

**SENSITIVITY ANALYSIS OF WAVE FORCES ON MONOPOD PLATFORM
UNDER METEOCEAN CONDITION BY SIMULATION**

by

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Dissertation submitted in partial fulfillment of
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CERTIFICATION OF APPROVAL

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project. The work is my own except as specified in the references and has not been done by unspecified sources or person.

JACKY TIONG KET CHIING

ABSTRACT

Stokes wave theory for deep water condition is employed in PETRONAS Technical Specification (PTS) for all water condition in Peninsular Malaysia Operation, Sarawak Operation and Sabah Operation. This research aims to address this through predicting wave forces from wave theories using simulation study and analyzing the sensitivity of wave forces to wave theories and deviated wave height criteria. In PTS, for 100-year storm event, the wave height criteria recorded shows fluctuation of 140% from the wave height criteria for operating event. With reference to this, it is feasible to adopt a deviation of $\pm 40\%$ with assumption that the wave height is reduced to 40% less is during the extreme low tidal waves. Wave theories are used in theoretical predictions of wave kinematic using the measured wave height, wave period and water depth. Extensive literature review has been done to better understand the wave forces, water condition and prediction wave kinematics by wave theories. Structural Analysis Computer Software (SACS) v3.5 is used as the main tools in the methodology. The monopod structure is modeled in SACS for PMO, SKO and SBO region. Stokes wave theory is identified as conservative for monopod structure design in PMO zone. For SKO zone, the wave force is recognized as sensitive towards wave theories. Based on analysis result, Cnoidal wave theory is identified as conservative over other applicable wave theories. Samarang in SBO zone is having transitional water condition. Wave force is recognized to be changing sensitively by Cnoidal wave theory. Analysis results reflected that Cnoidal is more conservative due to more critical values of wave forces predicted, higher order wave theory and best describe the transitional water condition.

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1. INTRODUCTION

1.1 Background

In the past ten years, the independents and majors have put a considerable effort into the development of minimum facilities in marginal shallow water. The fact that the oil production potential of the marginal field is low and may not produce enough net income encourages major oil and gas companies to resort to minimum platforms instead of the conventional offshore platforms.

A minimal platform sustains due to standardization of the facilities, which reduce the costs and increases the profit. Secondly, oil and gas companies are looking forward to unmanned, simplified and remote control operated platforms and these are fulfilled by minimal platforms.

Second worldwide survey of minimal facilities conducted by Mustang Engineering and Offshore Magazine identified approximately 150 minimum facilities or minimal platforms around the world. Locally, in Malaysian water, PETRONAS Carigali (PCSB) operates six minimal platforms in Peninsular and Sabah.

Minimal platforms owned by PCSB, specifically known as monopod platforms are designed based on PETRONAS Technical Specification (PTS) by Stokes or Stream wave theory. Reason of adopting these two wave theories is not specified. Wave theories are used in theoretical predictions of wave kinematic using the measured wave height, wave period and water depth. The adequateness of these two wave theories is questionable when shallow water condition applies. Sensitivity of wave forces to the wave theory is a gap that needs to be bridged.

1.2 Problem Statement

PETRONAS Technical Specification (PTS) Standard stated that wave kinematics (such as wave velocities) shall be developed using Stokes fifth order or Stream function. Since Stokes and Stream wave theories are only applicable for deep water condition (depth $>L/2$), therefore, it indicates that the PTS considered PMO, SKO and SBO zone are having deep water condition. In fact, SKO and SBO zone are having transitional water condition. There is a need to conduct sensitivity analysis on how the wave theories influence the computed wave forces, as well as, how the deviation of metocean criteria influence the wave forces.

1.3 Objectives

1.3.1 Objective One

To predict wave forces from wave theories and deviated metocean criteria using Structural Analysis Computer Software (SACS) v3.5.

In SACS v5.3, using the monopod model from OECU (three model for PMO, SBO and SKO), vary parameter as given in PTS (i.e.: wave height) by $\pm 40\%$ and simulating to obtain the corresponding wave forces for Stokes wave theory. The procedure is repeated for other wave theory.

1.3.2 Objective Two

To analyze the sensitivity of wave forces predicted by SACS v3.5 to wave theories

From the result obtained in SACS v5.3, sensitivity graph is plotted for wave theories from objective no. 1, with x-axis of percentage deviation of parameter (i.e.: wave height) and y-axis of wave forces.

1.3.3 Objective Three

To determine the most conservative wave theories for computation of wave forces for each zone

Analyze the sensitivity graph and determine the most conservative wave theory for each offshore site

1.4 Scope of Study

The scope of study are as follows but not limited to:

- Monopod Platform in Peninsular Malaysia Operation, Sarawak Operation and Sabah Operation
- Water condition; i.e. Shallow, Transitional and Deep
- Metocean Parameters
- Wave Height, Wave Theories and Wave Forces
- SACS v3.5
- Sensitivity Analysis

As a fundamental, wave theories, the wave kinematics of each wave theory and the condition to use particular wave theory will be studied. The wave theories involved in the studies are:

- Linear Airy Wave Theory
- Stokes Finite Amplitude Wave Theory
- Cnoidal Wave Theory
- Stream Function Wave Theory

2. LITERATURE REVIEW

2.1 Wave Force

For a monopod platform structure, it is obvious that this free standing single caisson piled to below the mud line does not guarantee a structural integrity in a storm conditions from a physical point of view.

This is evident through the journal by Lee and Liew (2014) named “Structural Sensitivity of Tarpon Monopods in Intermediate Water Depths for Marginal Field Development”. The result of this research revealed that a free standing monopod structure fails in all simulated storm conditions in the study and monopod is a structure whose integrity is highly dependent on its guying system.

However, this does not indicate that in non-storm or operating condition, the free standing single caisson will not experience any structural failure. The ability of free standing monopod in an operating condition to maintain its integrity is also questionable. In order to address this, it is vital to look into the magnitude of force that impacts the monopod structure, which is the wave force.

The total wave force imposed on a platform deck can be calculated with equation as follows (Isaacson and Prasad, 1992):

$$F_{tw} = F_b + F_s + F_d + F_l + F_i$$

where F_{tw} = total wave forces; F_b = buoyancy force (vertical); F_s = slamming force; F_d = drag (velocity dependent) force; F_l = lift force; and F_i = inertia force.

Slamming force is represented as the transfer of momentum from the water to the structure as the wave crest encounters the platform decks. Buoyancy, drag and inertia force are developed as the wave continues to inundate the deck.

Bea, Xu, Stear and Ramos (1999) stated in their published journal, “Wave Forces on Decks of Offshore Platforms”, that the maximum horizontal force developed on the platform deck by wave crest, will be formulated based on the horizontal drag force. In contrast, maximum horizontal force acting on the portions of structure below wave crest is based on the Morison equation.

