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Linear and nonlinear models to describe the lactation curve of Girolando cows

Abstract – The objective of this work was to compare the main linear and nonlinear models used to describe lactation curves and to evaluate the nonlinearity of the nonlinear models, in order to obtain the most adequate model to describe the lactation curves of the Girolando breed. Data from 165 lactations of 89 3/4 Holstein + 1/4 Gyr cows were used, and average yield was calculated every 20 days up to 310 days of lactation. Seventeen models of lactation curves available in the literature were compared. The selection of the best model was based on the curvature measures of Bates & Watts, the bias of Box, adjusted coefficient of determination, Akaike's information criterion, and residual standard deviation. The linear model of Cobuci estimated a yield peak of 16.7 kg at 40 days of lactation, whereas the nonlinear model of Wood estimated a yield peak of 16.8 kg at 41 days of lactation and a persistence of 6.82. Nonlinearity measures were the most appropriate for selecting the most suitable nonlinear model for the description of lactation curves. To describe the lactation curves of the Girolando breed, the most suitable linear model is that of Cobuci and the nonlinear model is that of Wood.

Index terms: dairy cattle, milk yield, modeling, regression.

Modelos lineares e não lineares para a descrição da curva de lactação da raça Girolando

Resumo – O objetivo deste trabalho foi comparar os principais modelos lineares e não lineares usados para descrever curvas de lactação e avaliar a não linearidade dos modelos não lineares, para obter o modelo mais adequado para a descrição das curvas de lactação da raça Girolando. Foram utilizados dados de 165 lactações de 89 vacas 3/4 Holandesas + 1/4 Gir, tendo-se calculado a produção média a cada 20 dias até 310 dias de lactação. Foram comparados 17 modelos de curvas de lactação disponíveis na literatura. A seleção do melhor modelo foi feita com base nas medidas de curvatura de Bates & Watts, no vício de Box, no coeficiente de determinação ajustado, no critério de informação de Akaike e no desvio-padrão residual. O modelo linear de Cobuci estimou um pico de produção de 16,7 kg aos 40 dias de lactação, enquanto o modelo não linear de Wood estimou um pico de produção de 16,8 kg aos 41 dias de lactação e persistência de 6,82. As medidas de não linearidade foram as mais adequadas para selecionar o modelo não linear mais adequado para a descrição das curvas de lactação. Para descrever as curvas de lactação da raça Girolando, o modelo linear mais adequado é o de Cobuci e o não linear é o de Wood.

Termos para indexação: gado leiteiro, produção de leite, modelagem, regressão.

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Introduction

Cow milk production is expressive worldwide. According to Food and Agriculture Organization (FAO), in 2020, milk yield was 718.03 million tons, of which 227.85 million tons were produced only in Europe (FAO, 2022). Currently, Brazil is the third largest milk producer, yielding 36.50 million tons in the same year, only behind USA and India (FAO, 2022).

Milk yield can be represented graphically during cow lactation by what is called a lactation curve. Glória et al. (2010) showed that the study of these curves is important because it allows of estimating total yield from partial yields, which makes it possible to carry out early culling and the evaluation of sires based on the incomplete lactations of their daughters. The study of lactation curves also facilitates the estimation of the milk volume needed by calves and aids in the strategic planning of calve nutrition and supplementation, from lactation to finishing for meat production (Henriques et al., 2011). However, the shape of the lactation curve can be modified by environmental factors that may interfere with milk yield, such as herd, calving year, calving season, and cow age at calving (Cobuci et al., 2000).

Lactation curves have been used to model the milk yield of different breeds and species, both through linear and nonlinear regression models. In linear models, estimation is exact, whereas, in nonlinear models, it is obtained through linear approximations. Despite this difference, nonlinear models have an excellent quality of fit and provide parameters with biological interpretation (Maia et al., 2009). However, according to Fernandes et al. (2015), the greater the nonlinearity of a model, the further from linear approximation it will be, which makes inferences about the studied parameters less reliable.

Expressions used to assess the adequacy of linear approximations and their effects on inferences are known as nonlinearity measurements, among which stand out the curvatures of Bates & Watts (1988) and the bias of Box (1971). Fernandes et al. (2015) concluded that measuring the nonlinearity of a nonlinear regression model is fundamental to assess the reliability of parameter estimates; however, these authors did not measure the nonlinearity of nonlinear lactation-curve models.

