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MULTIVARIABLE SYSTEM IDENTIFICATION OF A

CONTINUOUS BINARY DISTILLATION COLUMN

by

MOHAMMAD ADNAN BALOCH

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MULTIVARIABLE SYSTEM IDENTIFICATION OF A CONTINUOUS BINARY DISTILLATION COLUMN

by

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ABSTRACT

Distillation is a process that is commonly used in industries for separation purpose. A distillation column is a multivariable system which shows nonlinear dynamic behavior due to its nonlinear vapor-liquid equilibrium. In order to gain better product quality and lower energy consumption of the distillation column, an effective model based control system is needed to allow the process to be operated over a certain operating range. In control engineering, System Identification is considered as a well suited approach for developing an approximate model for the nonlinear system. In this study, System Identification technique is applied to predict the top and bottom product composition by focusing the temperature of the distillation column. The process in the column is based on the distillation of a binary mixture of Isopropyl Alcohol and Acetone. The experimental data obtained from the distillation column was used for estimation and validation of simulated models. During analysis, different types of linear and nonlinear models were developed and are compared to predict the best model which can be effectively used for designing the control system of the distillation column. Among the linear models such as; Autoregressive with Exogenous Input (ARX), Autoregressive Moving Average with Exogenous inputs (ARMAX), Linear State Space (LSS) model and Continuous Process Model were developed and compared with each other. The results of this comparison reveals that the performance of LSS model is efficient and hence it was further used to improve the modeling approach and compared with other nonlinear models. A Nonlinear State Space (NSS) model was developed by the combination of LSS and Neural Network (NN) and is compared solely with NN and ANFIS identification model. The simulation results show that the developed NSS model is well capable of defining the dynamics of the plant based on the best fit criteria and residual performance. In addition to this, NSS model predicted the best statistical measurement of the nonlinear system. This approach is helpful for designing the efficient control system for online separation process of the plant.

ABSTRAK

Penyulingan merupakan proses yang banyak digunakan oleh industri, untuk bertujuan pengasingan. Menara penyulingan; iaitu sistem yang mempunyai pelbagai pembolehubah berupaya menggambarkan perilaku dinamik tidak linear yang disebabkan oleh hubungan keseimbangan antara gas dan cecair. Dalam usaha untuk menghasilkan produk yang berkualiti tinggi dan mengurangkan penggunaan tenaga oleh menara penyulingan, satu model sistem kawalan diperlukan untuk memastikan proses yang beroperasi pada tahap operasi tertentu. Dalam kejuruteraan kawalan, teknik *'system identification'* dikenalpasti sebagai satu pendekatan yang bagus dalam membangunkan model ramalan untuk system tidak linear. Dalam kajian ini, 'system identification' digunakan untuk meramal komposisi produk dibahagian atas dan bawah dengan mengambil kira suhu menara penyulingan. Proses pengasingan yang dijalankan adalah berdasarkan pada campuran Isopropyl Alcohol dan Acetone. Data eksperimen yang diambil daripada menara penyulingan digunakan untuk membuat ramalan dan pengesahan kepada model simulasi. Semasa menganalisa, beberapa jenis model 'linear' dan model bukan 'linear' dibina dan dibuat perbandingan untuk meramal model terbaik yang boleh digunakan untuk mereka sistem kawalan menara penyulingan yang efektif. Antara model-model linear seperti ARX, ARMAX, model Linear State Space (LSS) dan model 'Continuous Process' dibina dan dibuat perbandingan antara satu sama lain. Hasil daripada perbandingan yang menunjukkan bahawa prestasi model LLS yang sangat efisyen, ia diadaptasi untuk memperbaiki model dan seterusnya dibandingkan dengan model-model tidak linear. Model Nonlinear State Space (NSS) yang dibina merupakan hasil gabungan antara LSS dan Neural Network dan seterusnya dibandingkan dengan model NN dan ANFIS. Hasil simulasi menunjukkan bahawa model ini berjaya mendefinasikan dinamik kepada 'plant' berdasarkan kepada criteria terbaik dan 'residual performance'. Sebagai tambahan, model NSS membuat anggakan pengukuran statistik yang terbaik untuk sistem tidak linear. Pendekatan ini sangat berguna untuk merangka satu sistem kawalan bagi proses pengasingan yang efisyen.

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LIST OF ABBREVIATIONS

Words	**Abbreviation**
Universiti Teknologi PETRONAS	UTP
System Identification	SI
Matrix Laboratory	MATLAB
Auto Regressive with Exogenous Input	ARX
Auto Regressive Moving Average with Exogenous Input	ARMAX
Linear State Space	LSS
Artificial Neural Networks	ANN
Multi Layer Perceptron	MLP
Gradient Decent Momentum	GDM
Lavernberg Marquardt	LM
Nonlinear State Space	NSS
Adaptive Neuro-Fuzzy Inference System	ANFIS
Membership Functions	MFs
Sum of Squared Prediction Error	SSPE
Root Mean Square Error	RMSE
Cross Correlation	CCR

NOMENCLATURES

τ	Time Constant
ζ	Damping Ratio
Φ	Static Mapping for NSS model
Ψ	Static Mapping for NSS model
y	Output
u	Input
θ	Gradient update for NN
μ	Optimization function for NN
α	Momentum for GDM algorithm
R	Search direction NN training algorithms
Z	LM training algorithm partial derivative vector
ε	Error vector

CHAPTER 1

INTRODUCTION OF STUDY

1.1 Introduction

The System Identification (SI), analysis and simulation has always remained an important field and pursued with vigor interest since early 1960s. The precedent literature shows a number of approaches and techniques that can be utilized for the System Identification. It can be as simple as a 'blind' approach using the concept of black box model and can be as complicated as applying techniques on top of an Artificial Neural Network [1]. The models of a real system have a fundamental importance in virtually all disciplines and can be useful for system analysis. It makes it possible to predict or simulate a system's behaviour. In the field of engineering, models are required for the design of new process and as well as for the analysis of existing process. The advance techniques for designing of controllers, optimization, supervision, fault detection and diagnosis components are also based on models of processes [2].

In control engineering, it is usually observed that the developed linear models are being used in different applications. However, in most of the cases, systems are of real time, ill-defined and uncertain in nature. Thus, system modelling by using primitive approaches is inappropriate for system modelling. Industrial plants and complex systems consist of different properties. They may be ill-defined and mostly non-linear systems. Moreover, the operations of such continuous process plants always involve various optional priority criteria such as product yield, cost, concentration and reliability [3, 4].

Continuous process operates at a stationary state for a very long period of time. In each steady state operating point the process functions at an equilibrium condition where the deviation of material composition is small, and the process conditions such as temperatures, pressures and flow rates are kept as constant as possible or within zone limits. In general, a continuous process can be effectively controlled using linear process models, at least for each mode (operating point). This is because any nonlinear function can be well approximated by a linear function around an equilibrium, which explains the success of linear model-based control technologies for continuous processes. The area where linear models are less or non-effective is during process start-up and shut-downs where nonlinear behaviour of the process becomes dominant [5].

The distillation columns are good examples of nonlinear processes. A distillation process is used to separate a given compound into products of higher value. The separation process is basically to vaporize the feed and draw the products from different locations (trays) of the column at different temperatures. The value of the products depends on their quality (purity). This makes quality control very important in operating a column. The operation must also be profitable and meet production goals. The role of distillation control is to meet these tightly interrelated objectives [5].

The nonlinear behaviour, ill-conditioned nature, hydraulic limits, separation limits, heat transfer limits, pressure constraint and temperature checks causes complications in designing the control system for the distillation column [6]. The designing of an effective control system is a prerequisite for sustainable processes as it improves product quality, process safety, product yield and reduces energy consumption. Therefore, this study focuses on developing multivariable linear and nonlinear models to represent the true nature of a binary distillation column for designing effective control strategy.

1.2 Problem Statement

Earlier researchers have revealed that controlling the dynamic process of a distillation column is a challenging task [7]. The use of model based control strategies such as Internal Model Control (IMC) and Model Predictive Control (MPC) have explicitly embedded a process model in the control algorithm. They show better responses in controlling distillation columns against the primitive methods because of their ability to satisfy strict performances and requirement [6, 8].

There are several modelling techniques that are used for the distillation column which can be categorized under fundamental and empirical modelling. The fundamental modelling techniques are based upon the basic elementary principles of the system such as mass, energy and momentum balance which are globally valid and are usually accurate with providing comprehensive process understanding. Nevertheless, primary models are complex for controller design and the process features for primary models developed are based on suppositions which possibly could be sometimes inaccurate [6, 9]. Empirical modelling techniques are based upon input output data measurement of the physical system. Such models are also considered as "black box" models due to the absence of its priori physical knowledge. The collected input-output data from the physical plant is considered as the most valuable information of its operation. Such models explain the practical relationship between the system inputs and outputs. Empirical models also represent the nonlinear relationship accurately in the area shown by the data although if the unmeasured disturbances are presented in the experiment. The results of the models not only depend upon precision of the measured values but also the relationship between the condition to be observed and the condition where the measurement has been performed [6, 10].

Nowadays, the requirement for process industries is fast and efficient modelling. Therefore, fundamental modelling is not attractive due to high manpower cost and long development time. Thus, empirical modelling may offer an attractive option for Malaysian process industries. Current practice in the process industries is to use sequence of single variable open loop testing's. These tests are performed manually. The advantage of these testing methods is that the operator will be able to analyze different step responses and can observe the behaviour of the system. The data from this single step tests may not contain fine process information about the multivariable system and these step signals do not stimulate enough dynamic information of the process. These issues can be rectified by the use of automatic multivariable test. In an open loop multivariable test many Manipulated Variables (MV) are perturbed using some test signals such as Pseudo Random Binary Signal (PRBS) signals [11].

1.3 Research Objectives

Process modelling is an important element for optimized control performance. Therefore, it is essential to investigate various modelling techniques that are suitable for multivariable steady processes. The primary aim of this research is supported by the following objectives;

1. Performing experimental work over a binary distillation column for collecting real plant data. A PRBS type structured input behaviour is given as input to the distillation column.

2. Evaluating linear system identification models, Autoregressive with Exogenous Inputs (ARX), Autoregressive Moving Average with Exogenous Inputs (ARMAX), Linear State Space (LSS) model and Continuous time Process Model and carrying out analysis using residual and correlation tests in application for online process control system.

3. Evaluating nonlinear models such as Neural Network (NN), Nonlinear State Space (NSS), Adaptive Neuro Fuzzy Inference System (ANFIS) models and carrying out analysis using residual and correlation tests in application for online process control system.

4. Examine comparative analysis between the linear and nonlinear models and predict the best model to be used for online applications.

1.4 Scope of Research

In this study, linear and nonlinear models are developed using SI technique and are compared. Linear and nonlinear modelling approaches are compared by measuring the best fit of the model, cross-correlation analysis and by observing the residual performance. For nonlinear identification, Neural Network based approach is more focused. All models in present case are developed completely based on the experimental data collected from an Advanced Process Control (APC) distillation column pilot plant, which will be discussed later in the upcoming chapter. All the models are developed using MATLAB/Simulink software.

1.5 Thesis Organization

This thesis is divided into six chapters

Chapter 2 provides the literature review on SI. It covers the linear and nonlinear methods such as intelligent techniques which have been used in the application of SI for modelling a highly nonlinear system such as distillation column. Some reviews related to this research are tabulated.

Chapter 3 explains the concept of distillation process. It outlines the experimental procedures which have been carried out on the plant using SI technique for data collection and the responses are shown.

Chapter 4 shows the result obtained for linear SI. Multivariable identification models are evaluated with each other. Autoregressive Exogenous input model (ARX), Autoregressive Moving Average with Exogenous input (ARMAX), Linear State Space (LSS) model and Continuous Process Model are compared and evaluated.

Chapter 5 shows the result obtained for nonlinear SI. It discusses Standard Neural Network approach using two different learning algorithms which are Gradient Decent with Momentum (GDM) and Lavernberg-Marquardt (LM). A new developed

Nonlinear State Space (NSS) model which is the combination of Linear State Space (LSS) model and Neural Network (NN) model and Adaptive Neuro-Fuzzy Inference System (ANFIS) models are also evaluated by comparing each other.

Chapter 6 concludes the research findings with discussion over the results and some recommendations for future work.

CHAPTER 2

SYSTEM IDENTIFICATION

2.1 Introduction

System Identification (SI) is an approach of modelling a system. It is a method to construct mathematical models for physical and dynamic systems based on experimental data. Based on an experimental procedure, a system is given certain input and the output response is measured. From the collected data of the input - output measured sequences, a model can be determined and its dynamic behaviour can be analyzed.

In this chapter, the concept of system identification is explained. Further, Linear system identification model and nonlinear system identification models are discussed. Some view over the applications of system identification over distillation column is also presented.

2.2 System Identification: Theory & Techniques

The dynamic behaviour of a system or process in time or frequency domain can be described by using mathematical expression. This mathematical expression is also called as an approximate mathematical model of the system and can be obtained based on basic fundamental theories from physics, chemistry and mathematics. But in many cases such mathematical models are excessively complicated and impossible to obtain in reasonable time due to its complex nature and process [11].

A common method to obtain an approximate mathematical model of a complex system starts by measuring the behaviour of the system with its external influences (system inputs) and then by determining a mathematical relationship of these influences with the system response (system outputs) without measuring the internal process of the complex system. This technique is known as System Identification. Therefore, System Identification can also be said to be an approach of developing approximate mathematical models of a system by means of input output data while performing an experimental process [11]. System Identification approach can be used for both linear and for nonlinear systems. Figure 2.1 summarizes many branches of active research work in this field. Further detail is described in section 2.2.1 and section 2.2.2.

The use of system identification in control engineering has been a popular alternative to physical modelling for obtaining model descriptions of given physical system. Applications of fundamental laws from mechanics, thermodynamics, chemistry etc, are often quite complex and time consuming tasks especially if large scale engineering systems are considered [12].

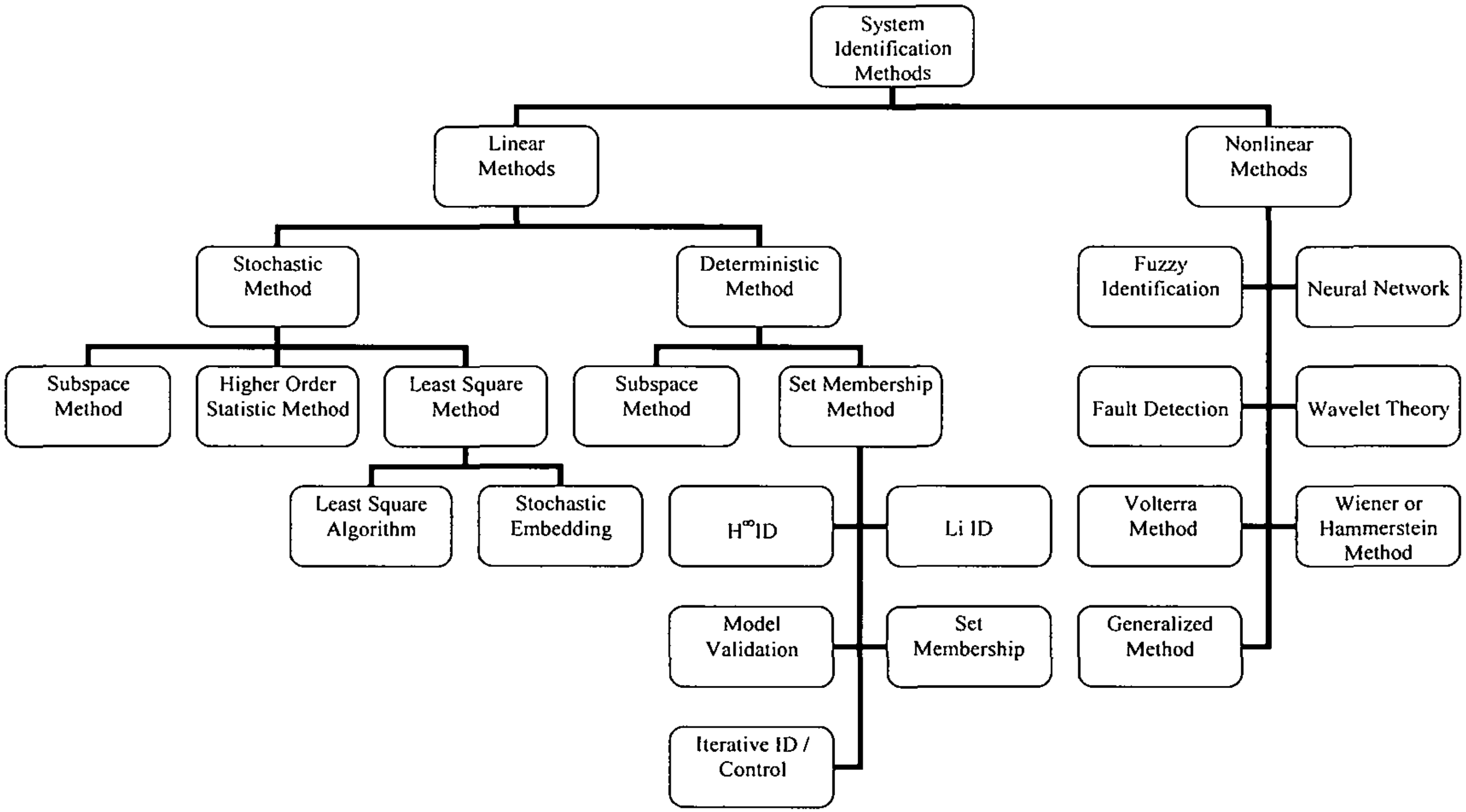


Figure 2.1: Various Methods of System Identification [13].

2.2.1 System Identification Procedure

The formation of an approximate mathematical model by means of input/output measurement involves four steps as explained below. Figure 2.2 shows the summary of the identification procedure.

2.2.1.1 Experimental Test

An experimental work is performed on the system and a set of input-output data is collected. The experiment has to be well designed which requires the ability to determine the input and output signals to be measured involving the sampling interval, which means systems characteristics are well reflected in the observed data. Thus, to capture useful data for system identification, it is essential to have some priori information or physical knowledge about the system.

A good identification test plays an important role in a successful identification. Current practice in the process industries is to use sequence of single variable open loop testing's. These tests are performed manually. The advantage of these testing methods is that the operator will be able to analyze different step responses and can observe the behaviour of the system. The disadvantages of these tests are that it costs high time and manpower. The data from this single step tests may not contain fine process information about the multivariable system and these step signals do not stimulate enough dynamic information of the process. These issues can be rectified by the use of automatic multivariable test. In an open loop multivariable test many Manipulated Variables (MV) are perturbed using some test signals such as PRBS signals [12].

Identification tests can also be done in closed loop operations with Control Variables (CV) under feedback control loop. Advantages of the closed loop tests can be;

1. Reducing the disturbance to the process operation and eliminate product off-specification. When a multivariable open loop test is used, some CVs may

float away and the operator needs to interfere in order to avoid product qualities from off specification. In closed loop test, however one can specify the amplitude of the set point movement and the controller will help to keep the CVs within their operation range [12].

2. Better model for control can be obtained using closed loop test. Under the CV variance limitations, the control performance degradation caused by model errors will be less if the closed loop test are carried out. High purity distillation columns are often ill-conditioned where top and bottom temperatures have strong relation [12].

The controller is used to keep important CVs within their operation restrictions during the identification tests. It can be done by using one or several PID control loops or other existing MPC controller in the plant. When testing the distillation column, it is often sufficient to control the top and bottom temperature using two PID controllers [12].

2.2.1.2 Model Structure

A set of selected models is obtained by identifying the properties. A suitable model is searched within this set. This is one of the most theoretical and difficult part of system identification. A model with some unknown physical parameters is constructed from basic physical laws and other well-established relationships. In other case linear models may be employed without reference to the physical background of the system. Since these models do not essentially imitate the information about the structure of the system, they are referred to as black box models. One of the most challenging issues is to find a good model structure, or to amend model orders, based on the input-output data.

2.2.1.3 Parametric Estimation

When the data is available and the model set is determined, the next step is to find the best model. The evaluation of the quality of the model is naturally considered upon the performance of the model while attempting to regenerate a set of new measured data which has not been used in the development of the model. This is considered the identification method. For model parameter estimation, an error criterion (loss function) is specified. Often the sum of the squares of some error signals (residuals) is used as the criterion. The values of the parameters are determined by minimizing the loss function [11].

2.2.1.4 Model Validation

This step is to examine if the estimated model is adequately good for the intended use of the system. First of all, a check is performed to see if the model is in accordance with the prior knowledge of the system. Then, a check if the model can fit the experimental data well, preferably by using a data sequence that has not been used in estimating the model. The final validation of the model is the application of the model [13].

2.2.2 Types of Models

There are three types of models which are common in the field of system identification [14]:

1. Black Box Model: A system which is analyzed exclusively in terms of its input-output and transfer characteristics without knowing its internal working. No priori knowledge of the system is available. The mathematical representation of black box model depends completely on the input sequence provided and the output is observed

2. White Box Model: It is the opposite of the black box model. The systems inner concept is available for examination. White box sometimes is also known as a glass box or a clear box. The mathematical representation of the model is dependable on the inner information available

3. Grey Box Model: In a grey box model some of the mechanisms describing the behaviour of the system are known, but are not fully represented in the model.

Grey box model specifies moderately known physical factors or a few limitations of the system

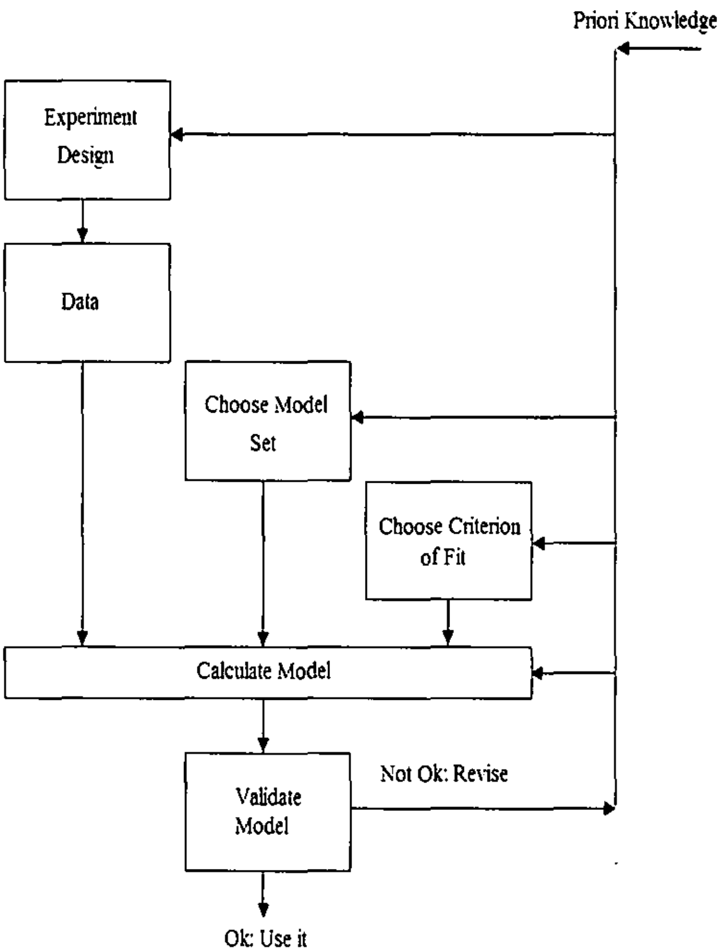


Figure 2.2: System Identification Procedure [11].

2.2.3 Linear System Identification

Linear system identification often refers completely to the identification of linear dynamic systems. It can be distinguished into parametric and non-parametric methods. Parametric models are capable of describing the process behaviour accurately with finite numbers of parameters such as the use of differential or difference equation model. These parameters often have a direct connection to physical measurements of the process such as mass, volume, composition, temperature etc. Nonparametric models generally require an infinite number of parameters to describe the process. One example of nonparametric model is an impulse response model [14].

Furthermore parametric methods determine a relatively small number of parameters. Usually these parameters are optimized according to some objectives. Parameter estimation is one typical example of parametric method. Parametric methods can also be used for determination of approximate non-parametric models whose number of parameters has been reduced to finite number. Finite Impulse Response (FIR) model is a good example of such cases [14]. In this thesis, parametric methods are discussed in the subsequent sections.

2.2.3.1 Autoregressive with Exogenous Input (ARX) Model

The ARX model is by far the most widely applied linear model in industries. Its parameters can be obtained by linear least square technique since the prediction error is linear in the parameters [14]. Equation 2.1 is the ARX model equation. Based on this equation, the ARMAX model block diagram is shown in Figure 2.3.

$$y[k]=\frac{B(q)}{A(q)}u[k]+\frac{1}{A(q)}e[k] \tag{2.1}$$

where $y(k)$ is the model output, $u(k)$ is the input to the model, $A(q)$ and $B(q)$ are the model polynomials and $e(k)$ is the noise.

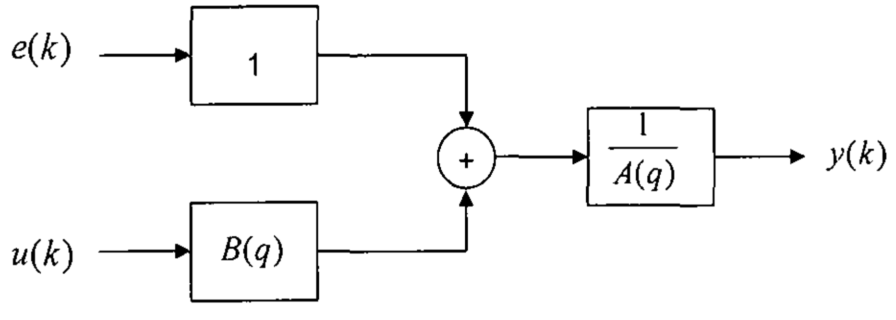


Figure 2.3: ARX Model.

2.2.3.2 Autoregressive Moving Average with Exogenous (ARMAX) Input Model

The ARMAX is the second most popular linear model after the ARX model. Compared to ARX model, ARMAX model is more flexible because it possesses an extended noise model. The identification method for ARMAX is similar to ARX model which is the prediction error method. Equation 2.2 is the ARMAX model equation [14]. Based on this equation, the ARMAX model block diagram is shown in Figure 2.4.

$$y[k]=\frac{B(q)}{A(q)}u[k]+\frac{C(q)}{A(q)}e[k] \tag{2.2}$$

where y(k) is the model output, u(k) is the input to the model, A(q), B(q) and C(q) are the model polynomials and e(k) is the noise.

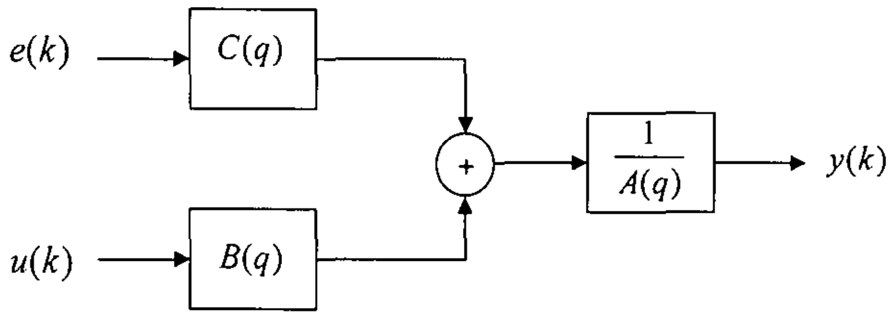


Figure 2.4: ARMAX Model.

2.2.3.3 Linear State Space Model

The Linear State Space (LSS) model presents an absolute image of the system, especially for MIMO systems against polynomial models. This is for the reason that the state space models are comparable to primary principle models. Nonlinear optimization is not involved in the identification process, therefore the initial guess is not necessarily to be considered while estimating the model [14]. Furthermore, adjusting the parameters for state space models are easier. The order or the number of states of the model is only selected. It also can determine the order by analyzing the singular values of the information matrix. The matrices are obtained based on the following two equations:

$$x(k+1) = Ax(k) + Bu(k) \tag{2.3}$$

$$y(k) = Cx(k) + Du(k) \tag{2.4}$$

x(k) is the state vector, *y(k)* is the system output, *u(k)* the system input. A, B, C and D are the system matrices. The dimension of the state vector *x(k)* is the only setting needed to provide for the state-space model. From equation 2.3 and 2.4, the linear state space model block diagram is shown in Figure 2.5.

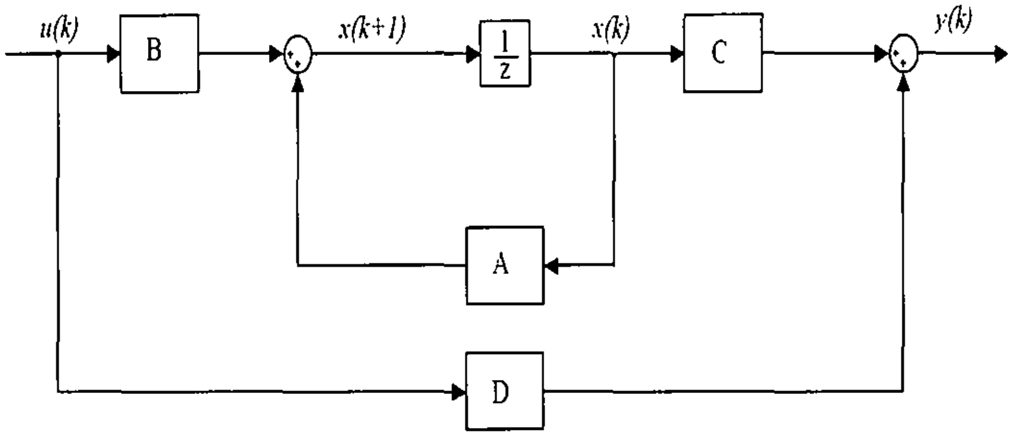


Figure 2.5: LSS Model Block Diagram.

