



Model Predictive Control with First-Order Hold Element

By

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
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Approved by,

(Marappagounder Ramasamy)

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

AHMAD SYAZMIN ISMAYUDDIN

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I deeply express my gratitude for all the related parties and individuals who help and guide me through the second half section of my final year project which is FYP 2.

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ABSTRACT

Nowadays, the size and complexity of petrochemical industries have increased considerably as the demand from the consumers have increased, while the profit margins are decreasing. The control systems play an important role in maximizing the profit margins by optimally utilizes resources and energy.

Model Predictive Control (MPC) handle multi-variable systems and constraints in a systematic way. The MPC has been used for about four decades and it provides a positive impact to the system which is why it's been mostly used in the petrochemical plant. However, in this report, it will discuss more on the way to improvise the performance of MPC by implementing the first-order hold element instead of zero-order hold element method. By using the single input and single output plant for MPC, the result will differentiate the zero-order hold and first-order hold. The tools that have been used for this project is the MATLAB simulation.

The expected result from the study is the first-order hold element method will show better result compared to the zero-order hold element method. However, there will be some cons if using the first-order hold and it will be discussed in the report. The function of this MPC is to get the production meet the required criteria by controlling the manipulated variables. The required criteria can be control by the system as it is the reference point.

In conclusion, the first-order hold element method will give a better result than zero-order hold element although nowadays many companies still using zero-order hold. This is because zero-order hold is easier to calculate and operate compared to the other method. However, the final production whether it is in a good quality or quantity is more important and it will increase the profit of that company. So, it is highly recommended the petrochemical company to use this equivalent to first-order hold element method in their MPC system as it will maximize the production of their product.

1.0 INTRODUCTION

1.1 Background of Study

The petrochemical industry is characterized as having a very dynamic and unpredictable marketplace conditions. As for now, we already witnessed the enormous variation in crude and product prices. For a company to generate the most profit out of their plant while responding to marketplace variations is by integration of all aspects of automation of the decision making process (Garcia and Prett, 1988). There are four layers of automation which is the measurements, control, optimization and logistic. Each one of these automation layers plays a unique and complementary role in allowing a company to react rapidly to changes. Hence, one layer cannot be effective without the others. In the petrochemical industries, control systems need to satisfy many practical performance criteria such as economic, safety & environment, product quality and human preference. Therefore, the only control process that can meet this criteria is the Model Predictive Control (MPC) which it can handle constraints in a systematic way during the design and implementation of the controller. This is the main reason for the success of these techniques in numerous applications in the chemical process industries.

Model Predictive Control (MPC) can offer some important advantages which are it can capture the dynamic and the static interaction of all variables. Other than that, the control calculation can be coordinated with the calculation of optimum set points and lastly it also can provide an early warning of potential problems. Based on the MPC that being used in industries, all of them are using discrete-time control systems (D. E. Seborg *et al.*, 2014). In this study, the investigation will be focused more on the implementation of MPC by using discrete-time control from two different methods. The first one is the zero order hold element which is already being implemented in now days MPC and second method is the first order hold element. From these two methods, the MATLAB simulation is being used in order to acquire the results.

The MPC will be introduced with a new set point and also the disturbances. From both methods, the differences can be clearly seen from the graph and we will see which one show a better result. Other than that, the performance of the result can also be calculated by the cost function. For this project, the cost function being used is the Integral Square Error (ISE). The simulation is being used to calculate the ISE and the data is taken from the graph which have been plotted from the both methods. Thus, from this cost function, we can clearly show the best method in the way of calculation instead of only show it in the graph.

1.2 Problem Statement

The last four decades have seen a rapidly increasing number of applications where control techniques based on dynamic optimization led to improved performance in maximizing the process output or minimizing energy use and emissions. It is the Model Predictive Control and it had been widely used in any industry regarding the process system and the result is magnificent. However, the MPC calculation is calculated by the zero-order hold element method which is very simple and is used most frequently in practice. According to (Qin & Badgwell, 2003), in his research regarding the survey of industrial MPC, they stated that the MPC will be keeping changing as the time has change.

There is a lot of algorithm which have been introduced in the last four decades and the problems are constraints, process nonlinearities, model uncertainty (robustness), unique performance criteria, and also cultural reasons. The problems face by the MPC is depend on the situation of the process. As for this project, the problem that needs to solve and improve is the formulation of the first order hold. The formulation that being used is by calculating the gradient instead of only calculate at the beginning point and maintaining the rest. Thus, the gradient calculation is very difficult to calculate and formulate. Although the first order holds already been introduced in the MATLAB simulation, yet it is not the formulation inside the MPC itself. The new design and coding need to be done to complete a new MPC which is equivalent to the first order hold. Other than that, by using the zero order hold, it will ignore the inter sample behavior of the system and the optimization will become non-convex.

1.3 Objective

The main aim and primary objective of this study is to formulate a new MPC by using first order hold element. Nowadays, a lot of companies are using MPC with zero order hold element method and it have helped the companies gain a lot of profit. Thus, with the new MPC been formulated by using the first order hold, it may improve much more profit than before. Other than that, the objective of this project is to explore and investigate the differences between the implementation of the zero order hold element and first order hold element in the MPC. The state-space model is being used in this MPC as it will indicate a simple system of single input single output (SISO) with no constraints been introduced to the input or output. By using the same plant design, both results from the zero order hold and first order hold will be plotted and it will be discussed.

2.0 LITERATURE REVIEW

2.1 Model Predictive Control

Model Predictive Control (MPC) is not a specific control strategy but a wide class of optimal control based algorithms that use an explicit process model to predict the behavior of a plant. There are varieties of MPC algorithms that have been formulated over past 40 years. For example, the Model Predictive Heuristic Control (MPHC) algorithm reported by Richalet *et al.* in 1976 which is using an impulse response model as its process model. Moreover, the most industrially popular Linear Model Predictive Control (LMPC) algorithm, the Dynamic Matrix Control (DMC) presented by Cutler and Ramaker (1979), the Generalized Predictive Control (GPC) by Clarke *et al.* (1987) which was intended to provide a new adaptive control alternative. The main differences for all these MPC algorithm are the types of models used to represent the plant dynamic and the cost function to be minimized. The MPC is an open nature, within which many works have been developed and are widely received by the academic world and industry. There are many applications of predictive control successfully in use at the current time, not only in the process industry but also applications to the control of other processes ranging from robots to clinical anesthesia. In practice, MPC has proved to be a reasonable strategy for industrial control, in spite of the original lack of theoretical results at some crucial points such as stability and robustness.

However, the MPC presents a series of advantages over other method, amongst which the following:

- It is particularly attractive to staff with only a limited knowledge of control because the concepts are very intuitive and at the same time the tuning is relatively easy.
- It can be used to control a great variety of processes, from those with relatively simple dynamics to more complex ones, including systems with long delay times or non-minimum phase or unstable ones.
- The multi-variable case can easily be dealt with.
- It intrinsically has compensation for dead times.
- It introduces feed forward control in a natural way to compensate for measurable disturbances.