Since the scope of study is the wave force at the free surface, which is the wave crest force, Morison equation is therefore not applicable. From the total wave force equation above, the drag force component which will be considered in this study, can be expressed in equation as follows (Bea et al, 1999):

$$Fd = 0.5pCdAu^2$$

There are four components to be determined to be able to calculate drag force, where p = mass density of seawater; Cd = drag coefficient; A = vertical deck area subjected to wave crest; and u = horizontal fluid velocity in the wave crest. Mass density of seawater is taken as 1027 kg/m^3 , drag coefficient is taken as 0.65 for clean member and 1.05 for fouled members according to PTS 34.19.10.30 and vertical area subjected to wave crest is taken from calculated deck surface area. The horizontal fluid velocities in the wave crest are determined based on wave theory (Bea et al, 1999).

Consequently, the wave force impacts the monopod structure can then be determined and the magnitude of it should be able to indicate whether the force is significant to cause failure to the monopod structure.

2.2 Water Condition

However, it is required to address selection of suitable wave theory for computation of wave velocities. Bea et al (1999) did mention that Stokes wave theory is employed for calculating wave crest velocities and addressed the reason is because the wave velocities is in deep water and transitional water depth. This is inconsistent with the PTS 34.19.10.30 where Stokes fifth order or Stream function is used for all water condition.

Each wave theory is applicable to particular water condition, either shallow or deep water condition. The distinction between shallow and deep water condition has nothing to do with absolute water depth. It is determined by the ratio of water's depth to the wavelength of the wave, d/L (Boss and Jumars, 2003).

As shown in Figure 1, the change from deep to shallow water waves occurs when the depth of the water, d , becomes less than one half of the wavelength of the wave. When d is much greater than $L/2$, it is a deep water wave or a short wave.

For deepwater waves, the wave's particle circular orbits become insignificant as it goes down to the bottom sea, no wave motion at bottom of the sea. For intermediate water waves, wave particles move in elliptical orbits and width of elliptical orbits reduces as the depth increases. For shallow water depth, the elliptical orbits of wave particles stay the same width down to shallow seabed.

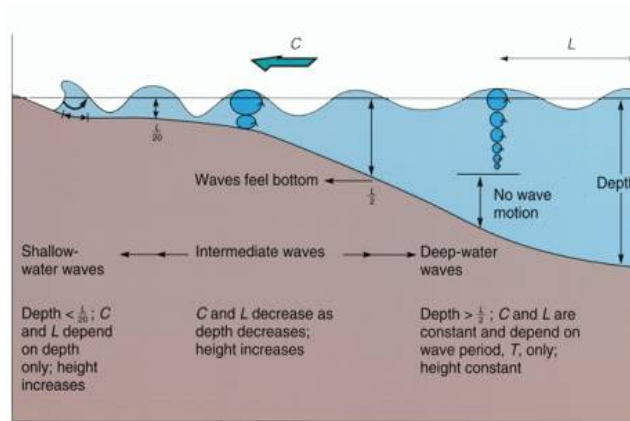


Figure 1 Changes in a wave as the depth shallows

2.3 Wave Theories

There are five wave theories to date and all these wave theories do not provide exact wave kinematics in real life but a close representative of the wave behavior based on assumptions.

Linear Airy wave theory is simplest and most useful of all wave theories because it has small amplitude, which represents most of the wave behavior. It is based on the assumption that the wave height is small compared to the wave length or water depth (Chakrabarti, 1986). This assumption allows the free surface oscillating conditions to be satisfied at the mean water level or deep water, rather than at the oscillating free surface.

Stokes wave theory is a non-linear and periodic surface wave on a fluid layer of constant mean depth. Comparing to Airy wave, Stokes wave's distance from MSL to wave crest and distance from MSL to wave trough are not equal as shown in Figure 2 below.

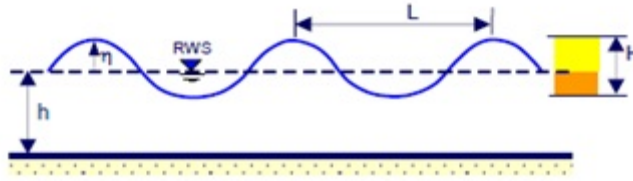


Figure 2 Stokes Wave

These conditions place restrictions on the wave heights in shallow water and thus Stokes theory is not applicable to shallow water (Chakrabarti, 1986). Stokes theory is valid for non-linear waves on intermediate and deep water that is for wavelength not large as compare with mean depth.

Cnoidal wave theory closely described finite amplitude long waves in shallow water. It is a periodic wave that usually has a sharp crests separated by wide troughs as shown in Figure 3 below. This theory accounts for long waves of finite amplitude and it is valid for $d/L < 1/8$. As the wave length becomes infinite, solitary wave theory is approached (Chakrabarti, 1986).

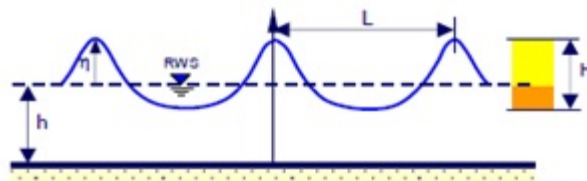


Figure 3 Cnoidal Wave

Chakrabarti (1986) explained that Stream wave theory is a non-linear wave theory related to that of Stokes and it is based on a stream function representation of the flow. Stream wave theory can be divided into two types:

- Regular Stream Function Theory
- Irregular Stream Function Theory

Regular Stream describes symmetric periodic waves while irregular Stream does not have any restrictions placed on the wave form.

Takagi, Ma and Stewart (2009) stated that solitary wave is a wave, which propagates without any temporal evolution in shape or size when viewed in the reference frame

moving with the group velocity of wave. It is a shallow water wave that consists of a single displacement of water above the mean water level as shown in Figure 4 below.

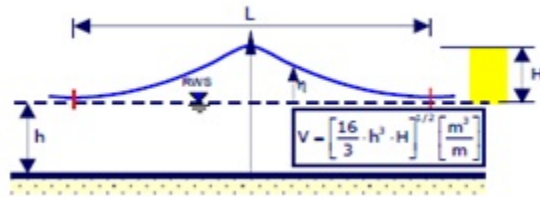


Figure 4 Solitary Wave

Solitary wave properties are as follows:

- Stable and travel over very large distances (normal waves tend to flatten out).
- Speed depends on the size of wave, its width depends on the depth of water.
- With property of self-retaining structure, it becomes soliton.
- High potential of generating Tsunami Wave.

The water condition and application of wave theories by Chakrabarti (1986) are summarized as follows:

Table 1 Summary of Wave Theory Application

Water Condition	Wave Theory
Deep Water $\left(\frac{d}{L} > \frac{1}{2} \right)$	Linear Wave Theory Stoke Wave Theory Stream Function Theory
Shallow Water $\left(\frac{d}{L} < \frac{1}{25} \right)$	Solitary Wave Theory Cnoidal Wave Theory

However, although these large number of nonlinear wave theories have been proposed and used for computing the wave kinematics, there are no theories applicable perfectly from deep water to very shallow water.