2.2.3.4 Continuous Process Model

Continuous Process Models are basic models which can be identified for any dynamic system. It is simple to perform and although it is the least general method, it provides adequate models for many applications. The form of the model is as follows, with X(s) as the input and Y(s) denoting as the output;

For 1st order process model, the equation is given as:

$$\frac{Y(s)}{X(s)} = \frac{K_P}{\tau s + 1} \tag{2.5}$$

For 2nd order process model, the equation is given as:

$$\frac{Y(s)}{X(s)} = \frac{K_P}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{2.6}$$

For 3rd order process model, the equation is given as:

$$\frac{Y(s)}{X(s)} = \frac{K_P}{(\tau_3 s + 1)(\tau_2 s + 1)(\tau_1 s + 1)} \tag{2.7}$$

where K_P is the process gain, τ is the time constant and ξ is damping ratio with respect to the input of the system.

2.2.4 Nonlinear System Identification

Modelling and identification of nonlinear dynamic systems is a difficult task because nonlinear processes are distinctive in the sense that they do not share many properties. A major goal for any nonlinear system modelling and identification scheme is universal: that is, the capability of describing a wide class of structurally different systems [14]. Figure 2.6 shows the task of nonlinear system identification.

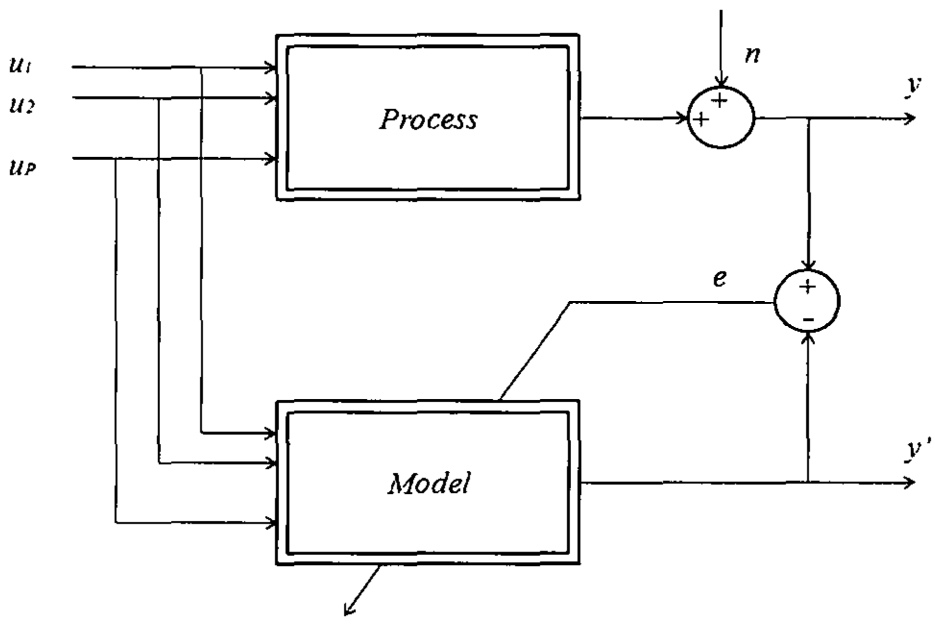


Figure 2.6: System Identification Model [15].

The concept of linear and nonlinear system identification is merely similar. In nonlinear system identification as shown in Figure 2.6 the model is adapted in order to represent the process behaviour. Process and model are fed with the same input $\underline{u} = [u_1\ u_2\ \ldots\ u_p]^T$, and their outputs y and y' are compared with the yielding the error signal e, which can be utilized for adapting the model. Note that the process output is usually disturbed by noise n [14].

Artificial Intelligence has played a significant role in nonlinear system identification. The application of Fuzzy Logic and Neural Network has a vast contribution for modelling not only linear but nonlinear systems. Fuzzy is mainly a rule based modelling technique while neural network updates its weights during computation. A similar concept is used by Adaptive Neuro Fuzzy Inference System (ANFIS) for updating its computational weights while being evaluated by the defined rules. In this study, three different nonlinear approaches have been discussed, the Artificial Neural Network, ANFIS technique and the Nonlinear State Space Model.

2.2.4.1 Artificial Neural Network

Artificial Neural Network (ANN) is a very common topic in the study of system identification. Due to its nonlinear computation, ANN provides a very much accurate measurement of a process system by means of its input-output data measurement. In terms of the ANN structure, it can be described in two: the Feedforward Network and the Recurrent Network.

Feedforward networks are very commonly used networks. An example of Feedforward network is the Multilayer Perceptron (MLP). Feedforward networks perform forward mappings between the input and output space which correlate the output and input of the identified mappings [16].

Training algorithm plays a very important role in computing the weights of the neural network in order to reduce the error as much as possible against the target data. The error of a particular arrangement of the network can be established by running several training algorithms through the network and later comparing the actual output generated with the target outputs. Throughout the training process, the weights of the network are iteratively adjusted in order to minimize the network performance function [17, 18]. The most common error function is the Mean Squared Error (MSE) where the individual errors of the output units on each case are squared and summed together. The mean square error is defined by the following equation:

$$E_N(\theta) = \sum_{i=1}^{N} \left(y_i - y(\theta, x_i)\right)^2 \tag{2.8}$$

where x_i is the input, y_i is the target output, $y(\theta,x_i)$ is the network output and θ is the parameter (weight).

A good approximation for the parameter θ is that it minimizes the Mean Square Error (MSE). The MSE is reduced by updating θ along the negative gradient of the MSE and this is given as follows:

$$\theta^{i+1} = \theta^{i} - \mu R V_\theta V_N(\theta) \tag{2.9}$$

In equation (2.9), the matrix R may change the search direction from the negative gradient direction to a more positive one. The function of parameter μ is an optimize based MSE criteria.

There are numbers of learning algorithm available for training a neural network. The very basic learning algorithm is the back-propagation learning algorithm in which the weights are updated according to the delta rule or also known as gradient decent. This algorithm is further upgraded with the addition of a constant value of momentum called as "Alpha (α)" for a faster convergence of the network output with respect to its weight update. This is shown by the following equation:

$$\Delta w = \alpha \times x_i \times \delta_i \tag{2.10}$$

Where x_i is the input to the particular neuron, δ_i is the error gradient and α is the momentum. If equation (2.9) is to be updated from equation (2.10), than it can be said that $R=\Delta w$.

Other learning algorithm such as Quasi-Newton, Resilient Back-propagation as well as conjugate gradient decent are much faster and advance training algorithms used in training the neural network. One another type of learning algorithm is the Lavernberg Marquardt (LM). As stated in [21], LM is an advanced non-linear optimization algorithm. Similar to as back-propagation network algorithm, LM is used to train the weights in a network. LM presumes that the basic function being modelled by the neural network is linear. Upon this computational approach, the minimum is settled closely in a single step. The computed minimum is evaluated and if the error is minor, the weights are shifted to a new point. Upon each generation this procedure is reiterated [22]. LM therefore negotiates between gradient descent approach and the linear model. It only makes the move when the error needs to be improved. When necessary the gradient decent model is used adequately in a small step to pledge a simple move [22]. LM uses the update equation:

$$\Delta w = -(Z^T Z + \lambda I)^{-1} Z^T \varepsilon \tag{2.11}$$

where ε is the vector of case errors, and Z is the matrix of partial derivatives of these errors with respect to the weights: if equation (2.9) is to be updated from equation (2.11), than it can be said that $R=\Delta w$.

Neural Network has been used by several researchers for the purpose of process identification. Due to its computational algorithms, ANN provides a close approximate model for systems especially when there is a presence of highly nonlinear behaviour. This makes it easier to design controllers, observers or even proper analysis can be performed for system enhancement.

2.2.4.2 Nonlinear State Space Model

The linear state-space model (LSS) presents an absolute image of the system, especially for MIMO systems against polynomial models. This is for the reason that the state space models are comparable to primary principle models. Nonlinear optimization is not involved in the identification process; therefore the initial guess is not necessarily to be considered while estimating the model. Furthermore adjusting the parameters for state space models are easier. The order or the number of states of the model is only selected. It also can determine the order by analyzing the singular values of the information matrix [23].

Since a LSS model is capable of representing a dynamic system, therefore the n^{th} - order multi input multi output (MIMO) time invariant nonlinear dynamic system can be written as follows [23]:

$$x(k+1) = \Phi[(x(k), u(k)] \tag{2.12}$$

$$y(k) = \Psi[(x(k)] \tag{2.13}$$

where Φ and Ψ are static nonlinear mappings. If the system is linear, than the linear state space model can be written as follows:

$$x(k+1) = Ax(k) + Bu(k) \tag{2.14}$$

$$y(k) = Cx(k) + Du(k) \tag{2.15}$$

where $x(k)$is the state vector, $y(k)$is the system output, $u(k)$is the system input. A, B, C, D, are the system matrices. The aspect of the state vector $x(k)$ is the only setting needed to provide for the state-space model.

As explained in [23] in equation 2.12 and 2.13, $\Phi(.)$ and $\Psi(.)$ are arbitrary functions which may or may not be linear. Therefore neural network is being used to approximate these mappings. Φ (.) maps the system states $x(k)$ and input $u(k)$ into the new state $x(k+1)$ while $\Psi(.)$ transforms the state $x(k)$ into the output $y(k)$. When using neural network to identify the system described by equation 2.13 and 2.12, two important assumptions are considered: (1) all the system states are measureable; (2) the system is stable. Based on these two important assumptions, Figure 2.7 shows the block structure of the nonlinear state space model. Significance of this block structure is for system identification as explained in chapter 6.

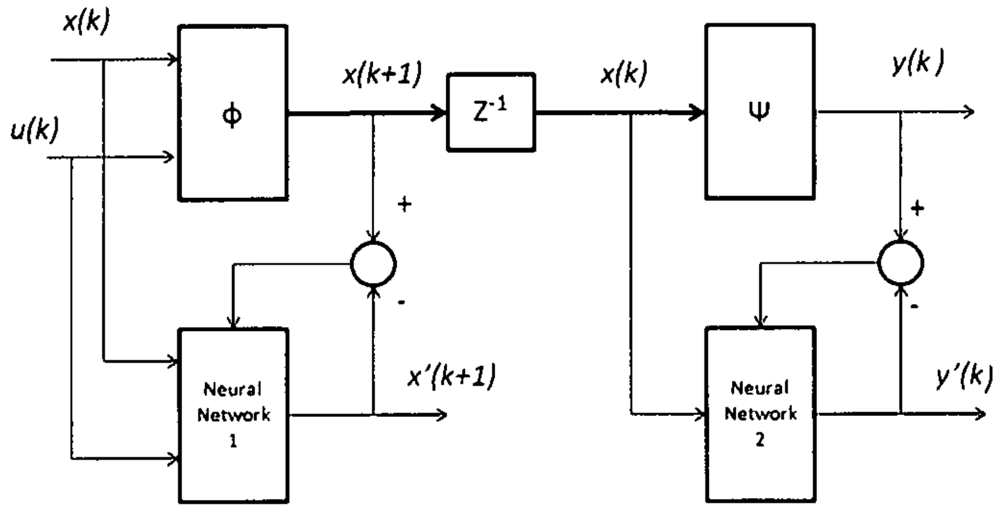


Figure 2.7: Identification Based on Measurable System States [23].

2.2.4.3 Adaptive Neuro-Fuzzy Inference System (ANFIS)

Neuro-Fuzzy networks are fuzzy models that are not exclusively designed by special understanding but are at least partially estimated from data. The relationship between fuzzy models and neural network forces the data-driven out by fuzzy modelling. Usually, the fuzzy model is structured in neural network architecture and learning techniques which are already established in the neural network framework are applied over the neuro-fuzzy networks [14]. Figure 2.8 shows the network architecture for the ANFIS model.

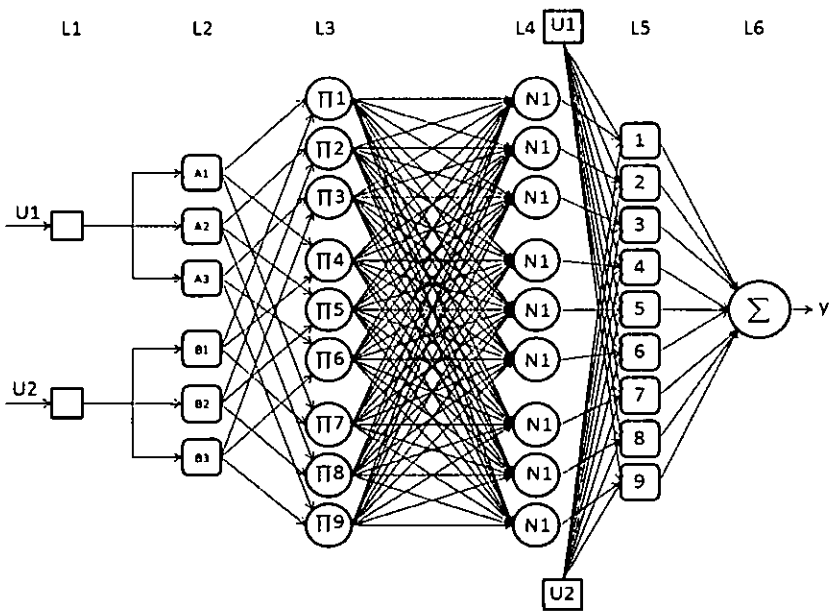


Figure 2.8: Adaptive Network Based Fuzzy Model.

ANFIS combines the least squares estimators and the gradient decent method [19,20]. Training algorithm of ANFIS composes a forward pass and a backward pass in each epoch. All through the forward pass, the ANFIS network receives a training set of input. Neurons outputs are computed on the layer-by-layer basis and the rules significant values are computed by least square estimation [19].

ANFIS is designed in several layers. It has an input and output layer and three hidden layers that represent functions and fuzzy rules [19]. Function of each layer of ANFIS as shown in Figure 2.8 for the case of distillation column is explained as below;

i. **Layer 1** is the input layer. Each neuron in this layer conveys external crisp signals directly to the next layer. That is;

$$y_i^{(1)} = u_i^{(1)} \tag{2.16}$$

where $u_i^{(1)}$ is the input of layer 1 and $y_i^{(1)}$ is the output of input neuron i in the layer. In case of 2 inputs for the distillation column, $u_i^{(1)}$ can be written as

$$u_i = \begin{cases} u_1; & reflux \quad flowrate \\ u_2; & steam \quad flowrate \end{cases}$$

ii. **Layer 2** is where the input signal computes with a membership/activation function. This process is known as fuzzification. The neurons can have any form of activation function, *i.e.* for a bell type activation function, it can be stated as;

$$y_i^{(2)} = \frac{1}{1+\left(\dfrac{u_i^{(2)}-a_i}{c_i}\right)^{2bi}} \tag{2.17}$$

where $u_i^{(2)}$ is the input for layer 2, $y_i^{(2)}$ is the output of layer 2, a_i, b_i and c_i are the parameters for control.

iii. **Layer 3** is the rule layer. Neuron in this layer corresponds to a single Sugeno-type rule [19]. The output of neuron i in this layer 3 is obtained as;

$$y_i^{(3)} = \prod_{j=1}^{k} u_{ji}^{(3)} \tag{2.18}$$

Following are examples of rules used for the identification of distillation column

- If **(reflux)** is **(MF1)** and **(steam)** is **(MF2)** than **(Top Temperature)** is **(MF3)**
- If **(reflux)** is **(MF1)** and **(steam)** is **(MF3)** than **(Top Temperature)** is **(MF2)**
- If **(reflux)** is **(MF2)** and **(steam)** is **(MF3)** than **(Top Temperature)** is **(MF1)**

iv. **Layer 4** is the normalization. The output of neuron i in this layer is determined as;

$$y_i^{(4)} = \frac{u_i^{(4)}}{\sum_{j=1}^{n} u_j^{(4)}} \tag{2.19}$$

where $u_i^{(4)}$ is the input and $y_i^{(4)}$ is the output of layer 4.

v. **Layer 5** known as defuzzification layer. This is where fuzzy techniques are applied to be part of neural network structure. The defuzzification neuron calculates the weighted consequent as;

$$y_i^{(5)} = \overline{u}_i[k_{i0} + k_{i1}u1 + k_{i2}u2] \tag{2.20}$$

where k_{i0}, k_{i1} and k_{i2} are set of consequent parameters of rule i.

vi. **Layer 6** represents a single summed neuron. All the defuzzified neurons are summed and the ANFIS output y *is* obtained by following equation

$$y = \sum_{i=1}^{n} \overline{u}_i[k_{i0} + k_{i1}u1 + k_{i2}u2] \tag{2.21}$$

ANFIS is nowadays widely being used not only for identifications but for different types of applications such as fault diagnosis, pattern recognition, medical diagnosis, control systems etc. In this study, ANFIS is used for system identification of a dynamic system, which is considered as a black box.

2.2.5 Multivariable Modelling

System Identification modelling techniques are also performed in multivariable approach. Process Industries are places where huge applications of multivariable systems can be found. Mostly these multivariable systems are nonlinear. Distillation Columns are good examples of multivariable process. Several variables in distillation columns are related to each other's process. Therefore single variable modelling is not much likely to be acceptable. Figure 2.9 shows an overall MIMO process system.

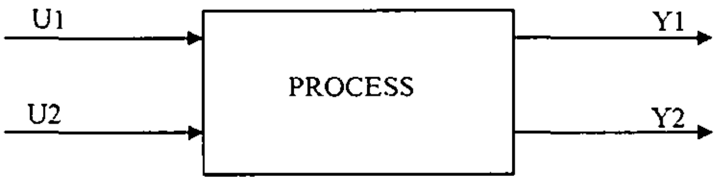


Figure 2.9: MIMO Process Block Diagram.

A Multi-Input Multi-Output (MIMO) can be decomposed into several Multi-Input Single-Output (MISO) one for each output. Modelling a system with several inputs may always be in different order with more than single output. Dividing MIMO systems into MISO systems makes it easier to analyze the system. Figure 2.10 shows a decomposed structure of Figure 2.9 into MISO process structure.

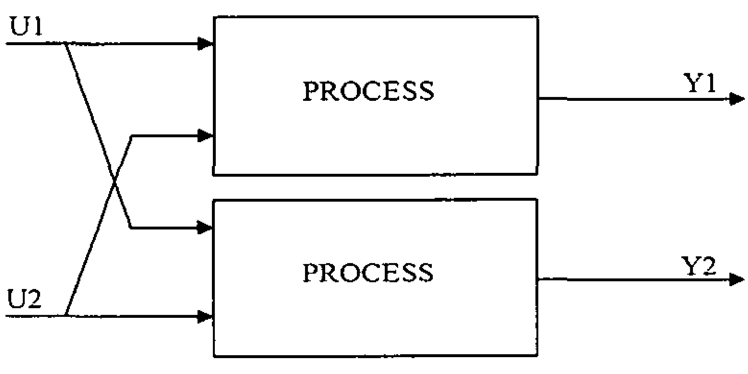


Figure 2.10: MISO Process Block Diagram.

2.3 A Review of System Identification Techniques in the Application of Distillation Columns

Several authors have discussed in their publications over the application of system identification for Distillation Columns. Some works were performed over continuous binary distillation columns; some are on batch type distillation column. Some application of ARX, ARMAX, and Linear State Space models over binary distillation column are summarized in the Table 2.1.

From Table 2.1 the study performed in [24], identification and temperature based control of a nonlinear distillation column was examined. Numbers of ARX models were created and based on different model validation tests best one was selected. The best nonlinear polynomial ARX model was used in nonlinear model predictive control with including the effects of nonlinear disturbance models. The controller performed well for set point tracking and rejecting the disturbance. Comparative analysis has been performed with the ARX model within the predictive control algorithm. Unsatisfactory response was observed from the performance of the linear model against the nonlinear model for the system until the parameters of the linear model are recursively updated for prediction errors.

From Table 2.1 the study performed in [25], a nonlinear ARX model (NARX) was developed for identifying a distillation column. The studies on the nonlinear dynamic characteristics of the distillation column using the experimentally validated first principle model have proven the need of nonlinear model for the distillation column. The study on the regressors of the NARX model has concluded that it has the capability of capturing the nonlinearity of the process.

From Table 2.1 the study performed in [26], an advanced technique of orthonormal basis filters is used in development of ARX model. Simulation studies on a staged binary distillation column have been performed. In the simulation studies, servo and regulatory performance of the models are analyzed. Proposed models are compared with conventional ARX and ARMAX models. The modelling approach proposed in this work can be used to identify higher order models linked with single unit operations. The developed models needed further modification to take into account the structural appearance of the system.

Table 2.1: System Identification Techniques Applied over Distillation Column

Model	Distillation Column Process	Type of Model	Discussion	Ref.
ARX	Methanol-Ethanol	NARX	• Simulation based model • Considered top & bottom temperature of the process column • Candidate models were generated and promising ones were selected based on different model validation tests	[24]
	Binary mixture of Methanol-water	NARX	• The study of effect of each regressor in the NARX model on nonlinearity of the process has concluded that the regressors of the past outputs having more effect on nonlinearity of the process compare to input regressors	[25]
	Simulation work / Methanol-Water	Reparametrized ARX	• An advanced technique using Orthonormal basis filters in development of ARX model	[26]
ARMAX	Binary mixture of Methanol-water	NARMAX	• Concept of GMC was used to identify the model for the process plant • Experimental data was used to develop the model and was compared with another developed linear model • The result obtained from the study is that the developed NARMAX model performed much better than the linear model terms of control	[27]
	Not stated	Quasi-ARMAX	• The modelling approach manages to capture the nonlinearity and the directionality of the process	[28]
	Gasoline/butane	NARMAX model	• A set of data was used to tune the parameters of a NARMAX structure implemented by using MLPs with appropriate lagged inputs • Dynamic nonlinear models were introduced to describe the relationship between a number of input variables and the estimated quantities	[29]
Linear State Space	Ethanol/1-propanol, ethanol/water	Time varying linear state space	• A linear time varying state space model was developed and tested • Comparing the model predictions with rigorous simulations, the state space model was able to predict the plant behavior accurately	[30]

From Table 2.1 the study performed in [27], concept of general model control (GMC) was used to develop the NARMAX model using experimental data and was applied for control purpose of the distillation column. GMC action based on NARMAX model performed successfully. Developed model was also compared to a linear model and has performed better in case of load disturbances for feed mole fraction and temperature.

From Table 2.1 the study performed in [28], a quasi-ARMAX model was developed for the purpose of modelling a multivariable nonlinear process. A binary distillation column was used as a case study. The developed model was capable of capturing the dynamics of the nonlinear plant. A nonlinear model predictive controller is presented for the control of the nonlinear process. The controller uses the quasi-ARMAX model for prediction and manages to perform a suitable control of the distillation column at different operation points.

From Table 2.1 the study performed in [29], a general procedure based on neural soft sensors has been proposed for real time estimation of variables obtained with large and unknown measuring delays and has been applied to estimate the concentrations of the top and bottom products in a debutanizer distillation column. Dynamic nonlinear models were introduced to illustrate the relationship between a number of input variables and the estimated quantities.

From Table 2.1 the study performed in [30], a linear time-varying state-space model has been developed and tested. The developed model was able to analyze the process plants behaviour. The results showed that for the same sampling period, the predictions are higher. It has been observed that the main disadvantage of the linear state space model is that it demands the knowledge of full state of the system. This issue could be solved if an observer is applied to it. The developed model is simple enough for studying the controllability and observability of the system, designing and implementation for an online control system.

Several researchers have performed system identification techniques using different nonlinear algorithms. ANFIS and Neural Network techniques have been the most widely used for modelling highly nonlinear systems such as Distillation Columns. Table 2.2 and Table 2.3 show few reviews of ANFIS and Neural Network identification approach of a Distillation Column.

Table 2.2: ANFIS Techniques Applied in Distillation Column

No	Distillation Column Process	Source of data	Discussion	Reference
1	Not mentioned	Simulation	• The study is aimed to estimate the compositions in the multi-component distillation column from temperature measurements • From the input -output data of the rigorous plant simulation ANFIS estimator is trained and from several simulation runs for different architectures of ANFIS estimator the estimator performance is optimized	[31]
2	C_3-C_4 : i-butane, n-butane, i-pentane	Experimental	• Estimators structures verification & generation capabilities are good in feed flow changes • Triangular structures are better than Gaussian	[32]
3	Methanol-Water	Simulation	• Varying feed flow, initial liquid composition values both in the column, boiler and condenser along with input values for the control actions were imposed on the model • As the model's dynamic will be modified with the unknown perturbations, the ANFIS model will be updated with real plant response	[33]

Table 2.3: Neural Network Techniques Applied in Distillation Column

Type of NN model / Training Algorithm	Distillation Column Process	Source of data	Discussion	Ref.
Feed-forward / GDM	buthylacetate-buthylalcohol water	Experimental	• Developed inverse dynamic model • Model applied to neural controllers successfully	[34]
	Propane/butane/ n-pentane/i-pentane/hexane	Experimental	• Model assessed against conventional method • Model outer performed	[35]
	Ethyl Benzene/ Methyl-Ethyl-benzene/di-ethyl-Benzene	Industrial	• Using input/output data of 2 present & past MPC model • Result showed good improvement	[36]
	Butadiene-1,3/ethyl acetylene/cis-butene-2/butadiene-1,2/C5	Experimental & simulation	• Using multipoint inputs to the NN model • NN soft sensors model became the based scheme inferential control & managed to strengthen the practicability of controller	[37]
	Crude oil	Experimental	• The NN model developed could represent & describe the process I/O relation	[38]
	5 mixture	Simulation	• Rigorous model data used for training & testing • Comparative analysis showed the ANN was in good agreement with results of simulations	[39]
Feed-forward / LM	Binary and multi-component	Simulation	• Several NN models developed with different numbers of hidden layers • Model developed compared against steepest decent BP Algorithm	[40]
	Not mentioned	Industrial	• Filter based NN • Model compared to Linear model • NN was able to accurately predict the dynamic step response	[41]
	Methanol - Water	Experimental	• Comparison between MISO & MIMO ANN model • Data used based on developed general code of 1st Principle model that can be used for binary & multi-component systems	[42]

From Table 2.2 the study performed in [31], two types of approaches are analyzed and compared, Extended Kalman Filter (EKF) and ANFIS. Using input-output data of a plant ANFIS network is trained and several simulation iterations were performed to obtain the optimize performance. Models were applied to an open loop control system where ANFIS performed better than the other developed approach.

From Table 2.2 the study performed in [32], ANFIS structures are trained and tested to gather one of the components of the top and bottom product compositions. The estimator's structures verification and generalization competencies are good enough especially in feed flow rate changes. Triangular structures are observed to be better than Gaussian structures that are used in membership functions (MF). Selected model of ANFIS is compared with NN and it has been observed that the ANFIS estimator performed better.

From Table 2.2 the study performed in [33], the design of the composition ANFIS is to use in conjunction with ANFIS-GA for dual control of the distillation column. Simulation based work has been carried out where the model performance was satisfactory. The performance of the control structure was verified for set-point and disturbance rejection.

From Table 2.3 the study performed in [34], feed-forward neural network approach was to develop an inverse dynamic model of a buthylacetate-buthylalcohol-water distillation column. The nonlinear dynamic relationship between the top product and the reflux flow rate was chosen as the inverse model. A distillation column plant input-output data was used as the training data and back-propagation learning algorithm was used to amend the weights of the network. Several input variables are used for the network. The reflux flow rate was selected as the output for the NN model. The inverse model was applied to the neural controller and the results showed successful controlling action.

From Table 2.3 the study performed in [35], three layered feed-forward neural network was used to model an 17 tray distillation column. Simulation data was used to train the neural network via back-propagation as learning algorithm. The performance of the neural controller was evaluated and compared with a conservative temperature control loop and neural inferential controller. It has been observed that the neural controller outperformed against the other controllers in terms of the transient and steady state response.

From Table 2.3 the study performed in [36], neural network model was developed for a multi-component high purity distillation column. A 52 valve trays industrial column was considered as a case study. A neural network with two hidden layers was used to construct the model. The training data was collected from an off-line experiment and back-propagation was used as learning algorithm. Results of the NN-MPC showed an enormous improvement in the control of the system over the linear MPC algorithm.

From Table 2.3 the study performed in [37], the neural network model was used for industrial distillation technology to develop a soft-sensor model. A three-layer back-propagation neural network model was developed which consisted of five input and two output variables. For the training data, 239 group data was collected from the plant data. The NN soft sensor model became the based scheme in inferential control and it managed to strengthen the probability of the controller.

From Table 2.3 the study performed in [38], the feed-forward neural network approach was used to build the operating model of a crude oil distillation unit (CDU). The CDU consisted of 44 trays distillation column. The neural network model with one hidden layer was used in order to predict the oil product qualities with respect to the system input variables. The neural network was trained using 232 sets of experimental data which was collected from the CDU operating system and back-propagation training algorithm was used to train the network. It has been observed that the neural network model was able to represent and describe the CDU process for the input and output relations.

From Table 2.3 the study performed in [39], ANN was used to model a 14 tray distillation column. The model was used to estimate five variable composition of the plant. The neural network model consisted of five layers with 116 input neurons, 10 output neurons and 34 neurons in all the three hidden layers. The output consists of liquid compositions and vapor compositions. Data gained from rigorous model is used for training and testing a neural network model. The training process was done by employing the back-propagation training algorithm. The comparison was done between the ANN based estimator and a semi rigorous model which showed that the predictions made by the neural network was in good agreement with the results of the simulation.