- The resulting controller is an easy-to-implement control law.
- Its extension to the treatment of constraints is conceptually simple, and these can be systematically included during the design process.
- It is very useful when future references (robotics or batch processes) are known.
- It is a totally open methodology based on certain basic principles which allows for future extensions.

There are three things which are known as the MPC elements and the first one is the prediction model, the objective function and lastly is obtaining the control law. The most crucial part is the prediction model. The model is the cornerstone of the MPC and it should be complete enough to fully capture the process dynamics and allow the prediction to be calculated, and at the same time to be intuitive and permit theoretic analysis. The different strategies of MPC can use various models to represent the relationship between the outputs and the measurable inputs. The prediction model can be divided into two parts which is the actual process model and the disturbance model. Both parts are needed for the prediction. The process model can be divided into 4 which is the state space, transfer function, step response and impulse response.

Some of the most popular methods will be reviewed in order to demonstrate their most outstanding characteristic. For example is the Dynamic Matrix Control (DMC), it uses the step response to model the process and only taking into account the first N terms, therefore assuming the process to be stable and without integrators. As regards the disturbances, their value will be considered to be the same as the instant all along the horizon. Model Algorithmic Control or also known as Model Predictive Heuristic Control (MPHC) is very similar to the DMC method with a few differences. Firstly, it uses an impulse response model where it is valid only for stable processes. Moreover, it makes no use of the control horizon concept so that in the calculation as much as control signals as future outputs appear. It introduces a reference trajectory as a first-order system which evolves from the actual output to the set point according to a determined time constant.

Another method being used is the state space model for Predictive Functional Control (PFC) for the case of fast processes. It allows for nonlinear and unstable linear internal model. Nonlinear dynamics can be entered in the form of a nonlinear state space

model and it has two distinctive characteristics which are the use of coincidence points and basis functions. The concept of coincidence points is used to simplify the calculation by considering only a subset of points in the prediction horizon. The desired and the predicted future outputs are required to coincide at these points, not in the whole prediction horizon. The controller parameterizes the control signal using a set of polynomial basis functions and this allows a relatively complex input profile to be specified over large horizon using a small number of parameters. Choosing the family of basis functions establishes many of the features of the computed input profile. These functions can be selected to follow a polynomial set point with no lag, an important feature for mechanical servo control applications

The methodology of all the controllers belonging to the MPC family is characterized by the following strategy, represented in Figures 1.

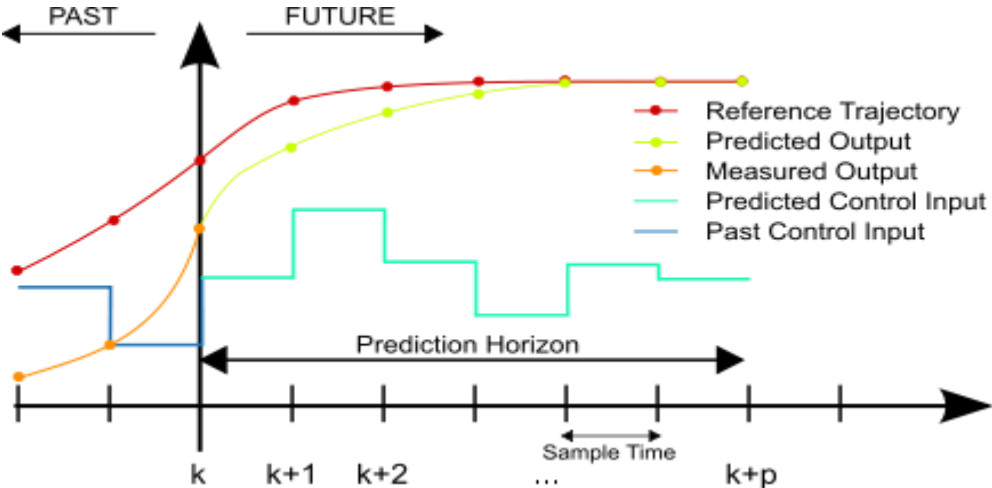


Figure 1: MPC strategy

The prediction horizon known as the predicted output behavior of a plant over future time interval. For a discrete time model, this means it predicts the plant output from the predicted output until sample time of $(k + p)$ based on all actual past control input $u(k), u(k - 1), \dots, \dots u(k - j)$ and the available current information of measured output. It means the input are calculated so that a set of predicted outputs reaches the set point in an optimal manner.

A sequence of control action adjustments to be implemented over a specified future time interval, which is known as the control horizon (m) is calculated by minimizing some specified objectives such as the deviation of the predicted output from set point over the prediction horizon and the size of control action adjustments in driving the process output to target plus some operating constraints. Although it will calculate at each sampling time, but only the first move is implemented and the other discarded and this procedure is repeated at each sampling instant. This theory is known as the receding horizon theory.

A nominal MPC is impossible as there is no model can constitute a perfect representation of the real plant. So, the prediction error, $\varepsilon(k)$ between the plant measurements $y_m(k)$ and the model prediction $\hat{y}(k)$ will always occur. The prediction error obtained is normally to update the future prediction. This error is also known as the residual. Figure 2 shows the block diagram of MPC.