Regarding dairy cattle, Facó et al. (2002) pointed out that the Holstein x Gyr crossbreed combines the most relevant characteristics between two breeds, i.e., the great ability to adapt to challenging environments of the Gyr breed and the rusticity and high milk yield of Holstein cows, resulting in an animal superior to those of other dairy crossbreeds. Therefore, it is important to compare the lactation curves of these two breeds to improve their crossbreed.

The objective of this work was to compare the main linear and nonlinear models used to describe lactation curves and to evaluate the nonlinearity of the nonlinear models, in order to obtain the most adequate model to describe the lactation curves of the Girolando breed.

Materials and Methods

The used data were obtained from 165 lactations of 89 3/4 Holstein + 1/4 Gyr (Girolando) cows reared in a milk production system on pasture with supplementation. In this herd, the number of lactations varied from one to three or more per cow.

Milk was weighed monthly between November 2012 and February 2018 at the Santa Lúcia farm, located in the municipality of São Simão, in the state of São Paulo, Brazil. A full lactation was considered as four weighings in sequential months, totaling 16 weighings. Therefore, only the weighings of a same cow that reached the total of 16 were used to describe the lactation curves, in order to avoid the loss of information when, for example, cows were sold. The average yield was calculated every 20 days until 310 days of lactation. Seventeen lactation-curve models – named after the authors of each study – were used to analyze the behavior of the lactation curves (Table 1), as adapted from Calvo Cardona et al. (2015).

Each model was partially derived in relation to its parameters and then, according to the result of these derivatives, were classified into linear or nonlinear. To estimate the parameters of the linear and nonlinear models, the 'lm' and 'nls' functions of the R statistical software (R Core Team, 2020) were used, respectively.

To assess the quality of fit of the model, the curvature measures of nonlinearity of Bates & Watts and the bias of Box were obtained using the IPEC package of the R software (R Core Team, 2020).

Nonlinearity measurements are based on the geometric concept of curvature, being decomposed

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into two components: intrinsic nonlinearity and nonlinearity due to the effect of parameters (Bates & Watts, 1988). Zeviani et al. (2012) concluded that intrinsic nonlinearity measures the lack of flatness of the expected surface, whereas parametric nonlinearity measures the nonexistence of uniformity of the surface coordinate system near the neighborhood of the solution's location. The authors highlighted that Box's bias allows of indicating which model parameters lead further away from a linear behavior.

Concomitantly to nonlinearity measures, quality estimators – i.e., adjusted coefficient of determination (R^2_{aj}), Akaike's information criterion (AIC), and residual standard deviation (RSD) – were used, as follows:

$$R^2_{aj} = 1 - \left[\left(1 - R^2\right)\left(n - i\right)/\left(n - p\right)\right];$$

$$AIC = -2 \ln L\left(\hat{\theta}\right) + 2p;$$

$$RSD = \sqrt{QME};$$

where R^2 is the coefficient of determination; n is the number of measurements; p is the number of parameters; i is related to the fit of the curve intercept, being equal to 1 if there is an intercept in the model and equal to 0 if there is no intercept; ln is the natural logarithmic operator; $L(\hat{\theta})$ is the maximum point of the maximum likelihood function of the models; and QME is the residual mean square. The best model was considered the one with the highest R^2 value and the lowest AIC and RSD values.

To verify the assumptions of normality, homogeneity, and independence of the models, the tests of Shapiro-Wilk, Breusch-Pagan, and Durbin-Watson were used, respectively, with their functions in R, at 1% probability. If p-value > 0.01, these tests are nonsignificant, i.e., the residuals present normality and constant variance and are also independent.

Confidence intervals were constructed for the estimates of the models selected as the most adequate to describe the lactation curves of Girolando cows. According to Draper & Smith (1998), when calculating the y-value corresponding to a given x-value in a linear model, the (x_0; y_0) pair is obtained and the confidence interval for y_0 is given by:

$$IC(y_0) = x_0^{'}\hat{\beta}_j \pm t_{(n-p,\alpha/2)}\sqrt{x_0^{'}(XX)^{-1}x_0 QME},$$

where x_0 is the line of the X incidence matrix of the linear model corresponding to the value of the independent variable (days in lactation) for which the Y estimate is being obtained, $\hat{\beta}_j$ is the estimate of the j^{th} parameter, and $t_{(n-p,\alpha/2)}$ is the upper quantile of the t-distribution.