From Table 2.3 the study performed in [40], a NN estimator was developed based on the LM learning algorithm and tested for 14 trays of binary and a multi-component mixture process. The training data was obtained from the simulation of mathematical modeling. The neural network models consisted of networks with 11 and 12 inputs for the binary process system and 116 and 19 inputs for the multi-component system. The output consisted of 4 and 10 neurons for the binary and multi-component systems respectively. The neural network was trained with the LM algorithm which was more accurate as compared to the Steepest Descent Back-Propagation algorithm.

From Table 2.3 the study performed in [41], filter-based neural network model was developed. This model is relatively similar to multilayer perceptron architecture. The data for the model was gained from an actual distillation column plant and Lavernberg-Marquardt algorithm was used to train the neural networks. Four variables which are the column feed, cooling water temperature, column vent and reboiler were chosen as inputs to the model. The output from the model was the column pressure. Developed NN model was compared with a linear model to examine the performance. It was found that the neural network model developed was able to precisely predict a dynamic step response of the system.

Based on Table 2.1, it can be observed that there are several nonlinear techniques used for the distillation column which possess a nonlinear behaviour by nature. Linear models are not much considered for system identification of nonlinear systems. Further observing Table 2.2, not a consistent model has been observed using ANFIS modelling structure and yet representing a MIMO system by MISO system is yet undiscovered. From Table 2.3, several NN structures are presented by researchers and it has been observed that no specific model has been observed to represent the best defined structure of a nonlinear system such as the distillation column.

One common analysis has been done is that most of the researchers construct the models based upon simulation data and developing the model using first order principles (model estimation). Furthermore, verifying the models using either simulation data by which the complete model tends to be a simulation based model or verify by experimental data (model validation). Some approaches do follow the sequence of performing model estimation and model validation by using completely experimental data. One most important point observed is that while the collection of data point from the plant, the standard input signal in system identification to be used is the PRBS which has not been properly practiced. PRBS input tends to show critical changes in the plant. The importance of experimental data using PRBS as input has been explained in section 2.2.1 of chapter 2.

One another analysis which has been observed is that for the purpose of control of a distillation column is based upon several inputs from the plant process column. Since all the dynamic effects of the inputs are interlinked with each other in the process column, therefore to make an easy control strategy for the plant is just to observe the main two data point which are simply be the two main inputs of the process column. Since these two input points are the nearest and are the most effective variables to the top and bottom temperature of the distillation process column.

Consequently, the present study will focus on the standard structure of system identification by using PRBS input on a multivariable scale over the two main input variables which are the reflux flow rate and the steam flow rate. These two variables are the most effective to the top and bottom temperature changes of the distillation process column. The standard system identification approach will be applied over a newly commissioned industrial sized binary distillation column and several linear and nonlinear models will be analysed.

2.4 Summary

In this chapter, a precedent literature is discussed about the concept of system identification. From basic understanding to intelligent techniques which have been used in the application of system identification for modelling a highly nonlinear system such as distillation column have been discussed. Linear system identification techniques have been explained and few reviews have been shown in tabular form which has been used for modelling distillation columns. Nonlinear system identification discussion in this chapter is mainly focused over the technique of neural networks and Adaptive Neuro Fuzzy Inference System. Comparatively neural network has been applied more for modelling a distillation column. This is due to the different computational algorithms which can be used for modelling a nonlinear system. Few literatures are discussed as well.

Although different techniques are used for modelling but it is still not clearly defined that which model is the only best to be used for a particular process. The major factor to be analyzed for modelling a system either linear or nonlinear is the residual of the model against the physical system. If the residual of the developed model is equally or well distributed among a limited range, therefore the model is considered a well developed model. Thus a proper analysis can be performed.

CHAPTER 3

EXPERIMENTAL SETUP AND PROCEDURE

3.1 Introduction

The process of distillation is frequently used to extract desired component from a chemical mixture. Among its type, the binary distillation is the process of separation between two mixtures from a component. This process takes place in a closed process columns where numbers of trays are placed inside. There are many types of distillation columns used in the process industry. In continuous distillation columns, the distillation column continually separates an incoming chemical mixture.

In this chapter, the concept of distillation column and the available process variables are explained. The concept of a process variable selection in distillation column is also discussed. The Advanced Process Control (APC) Continuous Distillation Column pilot plant which has been used for experimental work is modelled and explained. Plant descriptions and operating conditions are also shown in tabular form. SI procedure which has been applied for collection of the input – output data is discussed. The last section of this chapters shows the experimental input-output results collected from the plant which were used for system identification.

3.2 Distillation Column Process

Distillation Columns are widely used in industrial applications, especially in gas plants to make the distillation more efficient. Distillation Columns are specially designed columns in a tall cylindrical shaped column called as process column. This process column is internally fitted with numbers of trays horizontally to achieve efficient separation of a liquid feed which is inserted at one or various point of the process column [43].

There are two types of distillation column which are batch column and continuous column. For a Batch type distillation column the input feed to the column is introduced batch-wise. This means that the some amount of input feed is inserted in the process column, once the distillation process is completed a new fresh feed is sent for another round of distillation.

Another type of distillation column is the continuous distillation column. In continuous distillation column the input feed stream is continuously sent in the process column and the distillation is in continuous process. No interruption occurs in the process unless there is a problem in the surrounding process units. Continuous distillation columns are the most common type of distillation column. Continuous distillation column can be further classified into two: Binary and Multi-component. In Binary process, the input feed contains only two components whereas in multi-component feed contains more than two components [44]. In this work, the study is over Continuous Binary Distillation Column. Figure 3.1 shows a process flow diagram of a continuous binary distillation column.

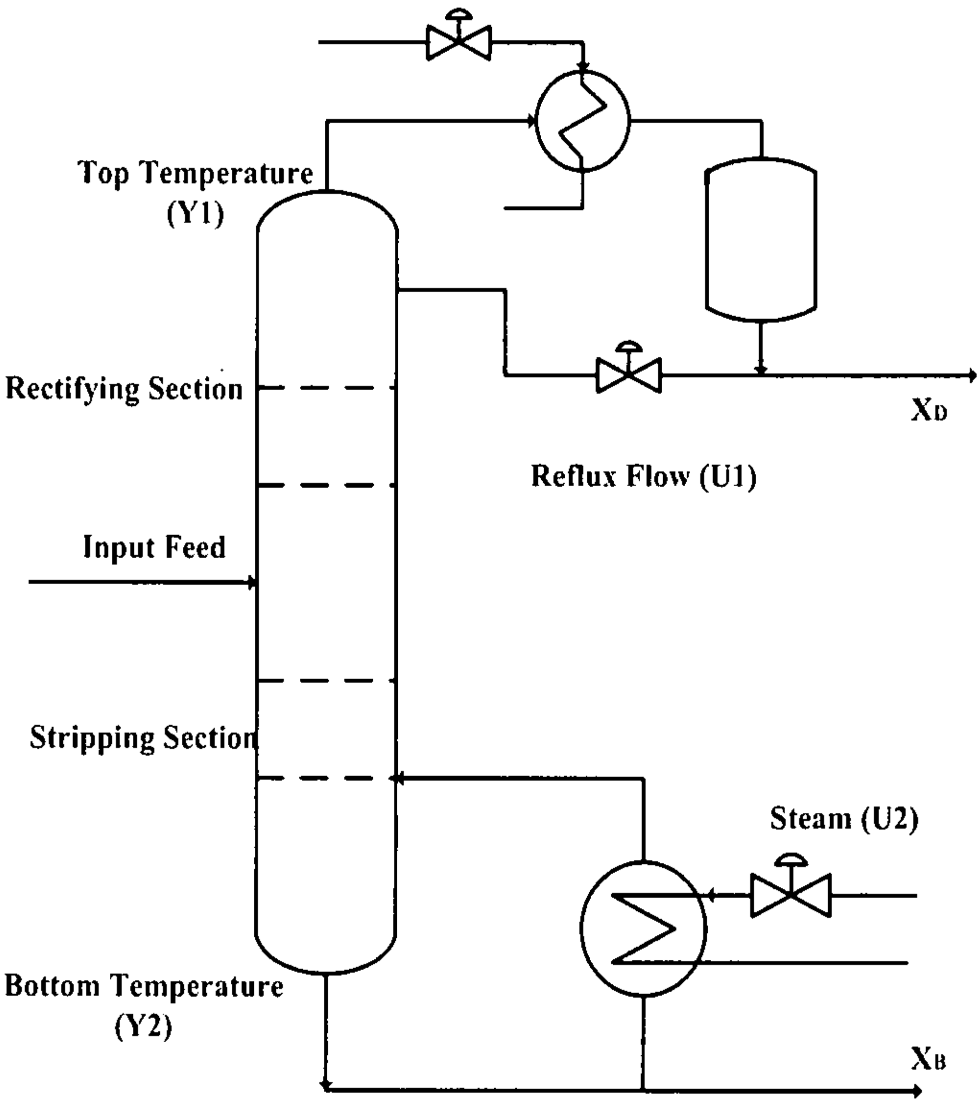


Figure 3.1: Process Flow Diagram of Distillation Column.

As shown in Figure 3.1, a continuous binary distillation column is divided into five parts which are the reflux, the rectifying section, the input feed, the stripping section and the rebolier. In the reflux is the condensed vapour collected from the top of the process column which is collected as the top product and also been recycled back to the column. The top section of the process column is the rectifying section. Here the rising vapour from the bottom passing through the trays, contacts the liquid flowing across them. Input feed point is where the fresh input feed is coming into the column.

Near the bottom of the process column is the reboiler. In this section the liquid is reheated or turned into vapour phase and feedback to the process column. At the bottom of the process column is called the stripping section. Here the reheated liquid in vapour form rises upward through the trays and contacts the down flowing liquid.

The operational process is as a binary mixture of liquid feed enters at one or more points in the process column which contains a series of stacked trays. The liquid flows over the plates and moves down the column. The liquid flowing over the plates comes in contact number of times with the vapour moving upwards. This is the most essential process in the distillation column. The bottom of the distillation column consists of a large volume of higher boiling point liquid. Some of the liquid flow is vaporized and fed back to the column and some are collected as the output product. This is the boil up process. Vapour at the top of the process column is stored in a condenser in liquid form. Ratio of the liquid is sent back into the process column which is known as the reflux and the remaining is the top collected distillate product [46].

3.2.1 Process Variables in Distillation Column

When a process plant is designed there are always some objectives to be considered. In the view of process control system, there are seven control objectives which are the followings:-

- Safety
- Environment Protection
- Equipment Protection
- Smooth operation and production rate
- Product Quality
- Profit
- Monitoring and Diagnostic

Process variables in a distillation column like temperature, pressure, flowrates and compositions should be monitored and controlled. These process variables inside distillation system affects one another, whereby a change in one process variables will result changes in other process variables. Therefore for controlling a distillation column, it is necessary for looking to all the variables but not only focusing to one variable. Each of the process variables has its own control loop and each of these control loops keeps track of the associated process variables.

3.2.2 Control & Manipulated Variable Selection for Distillation Column

Pressure is often considered as the prime distillation column variable, as it affects temperature, condensation, vaporization, composition, volatilities and almost any other process that takes place inside the column. Column pressure control is often integrated with the condenser control system. Increasing or decreasing the water flow rate will change the temperature of the condensing in the column. This in turn changes the pressure in the column [45].

Temperature is one of the important variables to be controlled in the process column. Online analyzers are rarely used for measuring the composition of the top and bottom of the product. This is because of its installation cost is very high. Composition if often regulated indirectly using temperature (at constant pressure in the column, there is a direct relationship of the temperature and composition for a binary mixture) [46].

Top product composition is regulated by adjusting the reflux flow and for bottom product composition the vapour flow is adjusted. Product composition can be controlled by fixing the process columns temperature. The top temperature controller manipulates the reflux flow and the bottom temperature manipulates the steam flow. The advantage of this control scheme is that it has a faster closed loop response and provides a better disturbance rejection [47].

3.2.2.1 Single Product Control

In this control strategy, the composition of one product is controlled while the composition of the other product is allowed to change. 90% of the distillation columns in industries are in use of single product control strategy instead of dual composition control, in which the top and bottom product compositions are controlled.

It is easy to use single composition control than dual composition control. The selection between single and dual composition control depends on the tradeoffs between the supplementary cost related with dual composition (maintenance for the controllers, analyzers cost, additional instrument costs, etc) and the financial advantage of dual composition (a rise in product recovery and reducing service costs) [49]. Moreover single composition control are mostly used for multiple staged distillation columns where the top product is collected and the bottom product is sent to another process column connected in series for further distillation process while dual composition are mostly used in binary distillation processes.

3.2.2.2 Dual Product Control

Dual product control configuration is mostly applied in binary distillation columns where the top and bottom product requires its independent control loop. The choice of the proper configuration for dual product control is a more difficult problem than for single product control because there are more feasible approaches and the analysis of performance is more complex. The most commonly used control configuration is the reflux/steam configuration because it provides good dynamic response, it is the least sensitive to feed disturbances and is the easiest to implement [49].

On the other hand, the reflux/top and vapour/bottom configuration is in general the least affected by disturbance and has good dynamic response but still there is no clear choice for the best configuration for dual composition control of distillation columns. For the cases when the overhead product is more important, reflux flow control is usually the best to be used. When the bottoms product is more important, steam flow is proper to be controlled [49].

Experimental work has been carried out for constant feed rate and feed composition intended for variation in reflux flow and steam flow. The pressure in the column is maintained at a constant value of 1 bar by controlling the cooling water flow rate to the condenser placed at the top side of the column. The liquid levels in the reflux drum and bottom of the process column are controlled by changing the flow rates of top and bottom product. The reflux and steam flow rates are varied by changing the set points value of the respective controllers in a PRBS sequence. In this study, for system identification the signals of the reflux flow rate (F_R) and steam flow rate (F_{ST}) controllers are used as the input data where as the temperature signals measured at tray 1 (T_1) and tray 14 (T_{14}) are taken as the output signals.

The inputs to the distillation column considered are the incoming reflux flow rate (U_1) and the steam flow rate at the reboiler (U_2). The outputs are the process column top temperature (Y_1) and the bottom temperature (Y_2). Figure 3.2 and Figure 3.3 shows the overall pilot plant (continuous binary distillation column) used for this study.

3.3 Advanced Process Control (APC) Distillation Column Pilot Plant

In this research study, the distillation column being used for experimental data collection and which is being modelled is a physical pilot plant located in the Chemical Engineering department of Universiti Teknologi PETRONAS. This continuous binary distillation column has a total number of 17 stages, considering the over head as the 1st stage and the reboiler located at the bottom of the column as the 17th stage. The feed to the column is at the 7th tray counting from the top. The feed to the tank is a mixture of Isopropyl Alcohol (IPA) and Acetone, while the outputs are the separation of both the IPA and the acetone concentration. Acetone is being collected as the top product and IPA being collected as the bottom product. Sampling points along with RTD sensors are available at every tray location, flow meters are installed in all flow lines such as the feed line and products steam lines and the reflux line. Differential pressure measuring devices are connected across the stripping and rectifying section of the process column and a pressure sensor on the top of the column. Suitable sized control valves are connected in all flow lines. A Distributed Control System (DCS) is set up for data acquisition and control of the plant. Table 3.1 shows the dimensions and operating conditions of the distillation column.

Table 3.1: Dimensions & Operation Condition of Distillation Column

Description	**Value**
Height	5.5 m
Diameter	0.15 m
Number of trays	15 trays
Type of tray	Bubble Cap
Feed Tray	7
Tray Spacing	0.35 m
Operating Conditions	**Value**
Feed Flow rate	0.15lt/min
Reflux Flow rate	0.5 lt/min
Steam Flow rate	20 kg/min
Distillate Flow rate	0.3 lt/min
Bottom Product Flow rate	0.2 lt/min
Feed Composition	0.1824
Column Bottom Temperature	80.5 °C
Column Top Temperature	72.7 °C
Column Pressure	1.013 bar

The values given in Table 3.1 were selected based on trial and error approach. This is because the APC plant is a new setup and has different specifications such as height and pipe line sizing. Before the real experiment was conducted step changes were given into the plant and changes were observed. The best magnitude effects were analysed and were used for actual experimental setup.

Figure 3.2: Distillation Column Unit.

(a) Reboiler Unit

(b) Steam Input Valve

(c) Reflux Input Valve

Figure 3.3: Reboiler Unit with Two Inputs Valves

3.4 System Identification Experiment over APC Distillation Column

System Identification experiment over the APC plant was conducted based upon the following approach as shown in Figure 3.4. An experimental setup was performed and approximately 4000 data's were collected. The collected data was divided into two for estimation of model and model validation. Several linear and nonlinear models were developed. Performance measurement was analysed by observing the best fit, sum of squared prediction error and cross correlation tests. The residual distribution of each model was also analysed.

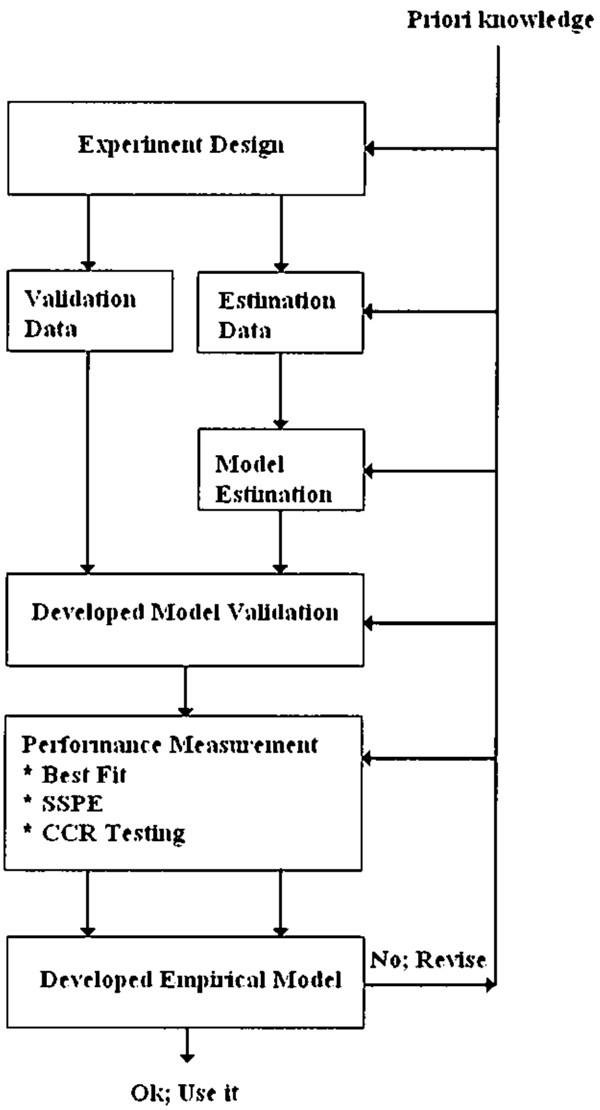


Figure 3.4: SI Approach over APC Plant.

For the system identification experiment, the input sequences are intended as a low frequency PRBS generated from MATLAB with band of 0 to 0.04 with level magnitude from 18kg/h to 22kg/h for steam and magnitude of 0.4lt/min to 0.8lt/min for reflux flow rates, respectively. The sequence of the PRBS was followed accordingly and the reflux and rebolier valves were manipulated. The magnitude for the input levels are chosen such that maximum excitation is obtained while allowing easy process running of the distillation column. Figure 3.5 and Figure 3.6 shows the experimental input data sequence for the plant.

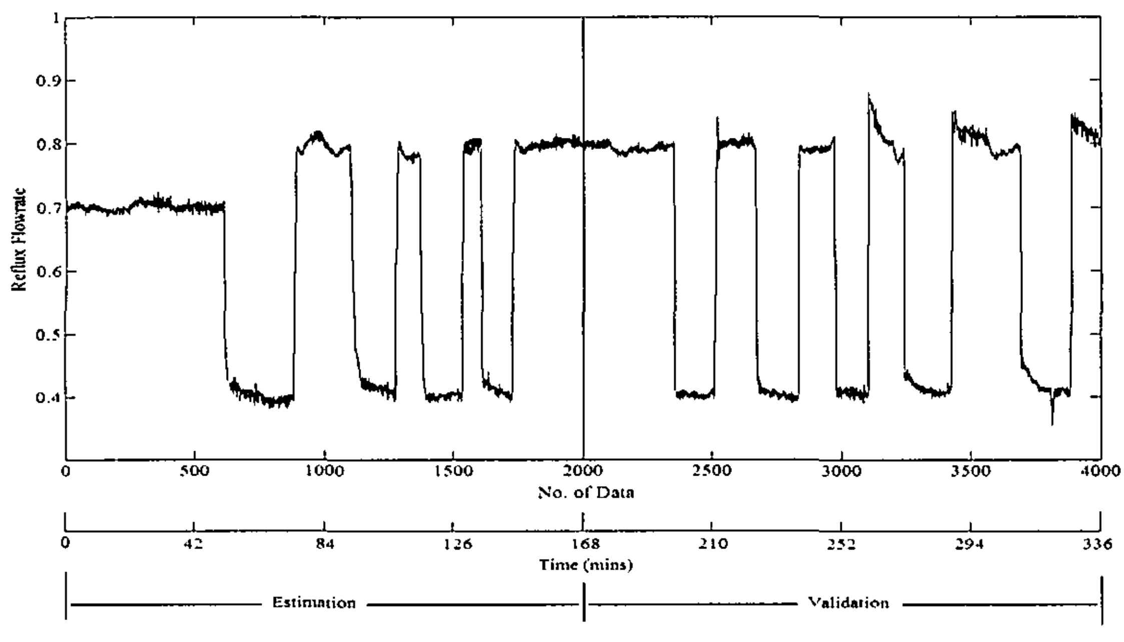


Figure 3.5: Input signal of Reflux Flowrate (U_1).

The input signals are an exact structure of the PRBS signal. Four thousand data points are collected with a sampling interval of 5 seconds from the APC plant. The first two thousand data points are used for model identification and the remaining data points are used for validation. The reflux flow signal is in a refined behaviour while the steam flow is very noisy. This is because steam itself has a very fluctuating behaviour.

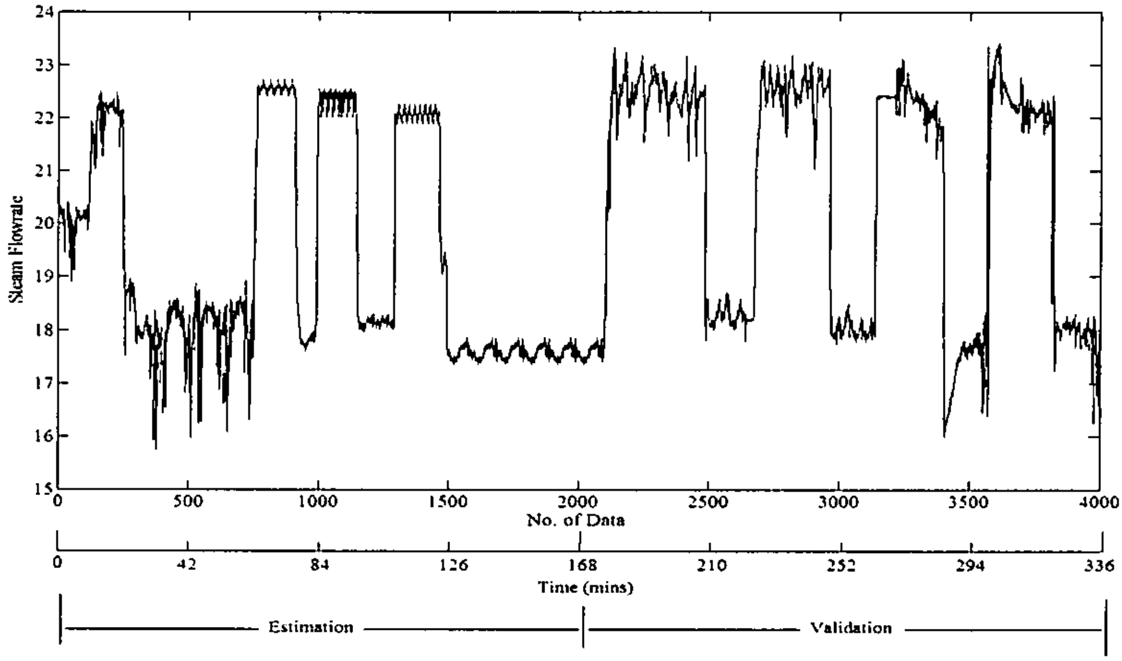


Figure 3.6: Input signal of Steam Flowrate (U_2).

Figure 3.7 and Figure 3.8 shows the output temperature response of the continuous distillation column. The top temperatur value varies in range from 72°C to 79°C and for botton temperature, it varies in range between 80°C to 83°C.

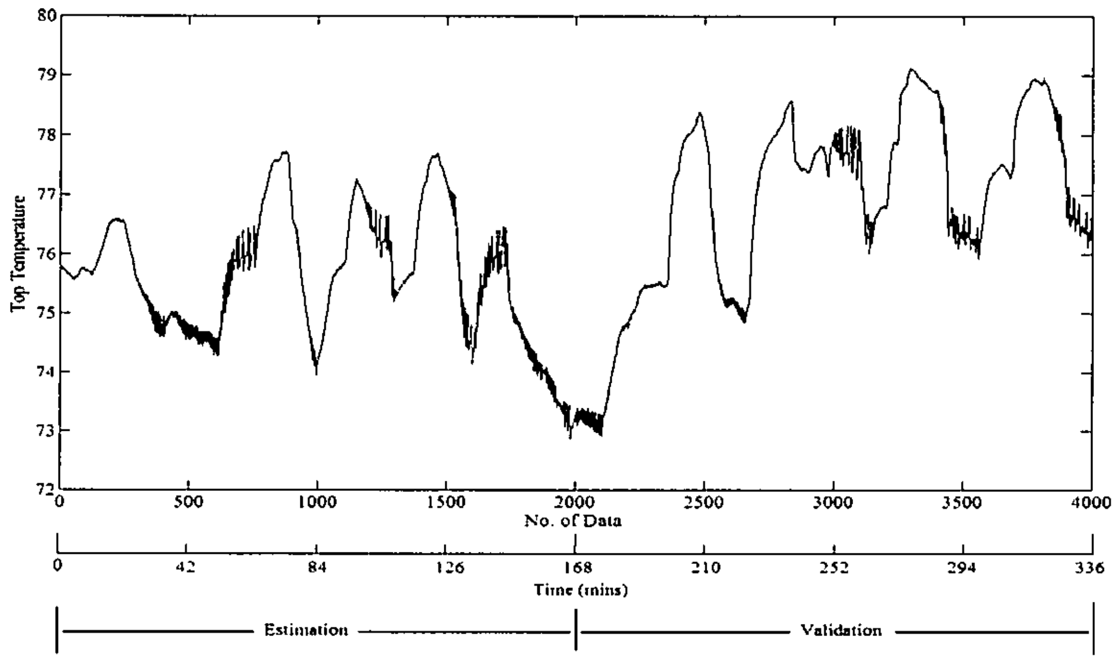


Figure 3.7: Output Signal for Top Temperature (Y_1).

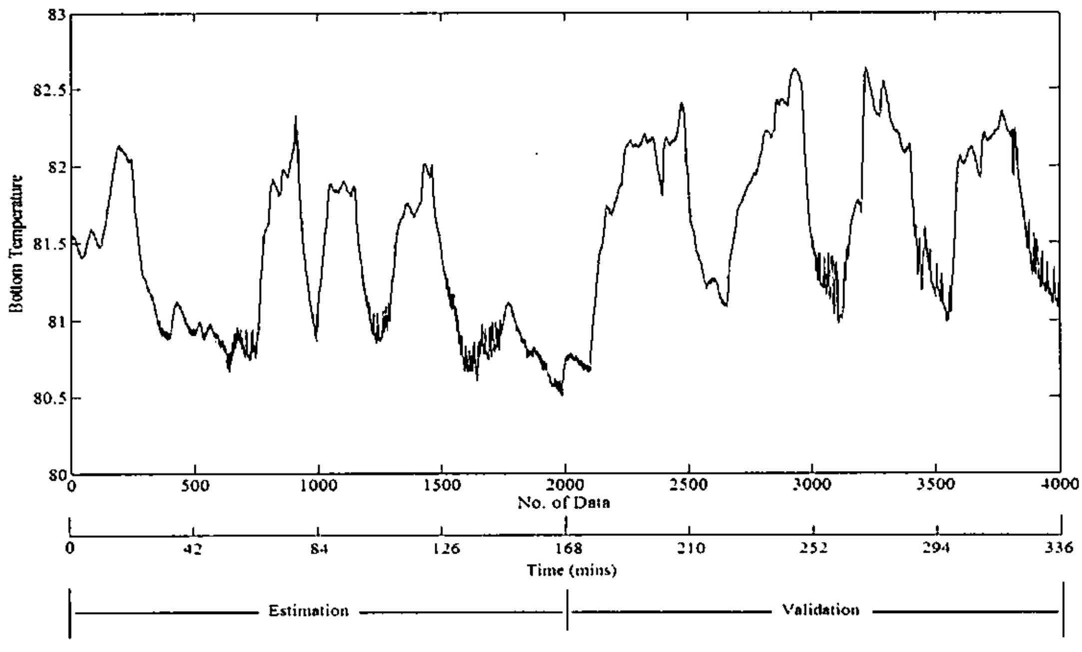


Figure 3.8: Output Signal for Bottom Temperature (Y_2).