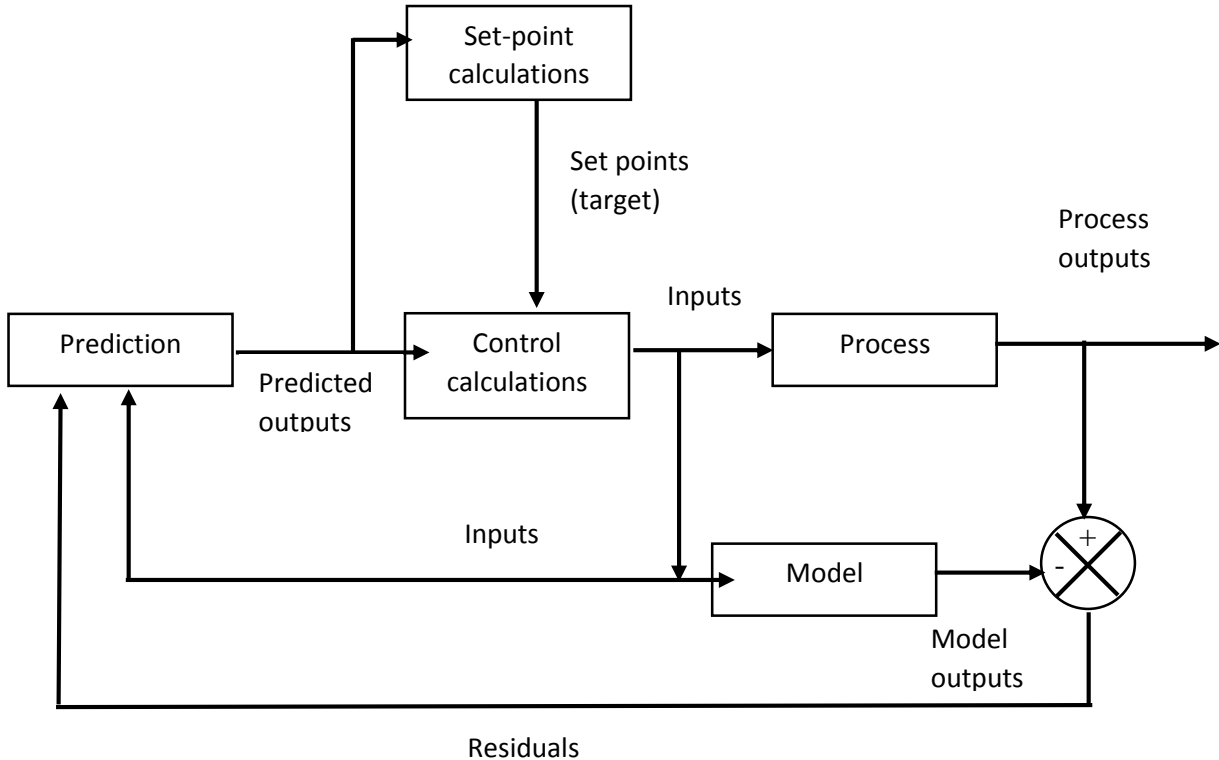


Figure 2: Block diagram for model predictive control.

2.2 Linear Model Predictive Control

MPC is actually a synonym to a Linear Model Predictive Control (LMPC) and the most MPC software available in the markets nowadays using linear models although the process are nonlinear. According to Qin and Bagwell (2003), the DMC from Aspen and HIECON from Adersa are using step convolution models and Finite Impulse model respectively. The reasons of using linear model is because it can be used in a straightforward manner from the process test data whereas it is very difficult to develop a generic nonlinear model from empirical data. Other than that, the Nonlinear Model Predictive Control (NMPC) will also counter a computational problem where it will become very complex, time consuming and sometimes non-convex. From a practical point of view, the conventional LMPC is acceptable in industries because most of the application of MPC to date are in refinery processing, where the process operates at a single set point and the primary use of controllers for disturbance rejection (Qin and Bagwell,2003).

2.3 State-Space Model

State space models can be used to formulate the predictive control problem. The main theoretical results of MPC related to stability come from a state space formulation, which can be used for both mono-variable and multi-variable processes and can easily be extended to nonlinear processes. The following equations are used in the linear case to capture the process dynamics:

$$x(t + 1) = Ax(t) + Bu(t) \tag{2.1}$$

$$y(t) = Cx(t)$$

In the single-input single-output (SISO) case, $y(t)$ and $u(t)$ are scalars and $x(t)$ is the state vector. A multiple-input multiple-output (MIMO) process has the same description but with input vectors u of dimension m and y of dimension n . An incremental state space model can also be used if the model input is the control increment $\Delta u(t)$ instead of the control signal $u(t)$. This model can be written in the general state space form taking into account that $u\Delta(t) = u(t) - u(t - 1)$. The following representation is obtained combining this expression with (2.1):

$$\begin{bmatrix} x(t+1) \\ u(t) \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x(t) \\ u(t-1) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u(t)$$

$$y(t) = [C \quad 0] \begin{bmatrix} x(t) \\ u(t-1) \end{bmatrix}$$

Defining a new state vector as $\bar{x}(t) = [x(t) \ u(t-1)]^T$, the incremental model takes the general form (2.1):

$$\begin{aligned} \bar{x}(t+1) &= M\bar{x}(t) + N\Delta u(t) \\ y(t) &= Q\bar{x}(t) \end{aligned} \tag{2.2}$$

Where the relationship between (M, N, Q) and the non-incremental form matrices (A, B, C) can easily be obtained by comparing (2.1) and (2.2). In order to minimize the objective function, output predictions over the horizon must be computed. If the incremental model is used, predictions can be obtained using (2.2) recursively, resulting in:

$$\hat{y}(t+j) = QM^j \hat{x}(t) + \sum_{i=0}^{j-1} QM^{j-i-1} N \Delta u(t+i)$$

Notice that the prediction needs an unbiased estimation of the state vector $x(t)$. If the state vector is not accessible, an observer must be included, which calculates the estimation by means of

$$\hat{x}(t|t) = \hat{x}(t|t-1) + K(y_m(t) - y(t-1))$$

Where $y_m(t)$ is the measured output. If the plant is subject to white noise disturbances affecting the process and the output with known covariance matrices, the observer become a Kalman filter and the gain K is calculated solving a Riccati equation. Now, the predictions along the horizon are expressed in vector form as

$$y = F\hat{x}(t) + Hu \tag{2.3}$$

Where $u = [\Delta u(t) \Delta u(t+1) \dots \Delta u(t+N_u-1)]^T$, the vector of future control increments, H is a block lower triangular matrix with its non-null elements defined by $H_{ij} = QM^{i-j}N$ and matrix F is defined as

$$F = \begin{bmatrix} QM \\ QM^2 \\ \vdots \\ QM^{N_2} \end{bmatrix}$$

Noticed that (2.3) is composed of two terms: the first depends on the current state and therefore is known at instant t , while the second depends on the vector of future control actions, which is the decision variable that must be calculated. The control sequence u is calculated minimizing the objective function, can be written as:

$$y = (Hu + F\hat{x}(t) - w)^T (Hu + F\hat{x}(t) - w) + \lambda u^T u$$

If there are no constraints, an analytical solution exists that provides the optimum as:

$$u = (H^T H + \lambda I)^{-1} H^T (w - F\hat{x}(t))$$

As a receding horizon strategy is used, only the first element of the control sequence, $\Delta u(t)$, is sent to the plant and all the computation is repeated at the next sampling time. Notice that a state observer is needed, since the control law depends on $\hat{x}(t)$. The optimal control sequence is generated by a static state feed-back law where the feedback gain matrix is computed via the solution of an algebraic Riccati equation.

Whatever kind of model is used, the control law is a static state feedback law that needs a state observer. In the case where constraints must be taken into account, the solution must be obtained by a Quadratic Programming algorithm.

2.4 Zero-Order Hold

For discrete-time control systems, it partially operates in discrete time and partially in continuous time. Therefore, in such a control system some signal will appear as a discrete time function and some appear as a continuous time function. According to Katsuhiko Ogata (1994), the data-hold is a process where it generates a continuous-time signal $h(t)$ from a discrete-time sequence $x(kT)$. The zero-order hold is the holding of amplitude of the sample from one sampling instant to the next. The other name for this zero-order hold is a clamper or staircase generator.

The figure 3 below show how the zero order hold operate.