Many intensive efforts have been made to examine the validity as well as the applicability of various wave theories. There are still no well-accepted guidelines for the application range of the wave theories. Researchers have been looking into more complicated validity of the wave theories using either analytical studies or experimental studies. Dean (1970) conducted analytical study and revealed that the

degree of mathematical satisfaction to the governing equations and boundary conditions for each wave theory. For transitional water condition, the Stream Function theory gave the best boundary condition fit over the entire range. Dean also claimed that Cnoidal theory was best in shallow water into intermediate depth.

A second-order Stokes was developed for finite amplitude waves using a power series based on H/L . Stokes claimed that the results require that H/d not be large and thus are applicable for deep water and much of the intermediate depth range. With reference to above, we can deduce that the Cnoidal wave theory, Stream fifth-order and Stokes second order are applicable to transitional water condition.

Stokes fifth-order wave theory is appropriate for engineering use primarily in deepwater where depth exceeds half the wavelength. Ippen (1966) stated that Airy theory ($d/L > 0.5$) is appropriate for deepwater waves. The Stream fifth order gave the best boundary condition fit over entire range includes deepwater except for steeper waves in very shallow water. For deep water condition, three wave theories, Linear Airy wave theory, Stokes fifth order and Stream fifth order are therefore can be claimed applicable.

Waves theories selection is a feature in SACS v3.5 that allows user to employ wave theory, which is identified by researcher as applicable, in performing analysis. SACS v3.5 is an integrated suite of software that supports the analysis, design and fabrication of offshore structures. SACS v3.5 is able to run analysis using wave theories selected by user to predict wave velocity, u and compute wave forces based on drag force equation.

3. RESEARCH METHODOLOGY

This research study consists of three major phases that include: extensive literature review; SACS modeling and simulation; interpretation of results and sensitivity analysis.

In SACS modeling, the monopod model as shown on Figure 5 is sourced from the Offshore Engineering Centre UTP (OECU) for PMO, SKO and SBO in the form of a generic model. The generic model does not include topside, facilities and appurtenances due to the reason that we are interested in the seastate loading that includes wave kinematics and current only. The monopod model was revised so that the computed wave forces at the free surface or wave crest forces are obtained.

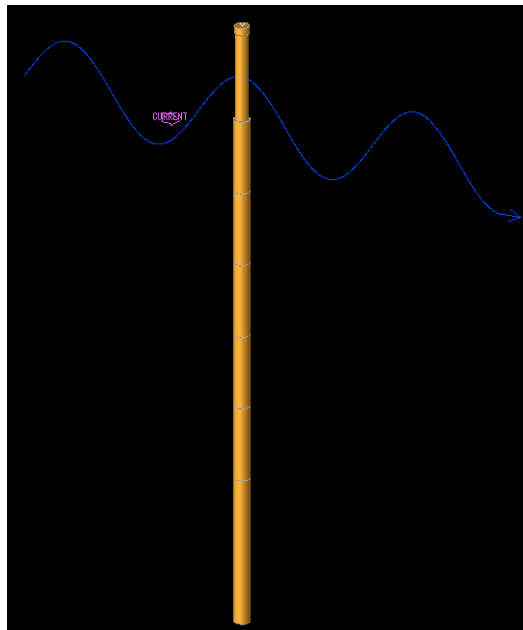


Figure 5 Monopod Modelling in SACS for PMO

Using the water depth for the zone, PMO for example, with 70m deep, calculate depth to wavelength ratio, d/L . From the value obtained, we can categorize it in a particular water condition and also determine the applicable wave theories.

In SACS v3.5, navigate to seastate load condition, select the desired wave theory and input the metocean criteria according to as given in PTS. As shown in Figure 6 and Figure 7, the Stokes wave theory is selected and the wave height is input as according to PTS. There are 72 crest positions with 5 degree of increment, which indicates circulation of 360 degree for wave crest angle. The water depth is input as

according to PTS and the mudline elevation is referencing to the bottom of the monopod model.

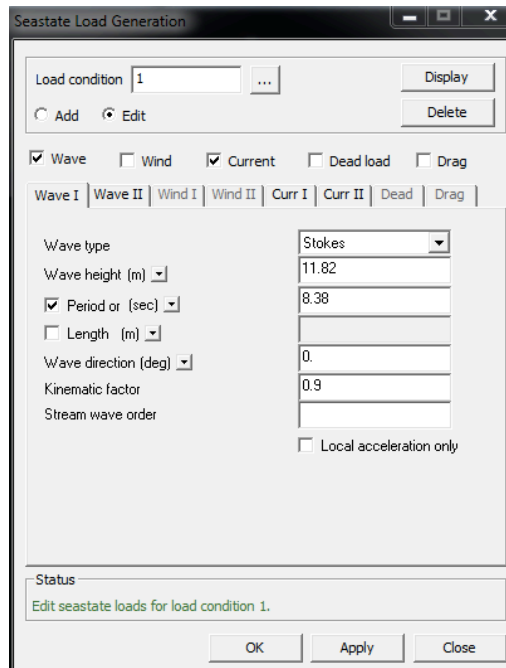


Figure 6 SACS Seastate Load Condition Input

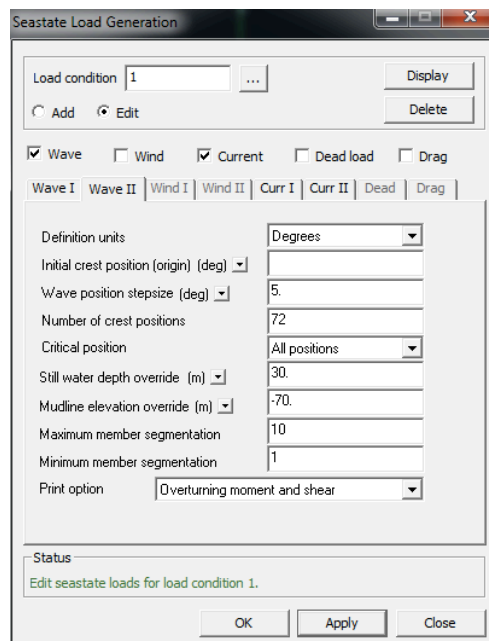


Figure 7 Seastate Load Condition Water Depth Input

Before performing analysis, it is important to select the analysis type and SACS section library to be referred to. Wave forces for Stokes with metocean criteria after the analysis is available as output.

In order to obtain a few more wave forces values so to be able to plot graphs and analyze the sensitivity, wave height is deviated by $\pm 40\%$ as shown in Table 2. Wave height is chosen for deviation because wave theory predicts wave kinematics based on measured wave height, in this case, deviated wave height. The theoretical predictions of wave kinematics were made using the measured wave height, wave period and water depth (Hattori, 1986). The adoption of wave height is also due to previous studies that determined the application range of the wave theories from intercomparisons of the validity limits for the water particle velocity, in terms of the relative water depth, d/L and relative water height, H/d .