Draper & Smith (1998) added that, in a nonlinear regression model, the approximate confidence interval for the y_0-value estimated for the x_0-value is given by:

Table 1. Seventeen models commonly used in the literature on lactation curves.

Model equation[(1)]	Model name	Reference
$Y = a\exp(-ct) + \varepsilon$	Brody, Ragsdale & Turner	Brody et al. (1923)
$Y = a\exp(-bt) - a\exp(-ct) + \varepsilon$	Brody, Turner & Ragsdale	Brody et al. (1924)
$Y = a\exp(bt - ct^2) + \varepsilon$	Sikka	Sikka (1950)
$Y = t/(a + bt + ct^2) + \varepsilon$	Nelder	Nelder (1966)
$Y = at^{b\exp(-ct) + \varepsilon}$	Wood	Wood (1967)
$Y = a + bt - ct^2 + \varepsilon$	Dave	Dave (1971)
$Y = a - ct + \varepsilon$	Madalena, Martinez & Freitas	Madalena et al. (1979)
$Y = a - bt + c\ln(t) + \varepsilon$	Singh & Gopal	Singh & Gopal (1982)
$Y = a + bt + ct^2 + d\ln(t) + \varepsilon$	Singh & Gopal	Singh & Gopal (1982)
$Y = a + bt + ct^{-1} + \varepsilon$	Bianchini Sobrinho	Bianchini Sobrinho (1984)
$Y = at\exp(-ct) + \varepsilon$	Papajcsik & Bodero	Papajcsik & Bodero (1988)
$Y = a - ct + \ln(t) + \varepsilon$	Cobuci	Cobuci et al. (2000)
$Y = a + bt + c\exp(-dt) + \varepsilon$	Wilmink	Wilmink (1987)
$Y=a(1/1+(b/c+t))\exp(-dt) + \varepsilon$	Rook, France & Dhanoa	Rook et al. (1993)
$Y = a\exp([b(1-\exp(-ct))/c - dt]) + \varepsilon$	Dijkstra	Dijkstra et al. (1997)
$Y = at^{bc}\exp(-ct) + \varepsilon$	Dhanoa	Dhanoa (1981)
$Y = at^{b\exp(-ct)} + \varepsilon$	Cappio-Borlino, Pulina & Rossi	Cappio-Borlino et al. (1995)

[(1)]a, b, c, and d, parameters of the lactation curves.

$$IC(\hat{y}_0) = \hat{y}_0 \pm t_{(n-p,\alpha/2)} \sqrt{g_0^{'}(MM)^{-1} QMEg_0},$$

where g_0 indicates the line of the M matrix of the partial derivatives of the nonlinear model (gradient matrix) corresponding to observation x_0.

For both the linear and nonlinear models, parameter estimation was performed using the least squares method as described in Seber & Wild (1989). In the case of the nonlinear models, before the ordinary least squares method, the Gauss-Newton convergence algorithm or method of linearization, in which a Taylor series expansion is used to approximate the nonlinear regression model to linear terms, was applied.

As in most iterative methods, the first step was to assign initial values to the vector of parameters, searching for estimates obtained for the lactation curves of other breeds in the available literature (Cobuci et al., 2000; Jacopini et al., 2016; Daltro et al., 2018).

Results and Discussion

The models classified as linear, whose partial derivatives did not depend on any parameter, were those of: Dave; Madalena, Martinez & Freitas; Singh & Gopal; Bianchini Sobrinho; and Cobuci. The nonlinear models, with at least one partial derivative dependent on the studied parameters, were those of: Brody, Ragsdale & Turner; Brody, Turner & Ragsdale; Sikka; Nelder; Wood; Papajcsik & Bodero; Wilmink; Rook, France & Dhanoa; Dijkstra; Dhanoa; and Cappio-Borlino, Pulina & Rossi.