3.5 Residual and Error Analysis

The models are analyzed by using different types of techniques such as best fit, sum of squared prediction error, root mean square error and cross-correlation. These techniques help to verify the best output and provide firm basis for a superlative model selection. *Sriniwas* [23] and *Ljung* [50] also used these techniques for common error analysis and justify their best selected model. The brief descriptions of each of these techniques are mentioned below;

3.5.1 Best Fit

One of the important characteristic for selecting the best modelling technique is by comparing the percentage of the output variation which is the percentage of comparison between the model output with the measured output. A higher percentage number means a better model. For N number data, the best fit criterion of a model is defined as:

$$Best\ Fit = \left[1 - \frac{\sum_{i=1}^{N} |y_i - \hat{y}_i|}{\sum_{i=1}^{N} |y_i - \bar{y}|} \right] \times 100\% \tag{3.1}$$

where $\bar{y}$ is the mean of the measured output.

3.5.2 Sum of Squared Prediction Error

Measuring the predictive ability of the model is another way of selecting the best model. The sum of squared prediction error is computed as;

$$SSPE = \sum_{i=1}^{N} (y - \hat{y}_i)^2 \tag{3.2}$$

where y is the actual output and $\hat{y}$ is the predicted output from the evaluated model.

3.5.3 Root Mean Square Error

RMSE gives the variance measurement of the residual which shows a complete fit of the model to the actual data. RMSE is a good measure of how exactly the developed model calculate the response. RMSE is an important measurement for the fit if the main purpose of modelling is for prediction. It can be computed as follows;

$$RMSE = \sqrt{MSE} = \left[\frac{\sum_{i=1}^{N} (y - \hat{y})^2}{N} \right]^{1/2} \tag{3.3}$$

where y is the actual output, $\hat{y}$ is the model output and N is the number of data points.

3.5.4 Cross-Correlation

Moreover for the independent test which is considered by analyzing the cross correlation, a good model has residuals uncorrelated with past inputs. Verification of cross correlation specifies that the model does not illustrate how part of the output relates to the consequent input. For instance, a peak outside the confidence interval for lag τ shows that the output $y(k)$ that originates from the input $u(k-\tau)$ is not appropriately described by the model [48]. This test is observed by estimating the cross correlation function of the residual error as follows;

$$R_{eu}^{N}(\tau)=\frac{1}{N}\sum_{k=1}^{N}e(k)u_i(k-\tau) \qquad (3.4)$$

where N is the number of the data samples of the residual error, $e(k)$ is the error sequence, $u_i(k)$*is* the input.

The whiteness also known as auto-correlation test does not provide a complete validation of the model because the whiteness does not guarantee that the parameterization was chosen appropriately. However when inconsistent estimation occurs the model cannot be validated and is thus rendered an outline [48].

3.6 Summary

In this chapter the concept and the dynamics of a continuous binary distillation column is explained. Types of process variables and its selection of a suitable control strategy is also discussed. Explaining the Advanced Process Control Pilot Plant and its description are given. System identification approach over the physical pilot plant and the capturing of the data are shown.

PRBS form of signal was used for the two inputs and the output response from the distillation column was captured. The reflux input magnitude was adjusted between the ranges of 0.4lt/min to 0.8lt/min and for the steam input the range was selected between 18kg/hr to 22kg/hr. Maximum excitation of the process was observed under smooth operation of the process column. The PRBS signal used as inputs showed a significant output observation for the top and bottom temperature of the process column. Four thousand data was captured from the process and are used for identification. Two thousand data are used for model estimation and the rest for model validation. Linear and Nonlinear models are developed using the real data collected from the plant which are discussed in chapter 4 and chapter 5.

CHAPTER 4

MULTIVARIABLE LINEAR SYSTEM IDENTIFICATION

4.1 Introduction

In this chapter results for system identification of linear models are shown and discussed. Estimation and validation errors for top and bottom temperature of the distillation column are given and are discussed. Moreover, error analysis of the linear models is also tabulated and discussed. Evaluations among the models are based upon the best-fit criteria and sum of squared predicted error. Prediction error response, histogram distribution response and Cross-Correlation Response (CCR) with respect to the two inputs for every model order are shown. All the developed linear model equations are given in Appendix B.

4.2 Linear System Identification

There are two categories of linear system identification models, parametric & non-parametric models. In the parametric system identification a standard structure for the model is assumed and the weights of the model are adjusted based on the observation for the system. Meanwhile in the non-parametric system identification, no standard structure for the model is assumed. In this chapter parametric models are used for identification of the distillation column. Linear models used for identification are the ARX, ARMAX, linear state space model and continuous-time process model. The evaluation of each models are measured up to 3rd order. Estimation and validation of the models are completely based on the experimental data. Models are observed based on the best fit criteria and prediction mean square error. Further analysis is done by observing the histogram distribution and cross correlation test from the two inputs.

4.2.1 First Order Models

System identification for the top and bottom temperature of the distillation column is performed for the 1st order of ARX model of structure 110, ARMAX model of structure 1110, Linear State Space and continuous time process model. Results for both top and bottom temperature are shown from Figure 4.1 to Figure 4.4.

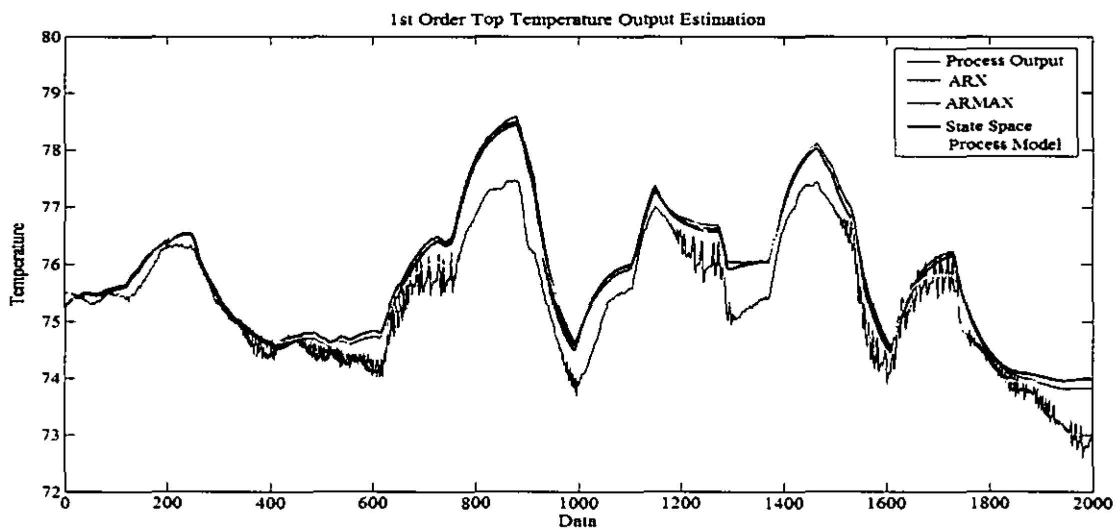


Figure 4.1: 1st Order Top Temperature Output Estimation.

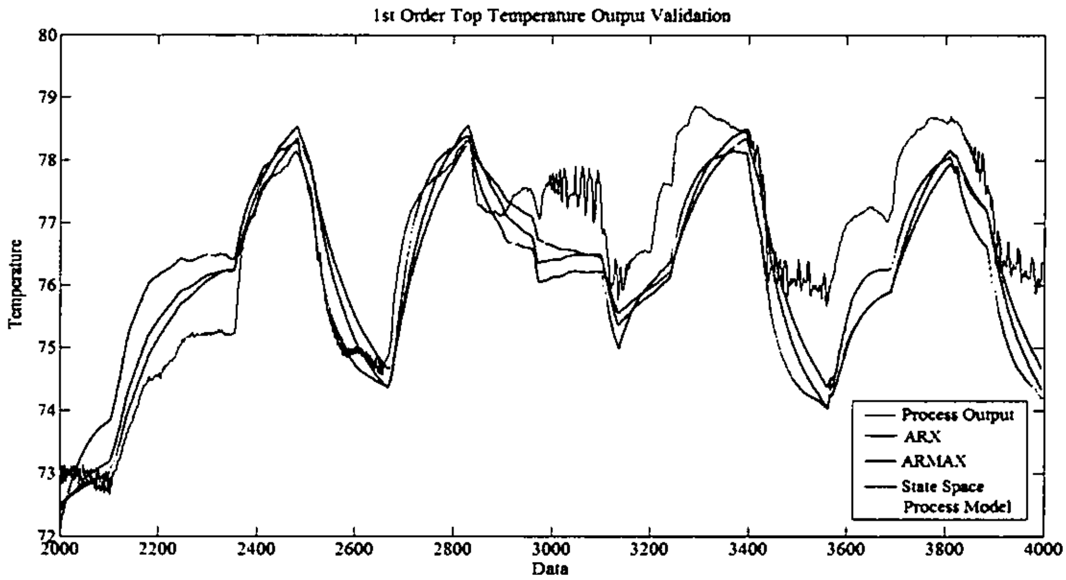


Figure 4.2: 1^{st} Order Top Temperature Output Validation.

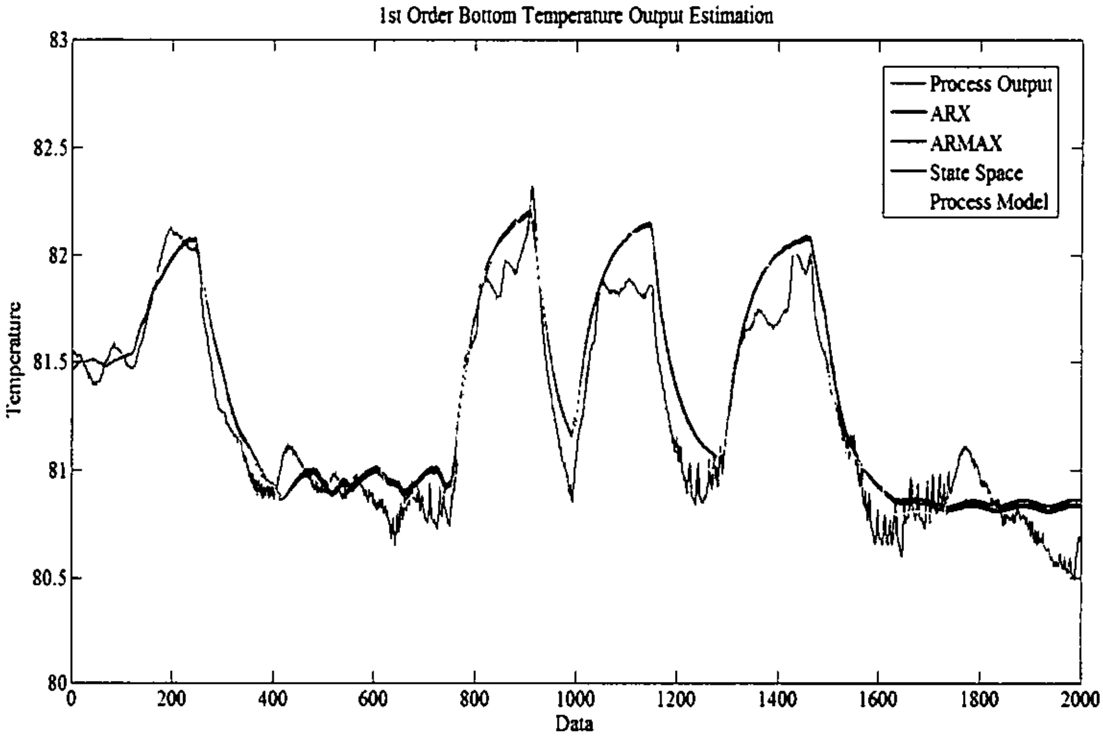


Figure 4.3: 1^{st} Order Bottom Temperature Output Estimation.

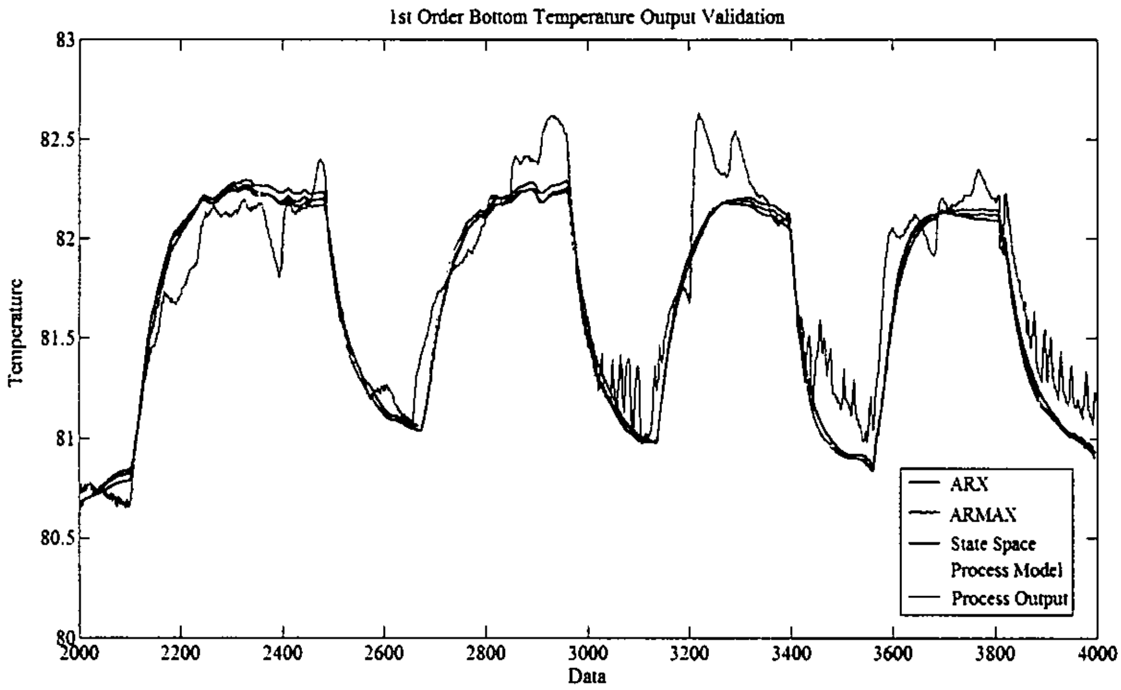


Figure 4.4: 1st Order Bottom Temperature Output Validation.

From Figure 4.1 and Figure 4.2, it can be observed that the estimation and validation of all the four linear models show best fit with large number of error to the actual process for the top temperature of the process column. Similarly, Figure 4.3 and Figure 4.4 also shows best fit with large number of error to the actual process for the bottom temperature of the process column. This shows that all the 1st order models are capable of capturing the dynamics of the nonlinear system but with the presence of a large number of error.

4.2.2 Second Order Models

System identification of the top and bottom temperature of the distillation column is performed for 2nd order of ARX model of structure 220, ARMAX model of structure 2220, Linear State Space and continuous process model. Results for both top and bottom temperature are shown from Figure 4.5 to Figure 4.8.

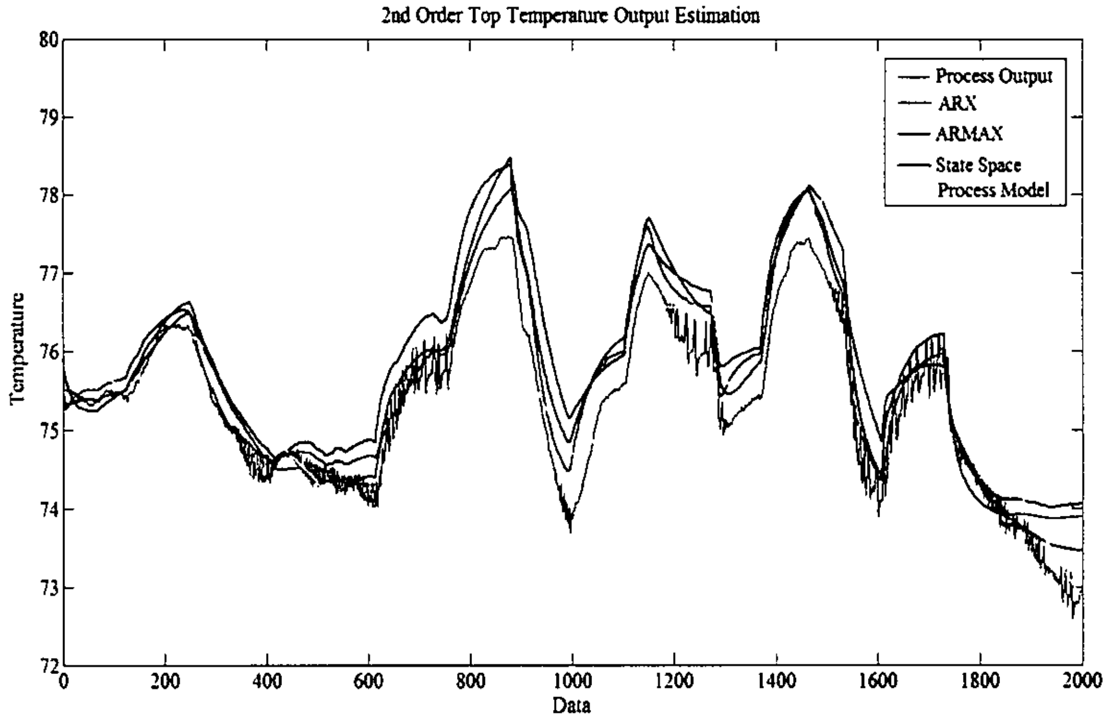


Figure 4.5: 2^{nd} Order Top Temperature Output Estimation.

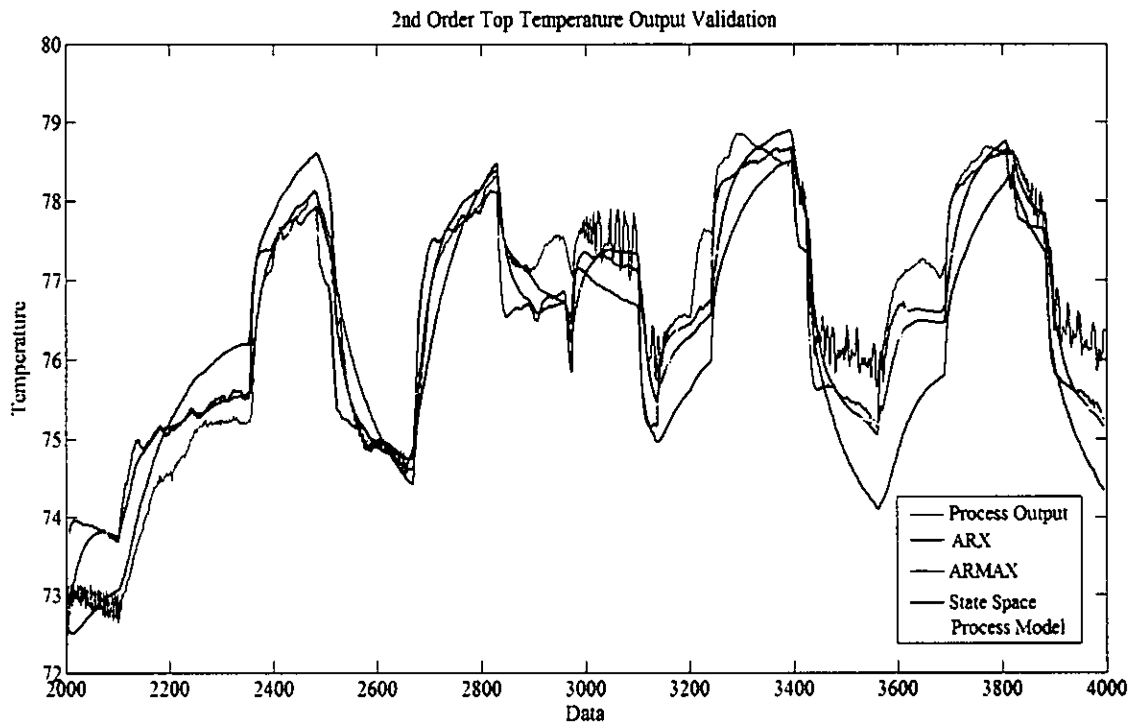


Figure 4.6: 2^{nd} Order Top Temperature Output Validation.

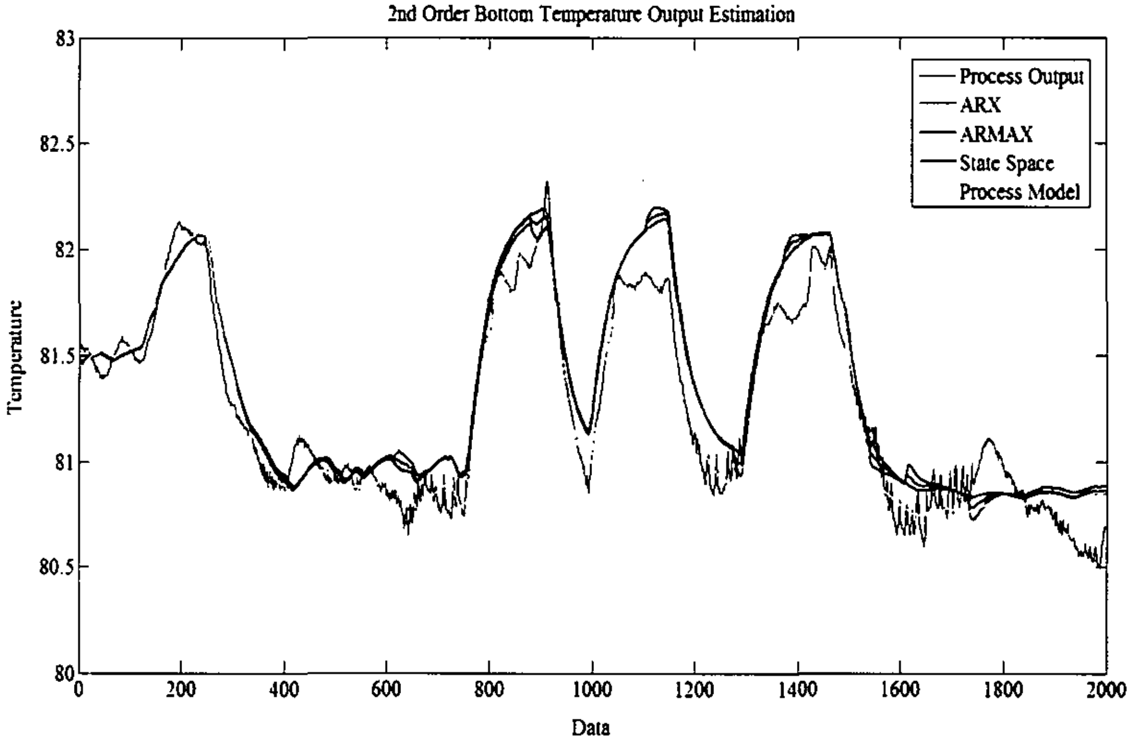


Figure 4.7: 2^{nd} Order Bottom Temperature Output Estimation.

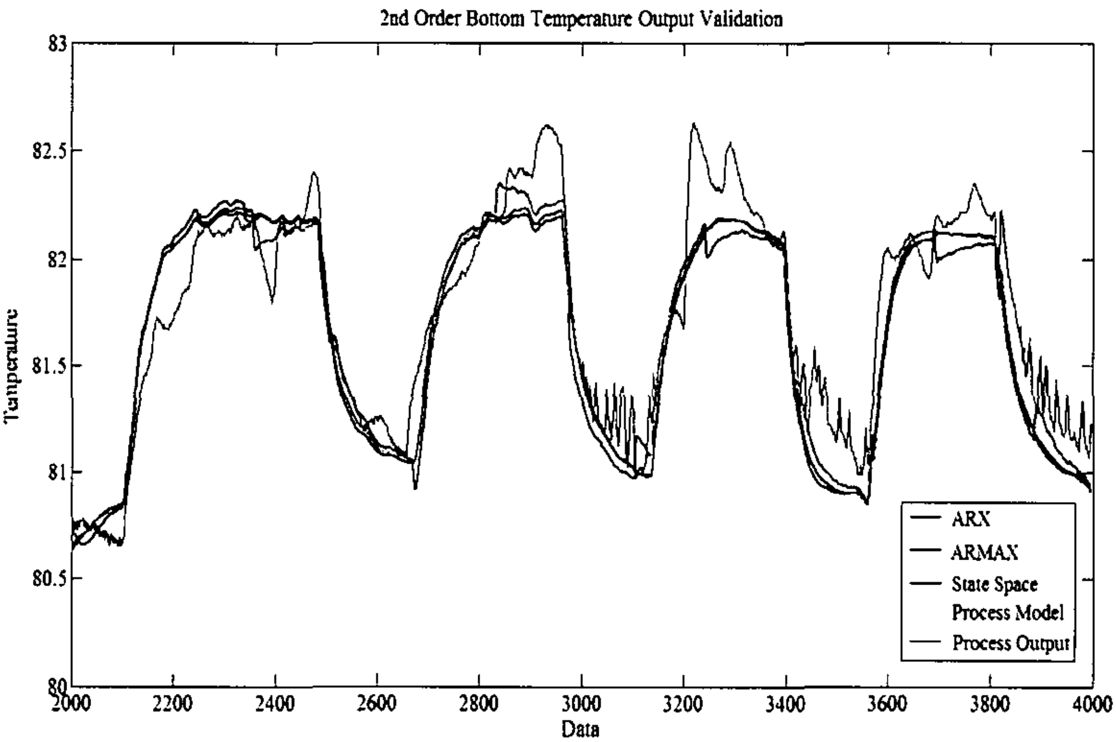


Figure 4.8: 2^{nd} Order Bottom Temperature Output Validation.

From Figure 4.5 and Figure 4.6, all the 2nd order linear models show a good best fit criteria result. The validation response shows a good capture of the actual process. But still there is a large number of error present in it. The process model shows a very low best fit percentage value for the top temperature of the process column. Meanwhile from Figure 4.7 and Figure 4.8, all the linear models show a good best fit results to the bottom temperature of the process column. This shows that the 2nd order linear models are capable of capuring the dynamics of the nonlinear system to some extent.

4.2.3 Third Order Models

System identification of the top and bottom temperature of the distillation column is performed for the 3rd order of ARX model of structure 330, ARMAX model of structure 3330, Linear State Space and continuous process model. Results for both top and bottom temperature are shown from Figure 4.9 to Figure 4.12.

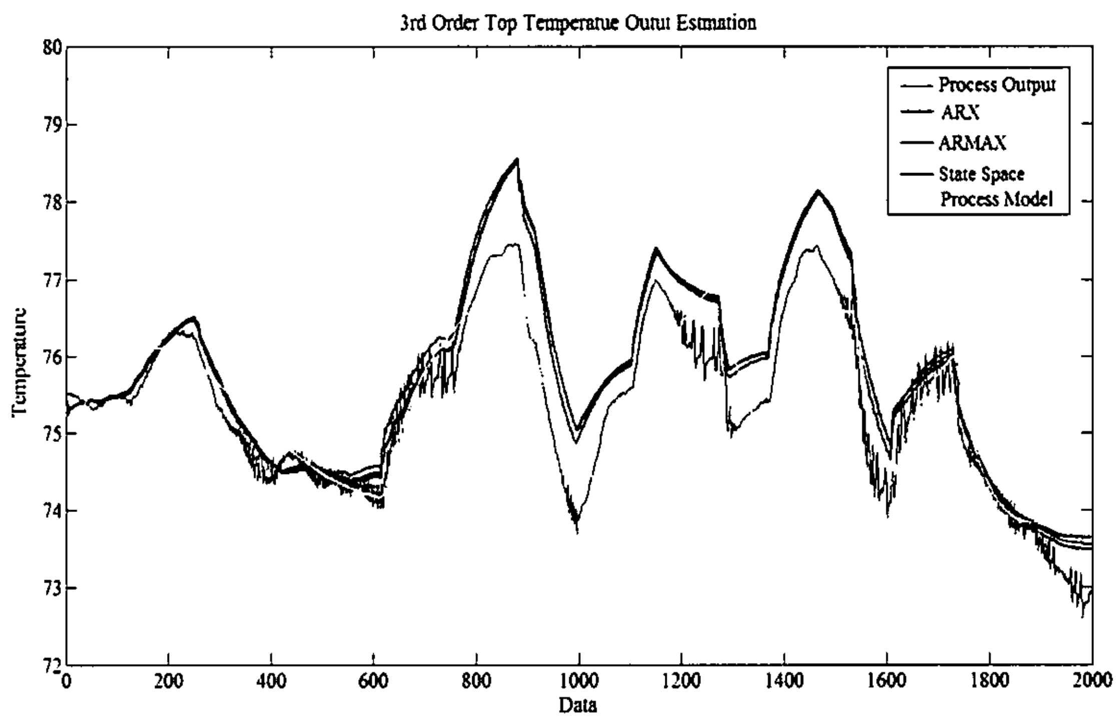


Figure 4.9: 3rd Order Top Temperature Output Estimation.