In PTS, for 100-year storm event, the wave height criteria recorded shows fluctuation of 140% from the wave height criteria for operating event. With reference to this, it is feasible to adopt a deviation of $\pm 40\%$ with assumption that the wave height is reduced to 40% less is during the extreme low tidal waves.

Table 2 Deviated Wave Height for PMO zone

Percentage Deviation	Wave Height, Hmax (m)
-40%	5.06
-20%	6.75
0%	8.44
+20%	10.13
+40%	11.82

A set of SACS computed wave forces is able to plot a graph. In order to analyze the sensitivity of wave forces to wave theories, the methodology is repeated with other applicable wave theories. The methodology is applicable to SKO and SBO.

Start Wizard Display Options Run Analysis View Summary Reset Analysis Subtype	
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Environmental Loading	✓ <Edit Environmental Loading Options>
Solve	✓ <Edit Solve Options>
Foundation	<Edit Foundation Options>
Gap Elements	<Edit Gap Elements Options>
Element Check	✓ <Edit Element Check Options>
Tubular Joint Check	<Edit Tubular Joint Check Options>
Graphical Post Processing	✓ <Edit Graphical Post Processing Options>
Section Library	
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User-Defined Section Library	
Input Files	
SACS Model File	pmo sko\sacinp.pmo

Figure 8 SACS Perform Analysis Options

4. RESULT AND DISCUSSION

The result included in this section is emphasizing on the interpretation and discussion of the sensitivity analysis of the monopod structure in PMO, SKO and SBO zones to wave theories and metocean parameters. The specifications of the zones are summarized as below:

Table 3 Specifications of the Zones

Zone	Depth (m)	d/L	Water Condition	Valid Wave Theories (from published research)
PMO	70	0.9876	Deep	Linear Airy (Ippen, 1966) Stokes 5 th order (Fenton, 1985) Stream 5 th order (Dean, 1970)
SKO (Balingian)	30	0.2569	Transitional	Stokes 2 nd Order (Fenton, 1985) Stream 5 th order (Dean, 1970) Cnoidal (Dean, 1970)
SBO (Samarang)	50	0.3696	Transitional	Stokes 2 nd Order (Fenton, 1985) Stream 5 th order (Dean, 1970) Cnoidal (Dean, 1970)

4.1 Peninsular Malaysia Operation (PMO)

From hand calculation attached in the appendix A, PMO zone with water depth of 70m is found to be in deep water condition. It is evident from the water depth to wavelength ratio, $d/L = 0.6391 (>1/2)$. As many previous studies have pointed out, Linear Airy, Stokes fifth order and Stream fifth order provides good prediction of the wave kinematics in deep-water region.

This is again proven to be true from the computer-simulation using SACS v3.5, when Cnoidal wave theory is selected for model simulation under PMO deep water condition, the simulation is not able to generate results. Therefore, Liner Airy wave theory, Stoke wave theory and Stream wave theory are considered in evaluating the sensitivity of wave forces to wave theories.

Table 4-6 show, for PMO zone, the wave forces computed from SACS v3.5, which correspond to five different wave heights: -40%, -20%, +20%, +40% of the metocean wave height criteria and also the metocean wave height criteria itself, using three applicable wave theories: Airy, Stoke and Stream. In PTS, for 100-year storm event, the wave height criteria recorded shows fluctuation of 140% from the wave

height criteria for operating event. With reference to this, it is feasible to adopt a deviation of $\pm 40\%$ with assumption that the wave height is reduced to 40% less is during the extreme low tidal waves.

For PMO, the still water depth is 70m and the monopod model is of 83m in length. From Table 4, it is noticeable that when the wave height is deviated from 40% less to 40% more, the wave force's magnitude changes significantly. In the case of using a different wave theory such as Stoke or Stream does not change the trend. As shown in Table 5 and Table 6, although different wave theory is employed, the wave force's magnitude changes significantly.

Another finding is that the magnitude of wave forces computed using Airy wave theory does not differ much from the magnitude of wave forces computed using Stoke or Stream wave theory. The largest difference in value between wave forces from Airy and wave forces from Stoke is 9.38kN. The largest difference in value between wave forces from Stoke and wave force from Stream is 17.62kN.

Table 4 SACS Computed Wave Forces using Airy

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (5.06m)	196.04	61
-20% (6.75m)	258.72	63
0% (8.44m – metocean criteria)	321.41	65
+20% (10.13m)	384.1	65
+40% (11.82m)	446.78	67

Table 5 SACS Computed Wave Forces using Stoke

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (5.06m)	196.83	61
-20% (6.75m)	258.38	63
0% (8.44m – metocean criteria)	326.00	65
+20% (10.13m)	403.56	65
+40% (11.82m)	494.57	67

Table 6 SACS Computed Wave Forces using Stream

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (5.06m)	194.90	61
-20% (6.75m)	253.42	63
0% (8.44m – metocean criteria)	318.42	65
+20% (10.13m)	391.31	65
+40% (11.82m)	476.95	67

A sensitivity graph is plotted as shown in Figure 9 to observe the sensitivity of the wave forces to wave theories. The first interpretation is that the wave force is sensitive to changing wave height, regardless of any of the three wave theories employed. The wave force increases linearly to 495 kN when wave height is increased from the criteria value to 40% more.

This sensitivity study also shows that the Airy, Stoke and Stream wave theory share the similar trend line. In addition, the magnitude of wave forces computed from each wave theory does not differ too much. To be more accurate, Stream wave theory has a closer resemblance of Stoke wave theory as observed from the values recorded in Table 4 to Table 6.

Plot for Airy in Figure 9 shows linear increment of wave forces from a linear equation is probably due to the nature of Airy as a linear wave theory. Airy is based on the assumption that the wave height is small compared to the wave length or water depth. This assumption allows the free surface boundary condition to be linearized by dropping wave height terms, which are beyond the first order (Chakrabarti, 1987).

Stokes and Stream function show non-linear increment of wave forces as seen on Figure 9. The equations of the graphs are in quadratic form of second order, which indicates that Stokes second order and Stream second order are sufficient and fifth order is not necessary. The non-linearity is due to the nature of these two wave theories that describes well for non-linear and periodic surface wave on a fluid layer of constant mean depth. It is observed that Stokes predicts a larger wave forces compared to Stream due to higher order, which considered wave steepness as expansion parameter.

It can be deduced that wave force is not sensitive towards the application of any of Airy, Stokes or Stream function from the similar slopes of the three plots. However, among these three wave theories, Stokes is more conservative and has a higher order which best describes steep waves in deep water.