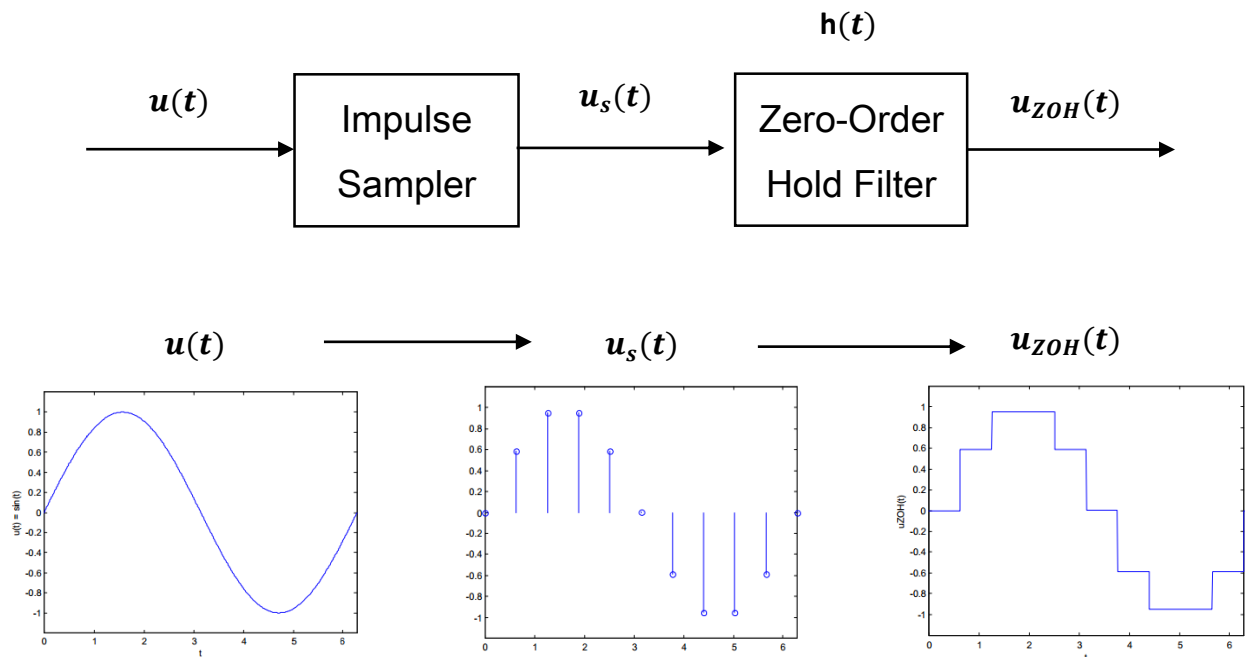


Figure 3: Zero Order Hold Graph

From the figure 3, the input, $u(t)$ is taken from the continuous time function and it will be discretize by the impulse sampler as $u_s(t)$. The function of the impulse sample is to hold each sample value for one sample interval. It is known as the sampling time. From this sampling time, the zero order hold will calculate the first point of each sampling time and make it constant until it reach the next one. The result will be like a stairs which can be seen in the final graph, $u_{ZOH}(t)$.

2.5 First-Order Hold

First-order hold element method is the same method as the zero-order for MPC except it is not usually used in control systems. However, according to Katsuhiko Ogata (1994), he said that it is worthwhile to see what the transfer function of the first-order hold may look like. In this first-order, the signal is reconstructed as a piecewise linear approximation to the original signal that was sampled. There are three types of first-order which is the basic first-order hold, delayed or causal first-order hold and lastly is the predictive first-order hold. For the algorithm that will be shown is the basic first-order hold.

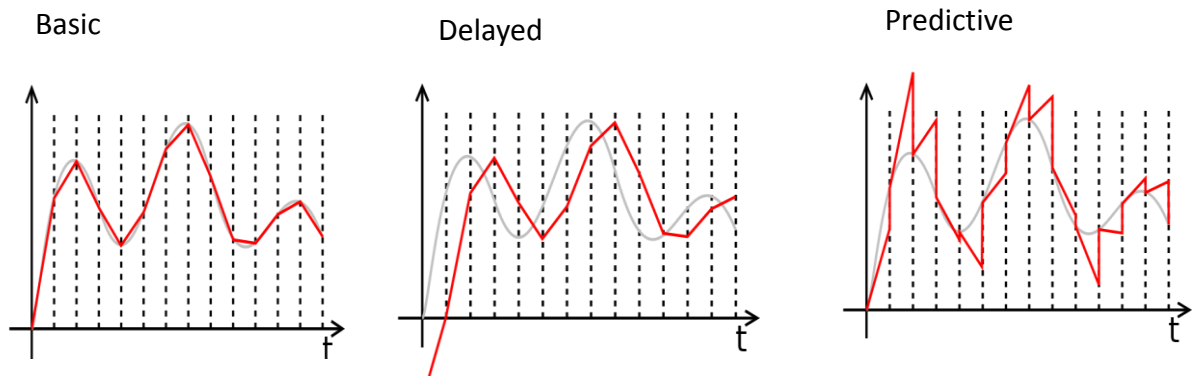


Figure 4: Three types of First Order Hold Graph

It needs to be started from the hold circuit which will convert the sampled signal into a continuous-time signal. The continuous-time input signal $h(t)$ obtained by use of the first-order hold is a piecewise-linear signal shown in figure 4.