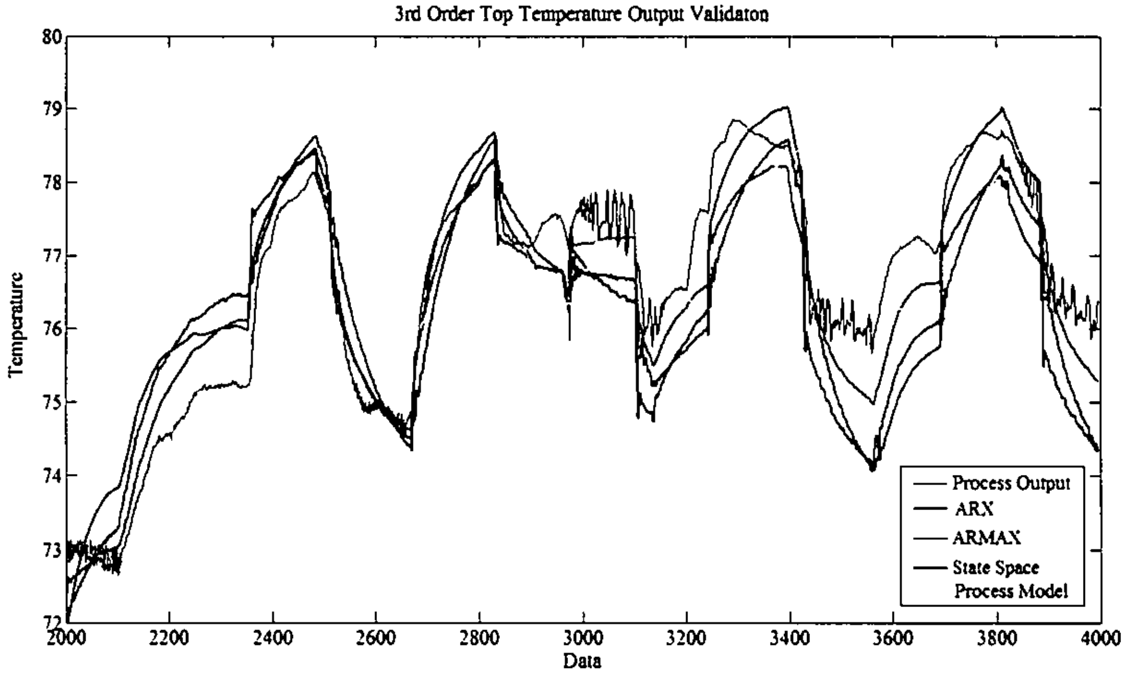


Figure 4.10: 3rd Order Top Temperature Output Validation.

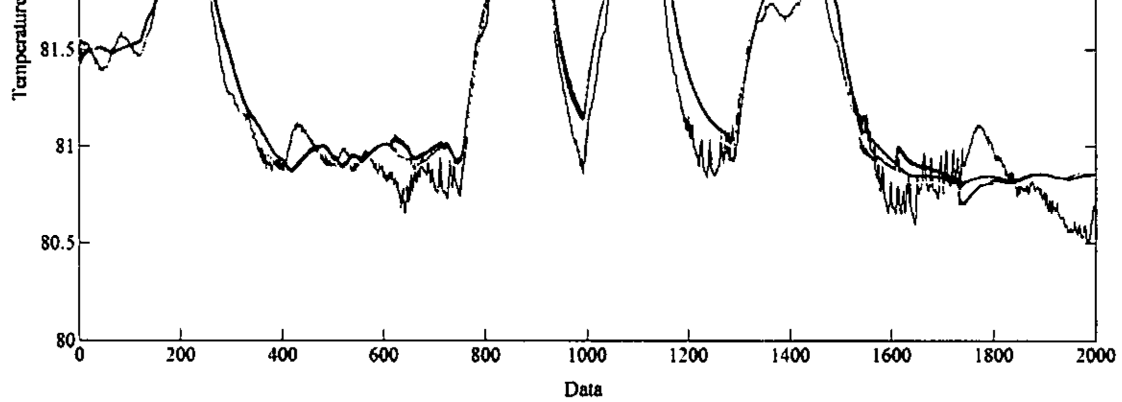


Figure 4.11: 3rd Order Bottom Temperature Output Estimation.

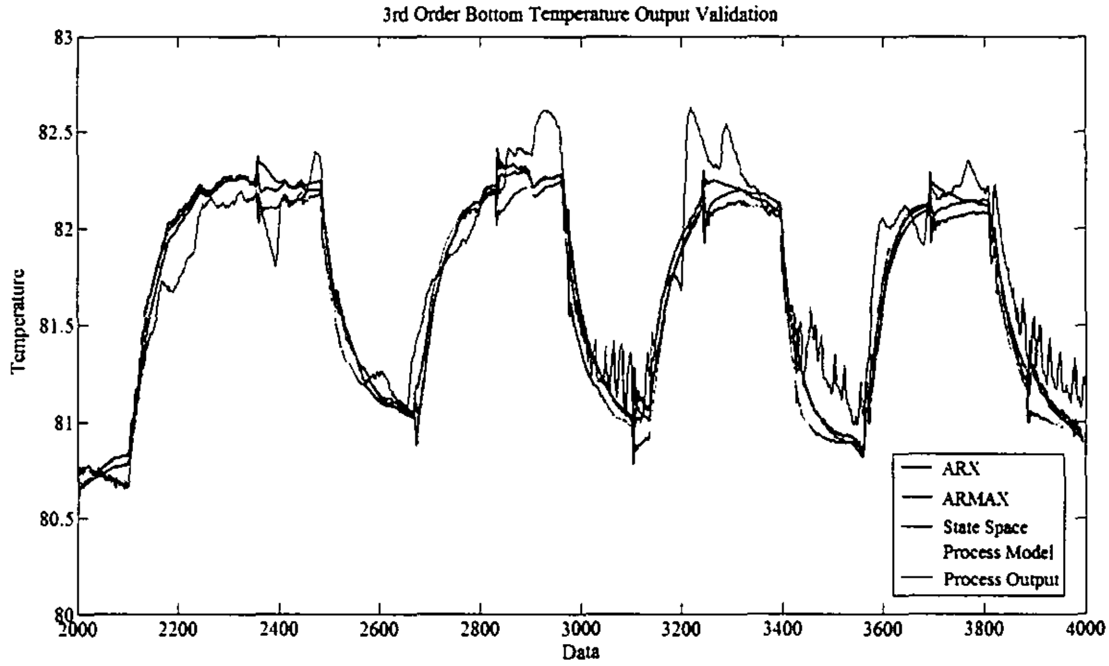


Figure 4.12: 3rd Order Bottom Temperature Output Validation.

From Figure 4.9 and Figure 4.10, all the 3rd order linear models show a good best fit criteria result but the process model shows a very low best fit percentage value for the top temperature of the process column. Meanwhile from Figure 4.11 and Figure 4.12, all the linear models show a good best fit results to the bottom temperature of the process column. 3rd order models also shows good compatible resuts of capturing the nonlinear systems dynamics. Equations of all the developed linear models are given in Appendix B.

4.3 Modelling Error Analysis

4.3.1 Best Fit Analysis for Top Temperature

Table 4.1 shows the model validation error analysis statistics for the developed linear models for the top temperature of the distillation column. Analyzing Table 4.1, it is shown that the ARX, ARMAX and Linear State Space model gives a better validation results compared to the process model. Overall the most suitable error analysis result has been given by the linear state space model of 3rd order with best fit of 66.18% and 0.0058 SSPE and 2nd order ARMAX model with best fit of 67.67% and 0.0044 of SSPE for the top temperature of the distillation column.

Table 4.1: Linear Model Best Fit Error Analysis Statistics

Model Performance Measurement			
		Validation	
Order	**Model Structure**	**Best Fit (%)**	**Sum of Squared Prediction Error**
1st Order	ARX	41.11	0.0180
	ARMAX	44.07	0.0378
	State Space	34.32	0.0067
	Process Model	39.22	0.2215
2nd Order	ARX	43.74	0.0114
	ARMAX	67.67	0.0044
	State Space	40.73	0.0057
	Process Model	23.45	0.6788
3rd Order	ARX	63.3	0.0086
	ARMAX	43.59	0.0057
	State Space	66.18	0.0058
	Process Model	40.75	0.6400

4.3.2 Top Temperature Residual Performance

4.3.2.1 First Order Residual Analysis

From Figure 4.13 to Figure 4.16 the predicted error or residual plot for the developed first order linear models are shown. These plots are based on the validation result of the model. Figure (a) shows the predicted error of the model, Figure (b) shows the histogram, Figure (c) and Figure (d) shows the Cross Correlation Response (CCR) from input U_1 and input U_2.

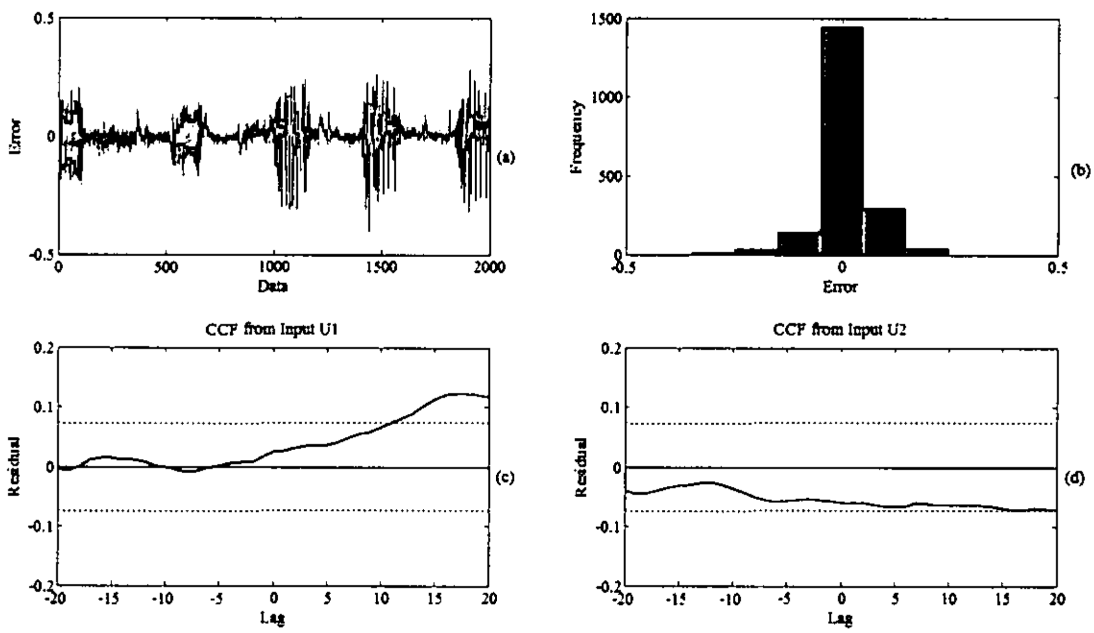


Figure 4.13: Residual Analysis for ARX Model Validation.

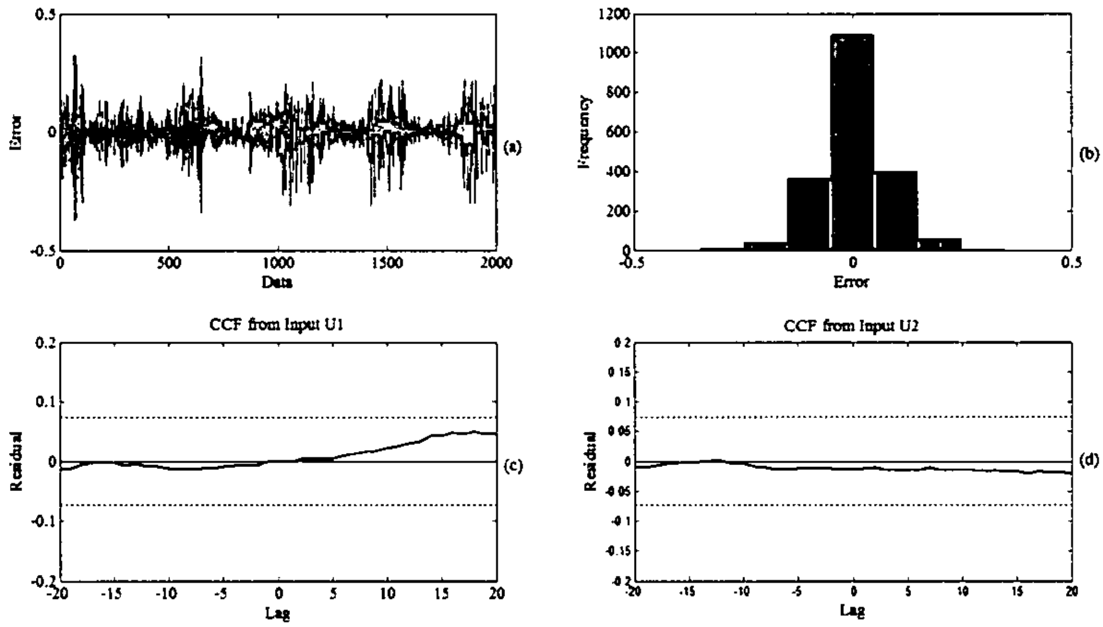


Figure 4.14: Residual Analysis for ARMAX Model Validation.

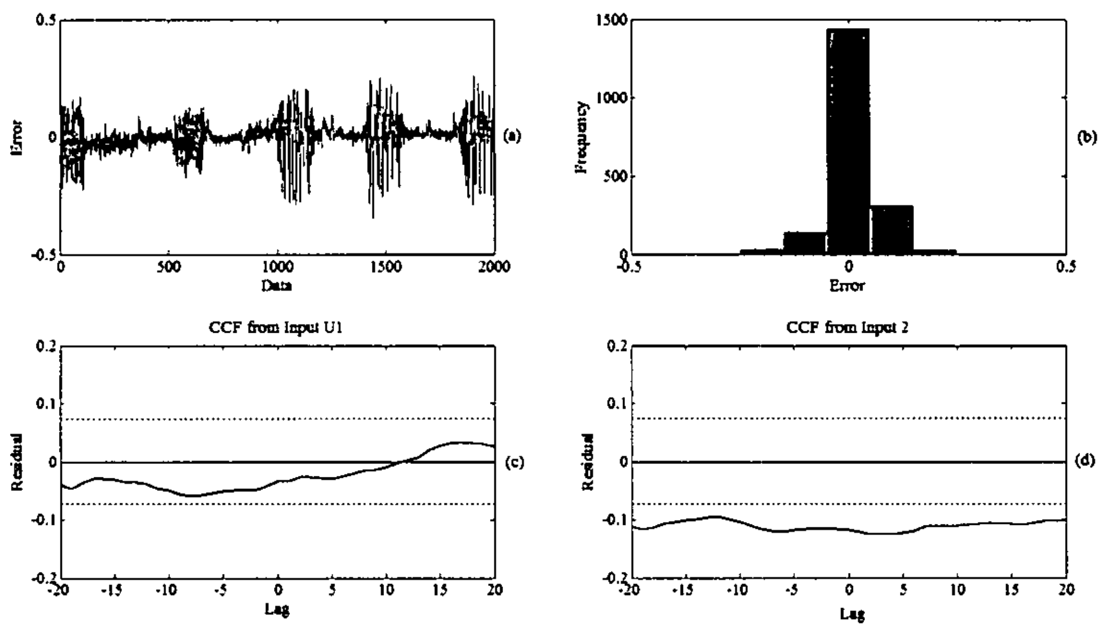


Figure 4.15: Residual Analysis for State Space Model Validation.

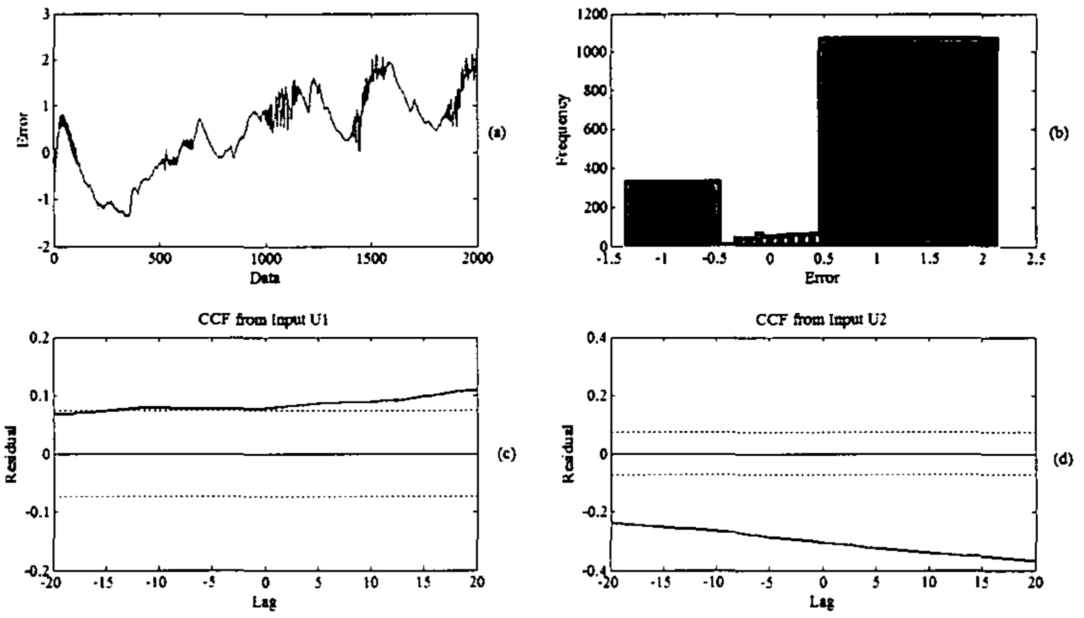


Figure 4.16: Residual Analysis for Continuous Time Process Model Validation.

Analysing the residual plots for all first order linear models, ARX, ARMAX and linear state space model shows the histogram plot well distributed of the prediction error with the centre at the origin whereas the continuous time process model does not show a well distribution. The cross-correlation from input U_1 and input U_2 for all models shows some relationship between the input and the residual, thus this means that not all the estimation data are modelled except for ARMAX model as the entire coefficient lies within the confidence interval of 95%.

4.3.2.2 Second Order Residual Analysis

From Figure 4.17 to Figure 4.20, the predicted error or residual plot of the developed second order linear models are shown. These plots are based on the validation result of the model.

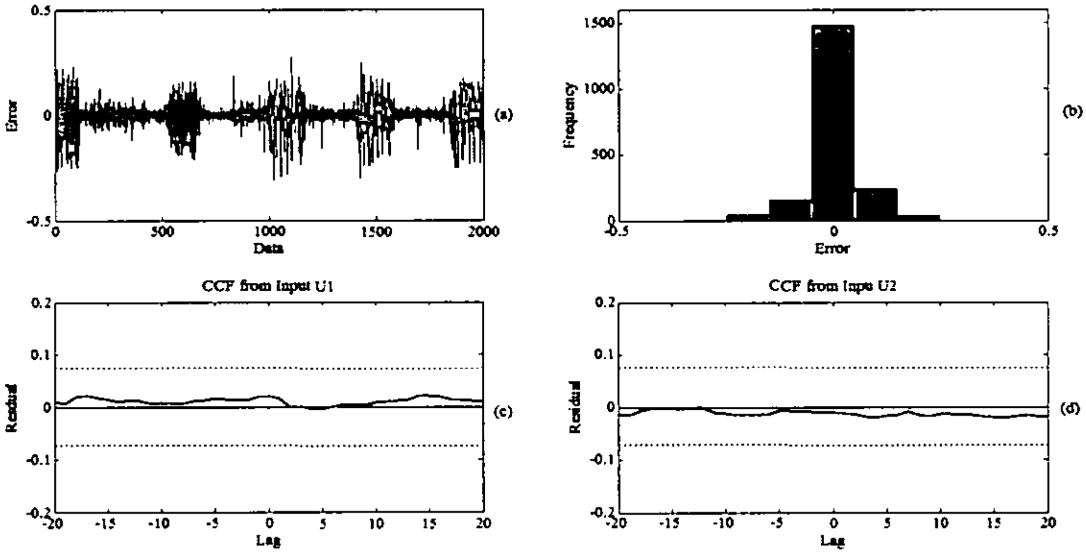


Figure 4.17: Residual Analysis for ARX Model Validation

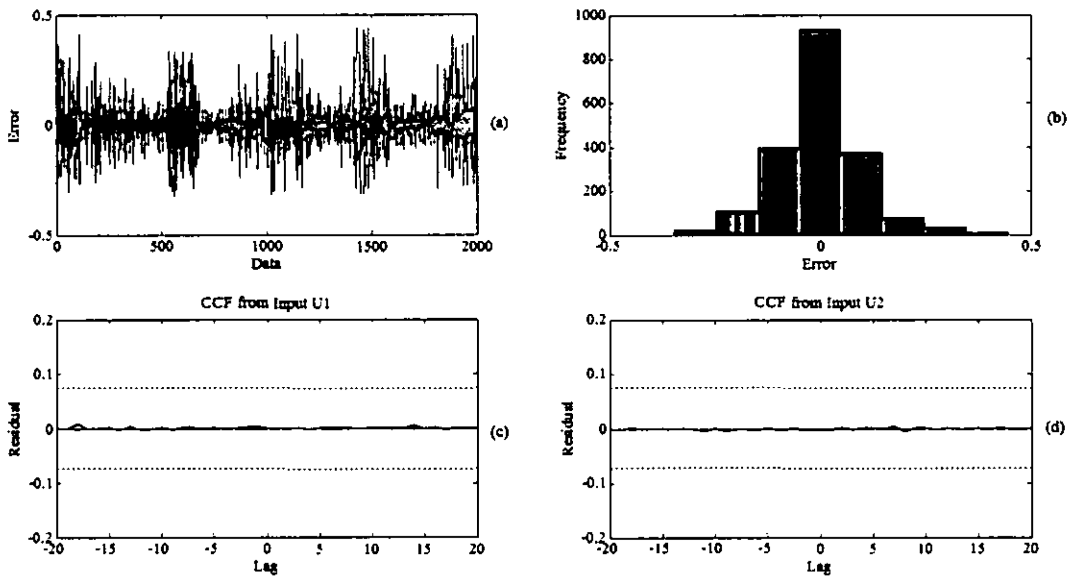


Figure 4.18: Residual Analysis for ARMAX Model Validation

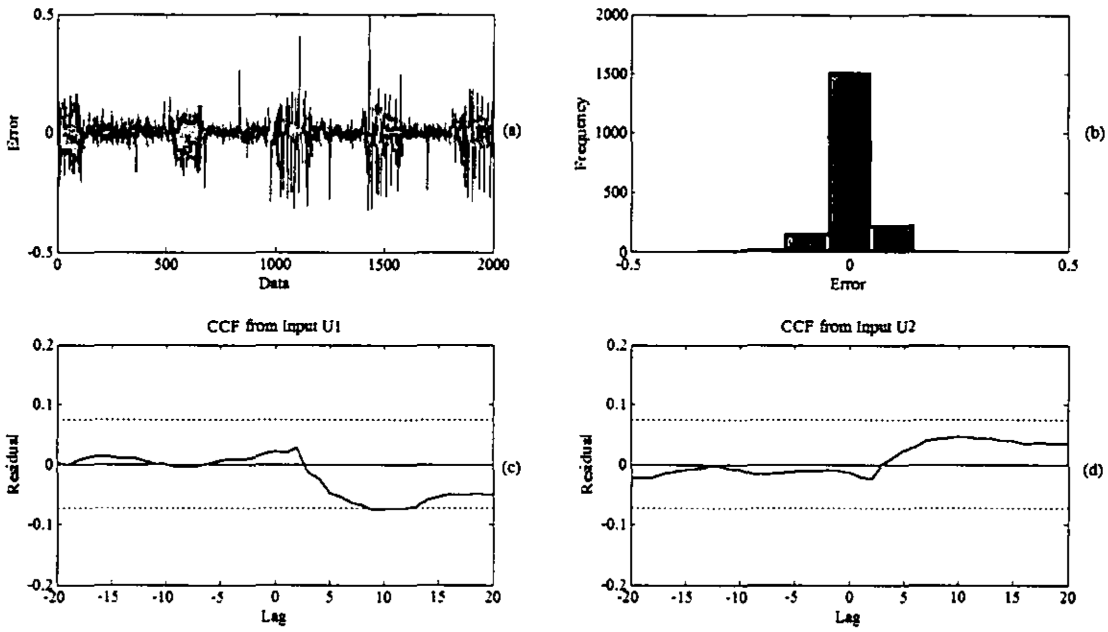


Figure 4.19: Residual Analysis for State Space Model Validation

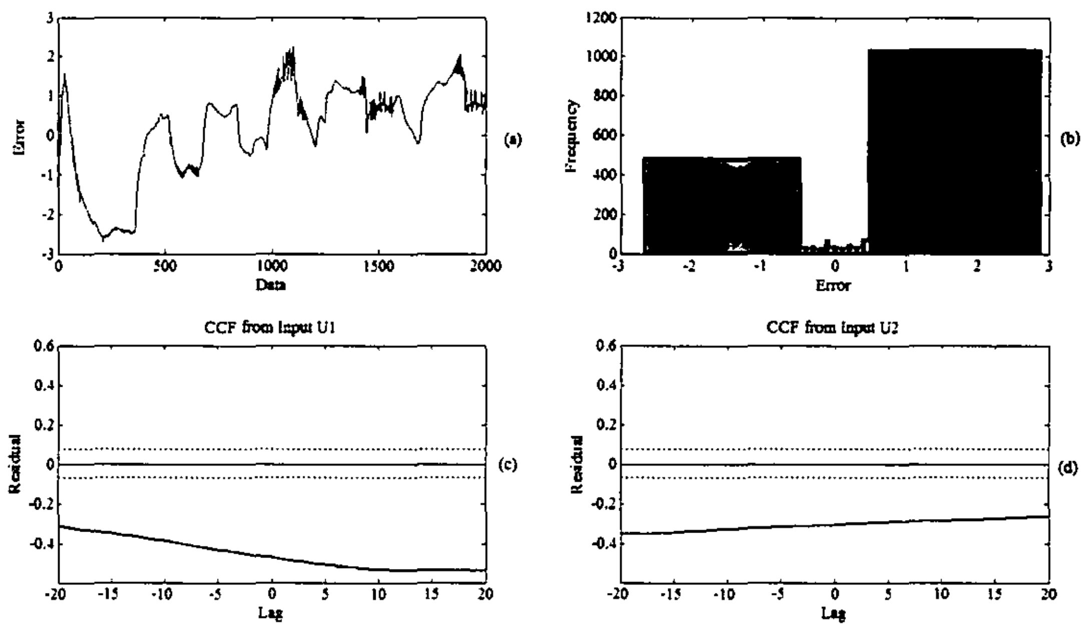


Figure 4.20: Residual Analysis for Continuous Time Process Model Validation

All the second order models show a well distributed histogram response of the model prediction errors with the mean at the centre except for continuous time process model. Cross-correlation response shows that ARX, ARMAX and linear state space model are well developed from the estimation data as no significant correlation is observed between the inputs and the residual whereas the continuous time process model does not show a good cross-correlation response.

4.3.2.3 Third Order Residual Analysis

From Figure 4.21 to Figure 4.24, the predicted error or residual plots of the developed third order linear models are shown along with the histogram. These plots are based on the validation result of the model.

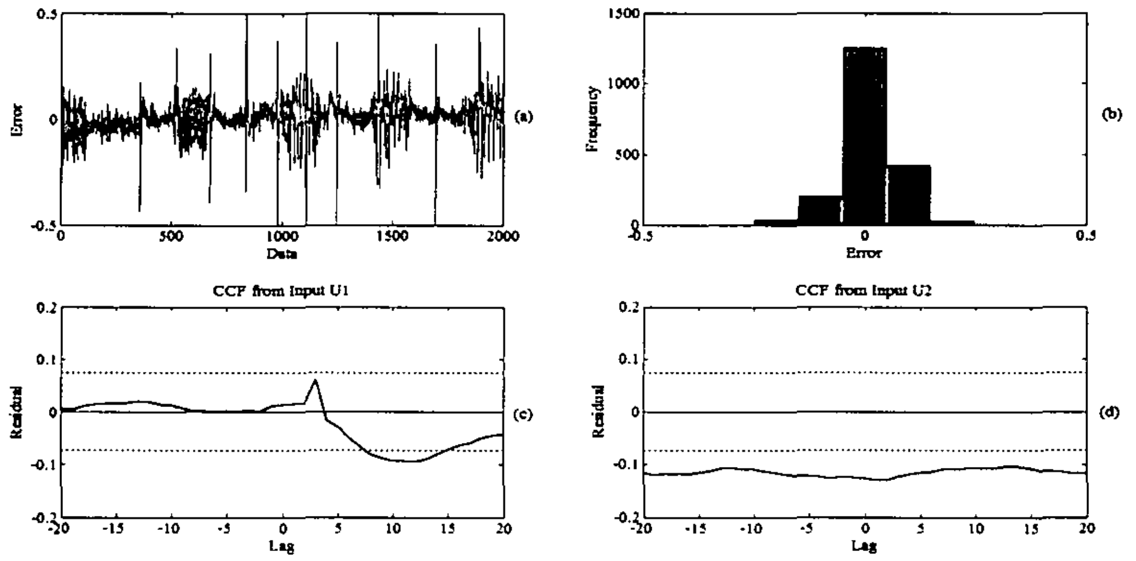


Figure 4.21: Residual Analysis for ARX Model Validation.