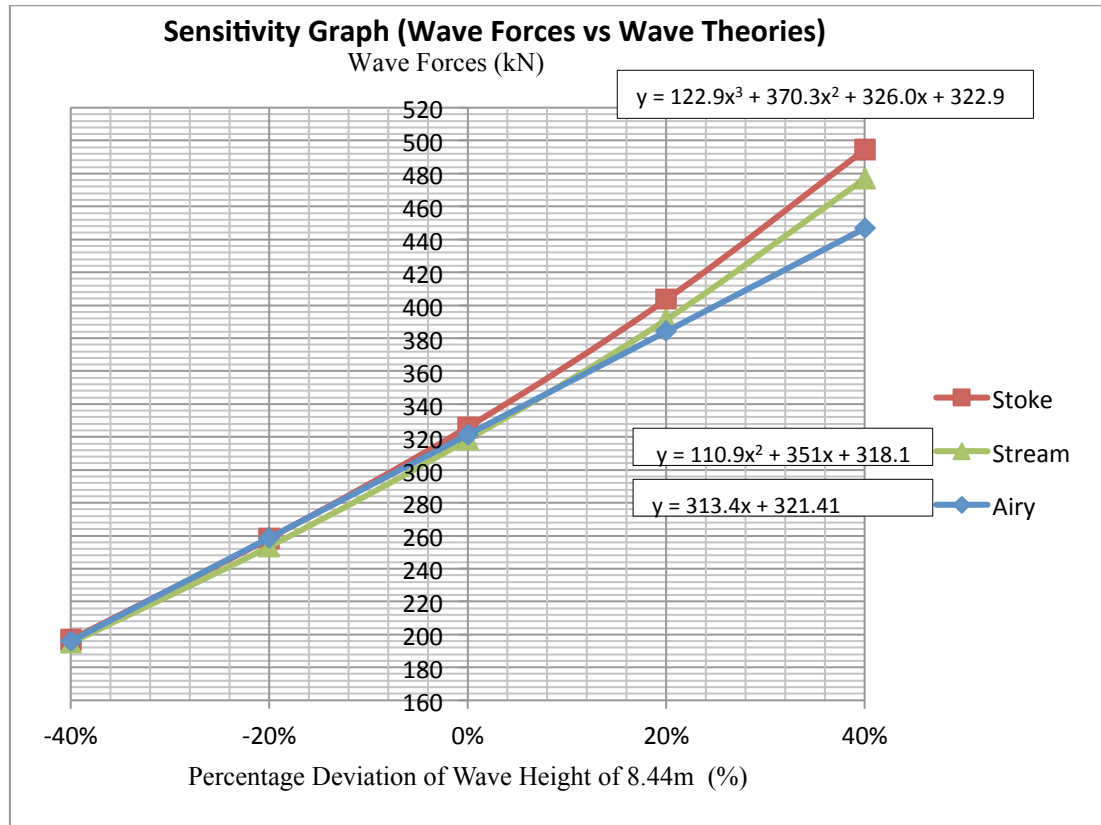


Figure 9 Wave Forces insensitive to Wave Theories

4.2 Sarawak Operation (SKO)

It is noted that for the SKO zone, Balingian offshore site is chosen as the study location among other sites in SKO. Balingian offshore site has a water depth of 30m with $d/L = 0.2569$ ($1/25 < d/L < 1/5$). It is categorized as transitional water condition.

Based on published research, three wave theories are identified as valid and applicable. Dean (1970) claimed that Stream Function Theory gives accurate approximations of wave kinematics in transitional water depth. Cnoidal wave theory provides best fit to the governing equations in transitional water depth (Dean, 1970). Stokes Second Order Theory describes wave of transitional depth. These three wave theories are considered in evaluating the sensitivity of wave forces to wave theories.

Table 7, Table 8 and Table 9 show the SACS computed wave forces, as a result of different wave heights deviated from the metocean criteria for the Balingian site in SKO zone using Cnoidal wave theory, Stokes wave theory and Stream Function. It is observed that by Cnoidal wave theory, the predicted wave forces' magnitude changes significantly when the wave height is deviated. In the case of using Stokes second order or Stream Function, the computed wave forces' magnitude shows the same behavior as that of Cnoidal wave theory.

Table 7 SACS Computed Wave Forces using Cnoidal Wave Theory

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (3.48 m)	78.70	67
-20% (4.64 m)	118.01	63
0% (5.80 m – metocean criteria)	154.97	67
+20% (6.96 m)	210.03	69
+40% (8.12 m)	299.85	69

Table 8 SACS Computed Wave Forces using Stokes Wave Theory

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (3.48 m)	97.04	61
-20% (4.64 m)	129.65	61
0% (5.80 m – metocean criteria)	162.74	63
+20% (6.96 m)	199.62	63
+40% (8.12 m)	243.70	65

Table 9 SACS Computed Wave Forces using Stream Function

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (3.48 m)	96.57	61
-20% (4.64 m)	128.50	61
0% (5.80 m – metocean criteria)	162.48	63
+20% (6.96 m)	199.21	65
+40% (8.12 m)	241.91	65

Another finding from the recorded result is that the magnitude of wave forces computed using Stream Function are similar when compare to the magnitude of wave forces computed using Stokes wave theory. The biggest difference is less than 2 kN. However, magnitude of wave forces computed using Cnoidal wave theory show significant difference to that of Stokes wave theory and Stream Function.

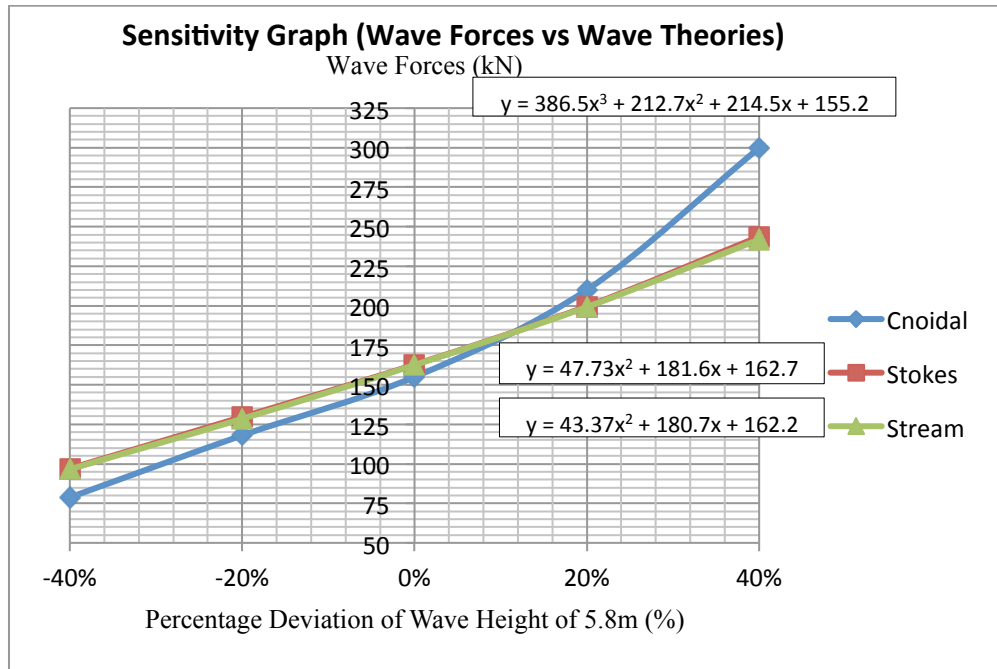


Figure 10 Wave Forces sensitive to Wave Theories

Figure 10 shows the sensitivity graph of wave forces versus percentage deviation of wave height against applicable wave theories. The first interpretation is wave forces changes in respond to changing wave height.

