DYNAMIC MODELING AND CLOSED-LOOP CONTROL OF A TWIN ROTOR MIMO SYSTEM

By

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FINAL PROJECT REPORT

Submitted to the Electrical & Electronics Engineering Programme
in Partial Fulfillment of the Requirements
for the Degree
Bachelor of Engineering (Hons)
(Electrical & Electronics Engineering)

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CERTIFICATION OF APPROVAL

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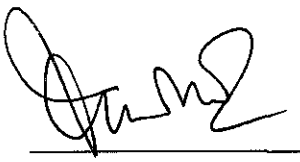
Approved:



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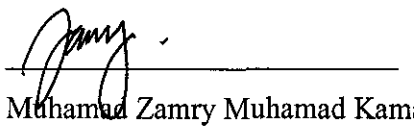
June 2008

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.



Muhamad Zamry Muhamad Kamaruzaman

ABSTRACT

The Twin rotor MIMO system (TRMS) is an aero-dynamical model of helicopter with significant cross-couplings between longitudinal and lateral directional motions. Its behavior in certain aspects resembles the real of a helicopter. Firstly, open loop control is implemented both for tail and main rotor to get the relationship of input and output of the system. Open-loop control is often the preliminary step for development of more complex feedback control laws. Next step was model identification as it is a well established technique for modeling of complex systems whose dynamics are not well understood or difficult to model from the first principles. State feedback controllers were designed by pole placement method for both rotors independently. The model then can be implemented in real-time experiments of the Twin Rotor MIMO System.

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank my supervisor Associate Professor Dr. Mohd Noh Karsiti for his encouragement, support and for being such a nice person to work with. Thank you for giving me advice and put faith in my capability to complete this project

My special thanks to Mr. Mahidzal Dahari, Teaching Assistant in Modern Control Engineering course for advice and guidance in my final year project. It is great pleasure to see this project succeed and can be completed within the given time.

I am indebted to Mr. Azhar Zainal Abidin, Technologist in Control System Laboratory course who supports me in technical area especially in handling the Twin Rotor MIMO System.

I would also like to thank my family for their support and never ending love since this project started. It was important internal support that I am really appreciated. Thank you also to those people who are not mentioned here.

TABLE OF CONTENTS

LIST OF TABLES ix
LIST OF FIGURES x
LIST OF ABBREVIATIONS xii
CHAPTER 1 INTRODUCTION 1
1.1 Background of Study 1
1.2 Problem Statement 2
1.3 Objectives and Scope of Study 2
1.3.1 Objectives of the Project 2
1.3.2 Scope of the Project 2
CHAPTER 2 LITERRATURE REVIEW 3
2.1 Twin Rotor MIMO System 3
2.1.1 Operation Modes 4
2.1.2 Helicopter PID 5
2.2 Control Systems 6
2.2.1 Closed-loop control 6
2.2.2 Open-loop control 7
2.3 Time response: second order systems 8
2.4 State space design 9
2.4.1 Pole placement using state feedback 10
CHAPTER 3 METHODOLOGY 11
3.1 System Modeling and System Identification 12
3.1.1 System Modeling 12
3.1.2 System Identification 13
3.2 State Feedback Controller Design 14
CHAPTER 4 RESULTS AND DISCUSSIONS 15
4.1 System Modeling and System Identification 15
4.1.1 Tail Rotor Modeling and Identification 15
4.1.2 Main Rotor Modeling and Identification 19
4.2 State Feedback Controller Design 22
4.2.1 Tail Rotor State Feedback Controller Design 22
4.2.2 Main Rotor State Feedback Controller Design 25

4.3 1-DOF Tail Rotor 28
4.3.1 Tail Rotor Model Simulation 28
4.3.2 Tail Rotor Real Time Experiment 29
4.4 1-DOF Main Rotor 31
4.4.1 Main Rotor Model Simulation 31
4.4.2 Main Rotor Real Time Experiment 32
4.5 2-DOF Tail and Main Rotor 34
4.6 Discussion and Findings 37
CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS 43
5.1 Conclusion 43
5.2 Recommendation 44
REFERENCES 45
APPENDICES 46
Appendix A Design formulas for second-order systems 47

LIST OF TABLES

Table 1 Parameter responses of tail rotor..... 16
Table 2 Parameter responses of main rotor..... 19

LIST OF FIGURES

Figure 2.1: Schematic diagram of TRMS 3
Figure 2.2: TRMS showing location of locking screws 4
Figure 2.3: Block diagram of Helicopter PID 5
Figure 2.4: Close loop control 6
Figure 2.5: Open Loop Control 7
Figure 2.6: Second-order underdamped response specifications 8
Figure 2.7: State space model 9
Figure 2.8: State space model with feedback 10
Figure 4.1 Tail rotor block diagram 15
Figure 4.2 System response of tail rotor 16
Figure 4.3: Tail rotor model identification 17
Figure 4.4: Model and system step responses of tail rotor before identification 17
Figure 4.5 Model and system step responses of tail rotor after identification 18
Figure 4.6: Main rotor block diagram 19
Figure 4.7: System response of main rotor 19
Figure 4.8: Main rotor model identification 20
Figure 4.9: Model and system step responses of main rotor before identification 20
Figure 4.10: Model and system step responses of main rotor after identification 21
Figure 4.11: Desired region in the complex plane for locating the dominant system poles of tail rotor model 23
Figure 4.12: Tail rotor state space model 24
Figure 4.13: Desired region in the complex plane for locating the dominant system poles of main rotor model 26
Figure 4.14: Main rotor state space model 27
Figure 4.15: Output tail rotor model simulation with saw tooth wave input 28
Figure 4.16: Output tail rotor model simulation with sine wave input 28
Figure 4.17: Output tail rotor model simulation with square wave input 28
Figure 4.18: Block diagram of tail rotor real time experiment 29
Figure 4.19: Output tail rotor real time experiment with saw tooth wave input 29
Figure 4.20: Output tail rotor real time experiment with sine wave input 30
Figure 4.21: Output tail rotor real time experiment with square wave input 30
Figure 4.22 Output main rotor model simulation with saw tooth wave input 31

Figure 4.23: Output main rotor model simulation with sine wave input 31
Figure 4.24: Output main rotor model simulation with square wave input 31
Figure 4.25: Block diagram of main rotor real time experiment 32
Figure 4.26: Output main rotor real time experiment with saw tooth wave input 32
Figure 4.27: Output tail rotor real time experiment with sine tooth wave input 33
Figure 4.28: Output tail rotor real time experiment with square wave input 33
Figure 4.29 Block diagram of TRMS controller 34
Figure 4.31 Output tail and main rotor real time experiment with sine wave input ... 35
Figure 4.32 Output tail and main rotor real time experiment with square wave input 35
Figure 4.33 Output tail and main rotor real time experiment with saw tooth wave input 36
Figure 4.34: Transient response of poles at -3±j2 39
Figure 4.35: Transient response of poles at -0.5±j2 40
Figure 4.36: Output main rotor simulation with step input (before using slider gain) 41
Figure 4.37: Main rotor state space model by using slider gain 41
Figure 4.38: Output main rotor simulation with step input (after using slider gain) .. 42

LIST OF ABBREVIATIONS

TRMS	-	Twin Rotor MIMO System
DOF	-	Degree of Freedom
PID	-	Proportional-Integral-Derivative

CHAPTER 1
INTRODUCTION

1.1 Background of Study

Nowadays, we can see recent advance in aircraft technology has led to various development of aircraft concept. The technology significantly is challenge to all control engineers. The design may start from modeling the model of the helicopter first. For this matter, we can model it from hardware that is called Twin Rotor MIMO System (TRMS).

Dynamic Modeling and Closed-Loop Control of a Twin Rotor MIMO System (TRMS) basically needs process of modeling and designing control strategy to control the behavior of the TRMS.

The TRMS is a laboratory set-up designed for control experiments by Feedback Instruments Ltd. TRMS provides us a high a high order, non linear system with significant cross coupling between longitudinal and lateral directional motions. It consists of two rotors; main and tail rotors with each rotors influencing both positions angles. Main rotor has movement in vertical plane and tail rotor responsible for the movement in horizontal plane.

TRMS is very useful in modeling as it gives us the relationship between input and output of the system. Since this project requires the development of a control strategy to control the behavior of the TRMS, hence the TRMS need to be modeled first before the control strategy can be introduced to the system. System modeling is a technique to express, visualize, analyze and transform the architecture of TRMS.

1.2 Problem Statement

The Twin Rotor MIMO System (TRMS) is a laboratory set up designed for control experiments. From the control point of view, it exemplifies a high order non-linear system with significant cross-couplings between longitudinal and lateral directional motions. It moves simultaneously in both the horizontal and vertical planes. It is said to have two degrees-of-freedom (DOF). Since there is no natural way to split 2-DOF complex model into two independent parts, 1-DOF models for both horizontal and vertical plane need to be build.

1.3 Objectives and Scope of Study

1.3.1 Objectives of the Project

By the end of this project, the expectations to be archive are:

- To obtain linearized models for 1-DOF main rotor and 1-DOF tail rotor of the TRMS
- To design a controller for TRMS based on the system modeling and identification.

1.3.2 Scope of the Project

The scope of study of this project gives focus on controller development of state-feedback controller using pole placement method.

CHAPTER 2
LITERRATURE REVIEW

2.1 Twin Rotor MIMO System

The Twin Rotor MIMO System (TRMS) is a laboratory set-up designed for control experiments. Its looks like a simple model of a helicopter. TRMS consists of tail rotor and main rotor. Tail rotor will give motion for TRMS in horizontal plane while main rotor moves the system in vertical plane. From the control perspective, TRMS exemplifies a high order non-linear system with significant cross-couplings.

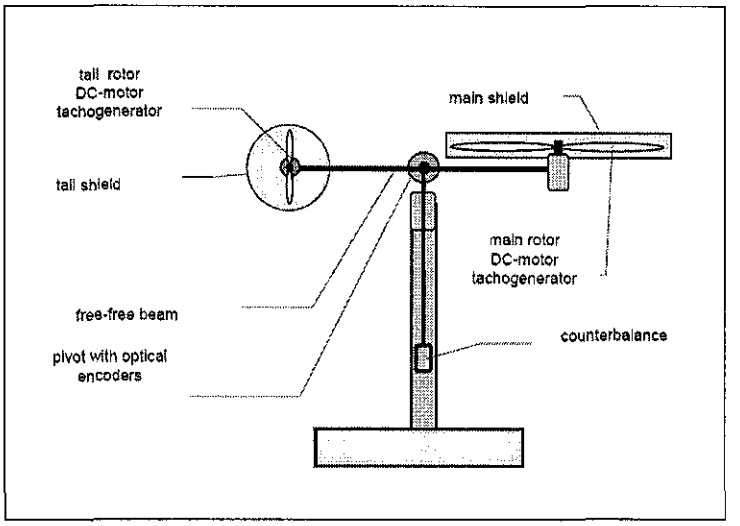


Figure 2.1: Schematic diagram of TRMS [4]

The system has a beam pivoted on its base so it can rotate freely both in the horizontal and vertical planes. The tail and main rotor are driven by DC motors.

The different between real helicopter and TRMS is the aerodynamic force of a real helicopter is controlled by changing the angle of attack while the aerodynamic of TRMS is controlled by varying the rotors speed. When there is change in the voltage

value applied to the DC motors, there will be a change of the rotation speed of the propeller which results in a change of the corresponding position of the beam. Besides, a real helicopter is free to move but the TRMS is anchored to its pillar.

2.1.1 Operation Modes

The TRMS can operate in three different modes which are:

- A 1-DOF system using only the tail rotor – horizontal motion
    - By manually tightening horizontal axis locking screw
- A 1-DOF system using only the main rotor – vertical motion
    - By manually tightening vertical axis locking screw
- A 2-DOF system using the tail and main rotor – horizontal and vertical motion
    - By releasing both vertical and horizontal locking screws

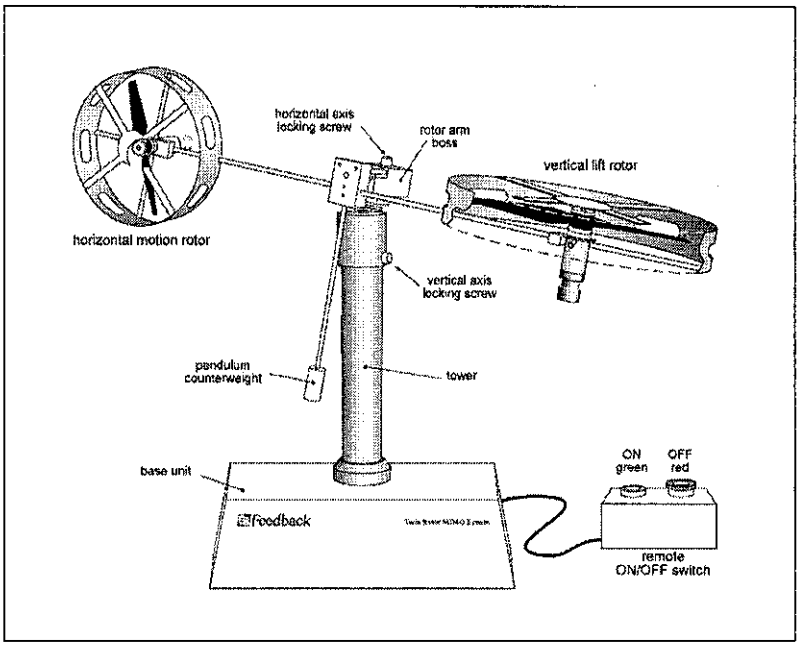


Figure 2.2: TRMS showing location of locking screws [4]

2.1.2 *Helicopter PID*

Helicopter PID is the simulink block diagram for the TRMS provided by Feedback Ltd. PID controller has been introduced to control the behavior of the system. The general PID form for the helicopter PID is

$$u(t)=K_p e(t)+K_i\int_0^t e(\tau)d\tau + K_d\frac{d}{dt}(e(t))$$

Where *u(t)* is the control output, and the error *e(t)* is the difference between desired value and measured value of quantity being controlled.