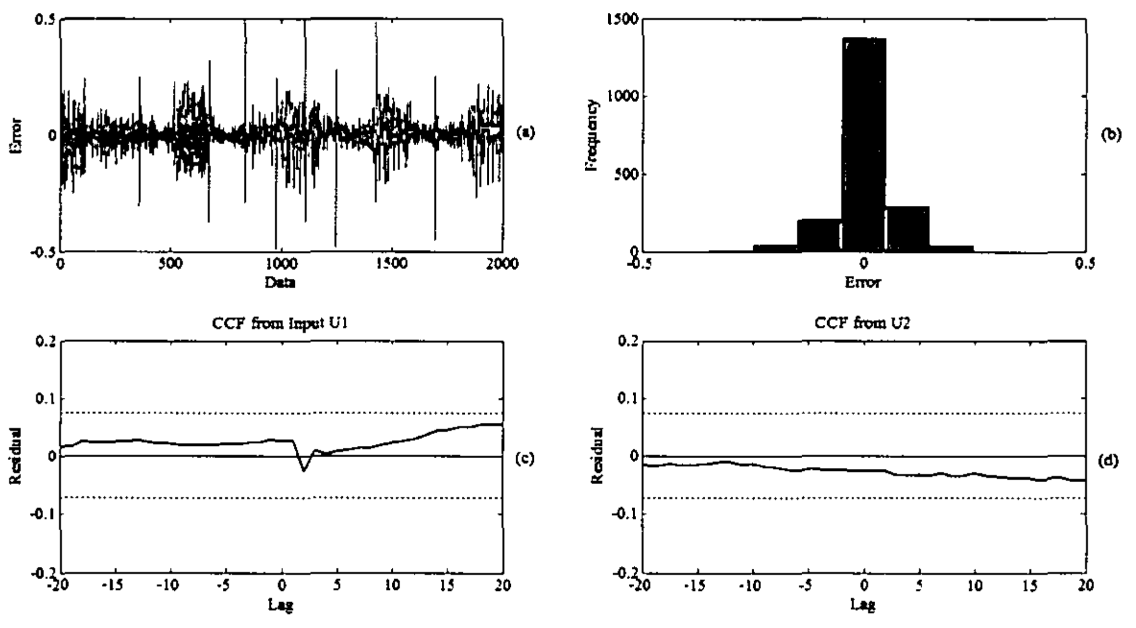


Figure 4.22: Residual Analysis for ARMAX Model Validation.

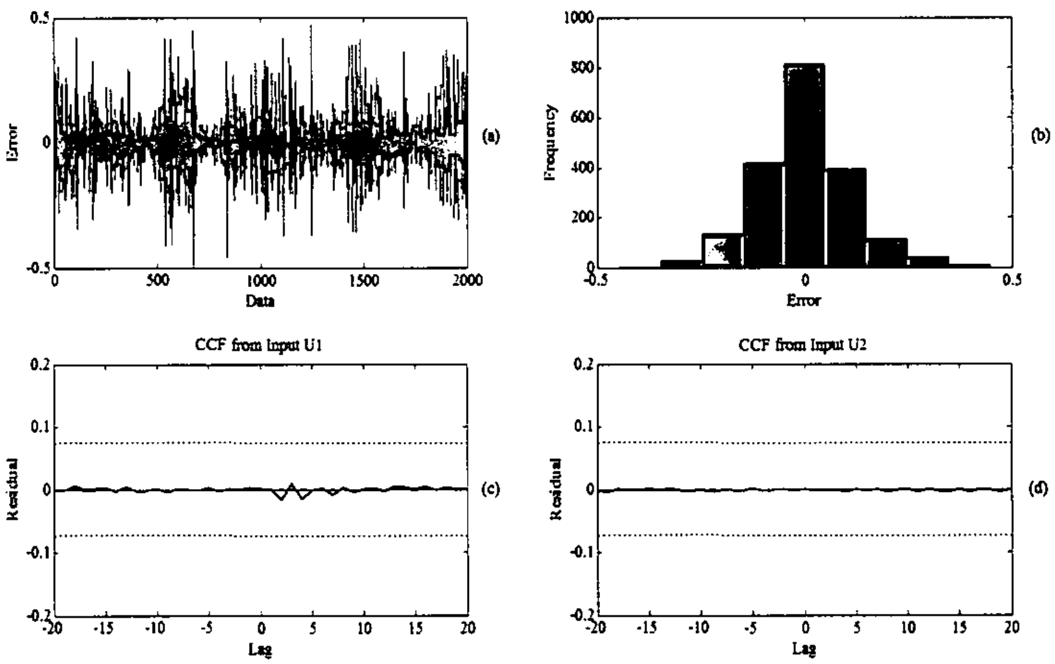


Figure 4.23: Residual Analysis for State Space Model Validation.

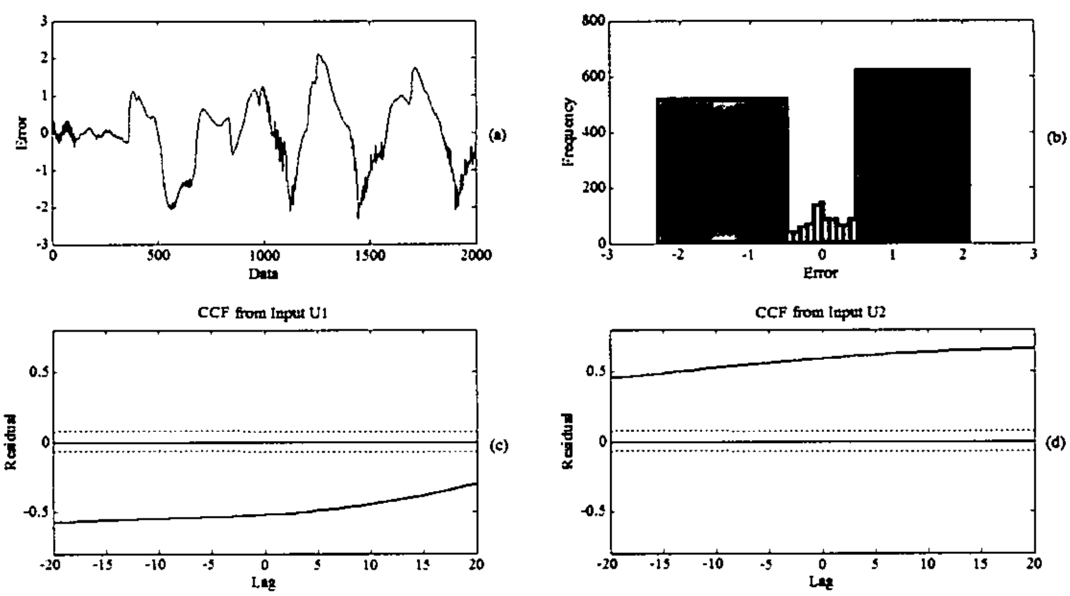


Figure 4.24: Residual Analysis for Continuous Process Model Validation.

All the third order models show a well distributed response with the mean at the centre except for continuous time process model. Analysing the cross-correlation response of the models, all models do not show a significant correlation between the input and the residual whereas for the continuous time process model, the cross-correlation response shows that the input has a relationship with the prediction error.

The mean and variance of the first, second and third order model are tabulated in Table 4.2.

Table 4.2: Linear Models Top Temperature Residual Histogram Performance Measurement

Model	Model order structure	Mean	Variance
ARX	1st order Structure - 110	0.0590	0.0440
	2nd order Structure - 220	0.0956	0.0400
	3rd – 330	0.0030	0.0540
ARMAX	1st order Structure - 1110	0.0023	0.0640
	2nd order Structure - 2220	0.0373	0.0100
	3rd - 3330	0.0900	0.0400
State Space	1st order	0.0840	0.0400
	2nd order	0.0116	0.0700
	3rd order	0.0040	0.0150
Continuous Process Model	1st order	0.4381	0.7220
	2nd order	0.1486	1.4317
	3rd order	0.0041	0.8705

Analysing the residual statistics of the developed linear models from Table 4.2, it shows that the 3rd order state space model and 2nd order ARMAX model gives the minimum mean value of 0.0373 and 0.004 and variance of 0.01 and 0.015. Analysing the residual histogram the graphs are well distributed and centered at the origin. The cross correlation response of the models shows that the estimation data was completely modelled as the model inputs does not show a considerable relation of the inputs with the models prediction errors.

4.3.3 Best Fit Analysis for Bottom Temperature

Table 4.3 shows the model validation error analysis statistics for the linear models developed for the bottom temperature of the distillation column. Analyzing Table 4.3, it is shown that the ARX, ARMAX and Linear State Space model gives a much better validation result compare to the process model. Overall the most suitable result has been given by the linear state space model or 3rd order with the best fit of 40.73% and 0.0004 SSPE and 2nd order ARMAX model with best fit 67.67% and 0.0004 SSPE for the bottom temperature of the distillation column.

Table 4.3: Linear Model Best Fit Error Analysis Statistics

Model Performance Measurement			
		Validation	
Order	Model Structure	Best Fit (%)	Sum of Squared Prediction Error
1st Order	ARX (110)	41.11	0.0019
	ARMAX (1110)	44.07	0.0004
	State Space	34.32	0.0004
	Process Model	39.22	0.0223
2nd Order	ARX (220)	43.74	0.0006
	ARMAX (2220)	67.67	0.0004
	State Space	64.18	0.0004
	Process Model	23.45	0.0137
3rd Order	ARX	63.30	0.0004
	ARMAX	43.59	0.0003
	State Space	40.73	0.0004
	Process Model	40.75	0.0235

4.3.4 Bottom Temperature Residual Performance

4.3.4.1 First Order Residual Analysis

From Figure 4.25 to Figure 4.28 the predicted error or residual plot of the developed first order linear models are shown. These plots are based on the validation results of the model. Figure (a) shows the predicted error of the model, (b) shows the histogram, (c) and (d) shows the cross correlation response from input U1 and input U2.

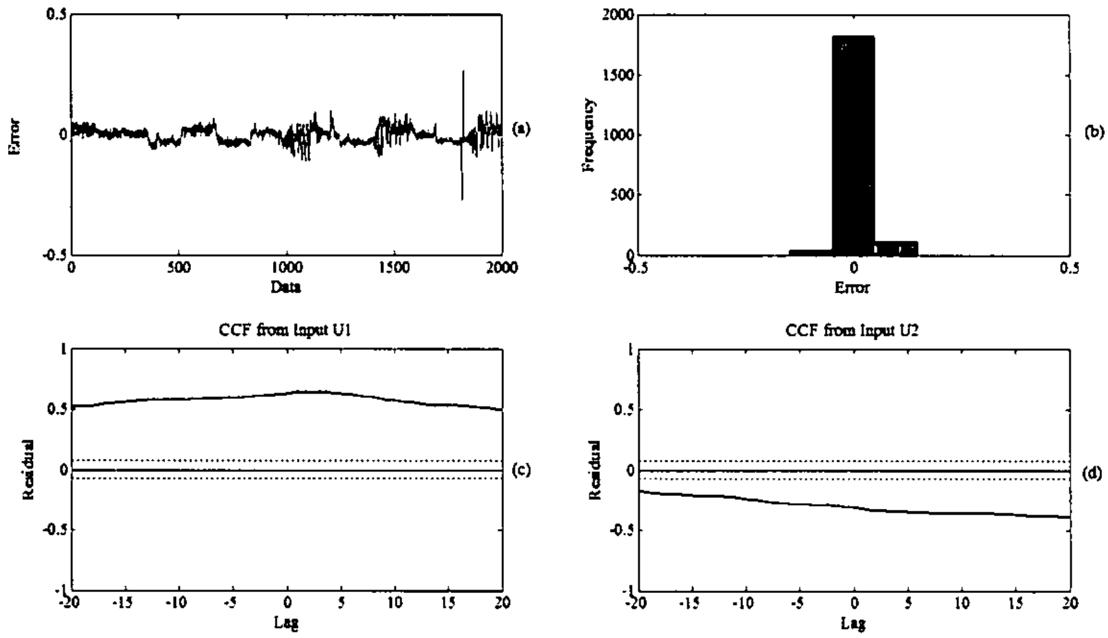


Figure 4.25: Residual Analysis for ARX Model Validation.

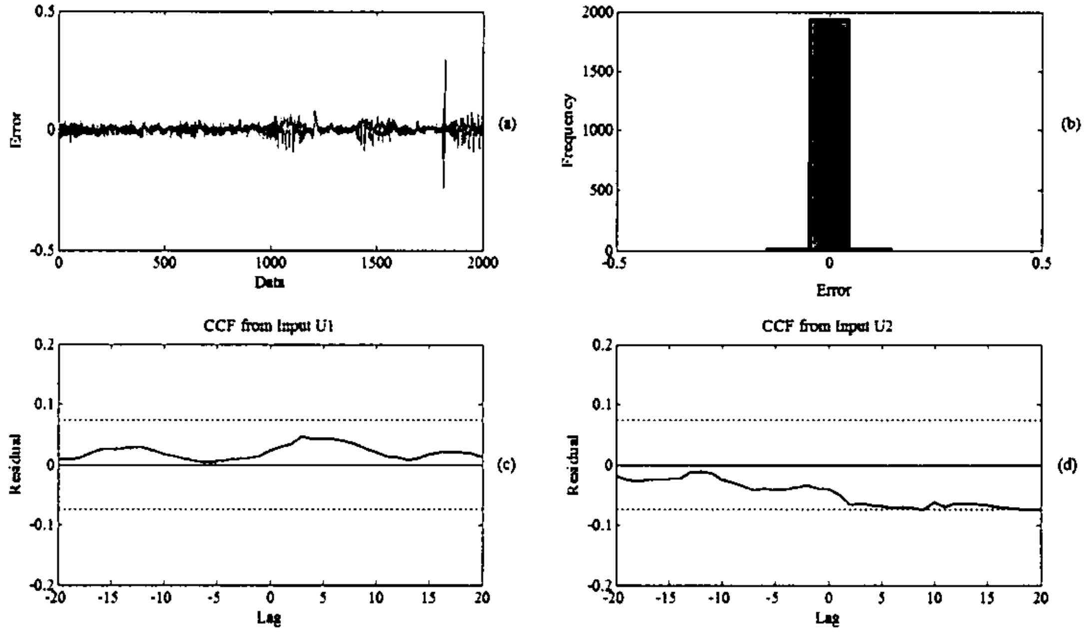


Figure 4.26: Residual Analysis for ARMAX Model Validation.

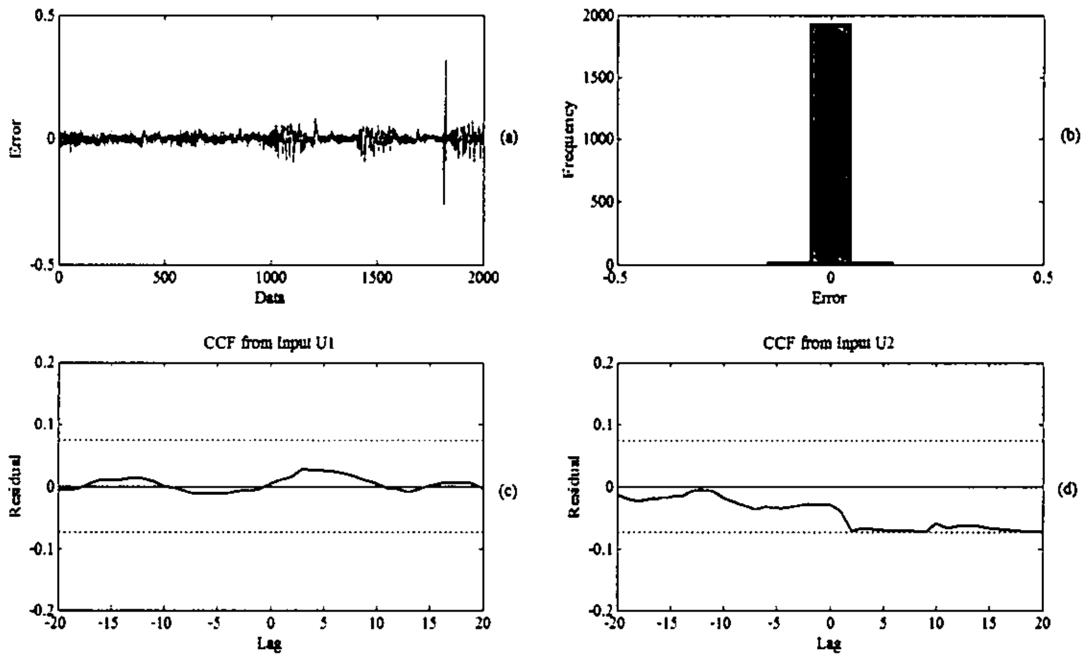


Figure 4.27: Residual Analysis for State Space Model Validation.

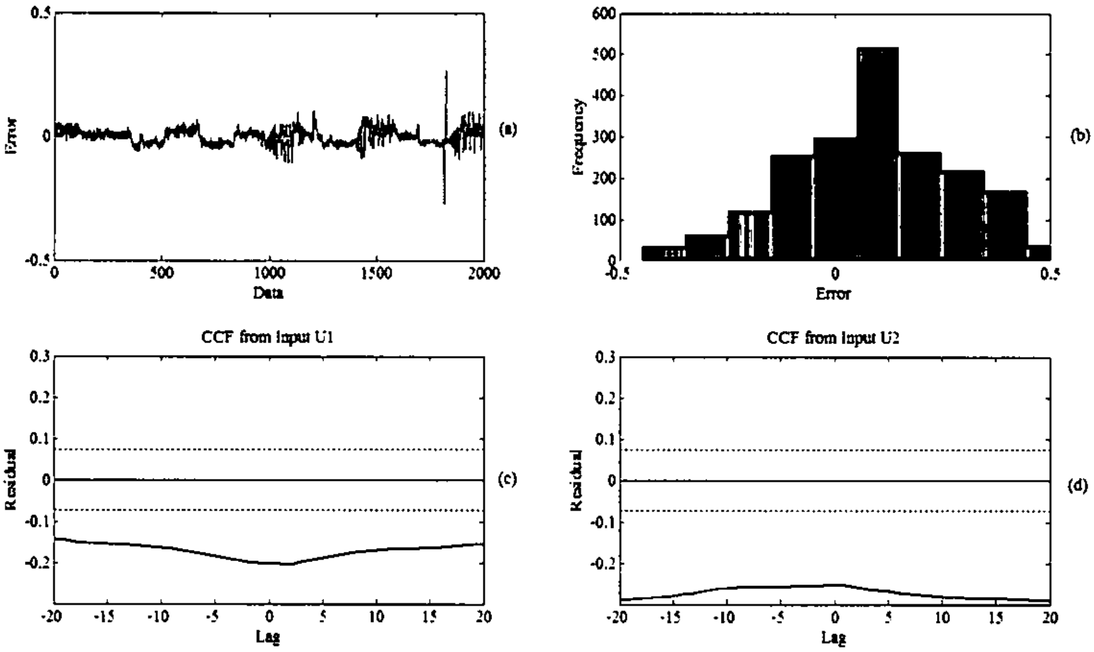


Figure 4.28: Residual Analysis for Continuous Process Model Validation.

Analysing the residual response of the developed linear model for the bottom temperature of the distillation process column, ARX, ARMAX, linear state space model show a well distributed prediction error response of the histogram while the continuous time process model does show a well distribution since the origin is not at the centre. Observing the cross correlation response for the ARX and continuous time process model response from the two inputs shows that not all the estimation date are used for developing the model since correlation is observed between the inputs and the residual. ARMAX and linear state space model response shows that the models are well developed from the estimation data since no significance correlation is observed between the inputs and the residual.

4.3.4.2 Second Order Residual Analysis

From Figure 4.29 to Figure 4.32 the predicted error or residual plot of the developed second order linear models are shown along with the histogram. These plots are based on the validation result of the model.

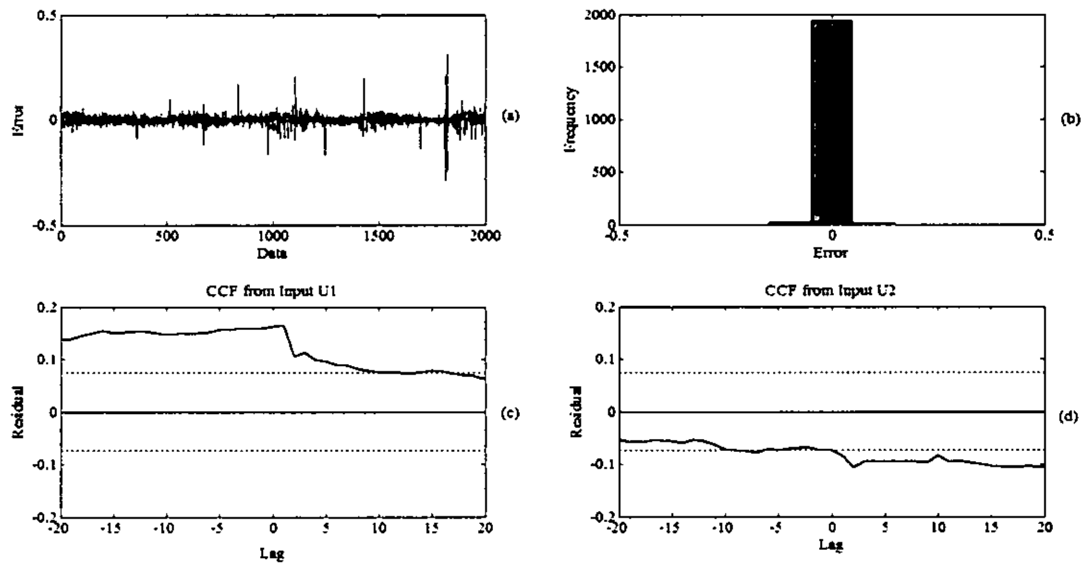


Figure 4.29: Residual Analysis for ARX Model Validation.

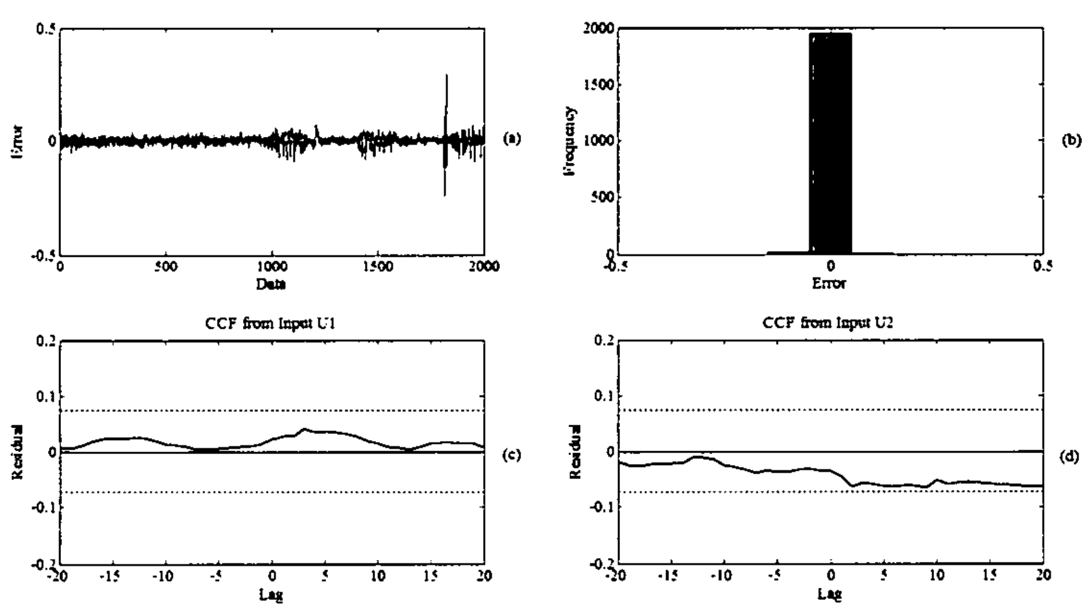


Figure 4.30: Residual Analysis for ARMAX Model Validation.

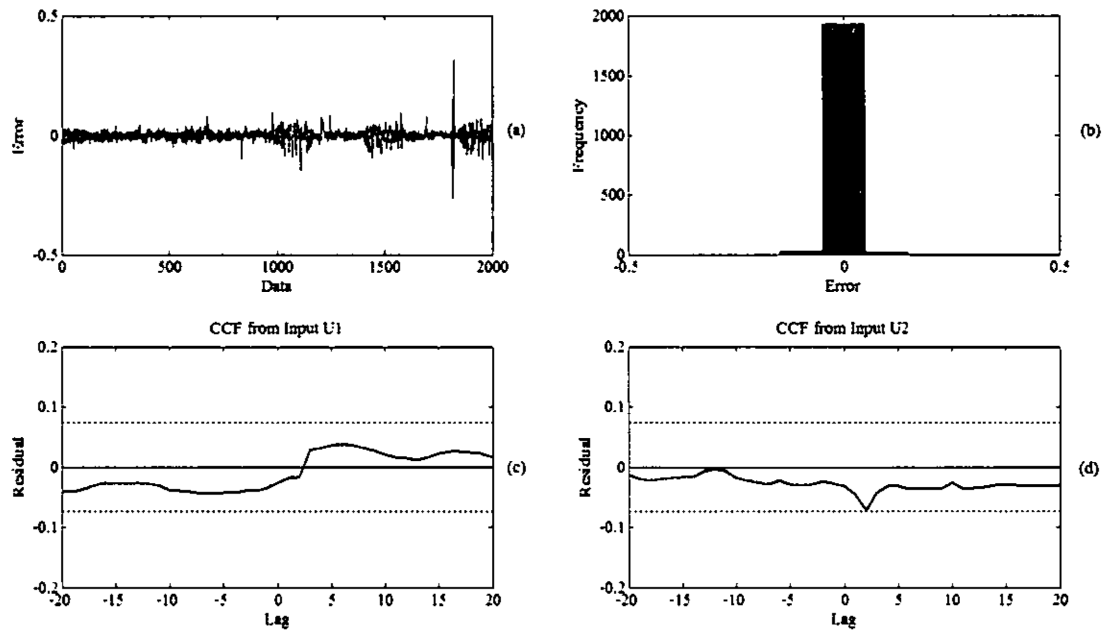


Figure 4.31: Residual Analysis for State Space Model Validation.

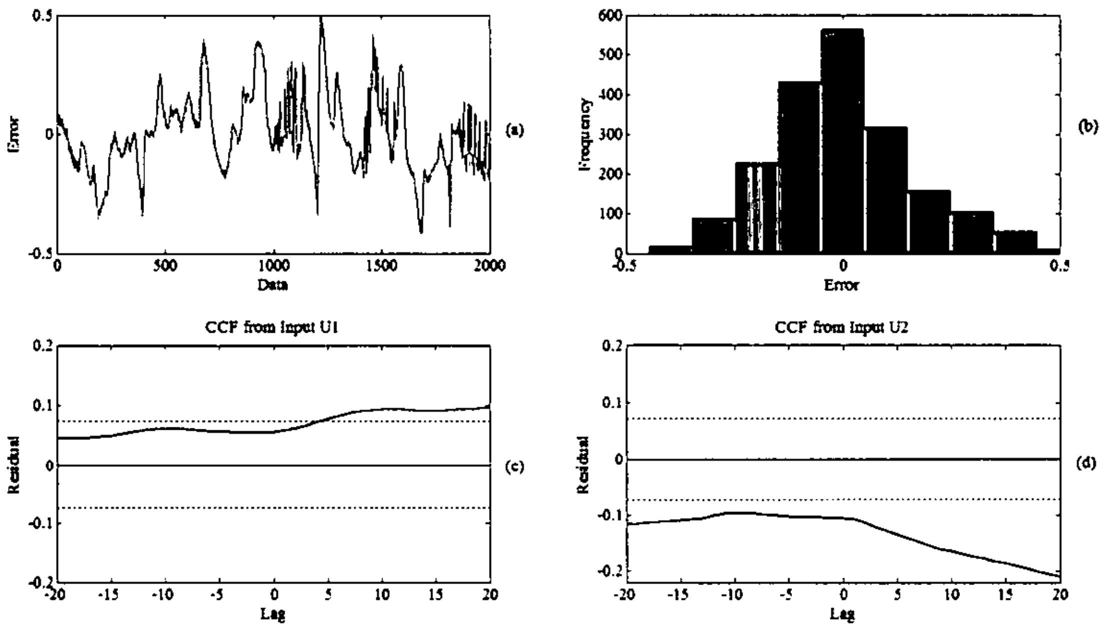


Figure 4.32: Residual Analysis for Continuous Process Model Validation.

Second order ARMAX and linear state space model shows a well developed response based upon their cross-correlation response and its well distributed histogram response of the prediction error. ARX model and continuous time process model shows a well distributed response but the cross-correlation response shows that the model is not well developed from the estimation data since correlation is observed between the inputs and the residual.

4.3.4.3 Third Order Residual Analysis

From Figure 4.33 to Figure 4.36 the predicted error or residual plot of the developed second order linear models are shown along with the histogram. These plots are based on the validation result of the model.

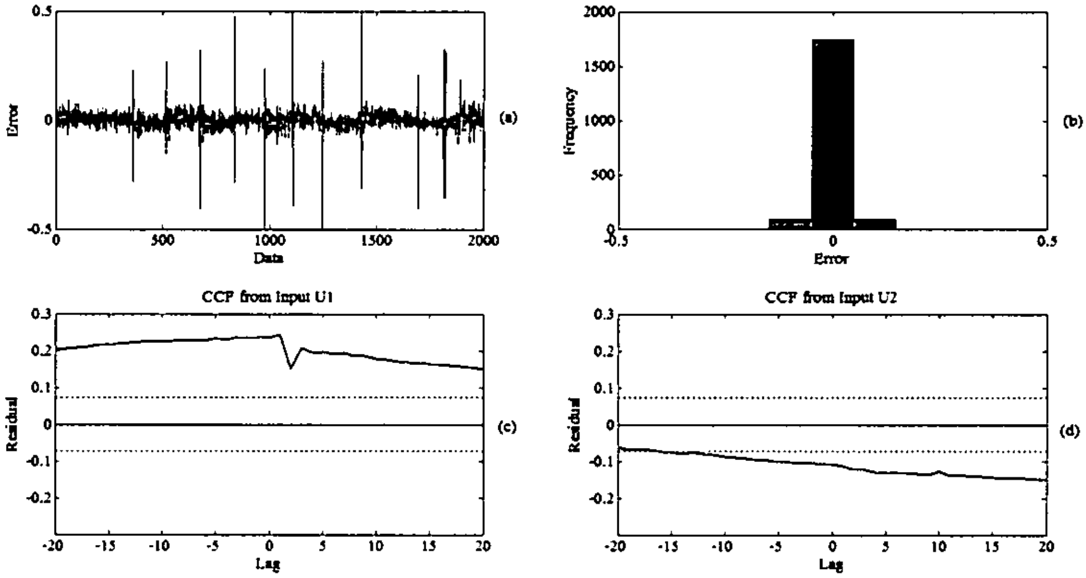


Figure 4.33: Residual Analysis for ARX Model Validation.