It is evident that only by Cnoidal wave theory, the wave forces change significantly while plot of Stokes and Stream wave theories show similar nonlinear trend line and less steep slope. This agrees well with second order nonlinear nature of these wave theories.

Cnoidal wave theory indicates that wave force has a higher sensitivity when wave height is increased to 112% and above as observed from Figure 10 on the significant gradient change.

Graph plot of Stokes and Stream show non-linearity which agree well with the non-linear nature of these two wave theories and the equations shown indicates second order is sufficient. Cnoidal plot is more sensitive in predicting wave forces because it is of third order of non-linearity as depicted by the equation shown, which accounts for a large class of long waves of finite amplitude (Chakrabarti, 1987).

It can be deduced that wave force is sensitive towards changing wave height when Cnoidal wave theory is employed. As the Cnoidal wave theory gives more critical

wave forces values, the application of Cnoidal wave theory over other wave theories is more conservative.

4.3 Sabah Operation (SBO)

It is noted that for SBO zone, Samarang offshore site is chosen as the study location among other sites in SBO. Samarang site has a water depth of 50m with $d/L = 0.3696$ ($1/25 < d/L < 1/5$). It is having a transitional water condition.

Samarang offshore site is having the same water condition as the Balingian offshore site. Similar set of wave theories will be considered as suggested by published research mentioned in the SKO result section.

Table 10 SACS Computed Wave Forces using Stream Function

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (4.14 m)	183.43	65
-20% (5.52 m)	243.38	67
0% (6.90 m – metocean criteria)	312.74	67
+20% (8.28 m)	395.28	69
+40% (9.66 m)	492.28	69

Table 11 SACS Computed Wave Forces using Stokes

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (4.14 m)	184.64	65
-20% (5.52 m)	244.11	67
0% (6.90 m – metocean criteria)	313.92	67
+20% (8.28 m)	397.19	69
+40% (9.66 m)	494.28	69

Table 12 SACS Computed Wave Forces using Cnoidal

Hmax (Wave Height)	Wave Forces (kN)	Load Case
-40% (4.14 m)	133.38	67
-20% (5.52 m)	208.12	69
0% (6.90 m – metocean criteria)	308.71	71
+20% (8.28 m)	493.55	71
+40% (9.66 m)	899.96	1

Table 10, Table 11 and Table 12 summarize the computed wave forces using Stream, Stokes and Cnoidal wave theories with different wave heights deviated from the metocean criteria for Samarang site in SBO zone.

It is observed that using Stream Function Theory, the computed wave forces' magnitude changes significantly when the wave height is deviated. In the case of using Stokes second order or Cnoidal wave theory, the computed wave forces' magnitude shows the same behavior as Stream Function Theory.

Another finding from the recorded result is that the magnitude of wave forces computed using Stokes wave theory are similar when compare to the magnitude of wave forces computed using Stream Function. The maximum difference is 2 kN. However, magnitude of wave forces computed using Cnoidal wave theory show significant difference to that of Stokes wave theory and Stream Function.

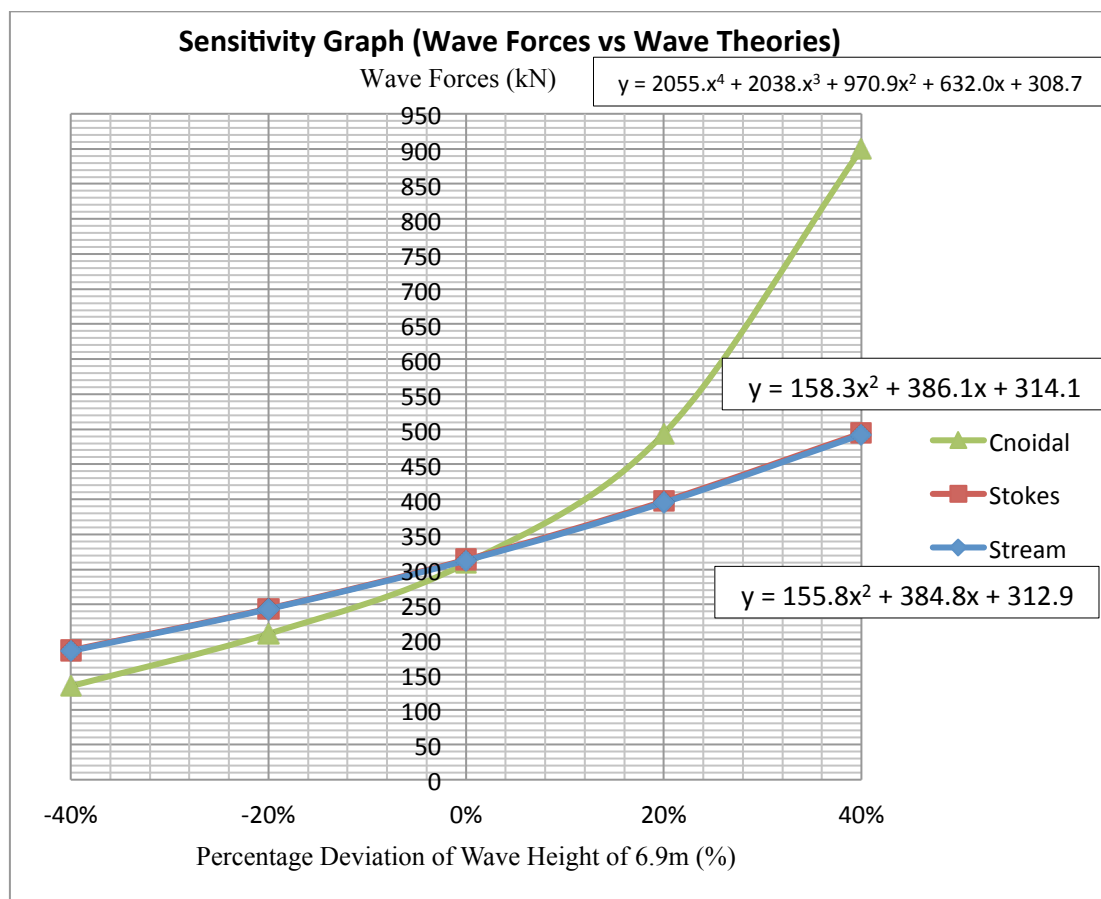


Figure 11 Wave Forces sensitive to Wave Theories

Figure 11 shows the sensitivity graph of wave forces versus percentage deviation of wave height against three applicable wave theories.