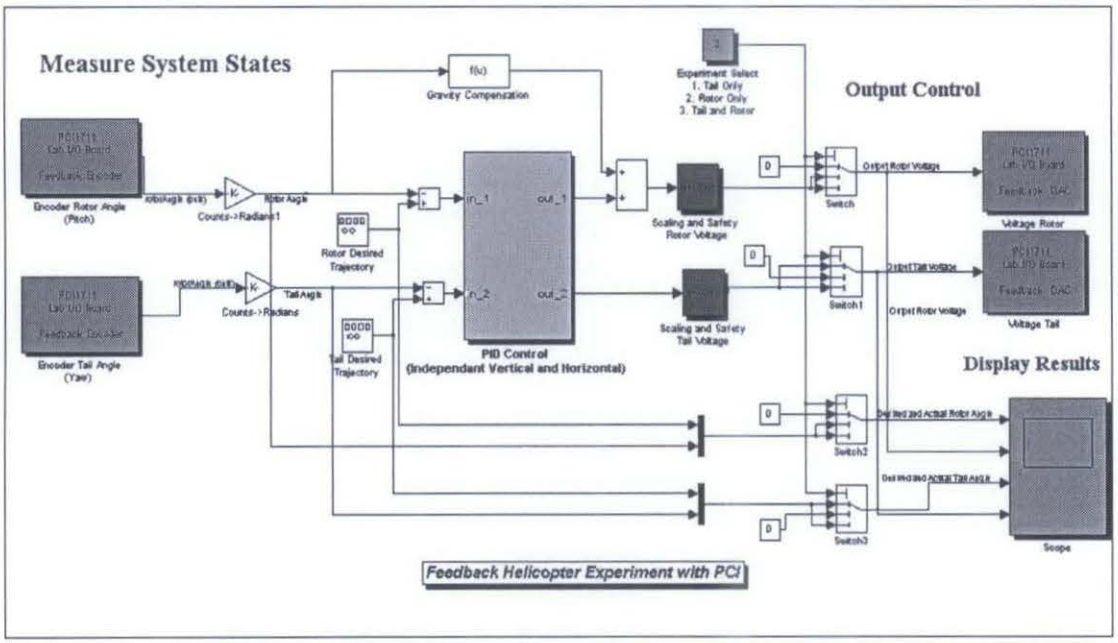


Figure 2.3: Block diagram of Helicopter PID

General description and operation of TRMS has been discussed in Feedback *Instrument Ltd,Twin Rotor MIMO System – User Manual 33-007-1C [4]*

2.2 Control Systems

A closed-loop control system is one in which an input forcing function is determined in part by the system response. The measured response of a physical system is compared with a desired response. The difference between these two responses initiates actions that will result in the actual response of the system to approach the desired response. This in turn drives the difference signal toward zero. Typically the difference signal is processed by another physical system, which is called a compensator, a controller, or a filter for real-time control system applications.

2.2.1 Closed-loop control

- Shows a closed-loop action (closed control loop);
- Can counteract against disturbances (negative feedback);
- Can become unstable, i.e. the controlled variable does not fade away, but grows (theoretically) to an infinite value.

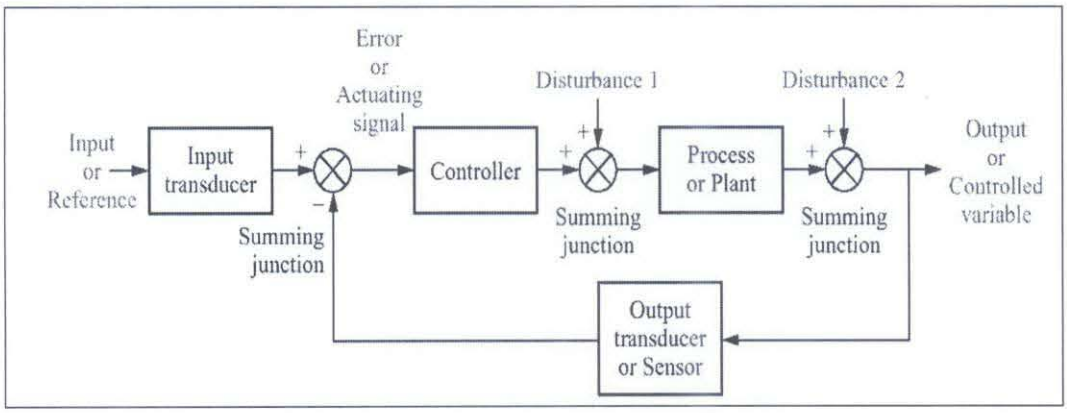


Figure 2.4: Close loop control [1]

2.2.2 *Open-loop control*

- Shows an open-loop action (controlled chain);
- Can only counteract against disturbances, for which it has been designed; other disturbances cannot be removed;
- Cannot become unstable - as long as the controlled object is stable.

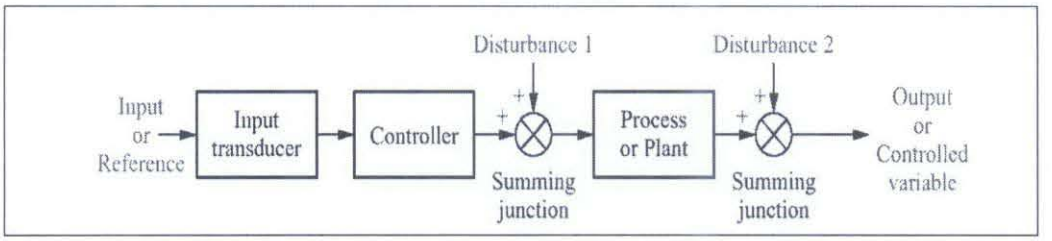


Figure 2.5: Open Loop Control [1]

To avoid the problems of the open-loop controller, control theory introduces feedback. A closed-loop controller uses feedback to control states or outputs of a dynamical system. Its name comes from the information path in the system: process inputs (e.g. voltage applied to an electric motor) have an effect on the process outputs (e.g. velocity or torque of the motor), which is measured with sensors and processed by the controller; the result (the control signal) is used as input to the process, closing the loop.

In some systems, closed-loop and open-loop control are used simultaneously. In such systems, the open-loop control is termed feed forward and serves to further improve reference tracking performance.

General description of control systems has been discussed in N.S.Nise, "Control Systems Engineering", 4th. Ed., Wiley, 2004. [1]

2.3 Time response: second order systems

There are two physically meaningful specifications for second-order system:

1. Natural frequency, ω_n: the frequency of oscillation of the system without damping
2. Damping ratio, ζ : exponential decay frequency over natural frequency

Then, we will have general second-order transfer function which is:

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

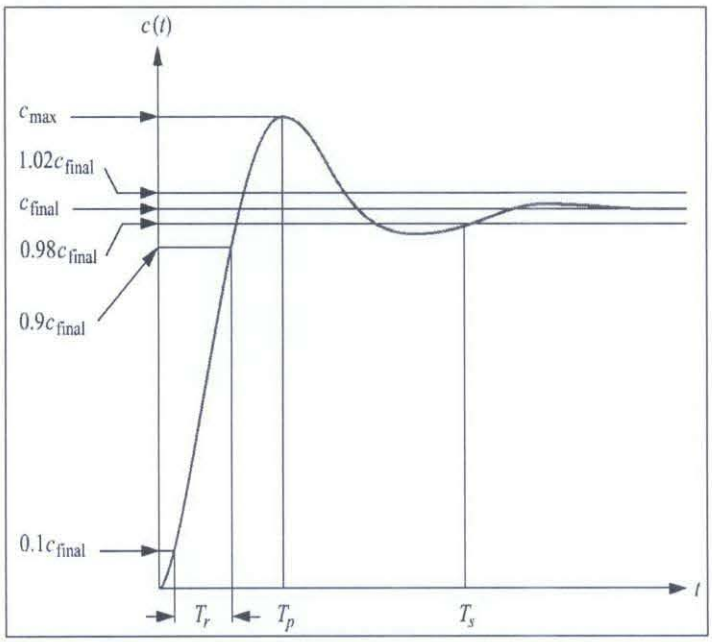


Figure 2.6: Second-order underdamped response specifications [1]

Performance Measures (for Underdamped Systems):

1. Peak Time, T_p: The time required to reach the maximum peak
2. Overshoot, %OS: The amount that the waveform overshoot the steady-state or final value at the peak time.
3. Settling Time, Ts: The time required for the oscillations to reach and stay within 2% of the steady-state value

2.4 State space design

Before this, in conventional approach, people designs a controller (compensator) which is has the desired damping ratio, ζ and an undamped natural frequency ω_n. For this method, it is only assume that the effect on the responses of nondominant closed-loop poles to be negligible. The possibility to be happened is the order of the system may get higher by 1 or 2 unless we do pole-zero cancellation.

In state space design, the dominant closed loop that has been discussing above will be replaced by feeding back all of the state variables.. One of the advantages is to get better measurement of all state variables.

In the state space representation, a system or plant is represented by:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

Where $\dot{x}$ = state vector

y = output signal

u =control signal

A= n x n constant matrix

B =n x 1 constant matrix

C = 1 x n constant matrix

D = constant

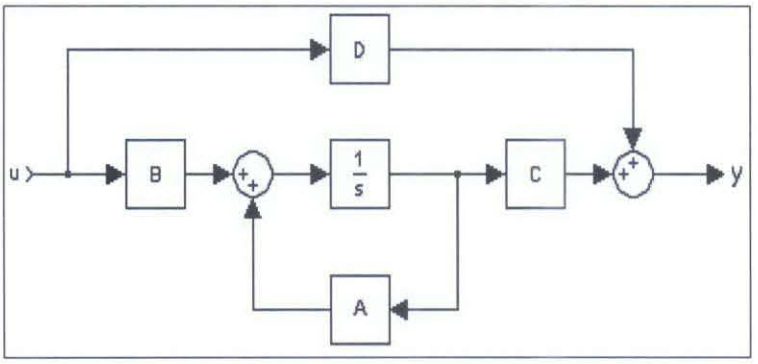


Figure 2.7: State space model

2.4.1 *Pole placement using state feedback*

We will use state variable feedback to archive the desired pole locations of the closed loop transfer function. To use this method, firstly we must have state space model that can be transformed from the transfer function. Then, we will have

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{C} = [c_1 \ c_2] \qquad \mathbf{D} = 0$$

If we have feedback gain represent by $\mathbf{K} = [k_1 \quad k_2]$ then our input will become

$\mathbf{u} = -\mathbf{Kx}$

By then, our new state space equation will be

$$\dot{\mathbf{x}} = \mathbf{Ax} - \mathbf{BKx} = (\mathbf{A} - \mathbf{BK})\mathbf{x}$$

The state feedback matrix is

$$[\mathbf{A} - \mathbf{BK}] = \begin{bmatrix} 0 & 1 \\ -a_0 - k_1 & -a_1 - k_2 \end{bmatrix}$$

And the characteristic equation is

$$\det[\mathbf{A} - \mathbf{BK}] = s^2 + (-a_1 - k_2)s + (-a_0 - k_1)$$

We can determine the value of **K** by comparing the equation above with

$$(s^2 + 2\zeta\varpi_n + \varpi_n^2)$$

The value for ζ and ϖ_n will be determined by the user.

Our new state space model can be visualized as in diagram below

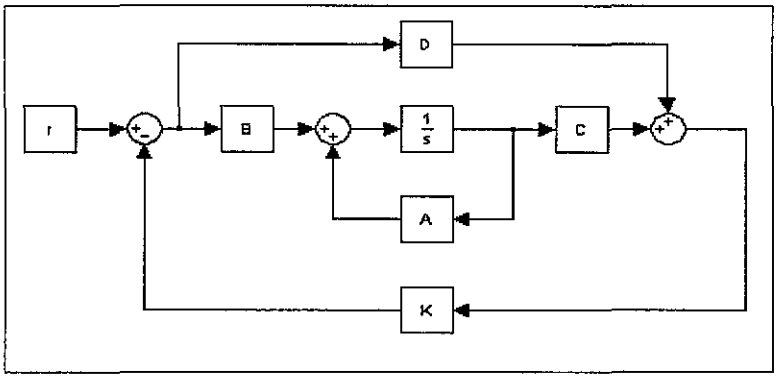


Figure 2.8: State space model with feedback

General description of state space design has been discussed in R.H. Bishop and R.C. Dorf, "Modern Control Systems", 9th Ed., Prentice Hall 2001 [2]