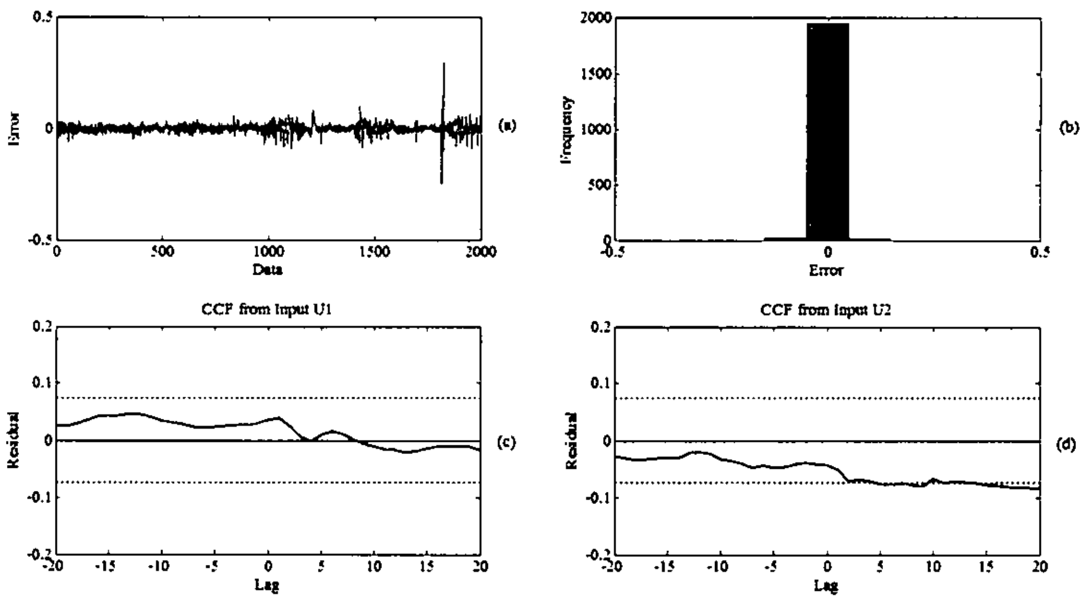


Figure 4.34: Residual Analysis for ARMAX Model Validation.

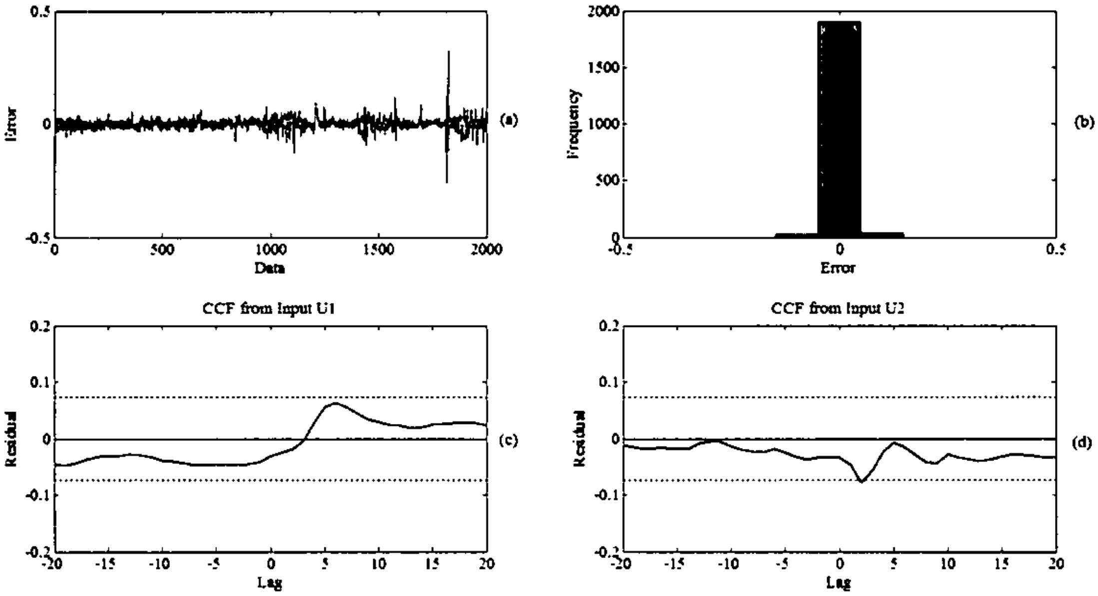


Figure 4.35: Residual Analysis for State Space Model Validation.

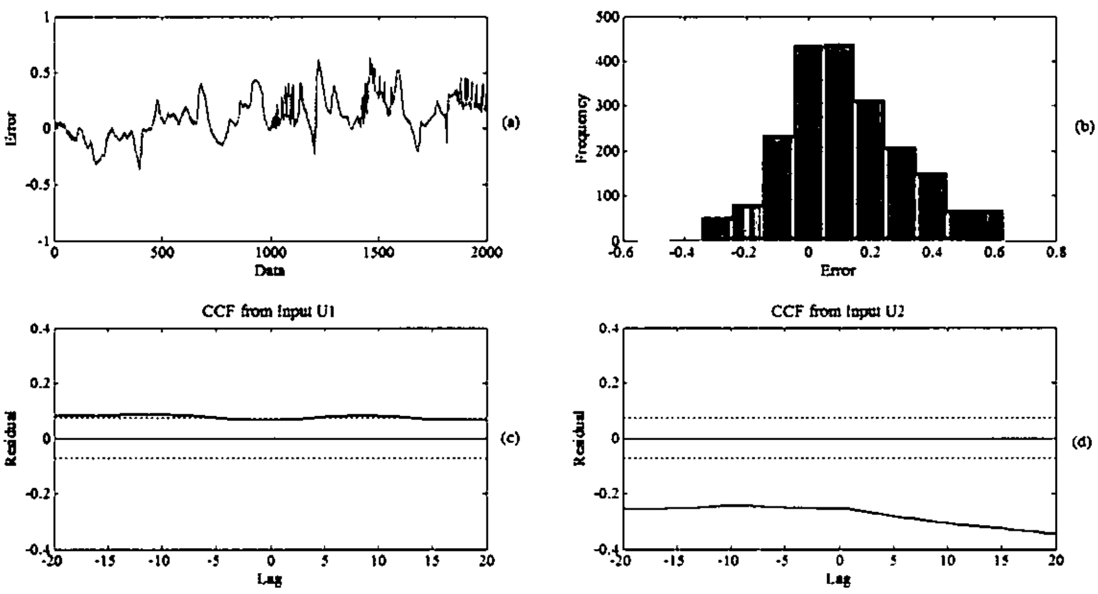


Figure 4.36: Residual Analysis for Continuous Process Model Validation.

Third order ARMAX shown in Figure 3.34 and linear state space model shown in Figure 3.35 demonstrates a well developed response based upon their cross-correlation response and their well distributed response of the histogram. ARX model shows a well distributed response but the cross-correlation response shows that the model is not well developed from the estimation data since inputs are having correlation with the residual. Similarly the continuous time process model is not well developed from its estimation data due to the correlation between the inputs and the residual and the histogram response of the model prediction error is well distributed but not centered at zero.

The mean and variance of the first, second and third order model are tabulated in Table 4.4.

Table 4.4: Linear Models Bottom Temperature Residual Histogram Performance Measurement

Model	Model order	Mean	Variance
ARX	1st order Structure - 110	0.0590	0.0440
	2nd order Structure - 220	0.0964	0.0400
	3rd – 330	0.0300	0.0540
ARMAX	1st order Structure - 1110	0.0890	0.0092
	2nd order Structure - 2220	0.0080	0.0380
	3rd - 3330	0.0600	0.0400
State Space	1st order	0.0290	0.0400
	2nd order	0.0800	0.0410
	3rd order	9.7e-004	4.3e-004
Continuous Process Model	1st order	0.5597	1.7034
	2nd order	0.7571	0.8708
	3rd order	0.7867	1.2087

Analysing the residual statistic of the developed linear models from Table 4.4, it shows that the 3rd order state space model and 2nd order ARMAX model gives the minimum mean value of 9.7e-4 and 0.008 and variance of 4.3e-4 and 0.038. Analysing the residual histogram of these two models, the graphs of these two models are well distributed and centred at the origin.

4.4 Summary

In this chapter, results have been shown and evaluated for different types of linear parametric models which are the ARX, ARMAX, State Space Model and the Continuous time process model. All models are simulated up until 3rd order. The simulation results show that the identifier performance for estimating the model output is acceptable to some extent since the models are able to capture the changes in the dynamics of the process. The analysis of the results as shown in Table 4.1 and Table 4.3 shows the best model results and are highlighted accordingly.

By analyzing the residual statistics from Table 4.1 and Table 4.3, it is observed that the linear state space model is to be considered as the most suitable model for the distillation column since its performance of best fit is higher and sum of squared prediction error is minimum than the other linear models. Furthermore, residual histogram plots of the model prediction error, cross-correlation graphs and statistics from Table 4.2 and Table 4.4 also verifies that the linear state space model and ARMAX model shows a compatible result. Since, it is said that State Space Model provides a more complete representation of a system not only for SISO systems but also for MIMO systems therefore, the 3rd order State Space model has been selected and has been enhanced in the development of Nonlinear State Space Model which has been discussed in Chapter 5.

CHAPTER 5

MULTIVARIABLE NONLINEAR SYSTEM IDENTIFICATION

5.1 Introduction

In this chapter the results of nonlinear system identification are presented. Three different modeling approaches have been used. Neural network trained with Gradient Decent with Momentum (GDM) and Lavernberg Marquardt (LM) algorithm, Nonlinear State Space model and Adaptive Neuro Fuzzy Inference System (ANFIS). Modeling errors have been analyzed and discussed. Modeling approach has been done considering MISO system.

5.2 Neural Network Approach

5.2.1 Gradient Decent with Momentum (GDM)

For nonlinear system Identification the first approach that has been used is the Backpropagation Feedforward network trained using the delta rule (also known as gradient decent, with the addition of momentum). Modeling for both top and bottom temperature of the distillation column has been done with respect to the two inputs which are shown in Figure 3.5 for the reflux flow and Figure 3.6 for the steam flow of chapter 3. Figure 5.1 and 5.2 show the estimation result for top and bottom temperature of the distillation column. 2000 data points are used for estimation. Network architecture used is 2-10-1 and the model achieved its target within 57 epochs.

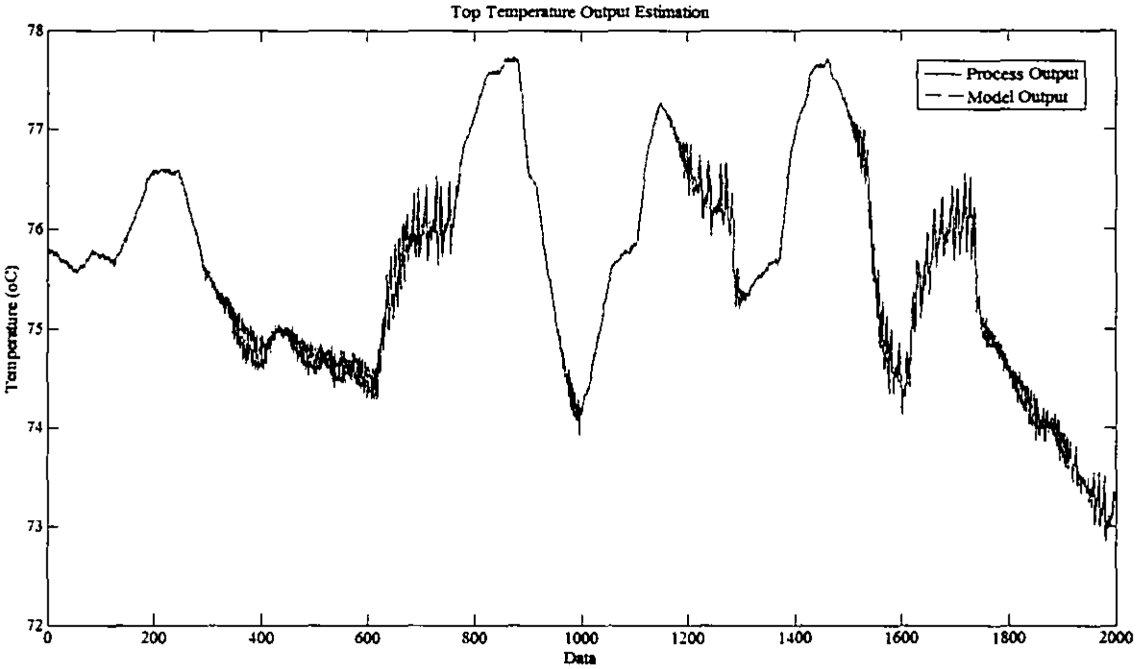


Figure 5.1: NN GDM Distillation Column Top Temperature Estimation.

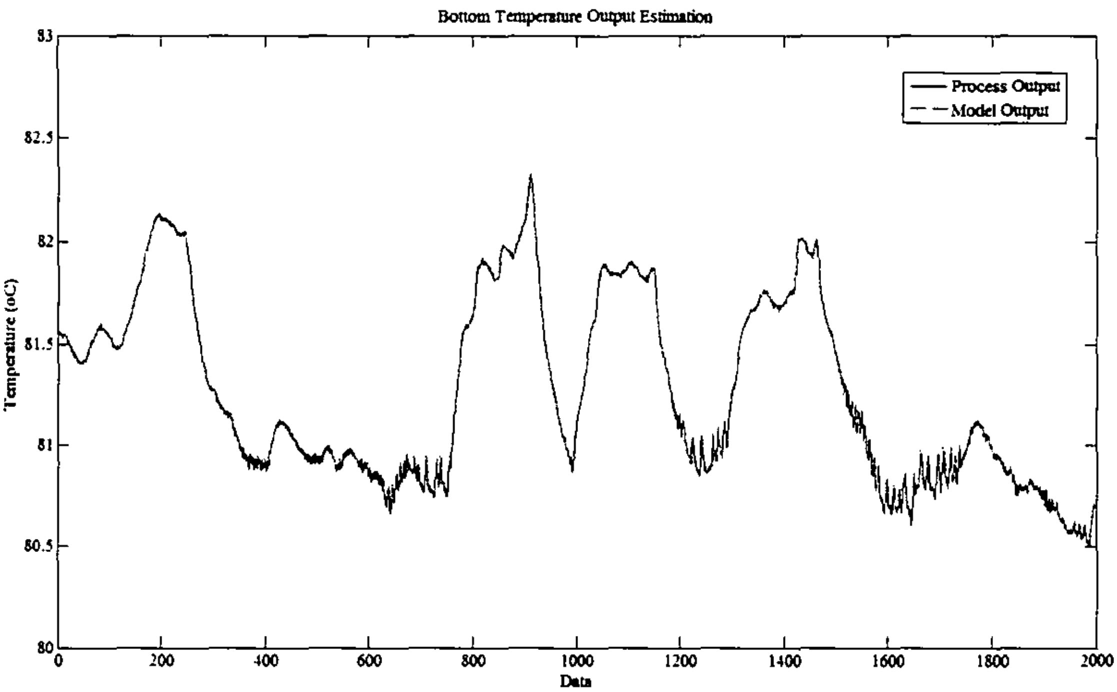


Figure 5.2: NN GDM Distillation Column Bottom Temperature Estimation.

Figure 5.3 and Figure 5.4 shows the validation result for top and bottom temperature of the distillation column. 2000 data points are used for Validation.

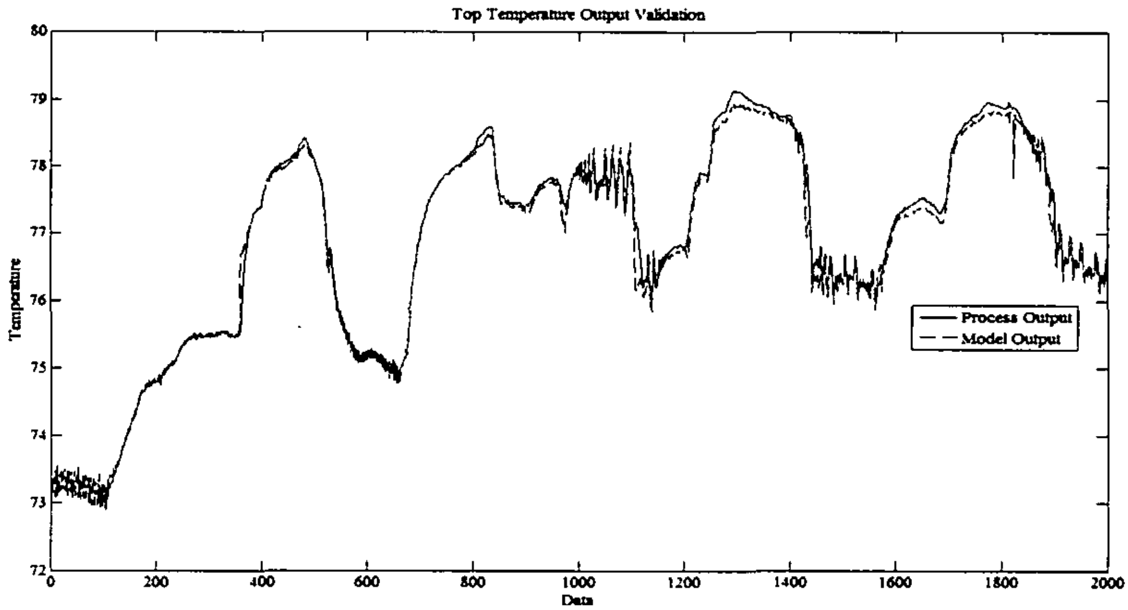


Figure 5.3: NN GDM Distillation Column Top Temperature Validation.

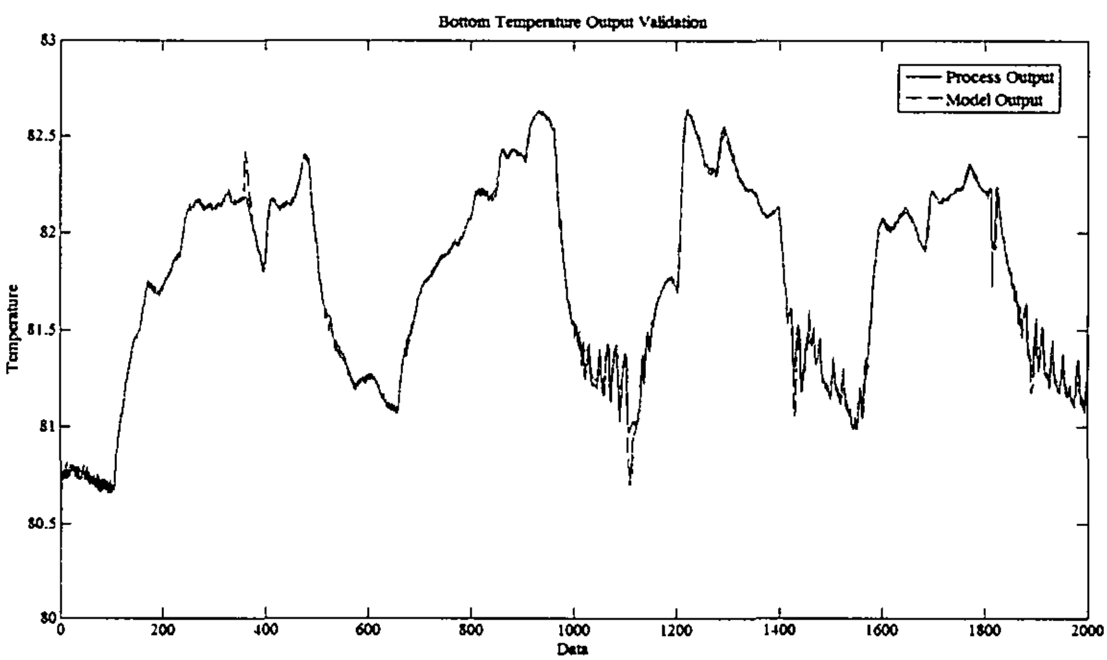


Figure 5.4: NN GDM Distillation Column Bottom Temperature Validation.

Identification of the APC plant by the NN trained by GDM algorithm shows a very significant result. The network is able to capture the dynamic changes of the process plant.

5.2.2 Lavernberg Marquardt (LM)

Lavernberg Marquardt algorithm for training a neural network is a very well known approach. Neural network trained with LM algorithm also is used for modeling the process plant. Modeling for both top and bottom temperature of the distillation column has been done with respect to the two inputs which are shown in Figure 3.5 for the reflux flow and Figure 3.6 for the steam flow of chapter 3. Figure 5.5 and 5.6 shows the estimation result for top and bottom temperature of the distillation column. 2000 data points are used for estimation. The models took 37 epochs to achieve its target with network architecture of 2-10-1.

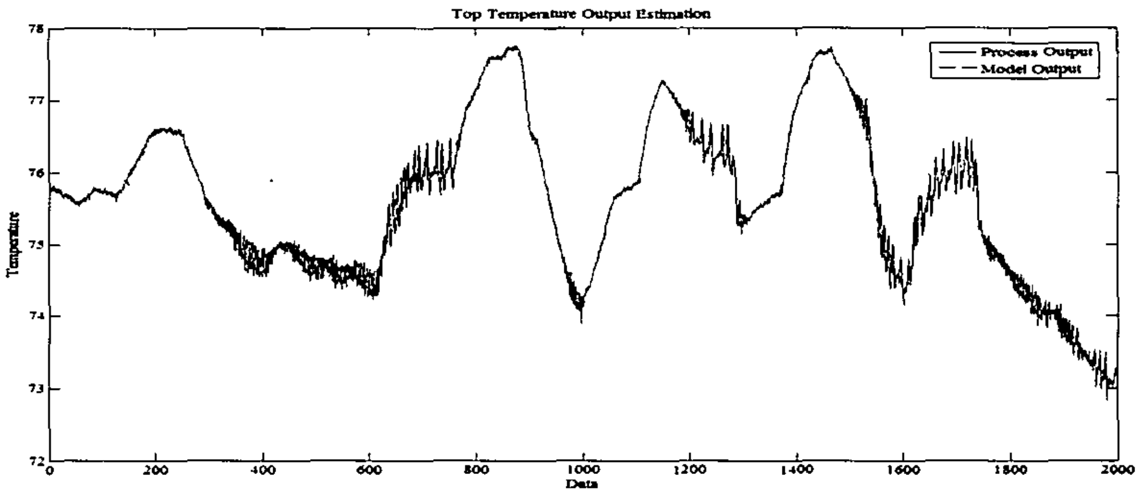


Figure 5.5: NN LM Distillation Column Top Temperature Estimation.

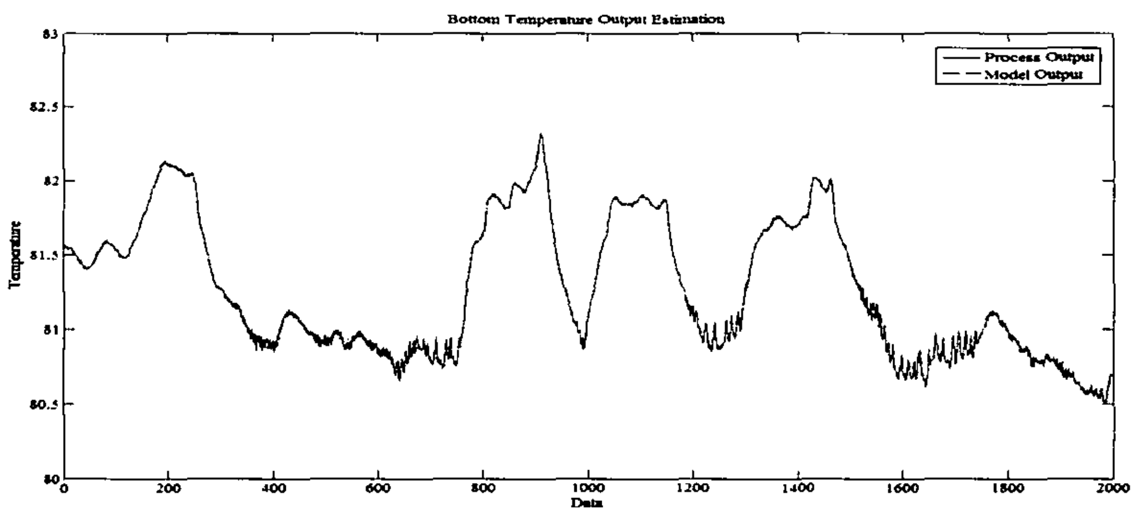


Figure 5.6: NN LM Distillation Column Top Temperature Estimation.

Figure 5.7 and Figure 5.8 shows the validation result for top and bottom temperature of the distillation column. 2000 data points are used for Validation.

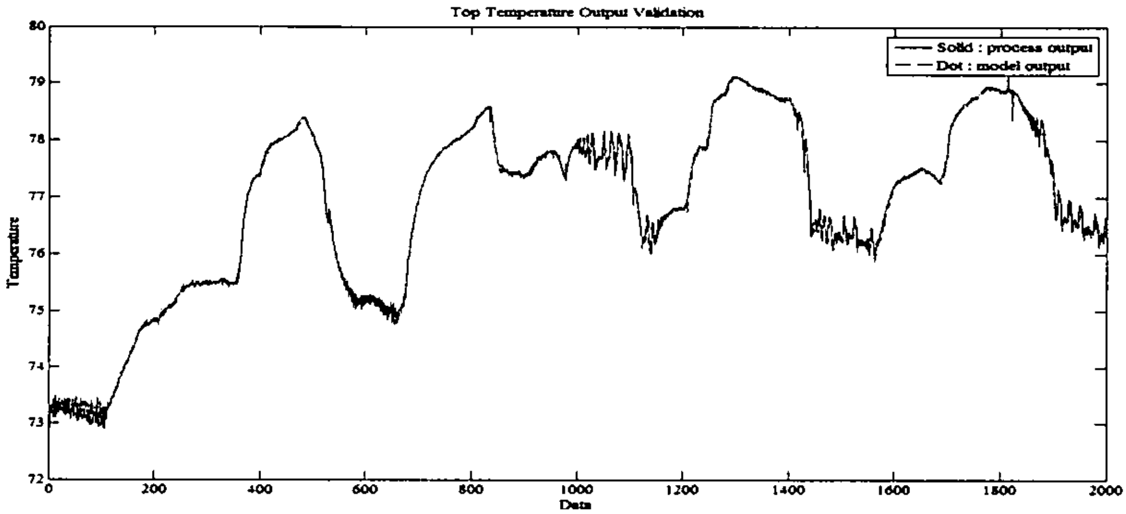


Figure 5.7: NN LM Distillation Column Top Temperature Validation.

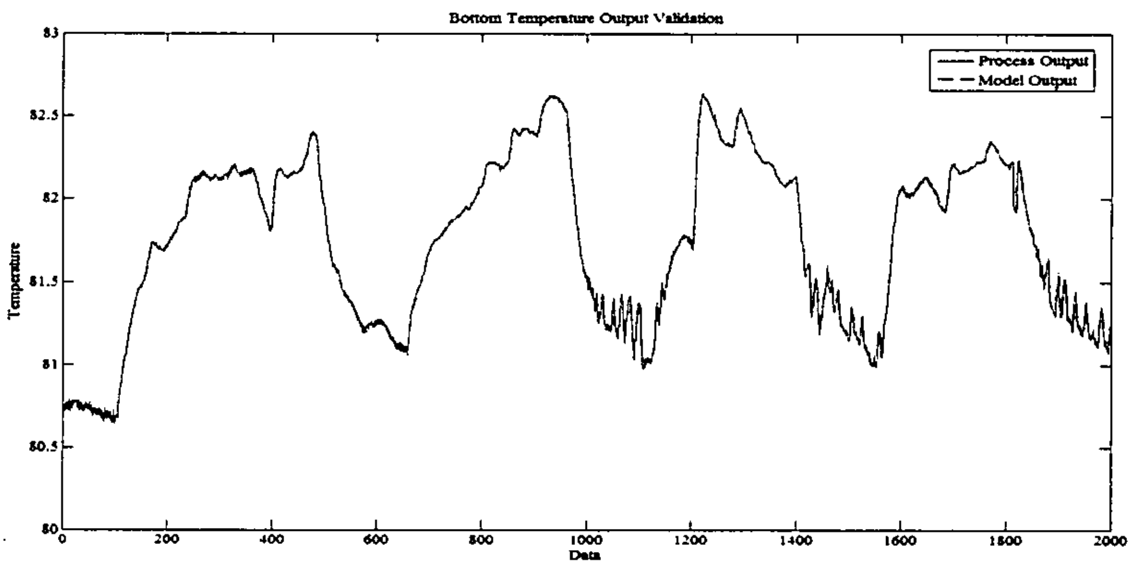


Figure 5.8: NN LM Distillation Column Bottom Temperature Validation.