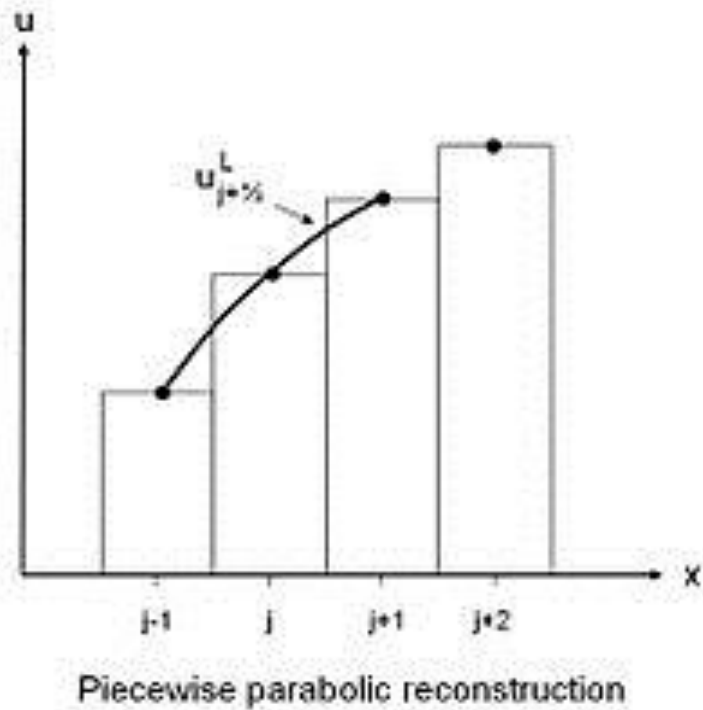
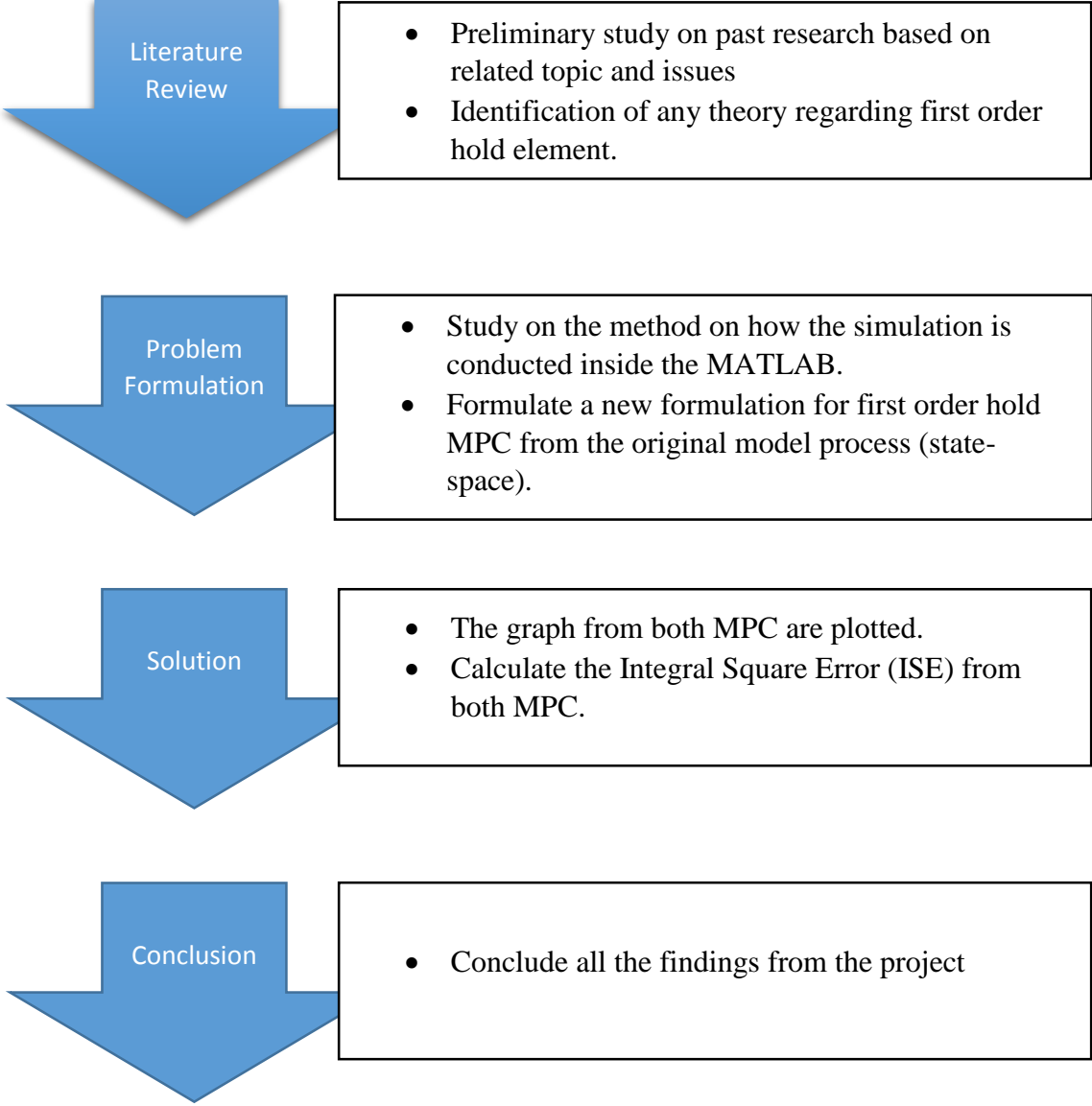


Figure 5: Piecewise-linear signal graph.

The first order hold element will have a better performance in the MPC because it will calculate the gradient from one sampling time to another. Therefore, the control valve will move smoothly into the next sampling time if it's been using the first order hold MPC instead of zero order hold MPC.

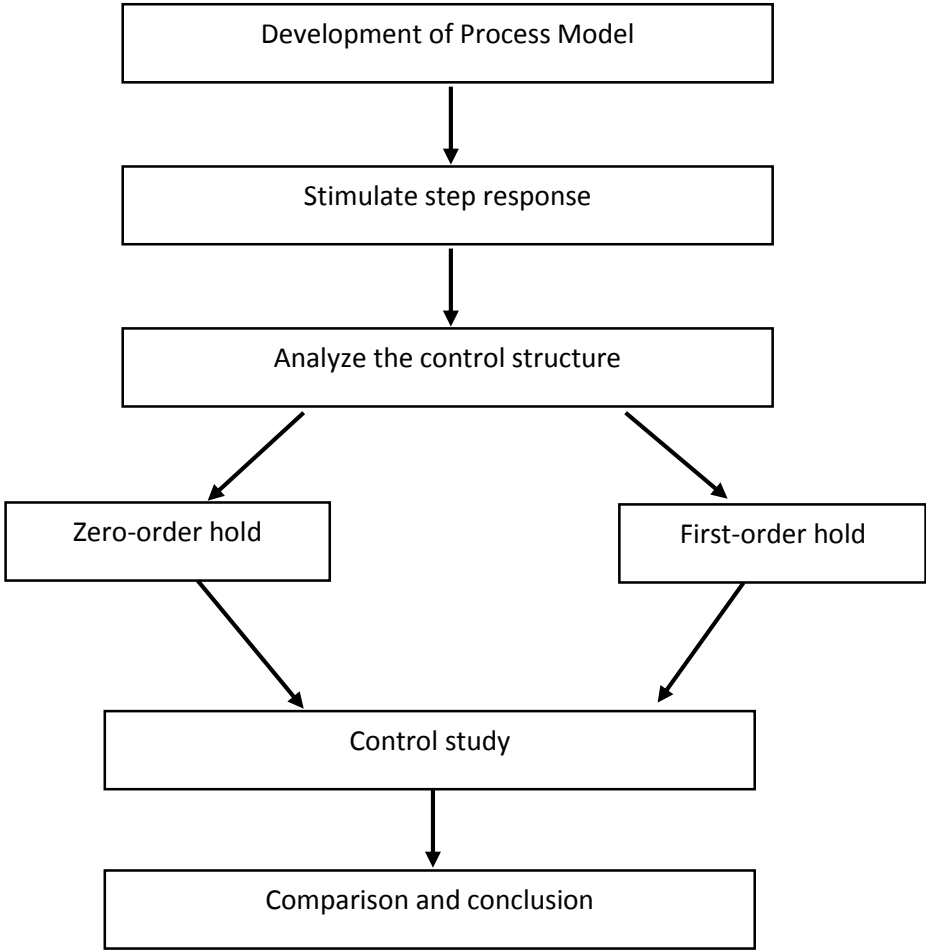
3.0 RESEARCH METHODOLOGY

3.1 Process Flow Chart



3.2 Research Methodology

The main purpose of this study is to compare the performance of different method in MPC calculation which is the zero-order hold and first-order hold. Firstly, the process model is developing for the first order hold MPC. Next is to stimulate the step response and analyze the control structure. From the zero order hold and first order hold, the graph will be plotted. Lastly, the comparisons of the result and give the conclusion based on the plotted graph and also the Integral Square Error (ISE) calculation. From this ISE calculation, we can determine which MPC will give a better performance. The figure below shows the methodology for the study to be done.