The nonlinearity measurements of the nonlinear models were also evaluated as recommended by Fernandes et al. (2015), Diel et al. (2019), and Fernandes et al. (2019). The model of Wood presented the lowest c^{τ} value and that of Brody, Ragsdale & Turner, the lowest c^{θ} value (Table 2). According to Fernandes et al. (2015), lower values of nonlinearity measures indicate a better fit of the model as more reliable estimates are obtained.

The model of Rook, France & Dhanoa and that of Dijkstra presented parametric nonlinearity measures (c^{θ}) with very high values (Table 2), indicating a low reliability of the parameter estimates. The high values obtained can be explained either by the arrangement of the parameters in the model or by the lack of suitability of the model for this data set (Fernandes et al., 2015, 2019), suggesting that a possible reparametrization of those models can be more efficient.

Box's bias was higher for the models with a number of parameters equal to four since the greater the bias of the parameter, the greater the deviation from linearity (Fernandes et al., 2019). Of the models with two parameters, the one that presented the lowest values for Box's bias was that of Brody, Ragsdale & Turner, and, of the models with three parameters, that of Wood. Considering all quality-of-fit criteria, the model of Wood showed better R^2_{aj}, RSD, and AIC values than that of Brody, Ragsdale & Turner.

Therefore, considering the results of all quality-of-fit criteria and also the measurements of the curvature of Bates & Watts and the bias of Box, the best nonlinear model to describe the lactation curve of Girolando cows is that of Wood. In alignment with the present study, Fernandes et al. (2019) highlighted that these two measurements are the most used to evaluate the nonlinearity of a nonlinear regression model. Furthermore, Oliveira et al. (2020) observed that the model of Wood shows the rigor and precision needed for the selection of cows based on their performance as to parametric estimates of lactation quality, such as peak milk yield, average milk yield, and ascending and descending rates.

The R^2_{aj} values were mostly above 0.94 (Table 2), emphasizing the good quality of fit obtained in the present work since R^2 is generally not very high in studies on lactation curves. Pereira et al. (2016), for example, found an R^2_{aj} value equal to 0.86 while using the model of Bianchini Sobrinho to describe the lactation curve of the *Bos taurus* x *Bos indicus* cross. Daltro et al. (2019) observed an R^2_{aj} of 0.57 when adjusting the model of Wilmink for 3/4 Holstein + 1/4 Gyr cows, whereas Pereira et al. (2016) reported an R^2_{aj} of 0.65 for the *B. taurus* x *B. indicus* cross using the same model. Applying the model of Wood to estimate the lactation curve of 3/4 Holstein + 1/4 Gyr cows, Daltro et al. (2019) found an R^2_{aj} of 0.57, whereas Jacopini et al. (2016) verified an R^2_{aj} equal to 0.76 and 0.91 for first- and second-lactation cows, respectively.

The linear model with the highest R^2_{aj} of 0.9597 was that of Cobuci (Table 2), which also presented the lowest RSD and AIC. Being a linear regression model, its nonlinearity measures are equal to zero, as they measure the approximation of the nonlinear model to the linear model. Therefore, the linear model that is most adequate to describe the lactation curves of Girolando cows is that of Cobuci, described as: Y = 14.0342 - 0.0252 × days + In(days).

The model of Madalena, Martinez & Freitas presented a low R^2_{aj} of 0.89 (Table 2); one of the reasons for this is that the expression of the model corresponds to a decreasing straight line, which is not the pattern of a lactation curve. Similarly Torquato et al. (2017) did not observe a peak in the lactation curve of Gyr daughters of the 1/2 Holstein-Gyr breed. Lazzari et al. (2013) also found that the lactation curve for Zebu cows tended not to peak or to peak only in the first few weeks.

The models of Wilmink, of Rook, France & Dhanoa, and of Dijkstra have an R^2_{aj} above 0.95 (Table 2); the first two were the nonlinear models with the highest R^2_{aj}. The lowest RSD and AIC values were found for the models of Wilmink and of Sikka, respectively. Considering only the R^2_{aj}, RSD, and AIC evaluators of quality of fit, the model of Wilmink would be the most adequate to describe the lactation curves of Girolando cows.