CHAPTER 3

METHODOLOGY

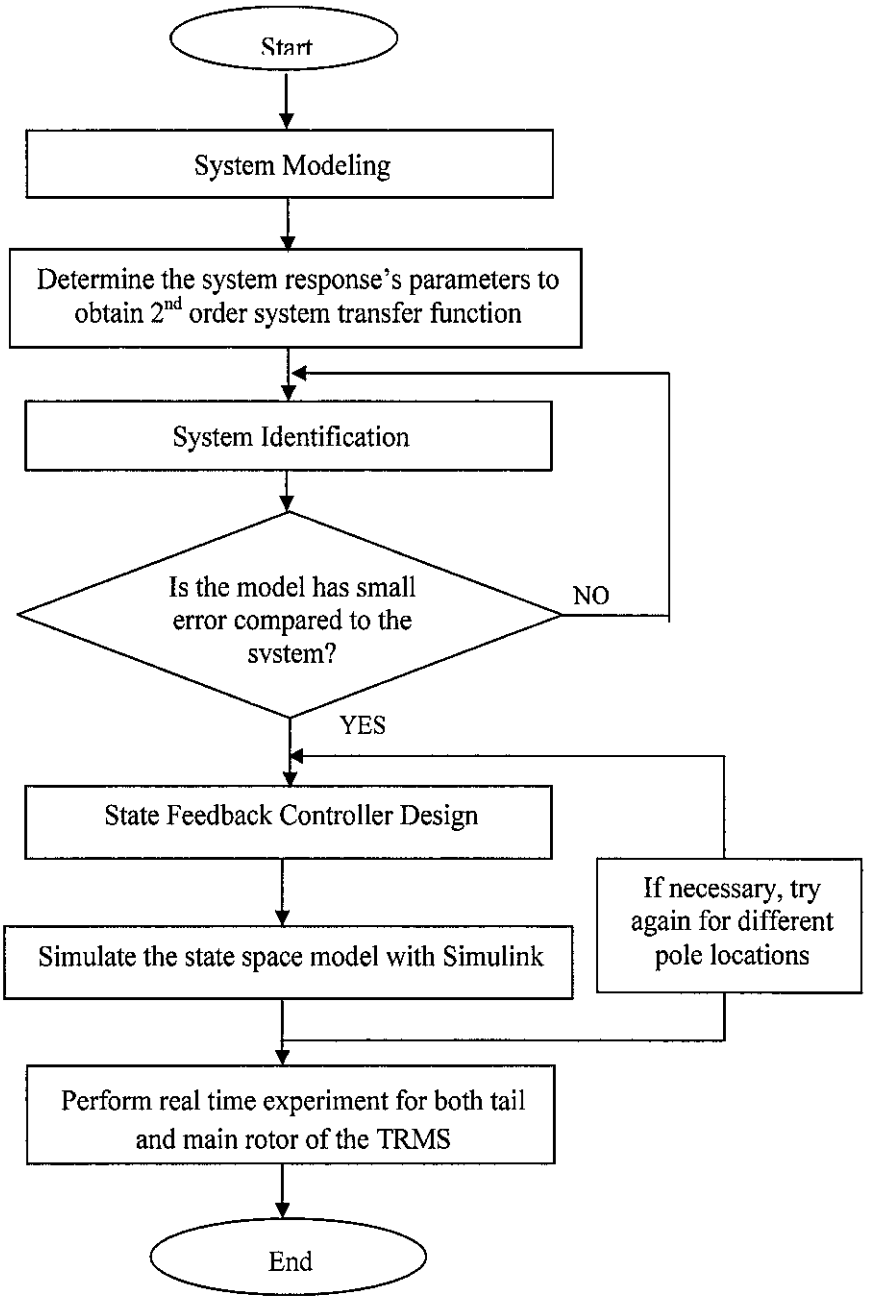


Figure 3.1: Project flow chart

3.1 System Modeling and System Identification

3.1.1 System Modeling

For the first step, both tail and main rotor have to be modeled to get the relationship of input-output of the system. Step input with certain value will be applied to the open loop control systems to get the system responses. In this TRMS case, the step input will be supplied to voltage block of each rotor, so the value of step input will control the voltage value of tail and main rotor.

For tail rotor, step input value is 0.36 while step input value for main rotor is 0.2.

Basically, here are steps taken for system modeling:

1. A step input with certain value is applied to the rotors
2. From the scope, collect the system responses
3. Analyze the system response to get relevant information parameters:
    a. Peak Time, Tp
    b. Percentage Overshoot, %OS
    c. Settling Time, Ts
    d. Damping Ratio, ζ
    e. Natural Frequency, ω_n
4. Generate 2nd order system transfer function by using parameters above.

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

3.1.2 System Identification

Once, we get the transfer function for each tail and main rotor, the models need to be identified to obtain a satisfactory degree of conformity of the model with the actual system. It can be done by tuning the model parameters until it approximately fit with the actual system responses. A good model is a model that can represent a small error compared to the actual system even though this process is quite time consumption process.

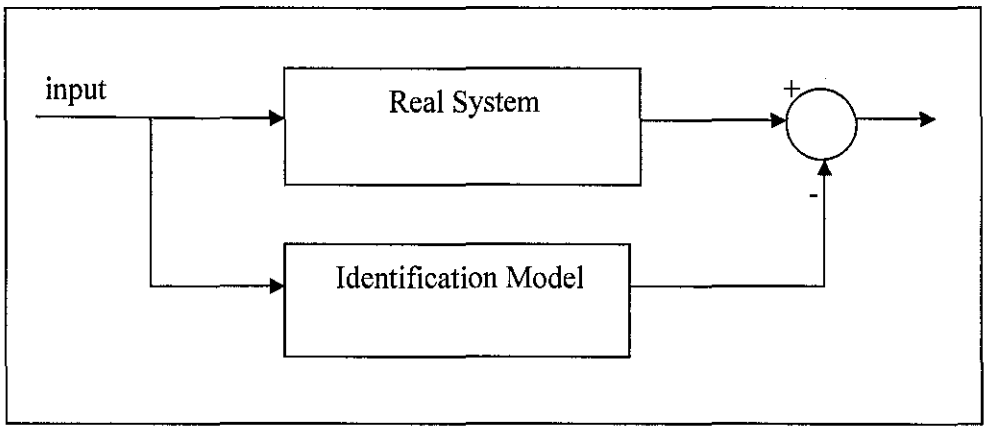


Figure 3.2: Identification diagram

3.2 State Feedback Controller Design

From the transfer function generated before, the process will be continued by designing state feedback controller by using pole placement design technique. Ackermann's formula can be used to find the state feedback gain for the desired pole locations. One thing to be considered is the pole placement technique only can be applied when the system is controllable.

State feedback controller design:

1. Convert the transfer function model into state space representation
2. Check the model's controllability
3. Find the pole locations of the model
4. Set the value of zeta, ξ and natural frequency, ω_n by set the design specification; percentage overshoot, %OS and settling time, Ts.
5. Select preferable new pole locations.
6. Find state space feedback gain by using Ackermann's formula.

Then, the state feedback controller is ready to be tested in simulation. We can have different value of state feedback gain with different pole locations. Once the desired state feedback gain have been chosen, the model now is ready to be implemented in real time experiment of Twin Rotor MIMO System.

To calculate the feedback gain of the tail and main rotor system, there are 3 methods that can be used: [3]

1. By using transformation Matrix T
2. By using direct substitution method
3. By using Ackermann's formula

To find the feedback gains in this project, method 3 is chosen because it is much easier compared to method 1 and method 2. Matlab is a very useful tool to solve mathematical equation. Feedback gain can be calculated by using Matlab command;

K = acker(A, B, poles)

Where: A is matrix of A of the state space representation
B is matrix of B of the state space representation
poles is desired poles placement

CHAPTER 4
RESULTS AND DISCUSSIONS

4.1 System Modeling and System Identification

4.1.1 Tail Rotor Modeling and Identification

Step input 0.36 has been introduced to the system. Tail rotor will start to rotate at 0° until it stops at steady state. From the system response, we can find all the parameters needed to calculate damping ratio, ζ and natural frequency, ω_n of the system.

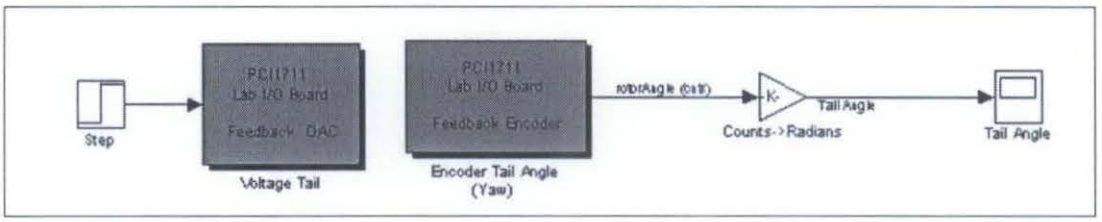


Figure 4.1 Tail rotor block diagram

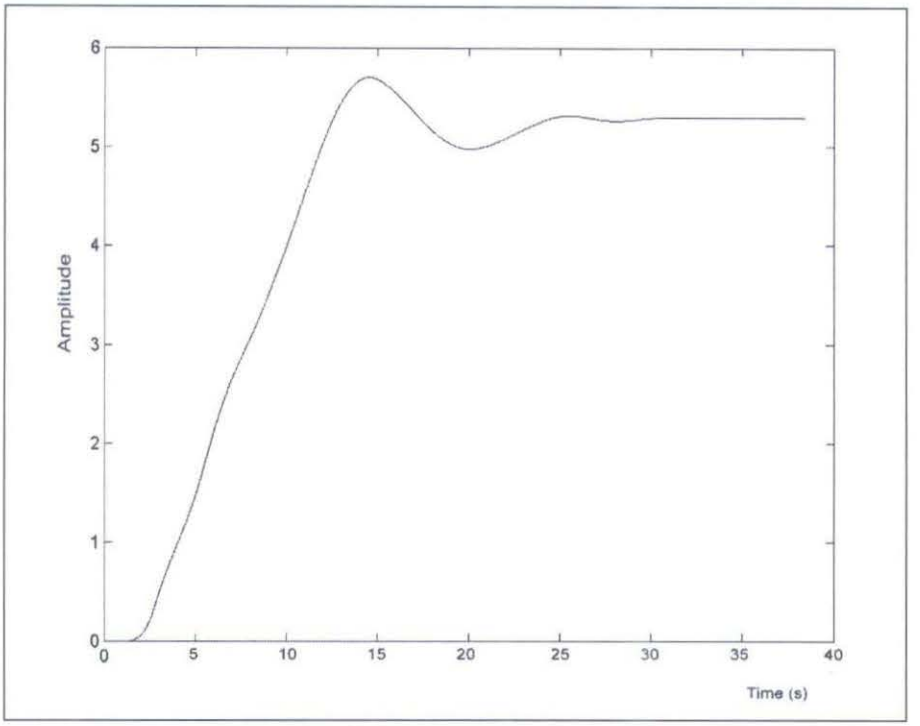


Figure 4.2 System response of tail rotor

Table 1 Parameter responses of tail rotor

Measured Parameter		Calculated Parameter	
Parameters	Value	Parameters	Value
Peak Time, Tp	14 s	Damping Ratio, ζ	0.6
Percent Overshoot, %OS	9.25 %	Natural Frequency, ω_n	0.2899
Settling Time, Ts	23 s		

From information above, we can generate 2nd order system equation for the tail rotor. The transfer function equation is:

$$T(s) = \frac{0.0840}{s^2 + 0.3479s + 0.0840}$$

However, the transfer function above is still not the transfer function for tail rotor model. Then, we go to model identification to find the approximately transfer function with small error compared to the actual system. Model identification can be done by connecting the transfer function generated above with the tail rotor actual system in parallel form.

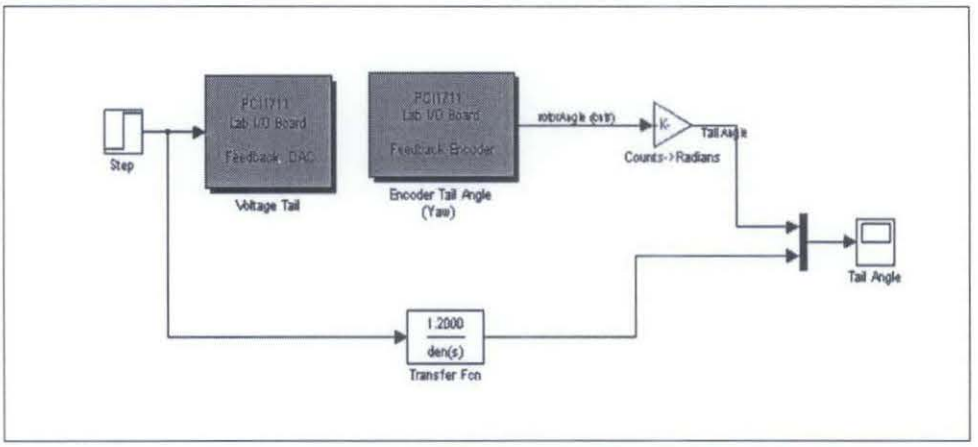


Figure 4.3: Tail rotor model identification.

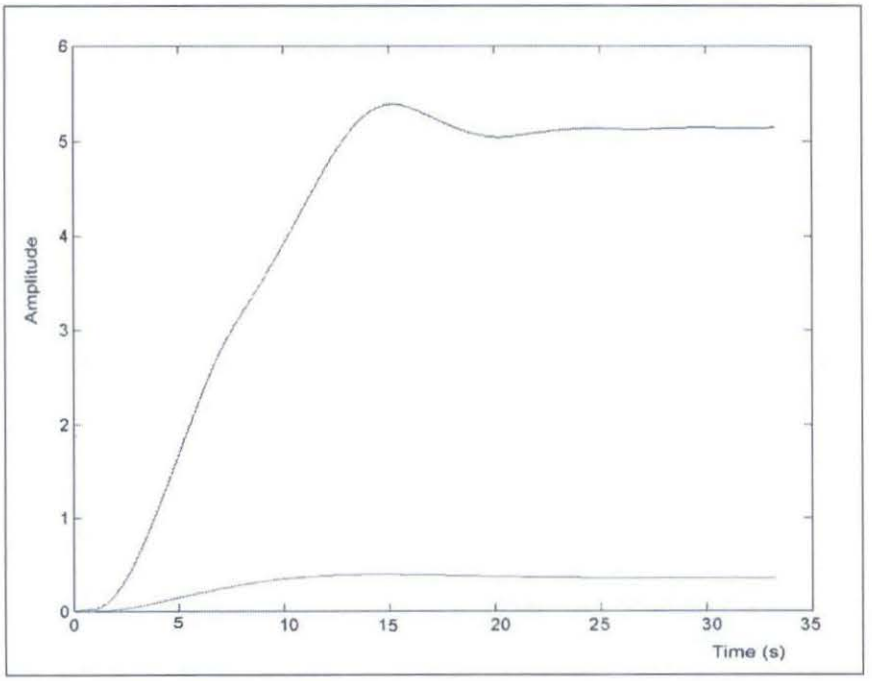


Figure 4.4: Model and system step responses of tail rotor before identification

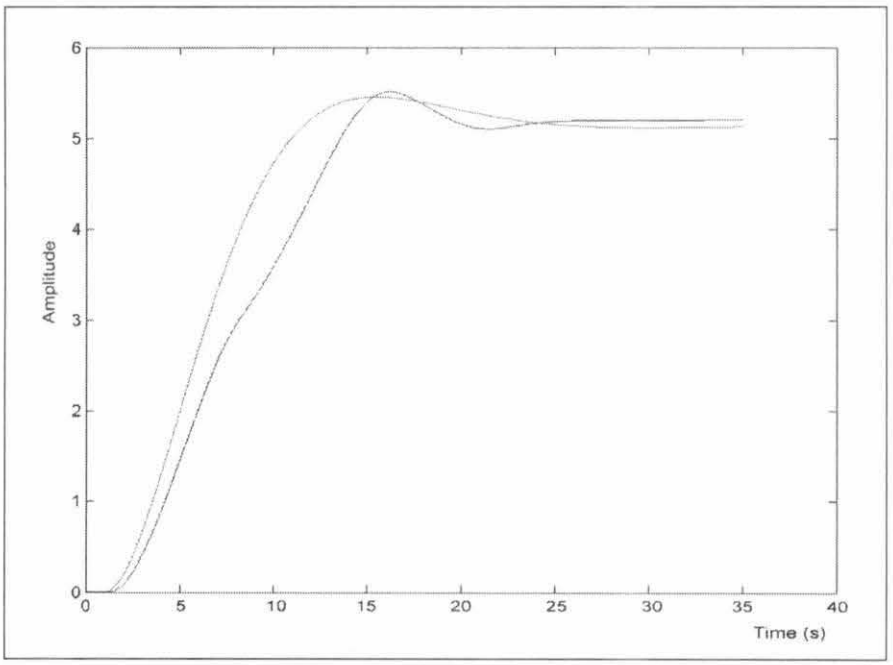


Figure 4.5 Model and system step responses of tail rotor after identification

Diagram in figure 4.4 show us step responses of tail rotor model and the actual system before identification. Only the denominator of the model transfer function can be tuned as it affects the nature of system response. After identification, the tail rotor transfer function is