3.3 Tools

In this study, MATLAB will be used to stimulate the performance for the first order hold MPC. MATLAB is a high-level language and interactive environment for numerical computation, visualization and programming. By using MATLAB, it can analyze data, develop a process model and calculate the ISE. The result from this MATLAB simulation will be the graph that shows the performance between first order hold MPC and zero order hold MPC.

3.4 Key Milestones

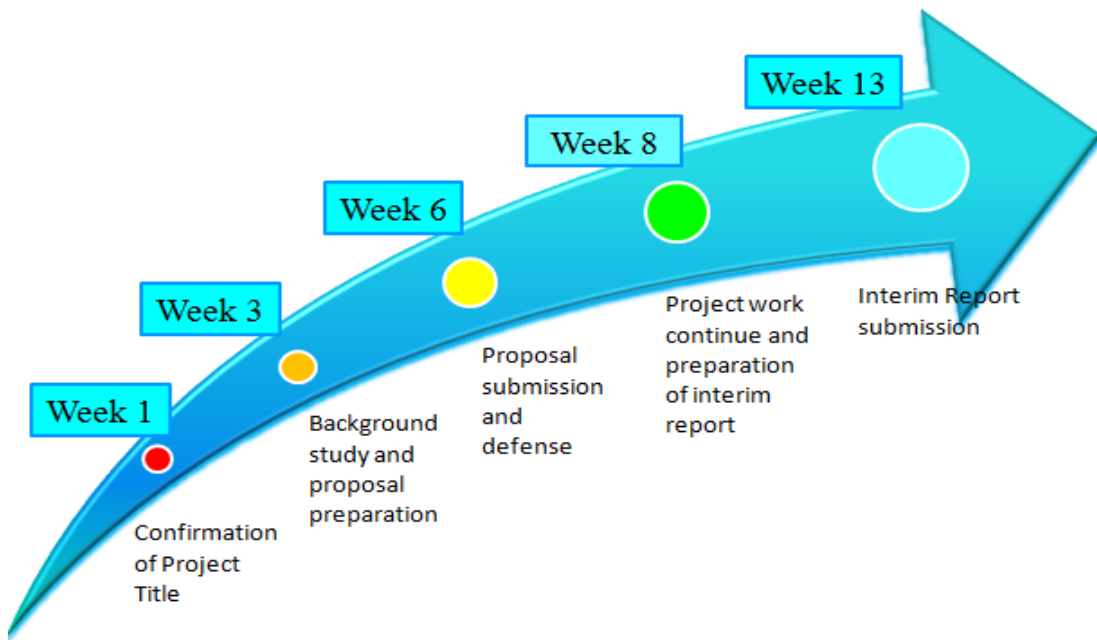


Figure 6: The milestone for this project.

3.5 Gantt-Chart

FYP 1		Week													
No	Detail Work	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Selection of project title	■	■												
2	Preliminary research work and proposal preparation			■	■	■									
3	Extended proposal submission						■								
4	Proposal defense							■							
5	Project work continue								■	■	■	■			
6	Submission of interim draft report												■		
7	Submission of final interim report													■	■
FYP 2		Week													
No	Detail Work	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Project work continues	■	■	■	■	■	■	■							
2	Submission of progress report								■						
3	Project work continues								■	■	■	■			
4	Pre-SEDEX											■			
5	Project work continue												■		
6	Submission of draft report												■		
7	Submission dissertation (soft bound)													■	
8	Submission of technical paper													■	
9	Oral Presentation														■
10	Submission of project dissertation (hard bound)														■

Figure 7: The Gantt chart

4.0 RESULT AND DISCUSSION

As what have been told in the previous chapter, the result will be discussed and compare between Zero Order Hold (ZOH) method and also First Order Hold (FOH) method. So, the design of the plant is simple which is the single input single output (SISO), linear equation and also assumed that there is no constrains available. The design of the plant will be discussed in the next subchapter.

For this project the new coding is being implemented in the simulation as to get the good result which is equivalent to the first order hold theory. The problem is that the MPC Toolbox inside the MATHLAB is already been set to use the zero order hold for the MPC system. Therefore, the new code need to formulate the first order hold MPC and then can compare the result with the zero order hold MPC.

4.1 Control Design

The control design that's been used in my project is by using the state-space. This is one other way that can be used in the simulation to show that the system is a linear differential equation. The advantage of using the state-space control is that it can be used in multi-variable processes as long as it is a straightforward manner. The state-space has been introduced to make the system to be SISO. However, it can also be used to be MIMO but in this project, it just needs to be SISO. For SISO LTI system, the state-space form is given below (Qin and Badgwell, 2003):

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

Where the x is a n by 1 vector representing the state, u is the scalar representing the input and y is a scalar representing the output. The things that determine the relationships between the state and input and output variables is the matrices of A (n by n), B (n by 1) and C (1 by n).

Other than state-space, there are also other process model such as the impulse response, step response and transfer function. Mostly the impulse and step response is being used in industries as it clearly reflects the influence of each manipulated variable on a determined output.

The original system is taken from the control design itself which is the state-space algorithm. To create the Model Predictive Control, the algorithm must be same with the control design so that it can operate smoothly with the plant. So, below is the original system from the control design.

$$x(k + 1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k)$$

For the augmented MPC, we have to introduce a new augmented inside the algorithm which is from the Ax, Bx and Cx with the A_{aug} , B_{aug} , and C_{aug} .

$$A_{aug} = \begin{bmatrix} A & 0_{nx \times ny} \\ C_m & 0_{ny \times ny} \end{bmatrix}$$

$$B_{aug} = \begin{bmatrix} B_m \\ 0_{ny \times nx} \end{bmatrix}$$

$$C_{aug} = [0_{ny \times nx} \quad I_{ny \times nx}]$$

Lastly, the augmented Model Predictive Control been produce with the equation below. Thus the new Model Predictive Control has been made and it will apply the First Order Hold method instead of Zero Order Hold.

$$\begin{bmatrix} x(k + 2) \\ y(k + 1) \end{bmatrix} = A_{aug} \begin{bmatrix} x(k + 1) \\ y(k) \end{bmatrix} + B_{aug} \Delta u(k)$$

$$y(k) = C_{aug} \begin{bmatrix} x(k + 1) \\ y(k) \end{bmatrix}$$

And the input signal is

$$u(k) = \Delta u(k)t + u(k - 1)$$

The algorithm of the augmented Model Predictive Control is a very crucial part of this research as it will determine the result of how does the first order hold MPC works. The reason of creating the augmented MPC is because the calculation for Δu which is known as the gradient. From the original system, it will not get the gradient from the system because it just only the input, u . Thus, this Δu will make the system to calculate in the first order hold.