Most studies on lactation curves use either R^2, R^2_{aj}, AIC, or Bayesian information criteria (BIC) as criteria to determine a model's quality of fit. Jacopini et al. (2016) and Hossein-Zadeh (2019), for example, used only R^2_{aj} and R^2, respectively, as a criterion, whereas Daltro et al. (2018) and Hossein-Zadeh (2019) applied AIC, BIC, and root mean square error. However, the literature on lactation curves is usually not concerned with the nonlinearity of the studied models, which are selected mistakenly via R^2, i.e., have unreliable parameter estimates. The results of the present work confirm that it is extremely important to consider nonlinearity measures in studies using nonlinear models as reported by Fernandes et al. (2015), Diel et al. (2019), and Fernandes et al. (2019).

Of the evaluated models, that of Wood is also used to describe the lactation curve of other bovine breeds, such as Holstein, Guzerá, Caracu, Holstein x Guzerá, Holstein x Nellore, Holstein x Zebu, Taurino x Zebu, as well as of other species, as buffaloes and goats. The model of Cobuci is used to characterize the lactation curves of the Guzerá and Holstein breeds.

Table 2. Quality-of-fit criteria for the linear and nonlinear models used to describe lactation curves of Girolando (3/4 Holstein + 1/4 Gyr) cows[(1)].

Author	R^2_{aj}	RSD	AIC	C^{τ}	C^{θ}
Linear model					
Dave	0.9451	0.3801	19.1302	-	-
Madalena, Martinez & Freitas	0.8985	0.5168	28.4652	-	-
Singh & Gopal	0.9533	0.3507	16.5557	-	-
Singh & Gopal	0.9509	0.3595	18.0698	-	-
Bianchini Sobrinho	0.9429	0.3878	19.7753	-	-
Cobuci	0.9597	0.3382	14.5801	-	-
Nonlinear model					
Brody, Ragsdale & Turner	0.8715	0.5690	31.2246	0.0075	0.0214
Brody, Turner & Ragsdale	0.9121	0.4737	26.1748	0.0114	0.3407
Sikka	0.9493	0.3666	17.9716	0.0065	0.0324
Nelder	0.9109	0.4698	25.9104	0.0129	0.0497
Wood	0.9479	0.3682	18.1157	0.0064	0.3485
Papajcsik & Bodero	0.7250	4.1490	94.8027	0.0685	0.2636
Wilmink	0.9512	0.3584	17.9722	0.2884	5.9632
Rook, France & Dhanoa	0.9512	0.3585	17.9802	0.2857	71.6451
Dijkstra	0.9510	0.3589	18.0148	0.2333	10.0834
Dhanoa	0.9479	0.3682	18.1157	0.0065	0.6584
Cappio-Borlino, Pulina & Rossi	0.8736	0.5661	31.8791	0.0320	0.6941

[(1)]R^2_{aj}, adjusted coefficient of determination; RSD, residual standard deviation; AIC, Akaike's information criterion; C^{τ}, intrinsic nonlinearity measures; and C^{θ}, parametric nonlinearity measures.

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Lazzari et al. (2013) and Cobuci et al. (2000) compared the adjustments of the models of Cobuci and Wood for the Holstein and Guzerá breeds, respectively. The authors concluded that the model of Wood, with a R^2 of 0.95, was better adjusted to the Holstein breed and that of Cobuci to the Guzerá breed.

As it is a nonlinear model, the parameter estimates of the model of Wood have practical interpretations: milk yield of 13.90 kg at the beginning of lactation (parameter a), a phase of increase in milk yield of 0.07 at the beginning of lactation (parameter b), and a phase of decline in milk yield of 0.001 after its peak (parameter c). For 3/4 Holstein + 1/4 Gyr cows, Daltro et al. (2019) found values of 14.04, 0.25, and 0.03 for parameters a, b, and c, respectively, whereas Jacopini et al. (2016) obtained values of 10.89, 0.16, and 0.003 for the same parameters in the first lactation and of 14.49, 0.12, and 0.003 in the second. For Girolando and Holstein cows in the first lactation, Oliveira et al. (2020) found values of, respectively, 15.5 and 17.4, 0.023 and 0.120, and 0.0021 and 0.0022 for parameters a, b, and c.