$$T(s) = \frac{1.200}{s^2 + 0.3850s + 0.0840}$$

4.1.2 Main Rotor Modeling and Identification

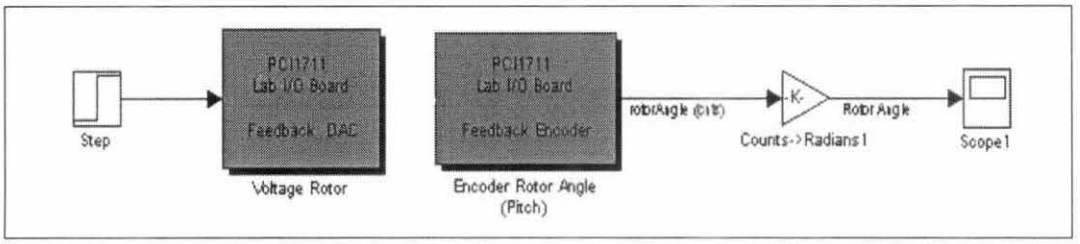


Figure 4.6: Main rotor block diagram

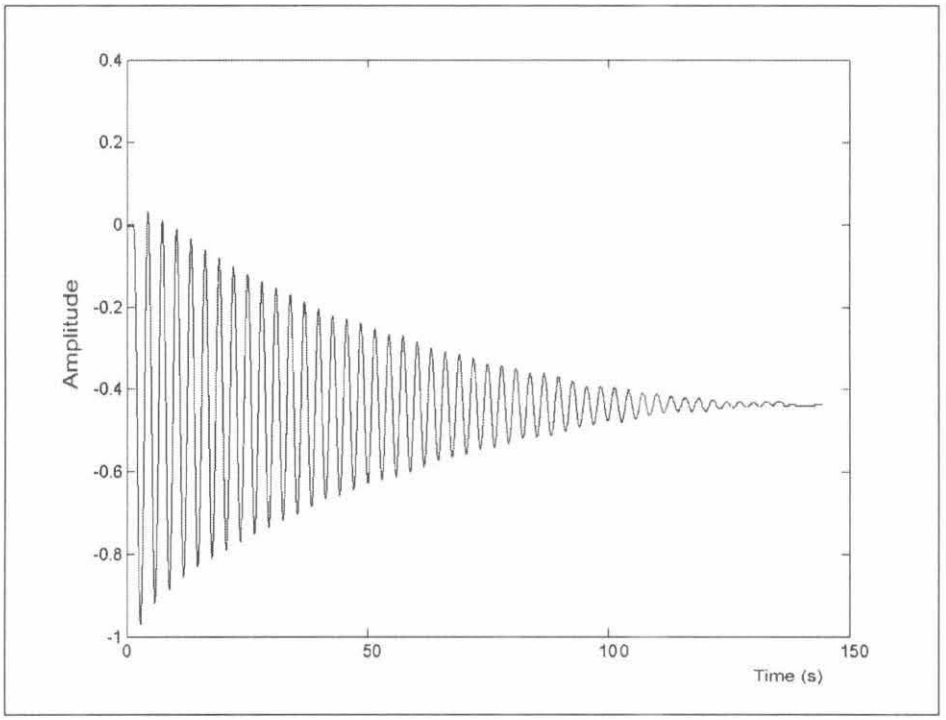


Figure 4.7: System response of main rotor

Table 2 Parameter responses of main rotor

Measured Parameter		Calculated Parameter	
Parameters	Value	Parameters	Value
Peak Time, Tp	3.1 s	Damping Ratio, ζ	0.0752
Percent Overshoot, %OS	126.74 %	Natural Frequency, ω_n	0.4433
Settling Time, Ts	120 s		

From calculated parameters above, the transfer function for the main rotor model is

$$T(s) = \frac{-0.1965}{s^2 + 0.0667s + 0.1965}$$

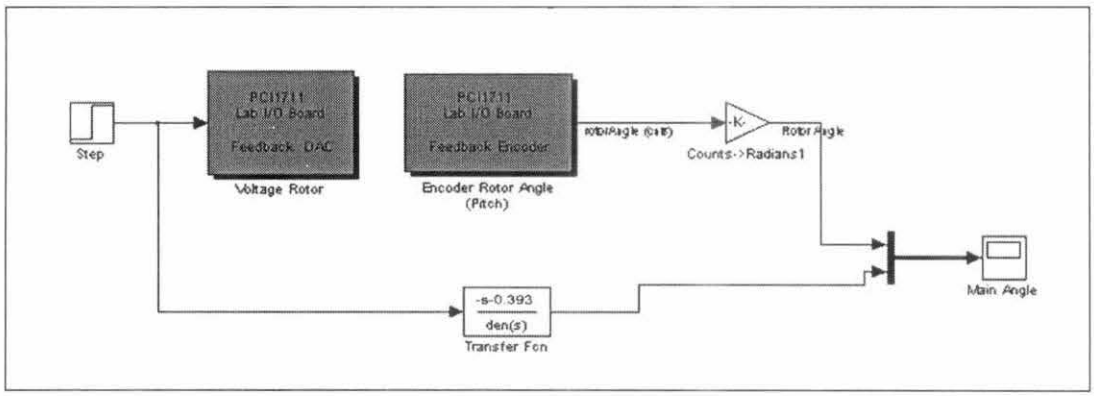


Figure 4.8: Main rotor model identification

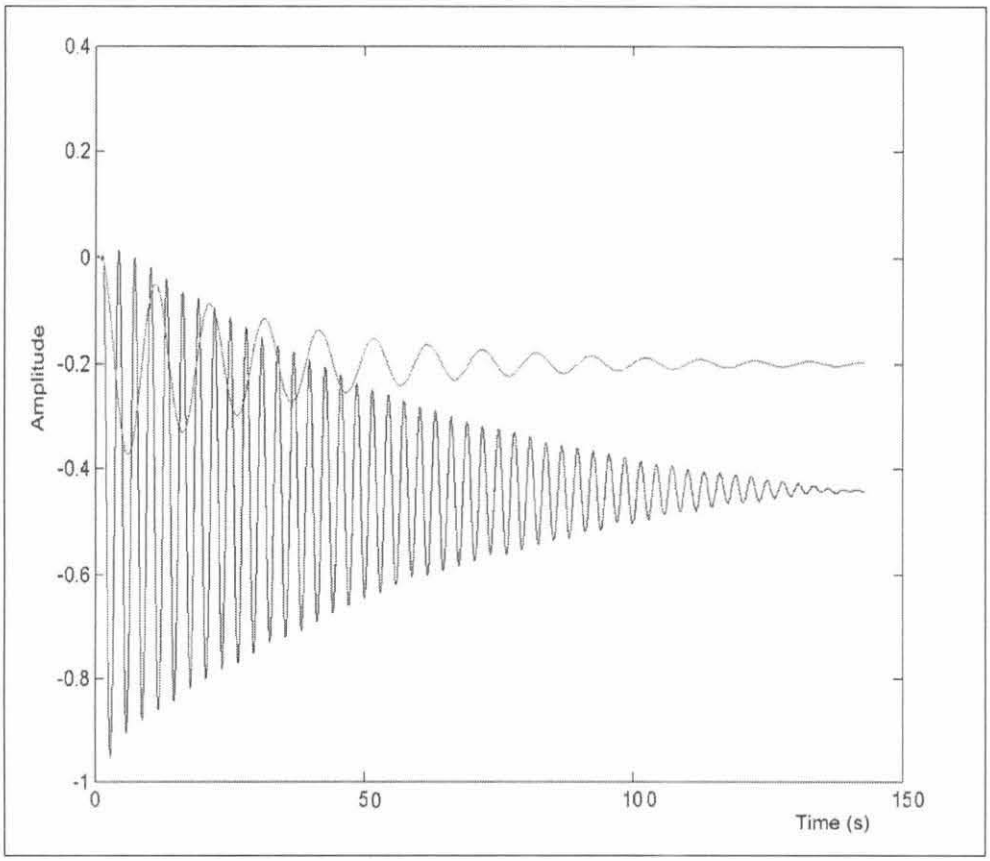


Figure 4.9: Model and system step responses of main rotor before identification

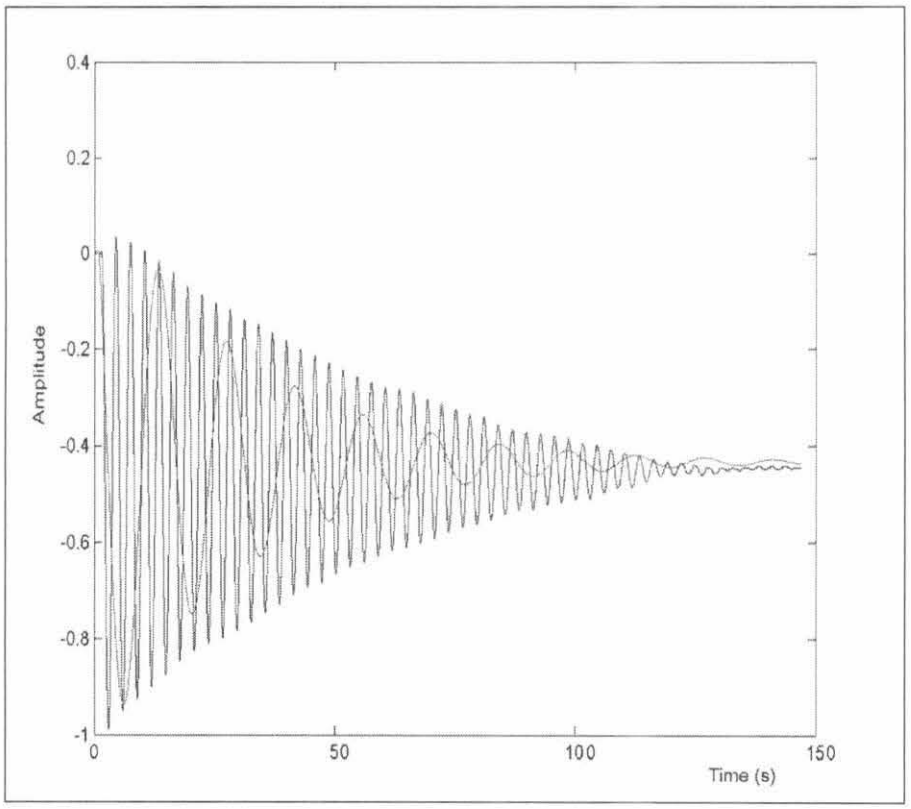


Figure 4.10: Model and system step responses of main rotor after identification

After identification process, the approximate transfer function for main rotor model is

$$T(s) = \frac{-s - 0.3930}{s^2 + 0.0667s + 0.1965}$$

Now system modeling and identification for both tail and main rotor model are done and the model transfer functions will be used for further process of designing state feedback controllers for each tail and main rotor.

4.2 State Feedback Controller Design

4.2.1 Tail Rotor State Feedback Controller Design

The state space representation of the tail rotor in control canonical form is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.3850 & -0.0840 \\ 1.000 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1.200 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The controllability of the system above can be determined by examining the controllability matrix of the system.

$$C_M = \begin{bmatrix} \mathbf{B} & \mathbf{AB} \end{bmatrix} = \begin{bmatrix} 1.000 & -3.850 \\ 0 & 1.000 \end{bmatrix}$$

The rank is 2 so the system is said to be *controllable.*

Next step is to find the poles for the system and the desired closed loop poles. The poles for the system is

$-0.1925 + j0.2167$

$-0.1925 - j0.2167$

The design specifications for the tail rotor model are:

1. Percentage overshoot, %OS is 20% to a step input
2. Settling time, Ts less than 5s percent to the step input

From the design specifications above, we will get

1. $\%OS \leq 20\%$ implies $\xi \geq 0.4559$
2. $Ts \leq 5s$ implies $\omega_n \geq 1.7546$

From the value of zeta and natural frequency above, we can generate second order transfer function which is

$$T_s = \frac{{\omega_n}^2}{s^2 + 2\xi\omega_n + {\omega_n}^2} = \frac{3.079}{s^2 + 1.6s + 3.079}$$

Then, we can have root locus diagram as shown as below. We can put our pole placement anywhere within the colored region to satisfy the design specifications.

