WFDC-RG for all benchmarks. From this table, we can see that the scale of WFDC-RG is much smaller than WFD-RG. Obviously, it spends less time to produce a WFDC-RG in comparison with the corresponding WFD-RG.

|  | WFD-nets |  |  |  | WFD-RG |  |  | WFDC-RG |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Benchmarks | $\|T\|$ | $\|P\|$ | $\|F\|$ | $\|D\|$ | $\|G\|$ | $\begin{array}{r}\text { Nos. of } \\ \text { States }\end{array}$ | $\begin{array}{r}\text { Nos. of } \\ \text { Arcs }\end{array}$ | $\begin{array}{r}\text { Time } \\ (\mathrm{s})\end{array}$ | $\begin{array}{r}\text { Nos. of } \\ \text { States }\end{array}$ |  | $\begin{array}{c}\text { Nos. of } \\ \text { Arcs }\end{array}$ | Time |$)$

Table 5. The test results

## 7 CONCLUSION

WFT-net is used to simulate a workflow system that operates on data tables, and the state reachability graph is defined to describe all its possible running behaviors. In fact, the data refinement method of WFT-net proposed in [27] has shortcomings due to its lacks of constraint relations between guard functions. Thus, it is easy to generate pseudo states (or illegal states) in the state reachability graph of WFTnet. In this paper, we propose an improved data refinement method. In our guarddriven state reachability graph, the constraint relation between guard functions are considered so that the running behaviors of workflow systems can be expressed more accurately. This paper also proposes some algorithms for generating reachability graphs for WFTC-net. Futhermore, we develop a modeling and analysis tool. The case study and experiments show the usefulness and effectiveness of our methods. In the future, we will consider the application of first-order computation tree logic (CTL) in the state reachability graph of WFTC-net, we want to verify whether there are logic errors in WFTC-net, and develop a new model checking tool based on WFTC-net.

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[^1]:    ${ }^{1}$ In order to distinguish, it is called the reachability graph in Petri net, and the state reachability graph in other nets.

[^2]:    ${ }^{2}$ A state of a WFT-net is usually called as a configuration.