Based on the estimates of the model of Wood, according to Glória et al. (2010), it is also possible to estimate lactation peak (p = (a*(b/c)^b)exp(-b)), peak time (d=b/c), and persistence (P=-(b+1)*ln(c)). In the present study, yield peak was 16.8 kg, peak time occurred approximately on day 41, and persistence was 6.82. Jacopini et al. (2016) obtained similar results for Girolando cows of the 3/4 Holstein + 1/4 Gyr genetic group in the first lactation, with a peak of 17.13 kg, approximate peak time on day 49, and persistence of 6.66. Oliveira et al. (2020) found a peak of 17.2 for Holstein cows in the first lactation and of 18.2 for the Jersey breed.

Although linear models do not have parameters with a practical interpretation, peak day and yield can be easily obtained through certain calculations, considering peak milk yield is the maximum peak of the lactation curve. By deriving the model of Cobuci with respect to t, the expression f(t) = (1/t)-c was obtained and, then, by equaling to zero and substituting the estimate of parameter c = 0.0252, it was used to determine peak day, which was approximately on the fortieth day. Moreover, by substituting the respective parameter in the expression of the model of Cobuci, peak yield was 16.7 kg. If the aim is only to determine the quality of fit and the predictive capacity of a model, the linear model, which has more easily estimated parameters and optimal properties, may be sufficient, but it makes it difficult to infer practical parameters of the lactation curve, such as persistence.

The tests of Shapiro-Wilk, Breusch-Pagan, and Durbin-Watson were nonsignificant at 1% probability (p-value >0.01) for almost all lactation curve models, except for that of Papajcsik & Bodero, i.e., all other models showed no violation of residual assumptions, indicating that the residuals presented normality, constant variances, and independence. Therefore, in the model of Papajcsik & Bodero, it is necessary to model and incorporate the residual autocorrelation.

The confidence intervals of the model of Cobuci, as highlighted by Draper & Smith (1998), are exactly obtained, without the use of approximations. In this model, from the first 10 days of lactation to 50 days of lactation, there was a slight increase in milk yield volume, ranging from an average of 16.0824 kg on the tenth day to 16.6831 kg on the fiftieth day. To facilitate the understanding of data, the lower and upper limits of the confidence intervals of 98, 80, and 60% were obtained (Table 3).

Based on the estimates, the approximate confidence interval of the model of Wood showed that, for 98% of the cows, milk yield was between 15.1767 and 16.9208 kg in the first 10 days of lactation, with an estimated average of 16.0487 kg. The same interpretation is valid for the confidence interval limits of 80 and 60% (Table 3). As the model of Wood model is nonlinear and interval estimates are only approximately obtained (Draper & Smith, 1998), its confidence interval limits are greater than those of the model of Cobuci.

Through the graphic representation, it was possible to visualize the behavior of the curve that adjusts milk yield data over the period under analysis (Figure 1). In this sense, the actual yield data are arranged around the average of the estimated values, showing the quality of fit of the model of Cobuci. In addition, the amplitude of the 98% confidence interval is greater for the model of Wood, compared with that of Cobuci, since the former is a nonlinear model and its intervals are obtained in an approximate way as pointed out by Seber & Wild (1989).

Table 3. Estimated mean yield and lower (LI) and upper (LS) limits of the confidence intervals of 60, 80, and 98% for milk yield of Girolando (3/4 Holstein + 1/4 Gyr) cows.