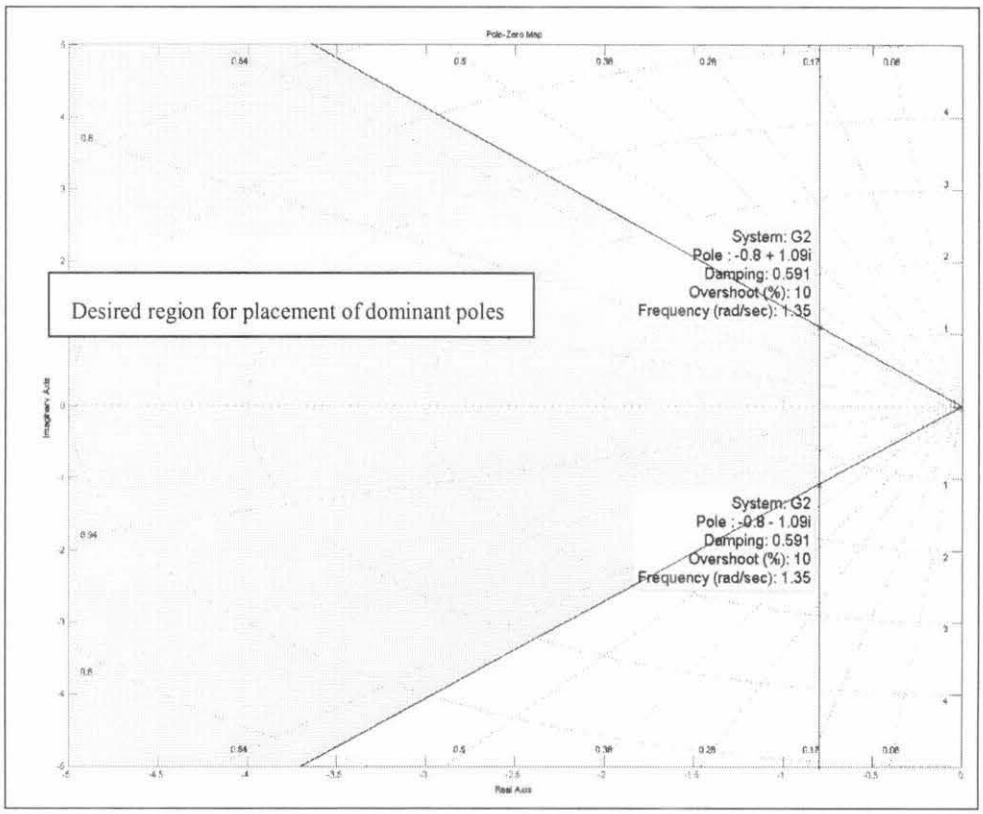


Figure 4.11: Desired region in the complex plane for locating the dominant system poles of tail rotor model

For the placement of dominant poles, we can try to place it at:

-1.75

-1.75

By using Ackermann's formula, then value of state feedback controller gain can be determined

$$K = \begin{bmatrix} 3.115 & 2.9785 \end{bmatrix}$$

Then, forward gain, N for the model need to be calculated such that:

$$N = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1}$$

$$N = \begin{bmatrix} 0 & 1.000 & 0 \\ 0 & 0 & 0.8333 \\ 1.000 & 0.3850 & 0.0700 \end{bmatrix}$$

$$N = \begin{bmatrix} 0.07 \end{bmatrix} + [K] * [0 \quad 0.8333]$$

$$N = 2.552$$

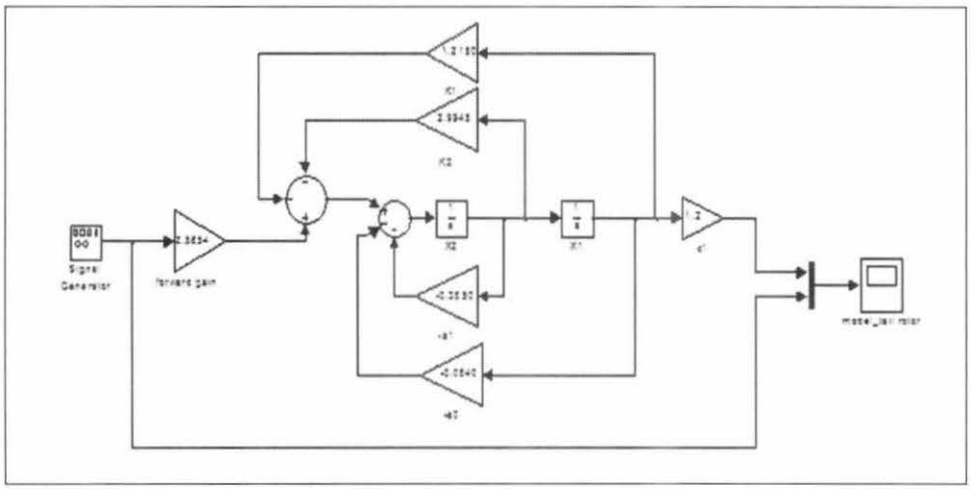


Figure 4.12: Tail rotor state space model

4.2.2 Main Rotor State Feedback Controller Design

The state space representation of the tail rotor in control canonical form is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x} + Du$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.0667 & -0.1965 \\ 1.000 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1.000 & -0.3930 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The controllability of the system above can be determined by examining the controllability matrix of the system.

$$C_M = \begin{bmatrix} \mathbf{B} & \mathbf{AB} \end{bmatrix} = \begin{bmatrix} 1.000 & -0.0667 \\ 0 & 1.000 \end{bmatrix}$$

The rank is 2 so the system is said to be *controllable.*

Next step is to find the poles for the system and the desired closed loop poles. The poles for the system is

$-0.0034 + j0.4420$

$-0.0034 - j0.4420$

The design specifications for the tail rotor model are:

1. Percentage overshoot, %OS is 10% to a step input
2. Settling time, Ts less than 5s percent to the step input

From the design specifications above, we will get

1. %OS $\leq$ 10% implies $\xi \geq 0.5912$
2. Ts $\leq$ 5s implies $\omega_n \geq 1.3533$

From the value of zeta and natural frequency above, we can generate second order transfer function which is

$$T_s = \frac{{\omega_n}^2}{s^2 + 2\xi\omega_n + {\omega_n}^2} = \frac{1.831}{s^2 + 1.6s + 1.831}$$

Then, we can have root locus diagram as shown as below. We can put our pole placement anywhere within the colored region to satisfy the design specifications.

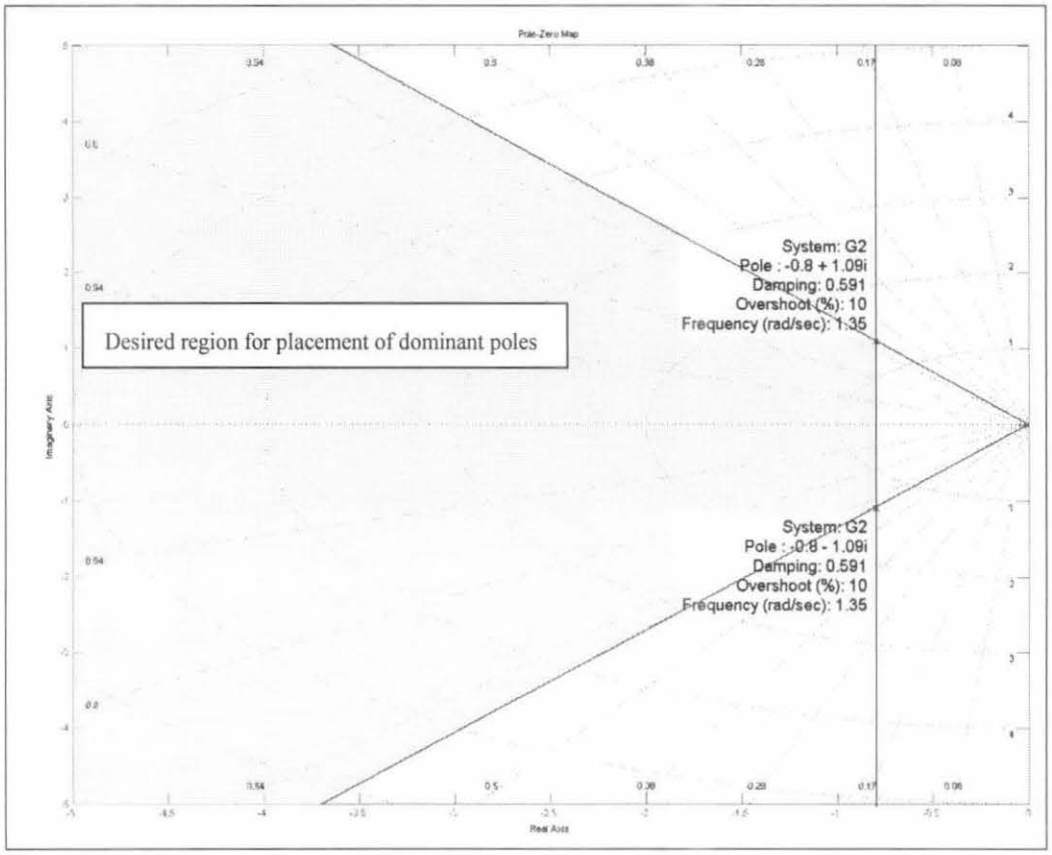


Figure 4.13: Desired region in the complex plane for locating the dominant system poles of main rotor model

For the placement of dominant poles, we can try to place it at:

-0.8

-0.8

By using Ackermann's formula, then value of state feedback controller gain can be determined

$$K = \begin{bmatrix} 1.5333 & 0.4435 \end{bmatrix}$$

Then, forward gain, N for the model need to be calculated such that:

$$N = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1}$$

$$N = \begin{bmatrix} 0 & 1.000 & 0 \\ 0 & -2.5445 & -2.5445 \\ 1.000 & -0.4333 & -0.5000 \end{bmatrix}$$