Under Cnoidal wave theory, the predicted wave forces changes significantly in respond to changing wave height. Graph plot of Stokes and Stream shows similar nonlinear trend line and less steep slope.

Cnoidal wave theory indicates that wave force is more sensitive when wave height is increased to 112% and above as observed from Figure 11 on the significant gradient change. While, plot of wave forces computed from Stokes and Stream shows similar trend.

Visual comparison provides fairly well nonlinearity for the three wave theories due to the nonlinear nature of them. Cnoidal plot shows different degree of linearity as it is of higher order than Stokes and Stream, which considers a large class of long waves of finite amplitude and also taking account of the bottom slope effect (Hattori, 1986).

It can be deduced that wave force is sensitive towards wave theories for SBO zone. As the Cnoidal wave theory gives more critical wave forces, the application of Cnoidal wave theory over other wave theories is more conservative.

5. CONCLUSION AND RECOMMENDATION

This research project address the need for analyzing the sensitivity of wave forces on a monopod structure under metocean condition for PMO, SKO and SBO.

The conclusions that can be made from the result findings are:

- Theoretical predictions of wave forces from wave height can be achieved using SACS v3.5.
- Predicted wave forces are not sensitive to wave theories for PMO zone.
- Predicted wave forces are sensitive to wave theories for SKO and SBO zones.
- Application of Stokes fifth order in PMO zone is more conservative.
- Application of Cnoidal in SKO and SBO are more conservative.
- The objectives have been achieved.

It is recommended that in this sensitivity analysis, other metocean criteria such as wave period, wave length, thickness of caisson and water depth be included in the analysis to obtain deeper insight on global sensitivity of wave forces on a monopod platform.

The author would like to envision that this research could benefit the offshore facilities industry as a valuable addition to the stock of literature on wave forces sensitivity.

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APPENDICES

Appendix 1: Water Condition Determination Hand Calculation (refer page 14)

Water Condition Determination

PMO (Depth = 70m)

$$d = 70\text{m}, T = 8.38\text{s}$$

$$L_0 = \frac{gT^2}{2\pi}$$

$$= 109.60\text{m}$$

$$\frac{d}{L_0} = 0.6387$$

From wave table, $\frac{d}{L} = 0.6391$ (deep water condition)

SKO (Balingian, depth = 30m)

$$d = 30\text{m}, T = 9\text{s}$$

$$L_0 = \frac{gT^2}{2\pi}$$

$$= 126.42\text{m}$$

$$\frac{d}{L_0} = 0.2373$$

From wave table, $\frac{d}{L} = 0.2569$ (transitional water condition)

SBO (Samorang, depth = 50m)

$$d = 50\text{m}, T = 9.4\text{s}$$

$$L_0 = \frac{gT^2}{2\pi}$$

$$= 137.91\text{m}$$

$$\frac{d}{L_0} = 0.3626$$

From wave table, $\frac{d}{L} = 0.3696$ (transitional water condition)

Appendix 2: SACS v3.5 generated result for PMO model with wave height of -
20% deviation from criteria in PTS using Stokes fifth order wave theory

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LOAD SUMMATION REPORT

Load Condition 1

The sum of forces at the origin are:

$F_x = 171.42$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -2458.47$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -14.342$

Load Condition 2

The sum of forces at the origin are:

$F_x = 151.82$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -2137.07$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -14.077$

Load Condition 3

The sum of forces at the origin are:

$F_x = 132.32$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -1815.41$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -13.72$

Load Condition 4

The sum of forces at the origin are:

$F_x = 110.41$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -1513.35$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -13.707$

Load Condition 5

The sum of forces at the origin are:

$F_x = 88.78$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -1157.96$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -13.043$

Load Condition 6

The sum of forces at the origin are:

$F_x = 65.88$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -805.49$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -12.227$

Load Condition 7

The sum of forces at the origin are:

$$F_x = 43.46 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = -464.3 \quad M_z =$$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -10.684$

Load Condition 8

The sum of forces at the origin are:

$$F_x = 20.94 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = -142.42 \quad M_z =$$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -6.802$

Load Condition 9

The sum of forces at the origin are:

$$F_x = -0.92 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = 224.19 \quad M_z =$$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -244.624$

Load Condition 10

The sum of forces at the origin are:

$$F_x = -21.79 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = 550.77 \quad M_z =$$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -25.277$

Load Condition 11

The sum of forces at the origin are:

$$F_x = -41.86 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = 871.82 \quad M_z =$$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -20.829$

Load Condition 12

The sum of forces at the origin are:

$$F_x = -60.0 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = 1164.33 \quad M_z =$$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -19.404$

Load Condition 13

The sum of forces at the origin are:

$$F_x = -77.27 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = 1453.7 \quad M_z =$$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -18.813$

Load Condition 14

The sum of forces at the origin are:

$$F_x = -91.98 \quad F_y = \quad F_z = \\ M_x = \quad M_y = 1703.31 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -18.519$$

Load Condition 15

The sum of forces at the origin are:

$$F_x = -105.8 \quad F_y = \quad F_z = \\ M_x = \quad M_y = 1948.41 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -18.415$$

Load Condition 16

The sum of forces at the origin are:

$$F_x = -116.59 \quad F_y = \quad F_z = \\ M_x = \quad M_y = 2144.05 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -18.389$$

Load Condition 17

The sum of forces at the origin are:

$$F_x = -126.56 \quad F_y = \quad F_z = \\ M_x = \quad M_y = 2334.07 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -18.442$$

Load Condition 18

The sum of forces at the origin are:

$$F_x = -133.26 \quad F_y = \quad F_z = \\ M_x = \quad M_y = 2467.59 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -18.517$$

Load Condition 19

The sum of forces at the origin are:

$$F_x = -138.57 \quad F_y = \quad F_z = \\ M_x = \quad M_y = 2594.45 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -18.723$$

Load Condition 20

The sum of forces at the origin are:

$$F_x = -140.57 \quad F_y = \quad F_z = \\ M_x = \quad M_y = 2660.87 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -18.929$$

Load Condition 21

The sum of forces at the origin are:

$$F_x = -142.03 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 2721.04 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -19.158$$

Load Condition 22

The sum of forces at the origin are:

$$F_x = -140.36 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 2718.36 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -19.367$$

Load Condition 23

The sum of forces at the origin are:

$$F_x = -138.33 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 2710.0 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -19.591$$

Load Condition 24

The sum of forces at the origin are:

$$F_x = -133.46 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 2640.16 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -19.782$$

Load Condition 25

The sum of forces at the origin are:

$$F_x = -128.39 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 2565.66 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -19.984$$