Days in lactation	LI98	LI80	LI60	Mean	LS60	LS80	LS98
Estimated confidence interval for the model of Cobuci							
10	15.7901	15.9335	15.9864	16.0824	16.1783	16.2313	16.3747
30	16.3220	16.4957	16.5598	16.6760	16.7923	16.8564	17.0301
50	16.3282	16.5023	16.5666	16.6831	16.7996	16.8638	17.0379
70	16.1797	16.3447	16.4056	16.5161	16.6265	16.6874	16.8524
90	15.9541	16.1062	16.1623	16.2641	16.3659	16.4220	16.5741
110	15.6803	15.8183	15.8692	15.9616	16.0539	16.1049	16.2429
130	15.3708	15.4957	15.5419	15.6255	15.7091	15.7553	15.8803
150	15.0311	15.1461	15.1886	15.2655	15.3425	15.3849	15.4999
170	14.6631	14.7733	14.8139	14.8877	14.9614	15.0020	15.1122
190	14.2676	14.3796	14.4209	14.4959	14.5708	14.6122	14.7241
210	13.8467	13.9675	14.0121	14.0930	14.1738	14.2184	14.3392
230	13.4044	13.5401	13.5902	13.6810	13.7718	13.8218	13.9575
250	12.9451	13.1003	13.1575	13.2614	13.3652	13.4225	13.5776
270	12.4727	12.6506	12.7163	12.8354	12.9545	13.0202	13.1981
290	11.9900	12.1930	12.2680	12.4039	12.5398	12.6148	12.8179
310	11.4991	11.7289	11.8138	11.9677	12.1215	12.2064	12.4363
Estimated confidence interval for the model of Wood							
10	15.1767	15.6045	15.7624	16.0487	16.3351	16.4930	16.9208
30	16.2789	16.5043	16.5875	16.7384	16.8892	16.9611	17.1978
50	16.3482	16.5474	16.6209	16.7543	16.8876	16.9724	17.1603
70	16.1664	16.3628	16.4354	16.5668	16.6983	16.7708	16.9673
90	15.8958	16.0863	16.1567	16.2842	16.4117	16.4821	16.6726
110	15.5838	15.7633	15.8296	15.9498	16.0700	16.1363	16.3158
130	15.2474	15.4133	15.4745	15.5855	15.6966	15.7578	15.9236
150	14.8933	15.0457	15.1021	15.2041	15.3062	15.3625	15.5150
170	14.5227	14.6653	14.7180	14.8134	14.9089	14.9615	15.1041
190	14.1354	14.2744	14.3257	14.4187	14.5117	14.5630	14.7020
210	13.7320	13.8750	13.9278	14.0235	14.1193	14.1721	14.3151
230	13.3154	13.4700	13.5271	13.6305	13.7340	13.7911	13.9456
250	12.8910	13.0630	13.1264	13.2415	13.3566	13.4201	13.5920
270	12.4639	12.6571	12.7285	12.8578	12.6255	13.0585	13.2517
290	12.0384	12.2553	12.3353	12.4804	12.2717	12.7056	12.9224
310	11.6177	11.8592	11.9484	12.1101	12.6255	12.3609	12.6025

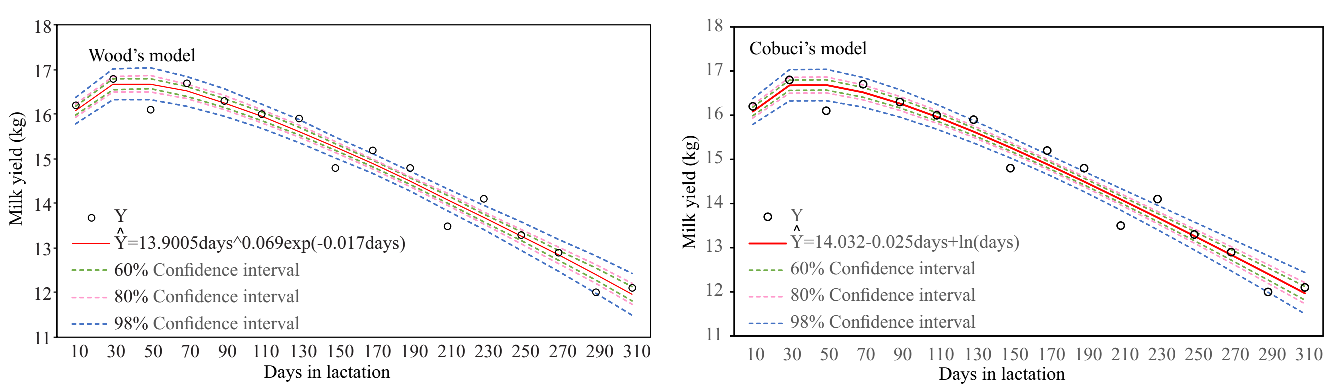

Figure 1. Confidence intervals and mean milk yield of Girolando (3/4 Holstein + 1/4 Gyr) cows estimated by the models of Cobuci and Wood.

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Conclusions

1. To describe the lactation curves of the Girolando breed, the most suitable linear model is that of Cobuci and the nonlinear model is that of Wood.

2. Nonlinearity measures are the most appropriate for selecting the most suitable nonlinear model for the description of the lactation curves of the Girolando breed.

3. The parameters of the nonlinear models allow of estimating the persistence in the lactation curves of the Girolando breed.

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