$$N = \begin{bmatrix} -0.5000 \end{bmatrix} + [K] * [0 \qquad -2.5445]$$

$$N = -1.6285$$

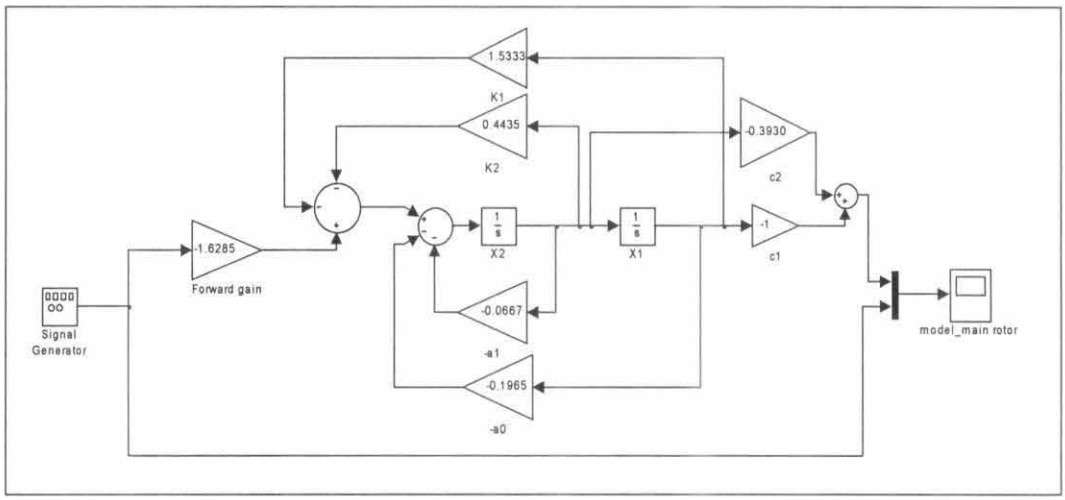


Figure 4.14: Main rotor state space model

4.3 1-DOF Tail Rotor

4.3.1 Tail Rotor Model Simulation

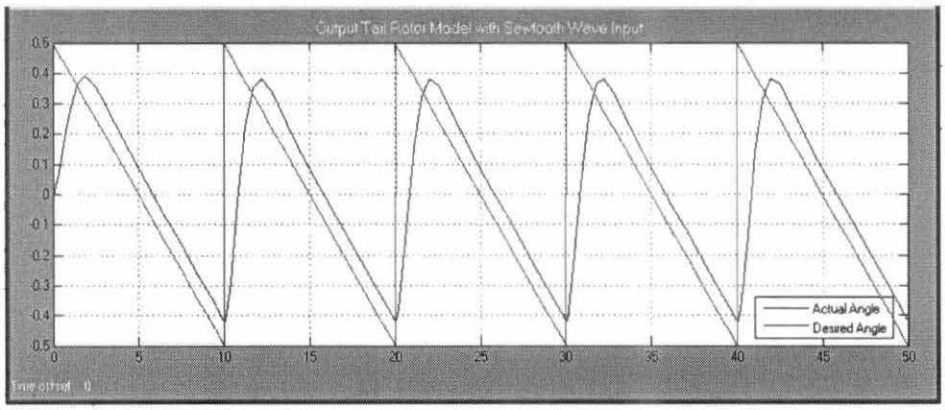


Figure 4.15: Output tail rotor model simulation with saw tooth wave input

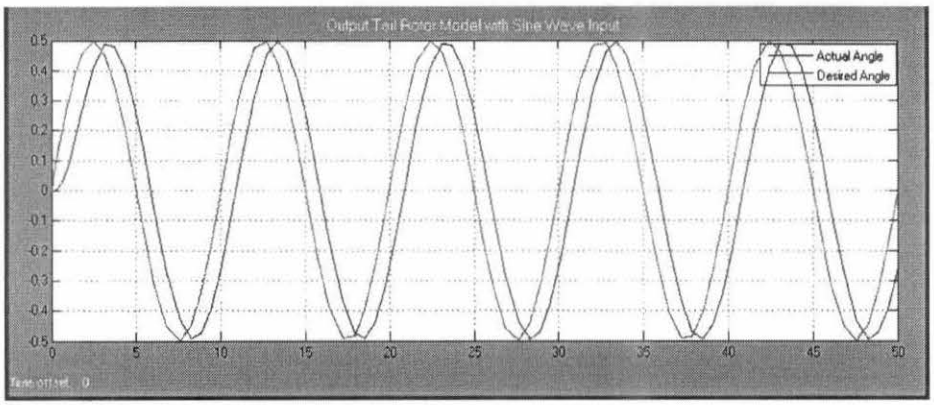


Figure 4.16: Output tail rotor model simulation with sine wave input

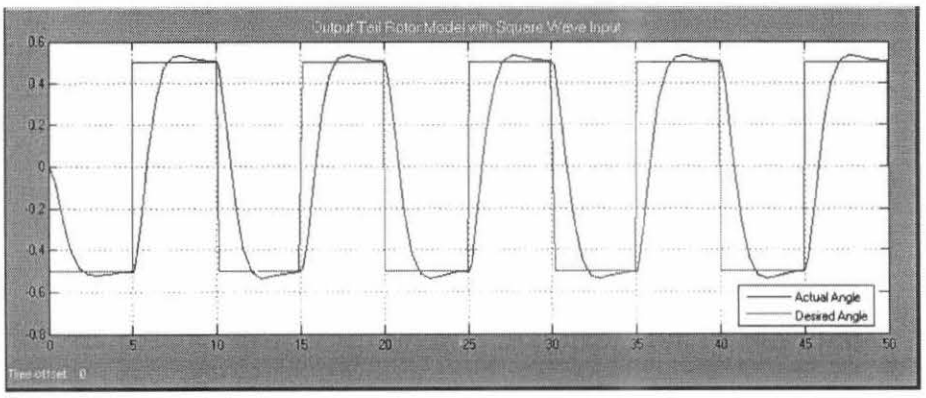


Figure 4.17: Output tail rotor model simulation with square wave input

4.3.2 Tail Rotor Real Time Experiment

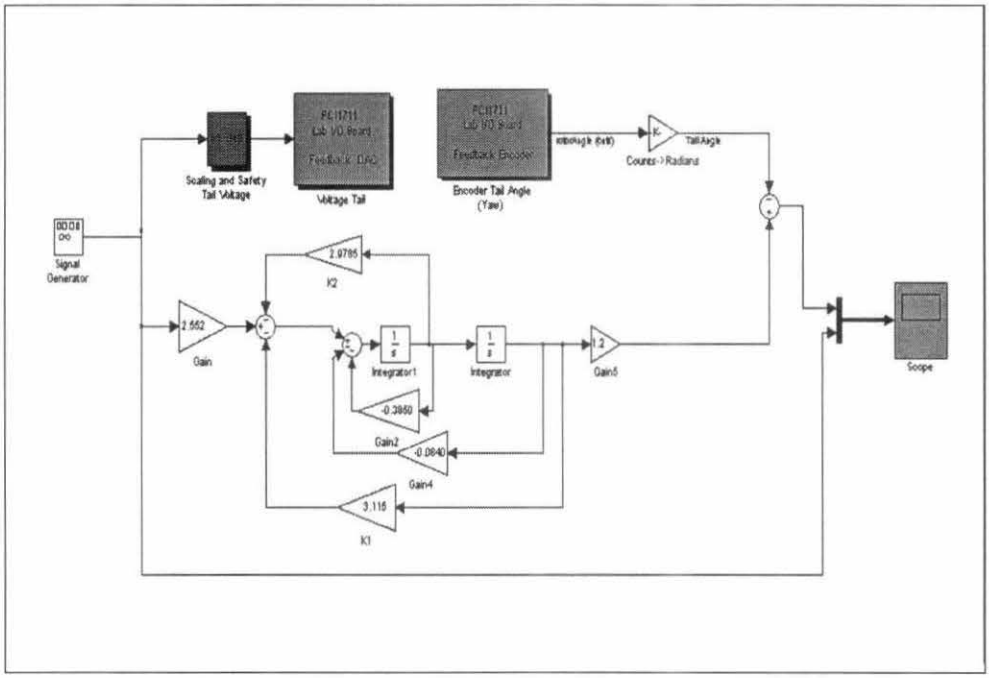


Figure 4.18: Block diagram of tail rotor real time experiment

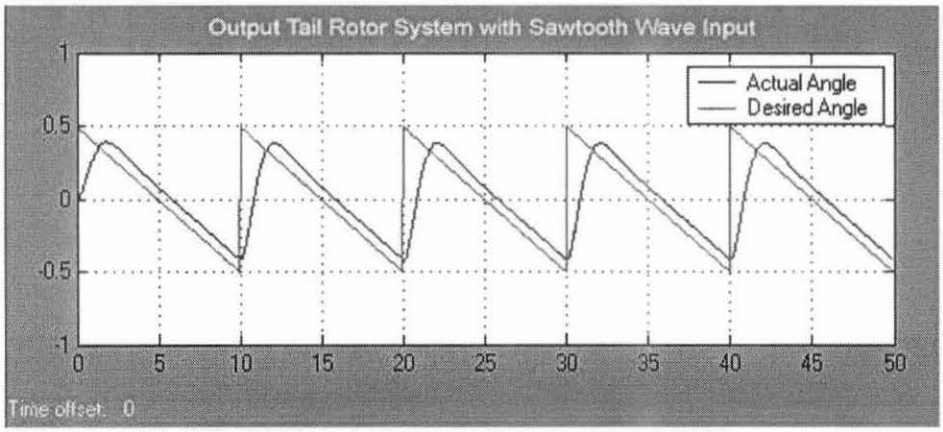


Figure 4.19: Output tail rotor real time experiment with saw tooth wave input

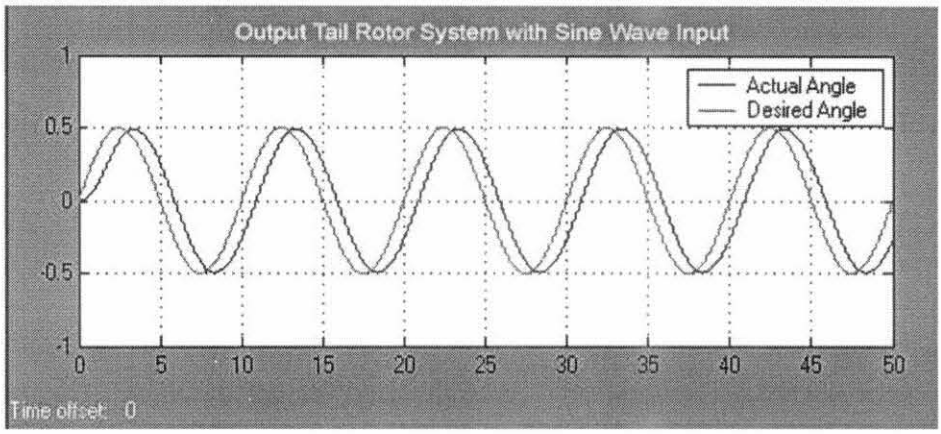


Figure 4.20: Output tail rotor real time experiment with sine wave input

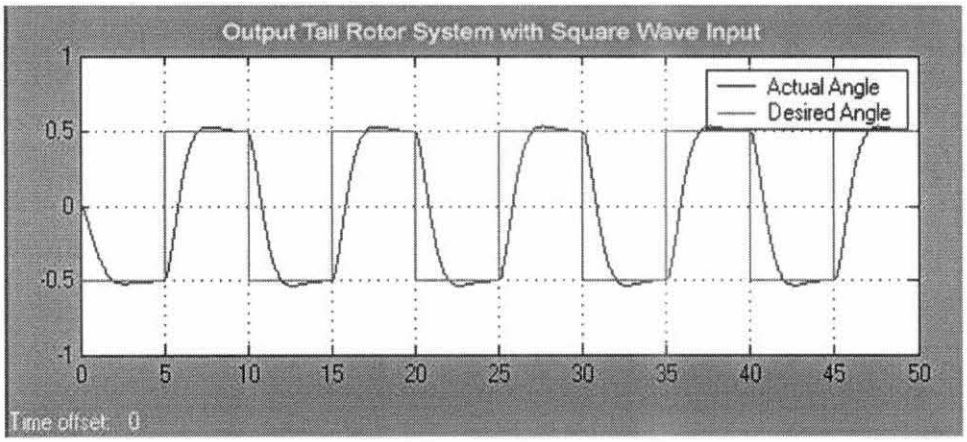


Figure 4.21: Output tail rotor real time experiment with square wave input

4.4 1-DOF Main Rotor

4.4.1 Main Rotor Model Simulation

Figure 4.22 Output main rotor model simulation with saw tooth wave input

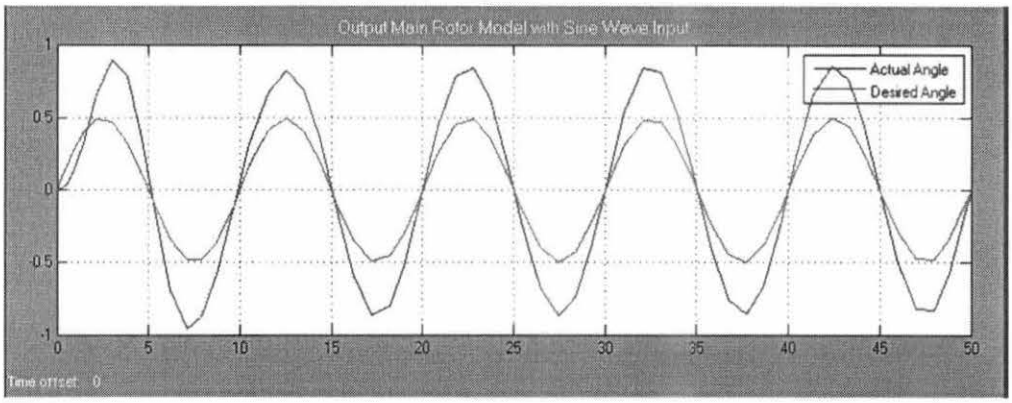


Figure 4.23: Output main rotor model simulation with sine wave input

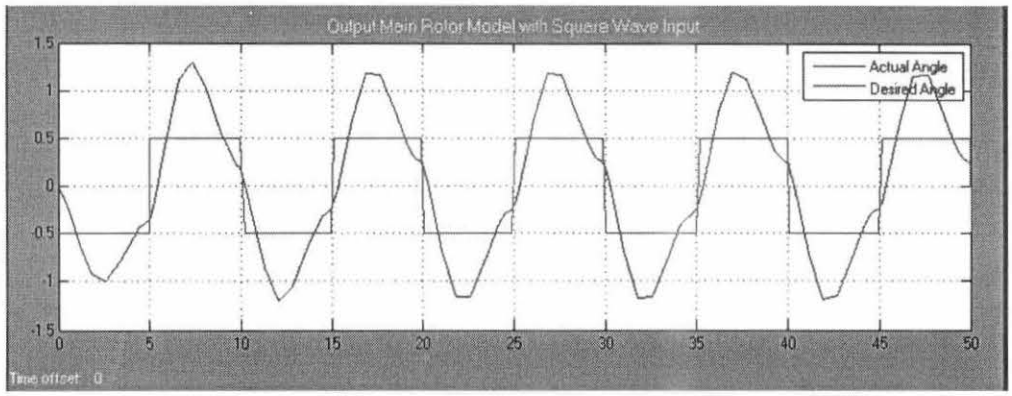


Figure 4.24: Output main rotor model simulation with square wave input

4.4.2 Main Rotor Real Time Experiment

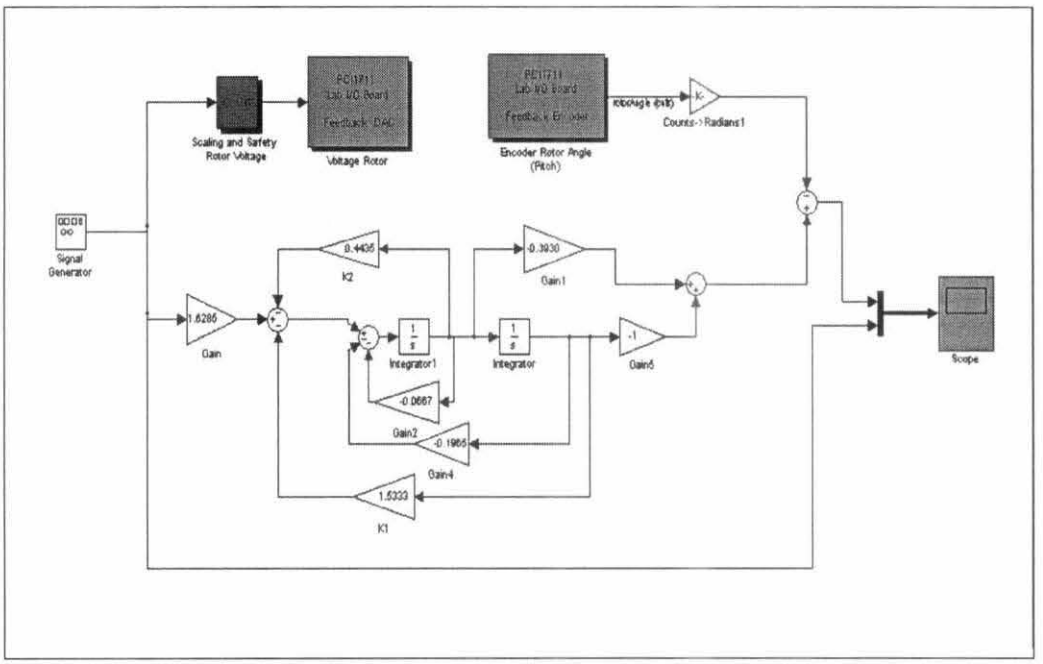


Figure 4.25: Block diagram of main rotor real time experiment

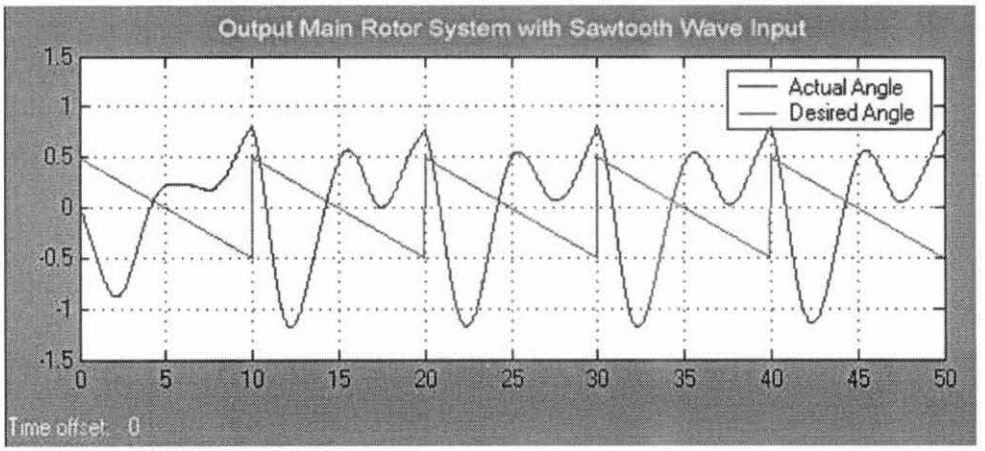


Figure 4.26: Output main rotor real time experiment with saw tooth wave input

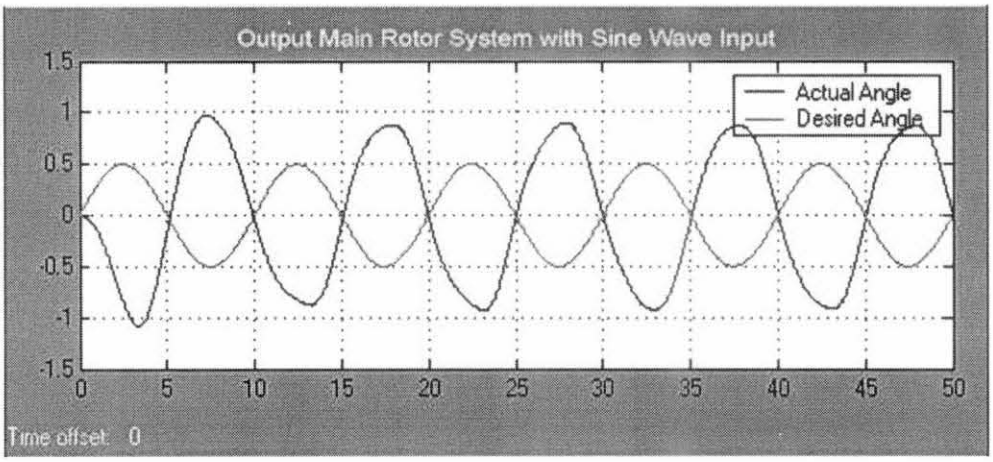


Figure 4.27: Output tail rotor real time experiment with sine tooth wave input

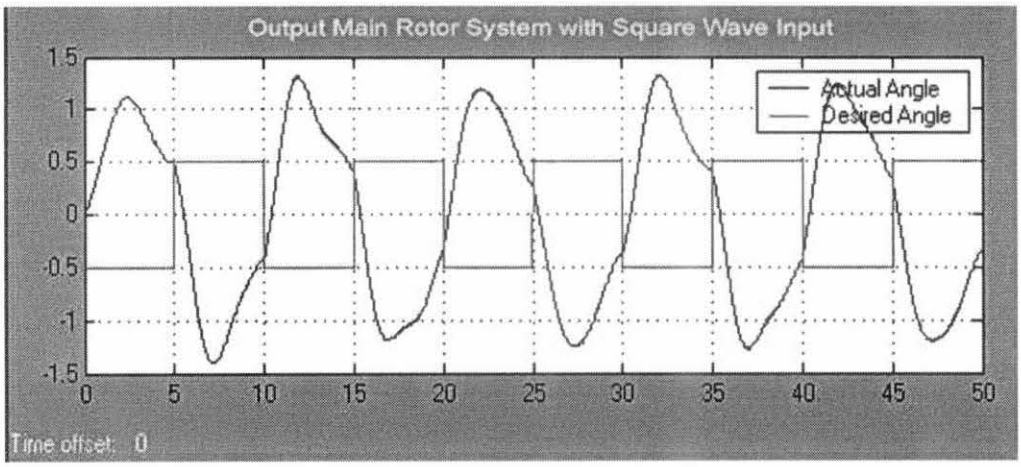