Load Condition 26

The sum of forces at the origin are:

$$F_x = -120.83 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 2433.74 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -20.142$$

Load Condition 27

The sum of forces at the origin are:

$$F_x = -113.2 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 2298.33 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -20.304$$

Load Condition 28

The sum of forces at the origin are:

$$F_x = -103.51 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 2111.81 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -20.403$$

Load Condition 29

The sum of forces at the origin are:

$$F_x = -93.68 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 1921.92 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -20.515$$

Load Condition 30

The sum of forces at the origin are:

$$F_x = -82.49 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 1690.56 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -20.493$$

Load Condition 31

The sum of forces at the origin are:

$$F_x = -71.71 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 1460.06 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -20.36$$

Load Condition 32

The sum of forces at the origin are:

$$F_x = -59.38 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 1193.08 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -20.093$$

Load Condition 33

The sum of forces at the origin are:

$$F_x = -47.18 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 926.06 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -19.628$$

Load Condition 34

The sum of forces at the origin are:

$$F_x = -34.08 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 633.77 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -18.596$$

Load Condition 35

The sum of forces at the origin are:

$$F_x = -21.08 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 341.72 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -16.209$$

Load Condition 36

The sum of forces at the origin are:

$$F_x = -7.49 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = 36.36 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -4.857$$

Load Condition 37

The sum of forces at the origin are:

$$F_x = 6.08 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = -273.31 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -44.966$$

Load Condition 38

The sum of forces at the origin are:

$$F_x = 19.92 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = -589.79 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -29.612$$

Load Condition 39

The sum of forces at the origin are:

$$F_x = 33.78 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = -904.96 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -26.789$$

Load Condition 40

The sum of forces at the origin are:

$$F_x = 47.58 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = -1217.65 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -25.592$$

Load Condition 41

The sum of forces at the origin are:

$$F_x = 61.45 \quad F_y = \quad F_z =$$
$$M_x = \quad M_y = -1530.69 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -24.909$$

Load Condition 42

The sum of forces at the origin are:

$F_x = 74.9$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -1832.02$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -24.46$

Load Condition 43

The sum of forces at the origin are:
 $F_x = 88.49$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -2134.32$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -24.118$

Load Condition 44

The sum of forces at the origin are:
 $F_x = 101.24$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -2416.18$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -23.866$

Load Condition 45

The sum of forces at the origin are:
 $F_x = 114.25$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -2700.17$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -23.633$

Load Condition 46

The sum of forces at the origin are:
 $F_x = 126.64$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -2959.84$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -23.372$

Load Condition 47

The sum of forces at the origin are:
 $F_x = 139.08$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -3220.64$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -23.157$

Load Condition 48

The sum of forces at the origin are:
 $F_x = 150.35$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -3448.81$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -22.938$

Load Condition 49

The sum of forces at the origin are:
 $F_x = 162.14$ $F_y =$ $F_z =$

Mx = My = -3681.12 Mz =
The center of forces is:
For X forces: X = 0.0 Y = 0.0 Z = -22.704

Load Condition 50

The sum of forces at the origin are:
Fx = 172.63 Fy = Fz =
Mx = My = -3875.64 Mz =
The center of forces is:
For X forces: X = 0.0 Y = 0.0 Z = -22.451

Load Condition 51

The sum of forces at the origin are:
Fx = 183.73 Fy = Fz =
Mx = My = -4074.99 Mz =
The center of forces is:
For X forces: X = 0.0 Y = 0.0 Z = -22.18

Load Condition 52

The sum of forces at the origin are:
Fx = 193.36 Fy = Fz =
Mx = My = -4231.87 Mz =
The center of forces is:
For X forces: X = 0.0 Y = 0.0 Z = -21.886

Load Condition 53

The sum of forces at the origin are:
Fx = 203.68 Fy = Fz =
Mx = My = -4394.0 Mz =
The center of forces is:
For X forces: X = 0.0 Y = 0.0 Z = -21.574

Load Condition 54

The sum of forces at the origin are:
Fx = 212.34 Fy = Fz =
Mx = My = -4509.57 Mz =
The center of forces is:
For X forces: X = 0.0 Y = 0.0 Z = -21.237

Load Condition 55

The sum of forces at the origin are:
Fx = 221.75 Fy = Fz =
Mx = My = -4630.48 Mz =
The center of forces is:
For X forces: X = 0.0 Y = 0.0 Z = -20.881

Load Condition 56

The sum of forces at the origin are:
Fx = 229.27 Fy = Fz =
Mx = My = -4702.4 Mz =

The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -20.51$

Load Condition 57

The sum of forces at the origin are:
 $F_x = 236.74$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4779.04$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -20.187$

Load Condition 58

The sum of forces at the origin are:
 $F_x = 241.98$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4803.4$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -19.85$

Load Condition 59

The sum of forces at the origin are:
 $F_x = 247.93$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4832.36$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -19.491$

Load Condition 60

The sum of forces at the origin are:
 $F_x = 251.32$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4806.83$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -19.126$

Load Condition 61

The sum of forces at the origin are:
 $F_x = 255.36$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4785.08$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -18.738$

Load Condition 62

The sum of forces at the origin are:
 $F_x = 256.6$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4708.4$ $M_z =$
The center of forces is:
For X forces: $X = 0.0$ $Y = 0.0$ $Z = -18.349$

Load Condition 63

The sum of forces at the origin are:
 $F_x = 258.38$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4634.58$ $M_z =$
The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -17.937$

Load Condition 64

The sum of forces at the origin are:

$F_x = 256.75$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4506.08$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -17.551$

Load Condition 65

The sum of forces at the origin are:

$F_x = 255.52$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4379.56$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -17.14$

Load Condition 66

The sum of forces at the origin are:

$F_x = 250.66$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4201.54$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -16.762$

Load Condition 67

The sum of forces at the origin are:

$F_x = 246.05$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -4024.79$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -16.357$

Load Condition 68

The sum of forces at the origin are:

$F_x = 237.5$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -3801.1$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -16.005$

Load Condition 69

The sum of forces at the origin are:

$F_x = 229.06$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -3578.19$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -15.621$

Load Condition 70

The sum of forces at the origin are:

$F_x = 216.56$ $F_y =$ $F_z =$
 $M_x =$ $M_y = -3314.59$ $M_z =$

The center of forces is:

For X forces: $X = 0.0$ $Y = 0.0$ $Z = -15.305$

Load Condition 71

The sum of forces at the origin are:

$$F_x = 204.09 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = -3051.38 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -14.951$$

Load Condition 72

The sum of forces at the origin are:

$$F_x = 187.75 \quad F_y = \quad F_z =$$

$$M_x = \quad M_y = -2754.88 \quad M_z =$$

The center of forces is:

$$\text{For X forces: } X = 0.0 \quad Y = 0.0 \quad Z = -14.673$$