4.2 Methodology

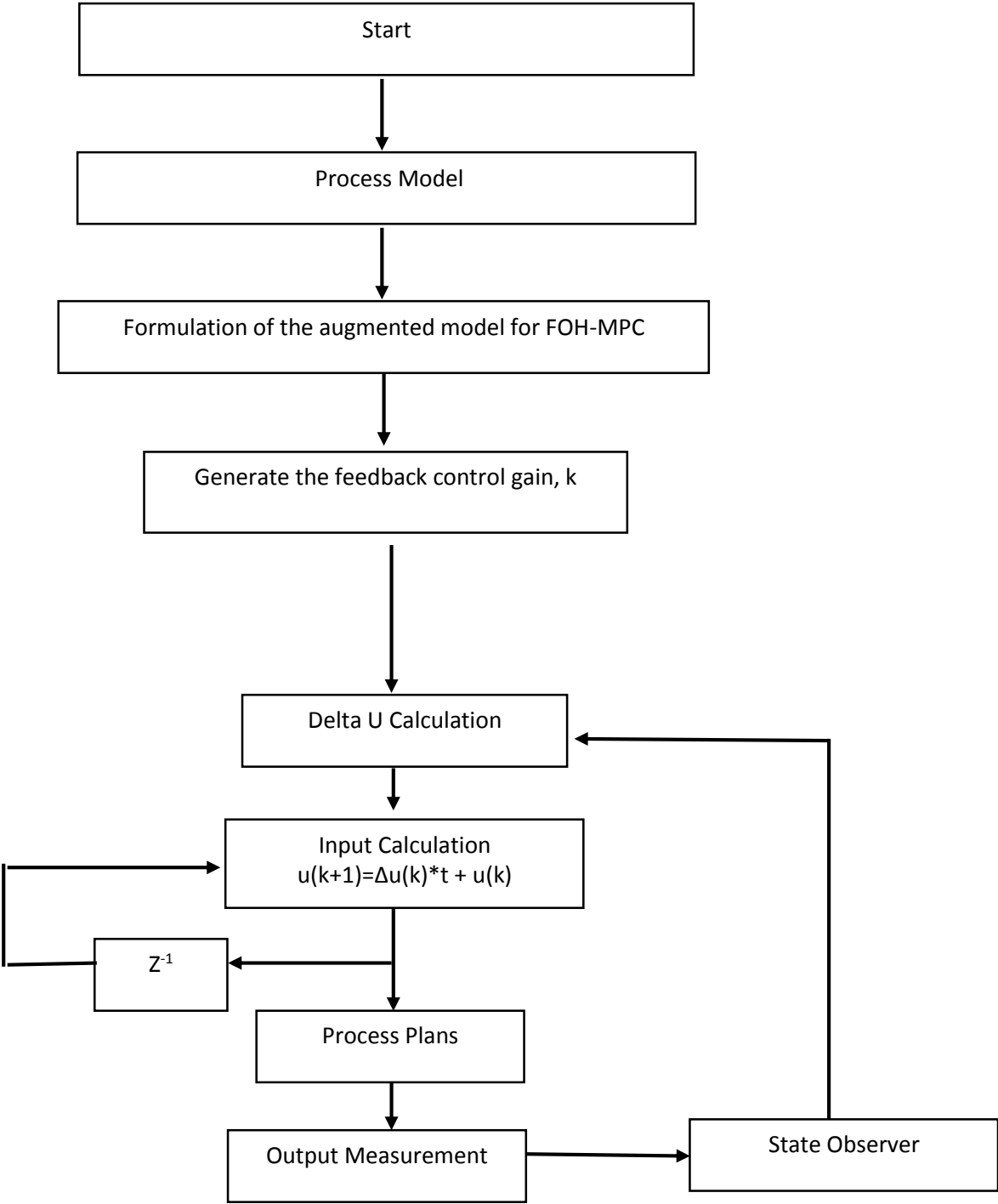


Figure 8: Methodology to create new Model Predictive Control

The methodology of the project starts with designing the process model which is the state-space control. For the process model, the algorithm of the state-space model has been used in the augmented model for new MPC of the first order hold. The new algorithm of augmented MPC first order hold will generate the feedback control gain (k). This process will occur in the offline mode. There are two modes which are offline and online where the offline is calculated outside of the system. If it is in the online, it will directly calculated and sent to the plant system and operate. The reasons that the generating of k value offline because the system for my project is a linear without any constraint.

After the k value has been calculated, the system inside the online mode will calculate the delta u. The delta u is the continuous signal of the first order hold. Other than that, it is also known as the gradient (m or t) for the first order hold order. Next step is to calculate the input of the first order hold which can be derive as $u(k + 1) = \Delta u(k) * t + u(k)$. The u(k) in the calculation is the previous input and the system will calculate all the new input for the first order hold order. The input calculation will be sent to the process plan as the u(k+1). Then the new input, u(k+1) will be sent back to the input calculation by introducing it to lag element, Z^{-1} . When the input has been calculated, the process plan will come out the output measurement and lastly is the state observer will calculate the state from the output measurement and sent it back to the delta u calculation. From the output measurement, it will plot the graph known as the output.