Figure 4.28: Output tail rotor real time experiment with square wave input

4.5 2-DOF Tail and Main Rotor

Both tail and main rotor can be combined together to produce 2-DOF mode which is movement of TRMS can be both in horizontal and vertical at the same time. On the "Experiment Select" box, we can choose the mode that we want by entering:

- constant value 1 for 1-DOF tail rotor
- constant value 2 for 1-DOF main rotor
- constant value 3 for 2-DOF tail and main rotor

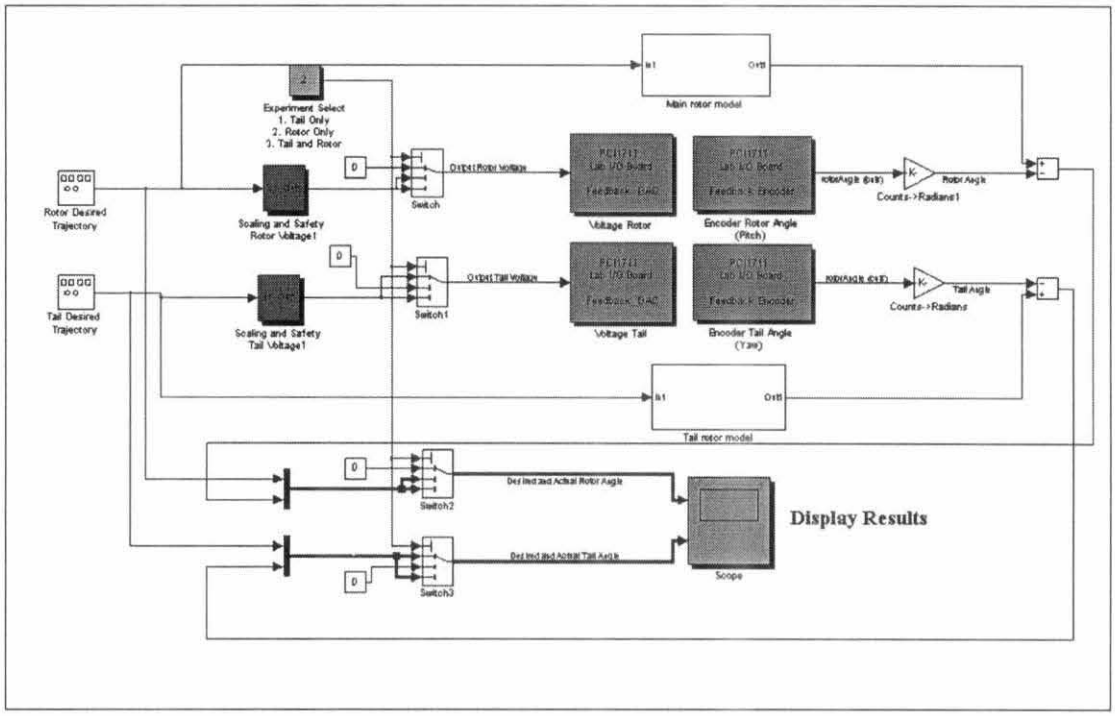


Figure 4.29 Block diagram of TRMS controller

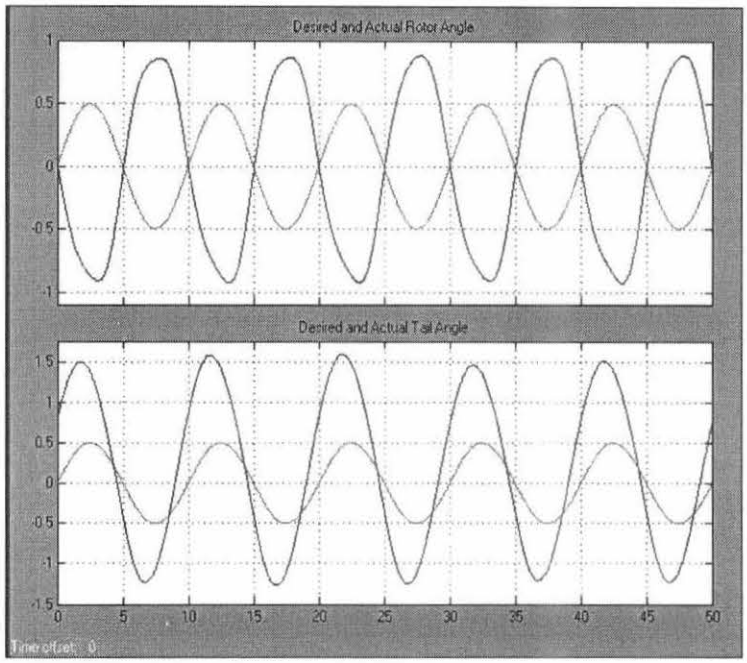


Figure 4.30 Output tail and main rotor real time experiment with sine wave input

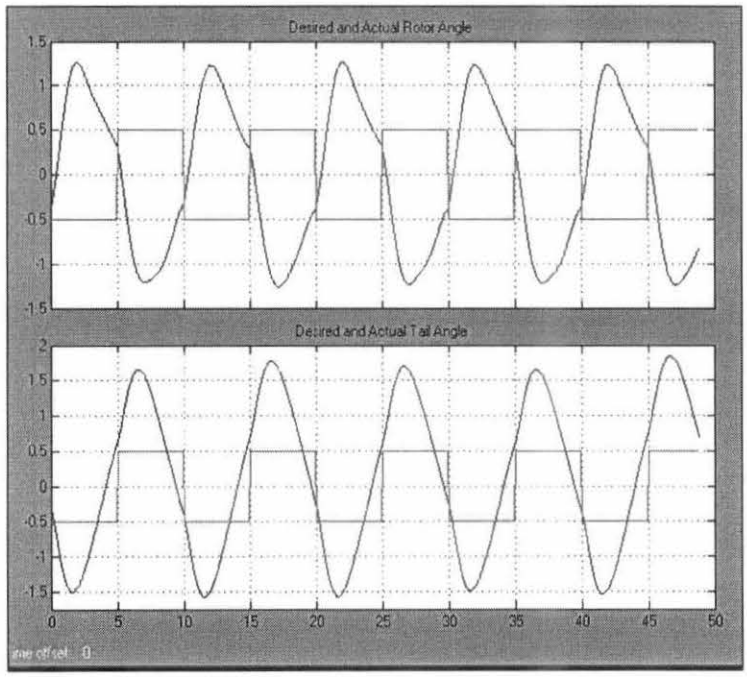


Figure 4.31 Output tail and main rotor real time experiment with square wave input

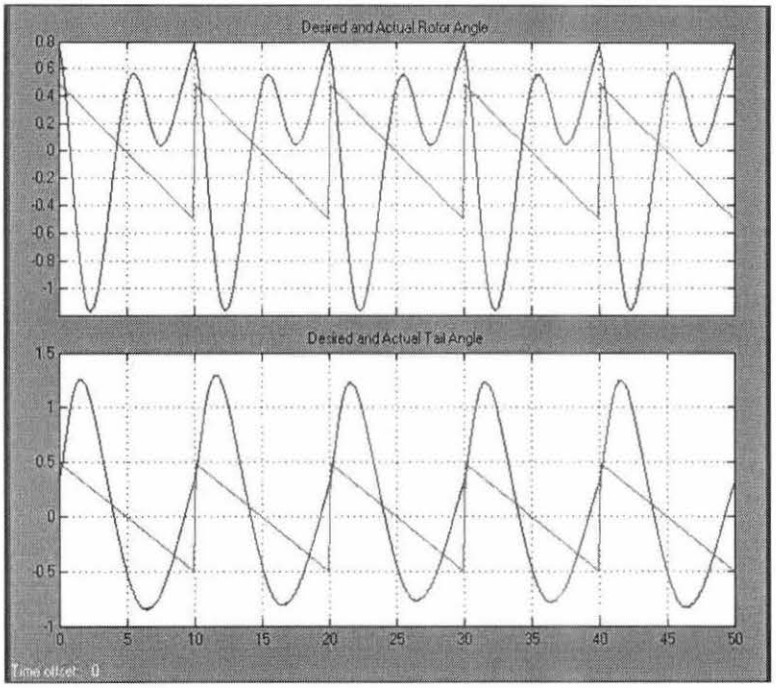


Figure 4.32 Output tail and main rotor real time experiment with saw tooth wave input

4.6 Discussion and Findings

Twin Rotor MIMO System is a system that can operate in 2-DOF mode (using tail and main rotor). This project has been carried by analyzing 1-DOF tail rotor and 1-DOF main rotor independently. System modeling and identification has been done for both tail and main rotor to get linear relationship of its input-output before designing controller for the both systems.

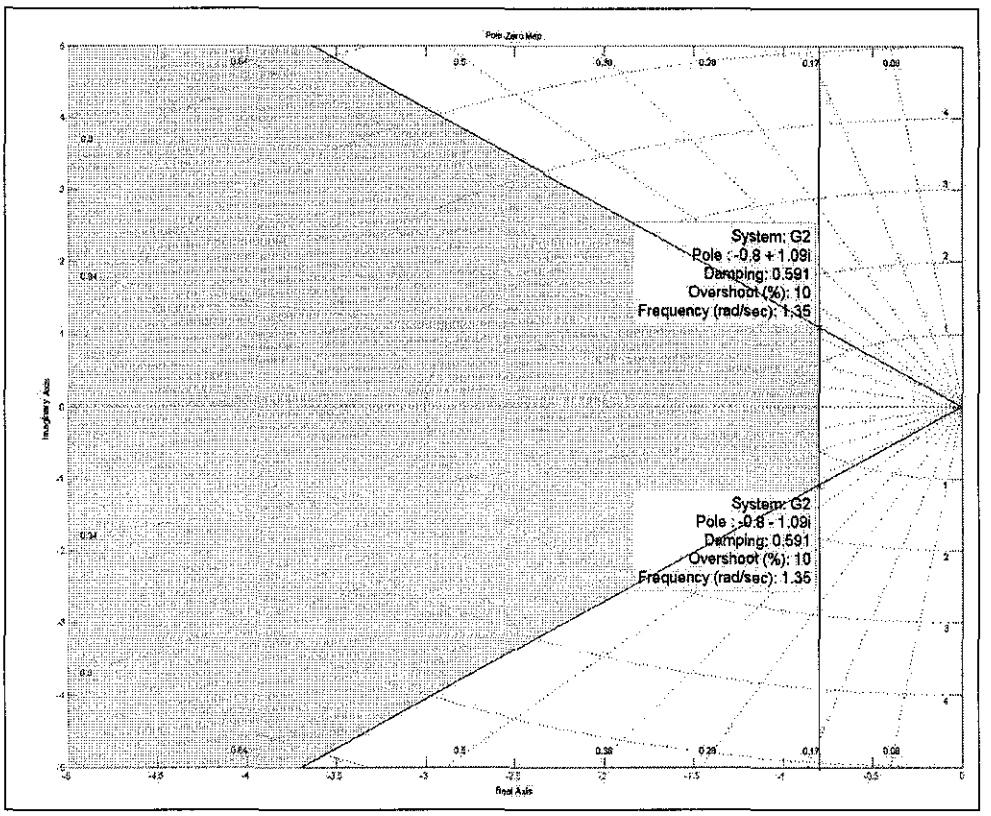
In control system, there are two methods to design feedback control of a system which are classical approach (transfer function method) and state variable approach. By having the feedback control, the performance of a closed-loop system can be improved. In control, we know that closed loop system is more complex and effective compared to the open loop system. For this project, the objective is to design the controller for this TRMS to control its behavior.