4.3 Input and Output Result

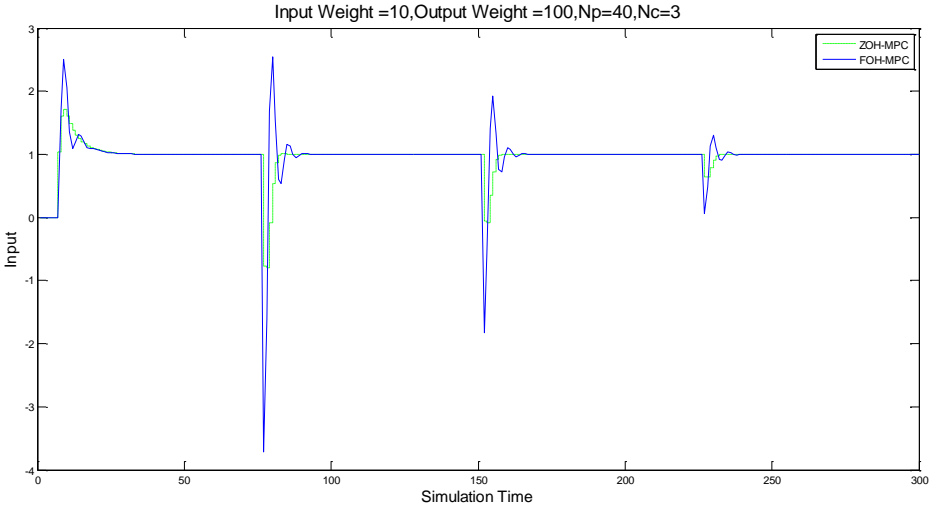


Figure 9: Input result for both First and Zero Order Hold

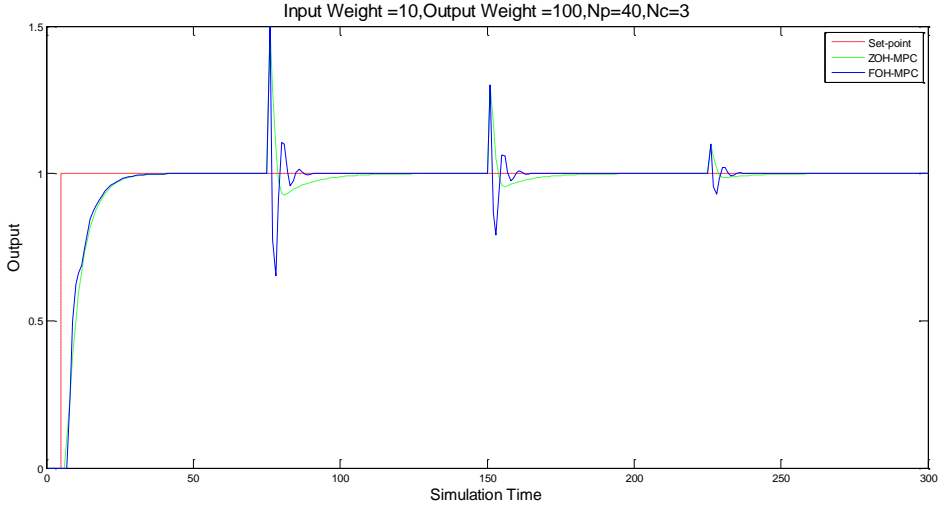


Figure 10: Output result for both First and Zero Order Hold

The result shows that the Model Predictive Control of First Order Hold show a positive behavior as it react faster than Zero Order Hold. The above result is generated at the control horizon, (N_c) of 3 and prediction horizon, (N_p) of 40. Most of the MPC are implemented the N_c from the range of 2 to 5 and the N_p is the (time constant x 4) and the time constant being used for augmented MPC is 10. The weight for input and output also been introduced to control the aggressiveness of the system. For the result shown, the input weight is 10 and output weight is 100. The input weight is lower than

the output weight because we want the aggressive work on the outcome which is the output. The data of this weight can be changed depends on the situation of our system.

The simulation time has been held for about 300. From the figure 11, we can see that the MPC is trying to operate by getting back to the set-point line at a time of 10, 75, 150 and 225. At 10, the set-point change has been introduced to the system which is from 0 to 1. So the new set-point line is 1 and the MPC is trying to reach it and maintain the result. Moreover, at the time of 10, 75, and 150, the disturbance has been introduced to the system. The disturbance has been introduced in different number. For example at the time of 10, the disturbance is 1.5, at the time of 75 the disturbance is 1.2 and lastly the disturbance is 1.1. From those changes made by the MPC in figure 11, it clearly states that the formulation for the new MPC which is using First Order Hold are functioning very well.

4.4 Discussion

4.4.1 Servo and Regulatory Problems

In the discussion, we will take a look at the figure 12 where the output graph has been plotted. From the figure 12, the blue line indicates the first order hold while the green line is the zero order hold which been simulate by the MPC inside the simulation itself. For your information, the input that has been introduced for both first order hold MPC and zero order hold MPC are the same as we want to discover which one produce a better result. The red line indicates the set-point that have been targeted which is from zero to one.

The servo problem can be determined when the set-point has been changed. For example in this project, the set-point has been changed from 0 to 1. The reasons for changing this set-point is to determine whether the both MPC can solve the servo problem or not. Thus, the base of the result figure 12, when the new set-point has been introduced which is 1, the MPC has shown a good result by move smoothly to the next new set-point. Moreover, the first order hold MPC shown a better result compared to the zero order hold MPC in term of the how fast the reaction taken to reach the new set-point. Thus, one of

the advantages of first order hold MPC is it solved the servo problems and also react faster than the zero order hold MPC.

Regulatory problem can be determined when we introduce any disturbances to the system and see whether the system can be brought back to steady state again which the set-point is. From the result of figure 12, there are three disturbances have been introduced at different number. The first one is being introduced about 1.5, followed by 1.2 and 1.1. Both MPC shows a good result where it does not ignore the changes that had been introduced by bringing back the output to its original set-point. From the result of figure 12, the first order hold MPC shows a much better result compare to zero order hold MPC in terms of how fast the reaction taken to come back to the original set-point after the disturbance been introduced. This advantages have given a big credit to the first order hold MPC.

4.4.1 Integral Squared Error (ISE)

The integral squared error (ISE) is the calculation of integrating the square of the system error over a fixed interval of time and this method are commonly used in linear optimal control and estimation theory (LING *et al*, 2011). To calculate the ISE, it need to get the Error vs Time data in the Simulink. The formula used to calculate the ISE is:

$$ISE = \int_0^{\infty} |e(t)|^2 dt$$

If take look closer to the figure 12, the graph of first order hold MPC is oscillating to reach the set-point line. Thus, the area under the graph is known as the e(t). For ISE calculation, the area under the graph need to be squared and the sum of it will determine the error of the system. Theoretically, according to LING *et al* the smaller the calculation of the ISE, the better and faster the performance of the system. For this project the calculation of the ISE has been calculated using the Simulink by taking the result of the output graph and apply the formula stated above. The ISE for first order hold MPC is 8.696 and for zero order hold MPC is 14.34. Thus, this shows that the first order hold MPC are better compared to zero order hold MPC in term of performance and error.

5.0 CONCLUSION AND RECOMMENDATION

As a conclusion, this project is important as it has produced a new Model Predictive Control using the first order hold element which can be used in industries and improve the performance of the controller. The objectives of this project have been completed successfully within the time limit.

From the MPC Toolbox, it has given me an important basic on doing the new code for the new MPC which is the first order hold. With the guide from the PHD student, Mr. Abdelraheem Faisal, we have accomplished a designing of the new MPC which operate the first order hold instead of zero order hold. The control design that's been use is the state-space and the plant is the SISO system.

The comparison from both MPC shows that the first order hold have more advantages than the other one. When the changes have been made to the set-point, the first order hold MPC reacts faster than zero order hold and have solved the servo problem. Moreover, it also reacts very fast when the disturbances has been introduced to the system and this have solved the regulatory problem of the new MPC. The Integral Square Error (ISE) also have been calculated for both MPC and the first order hold show lower result compare to zero order hold. Thus, the performances of first zero order MPC are better the other one.

By using the method of the first order hold, it calculated the gradient value until it reach next sampling time. However, the result from this new MPC with first order hold has been formulated and it is still not the real first order hold. The result that have been plotted is equivalent to the real result of the first order hold MPC theory.

For the future recommendation, the study and research on this paper need to move on as there is some problems regarding the coding of the new MPC. There are still some changes that can be done, mostly on the part of the formulation inside the simulation. However, there is still some modification that can be made to make the new MPC is better than everywhere.

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