In classical approach, we cannot apply the transfer function models to non-linear system. The transfer function models cannot be used effectively for multivariable systems (system with many inputs, many outputs). The transfer function approach confines to input-output behavior of linear systems only.

To analyze system that has many inputs and many outputs, we want to reduce the complexity of the mathematical expressions. State space approach can help us in this matter as well as solution for the problem above.

Pole placement is a method that we place closed-loop poles at the desired locations. To implement the pole placement method, we have to make sure that all the state variable are measurable and are available for feedback or known as controllable. When the system has controllability, the poles can be placed at any desired locations then state feedback can be designed for the system.

We must place the poles anywhere within the desired region as in Figure 4.11 and 4.13, or we will not meet the design specifications.



For example, in designing tail rotor model, the design specifications are:

$\%OS \leq 20\%$	implies	$\xi \geq 0.4559$
$T_s \leq 5s$	implies	$\omega_n \geq 1.7546$

If we choose the poles to be at -3±j2 whereas still in the desired region, then we will have:

$\xi = 0.8320$

$\omega_n = 3.6056$

So, our transfer function and transient response will be:

$$T_s = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2} = \frac{13}{s^2 + 6s + 13}$$

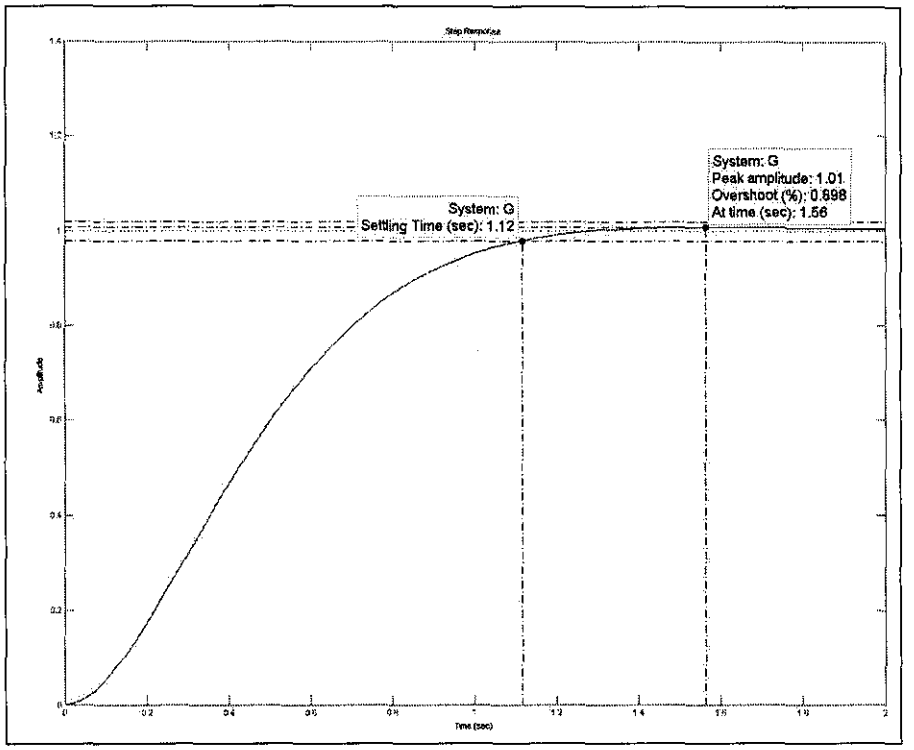


Figure 4.33: Transient response of poles at -3±j2

From the figure above, our new settling time,T_s and percentage overshoot, %OS are:

$\%OS = 0.898$ $\%OS \leq 20\%$

$T_s = 1.12s$ $T_s \leq 5s$

The selected poles are acceptable since they are in desired region and give value of settling time,T_s and percentage overshoot, %OS that meet the design specification.

If we choose our poles to be at -0.5±j2 whereas beyond the desired region, then we will have

$\xi = 0.8320$

$\omega_n = 3.6056$

$$T_s = \frac{\omega_n{}^2}{s^2 + 2\xi\omega_n + \omega_n{}^2} = \frac{4.25}{s^2 + s + 4.25}$$

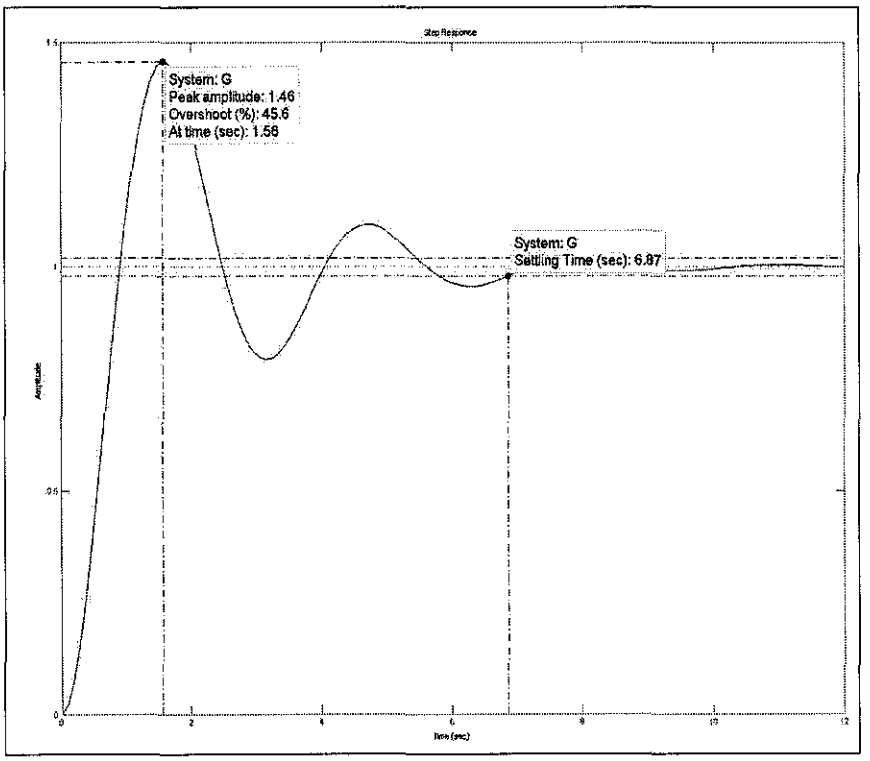


Figure 4.34: Transient response of poles at -0.5±j2

The system has percentage overshoot and settling time that are beyond the design specification

%OS = 45.6 %	%OS ≥20%
T_s = 6.87s	$T_s \geq$ 5s

So poles at -0.5±j2 are not acceptable for this tail rotor design.

The simulation of the models of tail and main rotor in Simulink show us that the outputs are able to follow the given trajectory sine wave, square wave and saw tooth wave with significant error. The steady state error can be reduced by replace the forward gain with slider gain block. Then, we can adjust the gain value until it reaches zero steady state error. As example, we consider the main rotor model:

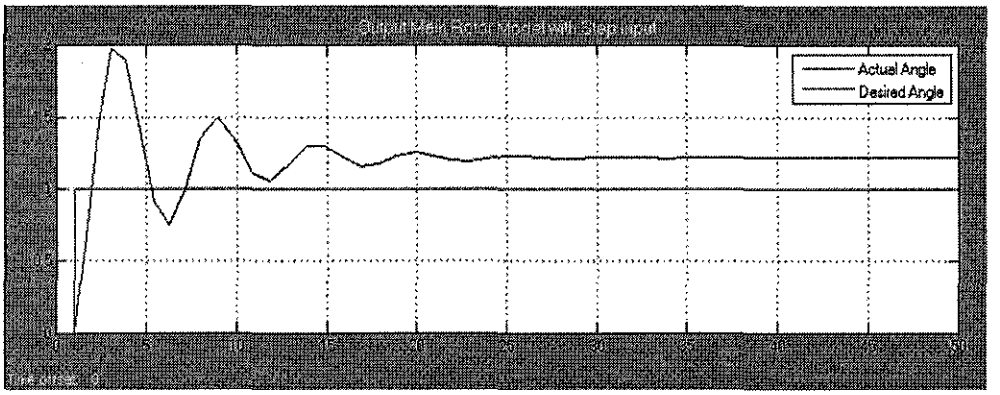


Figure 4.35: Output main rotor simulation with step input (before using slider gain)

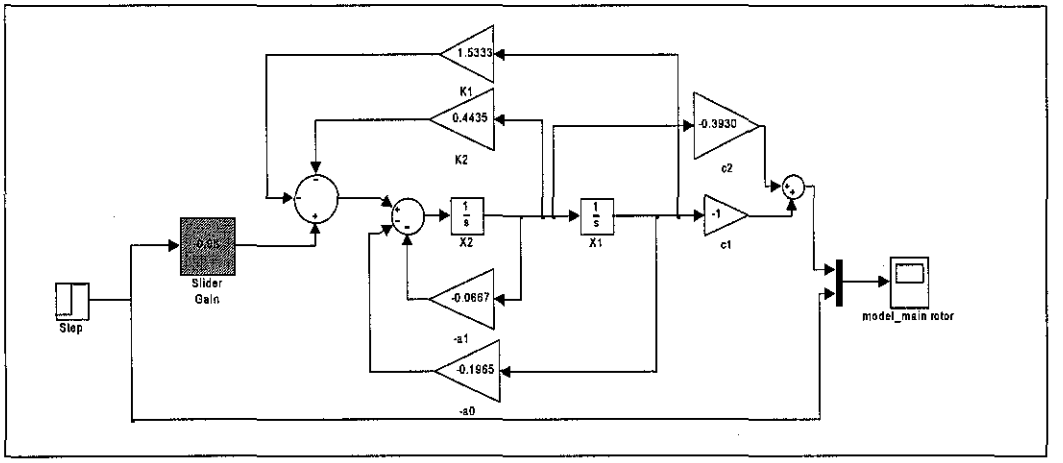


Figure 4.36: Main rotor state space model by using slider gain

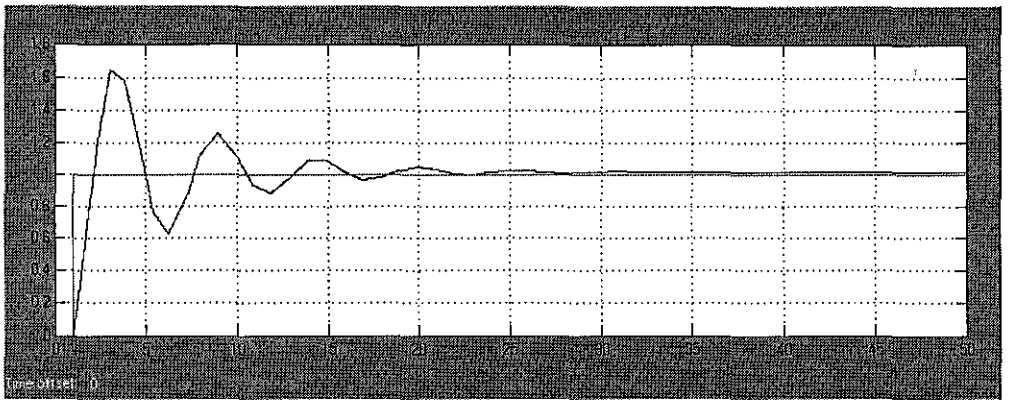


Figure 4.37: Output main rotor simulation with step input (after using slider gain)

From the results in real-time experiments, the behavior of tail and main rotor of TRMS have been controlled by tail and main rotor state feedback model. The output of tail and main rotors will follow the given trajectory signal i.e. sine wave, square wave and saw tooth wave input.

Eventually, there are limitations of state variable feedback application. State feedback leads to PD-type or PID compensators which have infinite bandwidth and it is not applicable to sense all the states and feed them back. State space method also is very sensitive whenever there is a change in parameters.

CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

The focus in completing this project is only about how to archive the objectives. From the results obtained, the state feedback controller for both 1-DOF tail and main rotors are able to follow the given trajectory but still with a significant steady state error.

This project had been done by using state feedback pole placement method. At first, the system modeling and identification need to be done before state feedback can be designed. The models had been designed and simulated by using Simulink before the models were tested in real-time experiments.

In a nutshell, objectives of this project are successfully achieved and here is the end of project entitled 'Dynamic Modeling and Closed-loop System of a Twin Rotor MIMO System'

5.2 Recommendation

There is many more control strategy that can be implemented to this TRMS. Among of the control strategy are adaptive control, hierarchical control, optimal control and intelligent control. [6][7]

This project also can be tried by using PID method instead of state feedback method. There are many type of PID controller designs can be implement for a plant in control system. Further study on PID controller designs is highly recommended for the development of this project. Common PID controllers in control systems are Ziegler-Nichols and Root Locus Method. [5]

Besides, the performance of lab computer for this project still can be improved as its performance is out-dated and it will slow down the pace of project process.

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APPENDICES

APPENDIX A

DESIGN FORMULAS FOR SECOND-ORDER SYSTEMS

These formulas are valid for second order systems of :

$$\text{Ts} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Peak Time $$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

- Settling time $$T_s \cong \frac{4}{\zeta\omega_n}$$

- Percent Overshoot $$\%OS = 100\, e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$