Observing the resposne of the neural network trained with Laverberg Marqurdt algorithm, the nework is capable of capturing the changes in the dynamics of the nonlinear process.

5.3 Nonlinear State Space Model

A different type of approach using neural network, nonlinear state space model was developed in conjunction of linear state space model along with neural network as has been explained in Chapter 2. Modeling for both top and bottom temperature of the distillation column has been done with respect to the two inputs which are the reflux flow and the steam flow. The linear state space model of 3rd order discrete time system is used for developing the nonlinear state space model. The MISO forms of linear state space models obtained for top temperature of the system is given by equation 5.1 to 5.3. The equation is taken from the linear model discussed in chapter 4.

$$x(k+1)=\begin{bmatrix}0 & 0 & 0.4375\\ 1 & 0 & -1.3226\\ 0 & 1 & 1.881\end{bmatrix}x(k)+\begin{bmatrix}-0.04537 & -0.001\\ 0.6431 & 0.01\\ -0.646 & -0.00521\end{bmatrix}\begin{bmatrix}U_1 & U_2\end{bmatrix} \qquad (5.1)$$

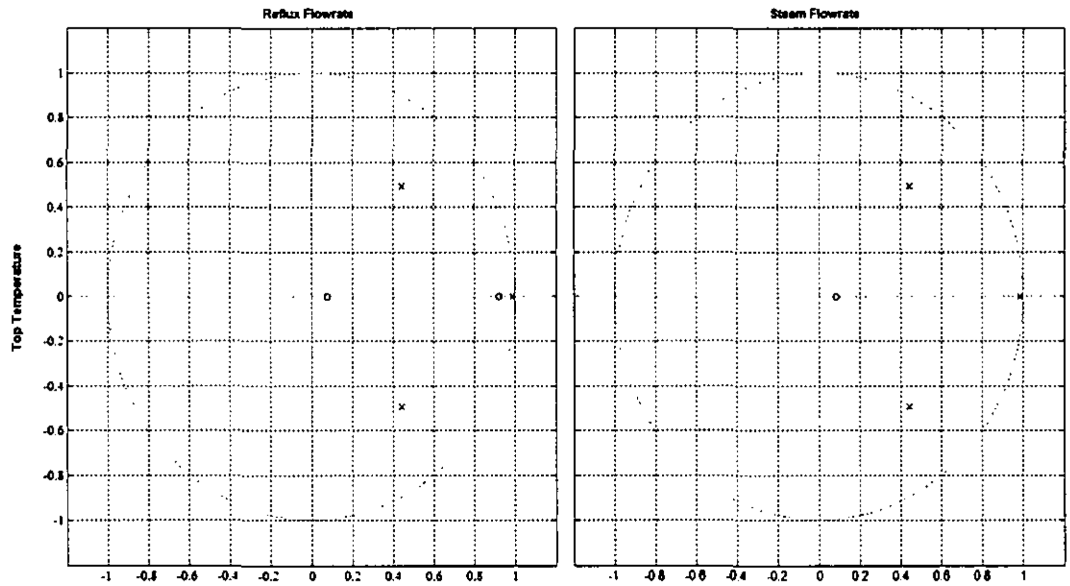
The MISO linear state space model output equation obtained for top temperature of the system is given as;

$$y_1(k)=\begin{bmatrix}0 & 0 & 1\end{bmatrix}x(k)+\begin{bmatrix}0\end{bmatrix}\begin{bmatrix}U_1 & U_2\end{bmatrix} \qquad (5.2)$$

Where $x(k)$ is the system state, $y_1(k)$ is the output of the system. The initial state $x(0)$ of the system is given as;

$$x(0)=\begin{bmatrix}-0.00355\\ -0.017\\ -0.003\end{bmatrix} \qquad (5.3)$$

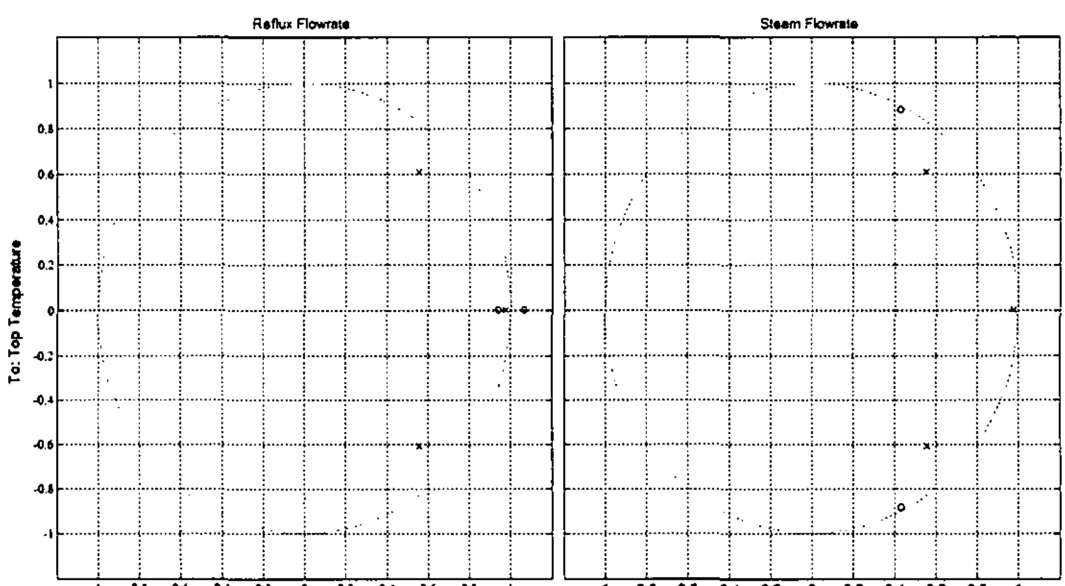
As stated in [24], when using neural network to identify the system two important assumptions are considered: (1) all the system states are measureable; (2) the system is stable. States for the physical system are considered to be measured and the stability of the system is observed by the pole-zero plots as shown in Figure 5.9 for top temperature process and Figure 5.10 for bottom temperature process. Observing the graphs from Figure 5.9 and Figure 5.10, all the poles positions appears to be within the unity circle which shows the stability if the system.



(a) (b)

Figure 5.9: (a) I/O Poles & Zeros from Input U_1 to Output Y_1

(b) I/O Poles & Zeros from Input U_2 to Output Y_1



(a) (b)

Figure 5.10: (a) I/O Poles & Zeros from Input U_1 to Output Y_2

(b) I/O Poles & Zeros from Input U_2 to Output Y_2

Figure 5.11 and Figure 5.12 shows the estimation and validation result for top temperature of the distillation process column. 2000 data points are used for both estimation and validation.

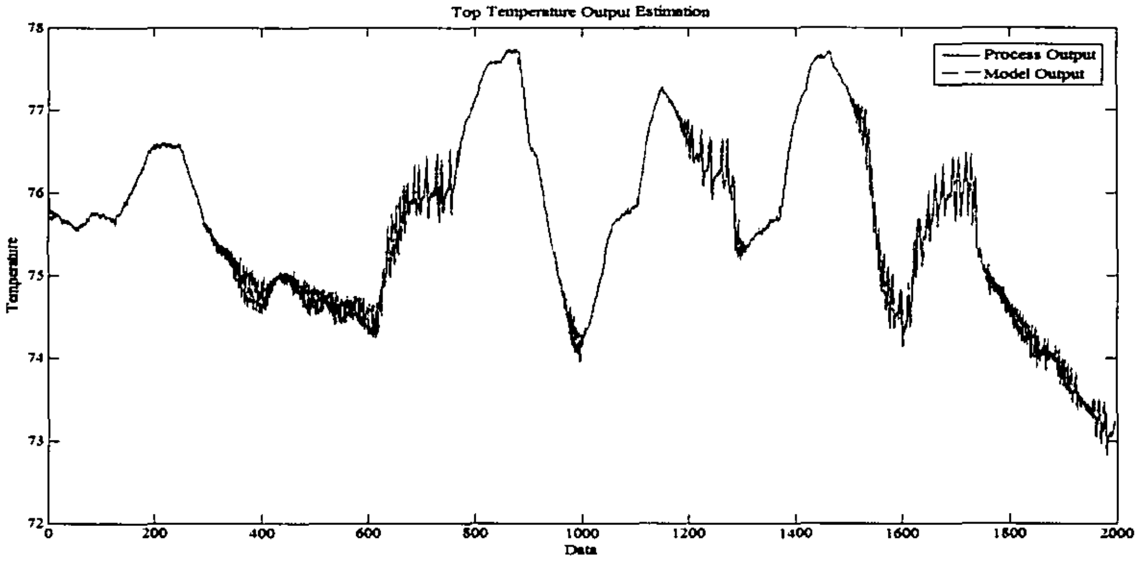


Figure 5.11: Nonlinear State Space Model for Top Temperature Estimation.

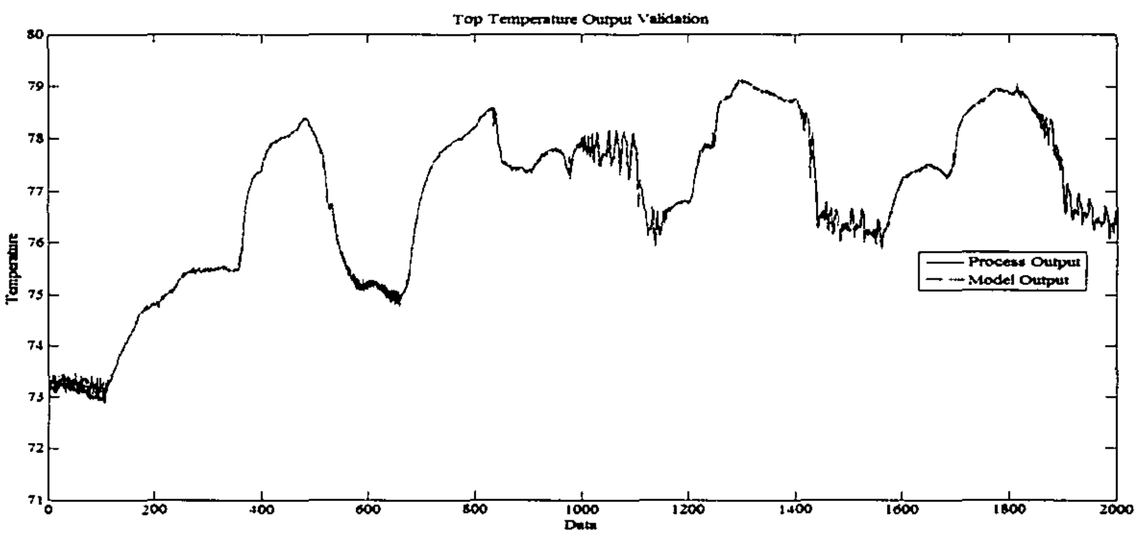


Figure 5.12: Nonlinear State Space Model for Top Temperature Validation.

The MISO forms of linear state space models obtained for bottom temperature of the system is given by equation 5.4 to 5.6. The equation is taken from the linear model discussed in chapter 4.

$$x(k+1)=\begin{bmatrix} 0 & 0 & 0.67 \\ 0.5 & 0 & -0.8872 \\ 0 & 2 & 2.1 \end{bmatrix} x(k)+\begin{bmatrix} -0.3861 & 0.00606 \\ 0.4 & -0.002705 \\ -0.4 & 0.0063 \end{bmatrix} \begin{bmatrix} U_1 & U_2 \end{bmatrix} \quad (5.4)$$

The MISO linear state space model output equation obtained for bottom temperature of the system is given as;

$$y_2(k)=\begin{bmatrix} 0 & 0 & 0.5 \end{bmatrix} x(k)+\begin{bmatrix} 0 \end{bmatrix}\begin{bmatrix} U_1 & U_2 \end{bmatrix} \quad (5.5)$$

Where $x(k)$ is the system state, $y_2(k)$ is the output of the system. The initial state $x(0)$ of the system is given as;

$$x(0)=\begin{bmatrix} -0.013771 \\ -0.004 \\ -0.0033 \end{bmatrix} \quad (5.6)$$

Figure 5.13 and Figure 5.14 shows the estimation and validation result for bottom temperature of the distillation column. 2000 data points are used for both estimation and validation.

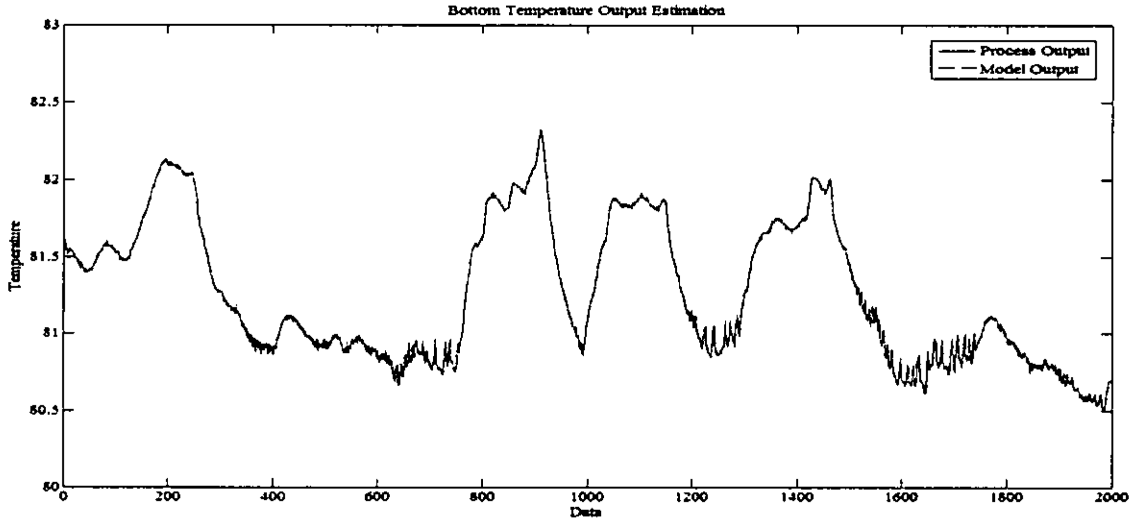


Figure 5.13: Nonlinear State Space Model for Bottom Temperature Estimation.

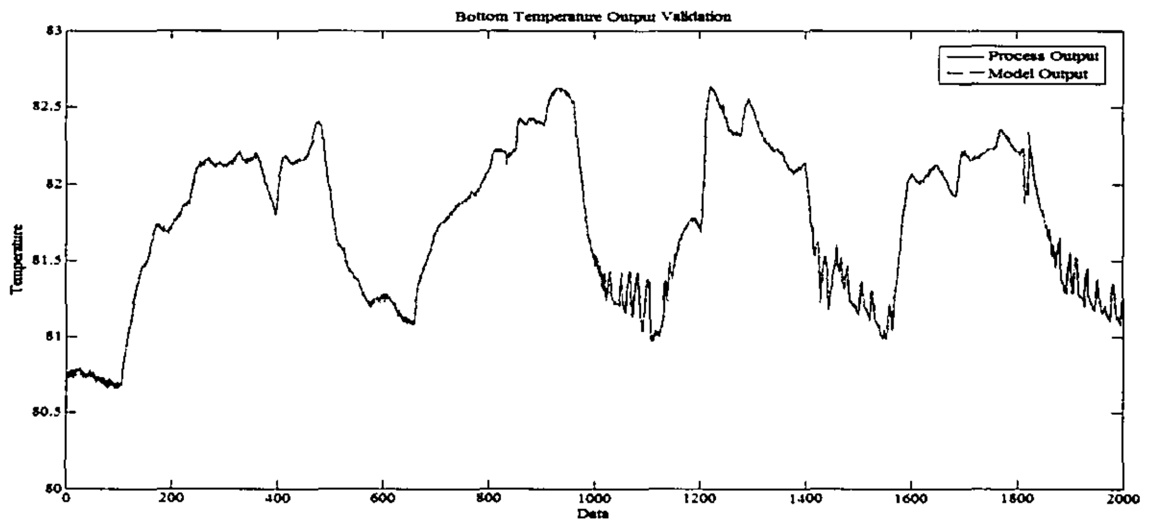


Figure 5.14: Nonlinear State Space Model for Bottom Temperature Validation.

Analysing the response of nonlinear state space model, the model is capable of capturing the nonlinear dynamic changes of the system. The error analysis for both top and bottom of the model are tabulated in Table 5.3 and Table 5.4.

5.4 Adaptive Neuro-Fuzzy Inference System

Adaptive Neuro Fuzzy Inference System is one another approach can be used for identifying nonlinear systems. In construction of ANFIS structure, parameters are determined. There are quite a few MFs such as Triangular, Trapezoidal and Gaussian can be used as an input MFs. Commonly used MFs in literature are the Triangular and Gaussian. For this reason, Sigmoid, Gaussian and Triangular are chosen as input MF type in this study. Number of MFs on each input can be chosen as 3, 5, and 7 to define the linguistic labels significantly.

Since, there is no typical method to employ the expert knowledge; automatic rule generation (grid partition) method is usually preferred [51]. According to this method, for instance, an ANFIS model with two inputs and three MFs on each input would result in 3^2=9 Takagi-Sugeno fuzzy if-then rules automatically. Although this method can require much computational knowledge especially in systems that have to be defined with many inputs, it is used in this study due to advantage of MATLAB software. Therefore, rule bases of the estimators are formed automatically with the number of inputs and number of MFs. After the ANFIS structure is constructed, learning algorithm and training parameters are chosen. As mentioned in chapter 2, the hybrid learning algorithm is used in this study. Simulation has been performed for the top temperature and bottom temperature of the distillation column using ANFIS structure. Table 5.1 and Table 5.2 show the RMSE statistics of the ANFIS model.

Table 5.1: RMSE Estimation Performance Measurement for ANFIS structure

	Top Temperature			**Bottom Temperature**		
Membership Functions	3MFs	5MFs	7MFs	3MFs	5MFs	7MFs
Sigmoid	0.7578	0.7074	0.6781	0.2696	0.2597	0.2464
Gaussian	0.7490	0.6930	0.6770	0.2695	0.2569	0.2470
Triangular	0.7841	0.7111	0.7064	0.2774	0.2611	0.2501

Table 5.2: RMSE Validation Performance Measurement for ANFIS structure

	Top Temperature			**Bottom Temperature**		
Membership Functions	3MFs	5MFs	7MFs	3MFs	5MFs	7MFs
Sigmoid	1.0746	1.0107	1.0037	0.3115	0.304	0.2902
Gaussian	1.0834	1.0158	0.9943	0.3122	0.3028	0.2903
Triangular	1.1589	1.0451	0.9982	0.3299	0.3026	0.2918

Analyzing Table 5.2, 5 Gaussian type MF shows a good result with RMSE value of 1.0158 for top temperature and 0.3028 for bottom temperature. Although the other type MF shows similar result but due to higher number of rules, the computation for both learning and training phase could take a much longer time. Figure 5.15 and Figure 5.16 show the estimation and validation result of the ANFIS structure of the

Gaussian type with 5MFs modelled for top temperature of the distillation column. Membership functions and fuzzy inference rules are given in Appendix C.

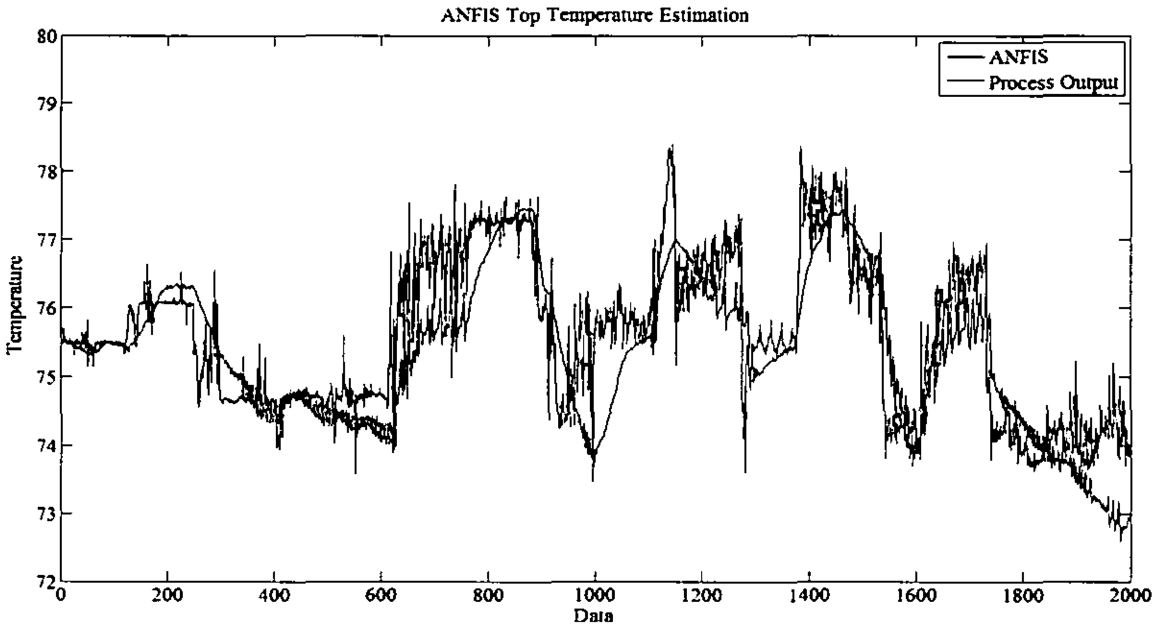


Figure 5.15: ANFIS Top Temperature Estimation.

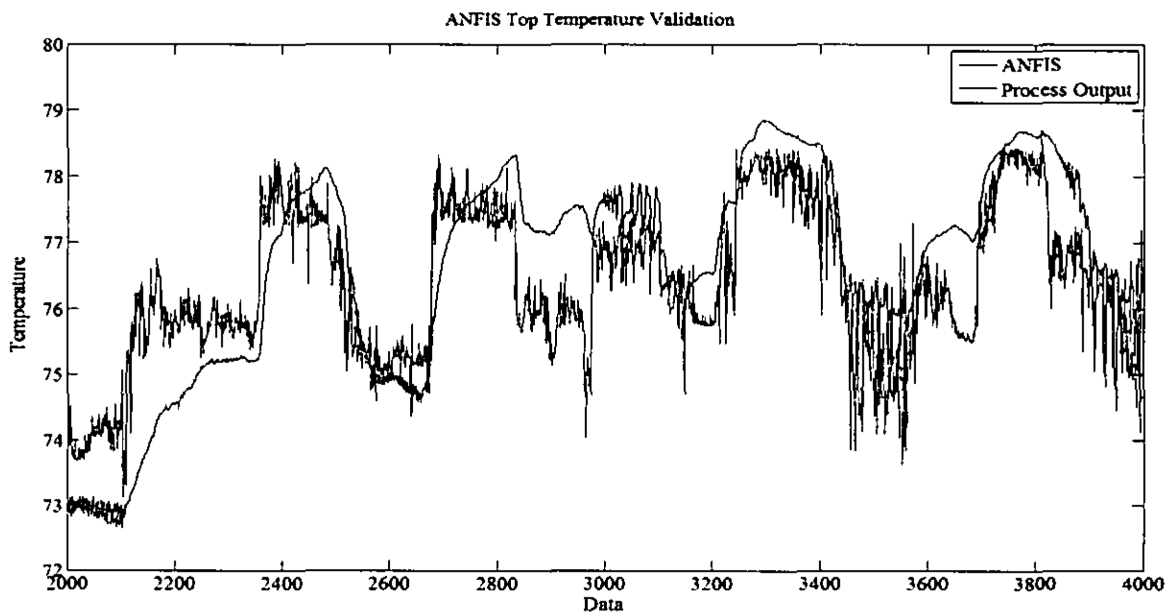


Figure 5.16: ANFIS Top Temperature Validation.

Analyzing Table 5.2, 5 Gaussian type MF shows a good result compared to other ANFIS approaches. Although 7 Gaussian and Triangular type MF shows similar result but due to higher number of rules, the computation for both learning and training phase could take a much longer time. The following Figure 5.17 and Figure

5.18 shows the estimation and validation result of the ANFIS structure of the Gaussian type with 5MFs modeled for bottom temperature of the distillation column. Membership functions and fuzzy inference rules are given in Appendix C.

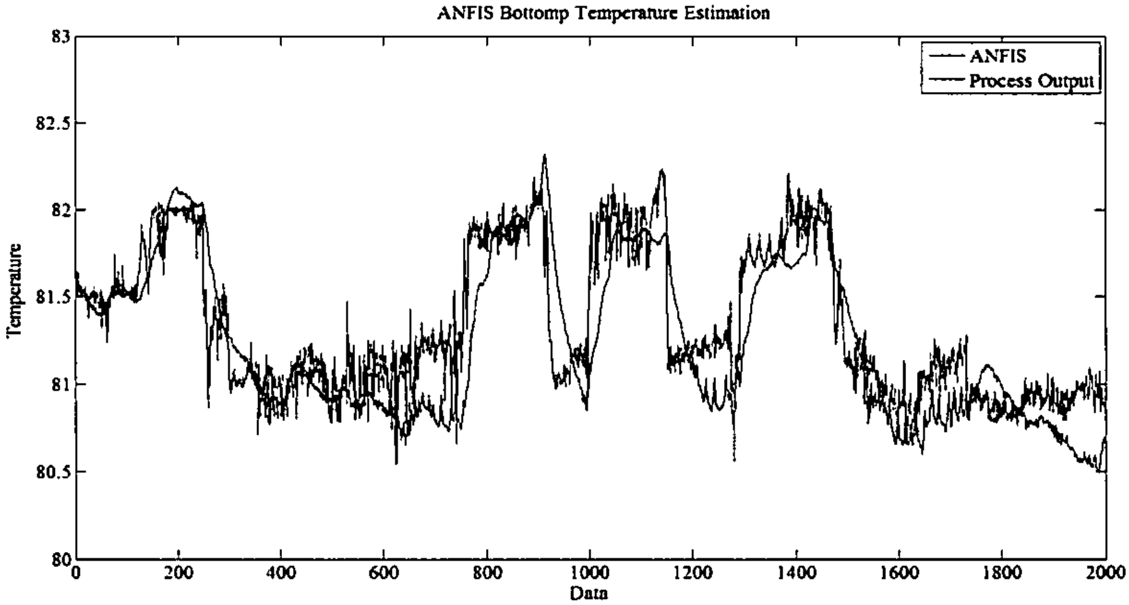


Figure 5.17: ANFIS Bottom Temperature Estimation.

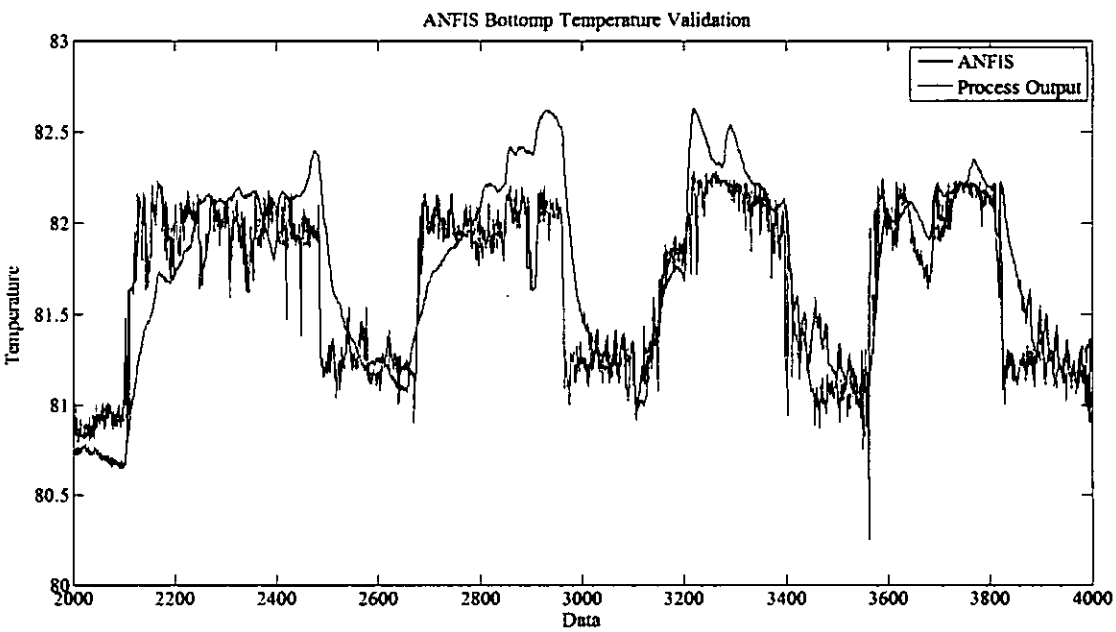


Figure 5.18: ANFIS Bottom Temperature Validation.

Looking into the ANFIS response both for top and bottom of temperature, the result is very noisy. This is because ANFIS is sensitive to noisy signals when given at the input. This is one reason that the ANFIS has not accurately modelled the input-output data's which in result shows that big error is present in the response.

5.5 Modelling Error Analysis

5.5.1 Best Fit Error Analysis

Following Table 5.3 shows the performance measurement of different types of nonlinear models.

Table 5.3: Numerical Results Performance Measurement for Top Temperature

Model Performance Measurement				
	Validation for Top		Validation for Bottom	
Model Structure	Best Fit (%)	Sum of Squared Prediction Error	Best Fit (%)	Sum of Squared Prediction Error
Gradient Decent Momentum	97.43	0.0120	98.44	0.0216
Lavernberg Marquardt	97.77	0.0312	98.39	0.0134
Nonlinear State Space	97.96	0.0162	99.38	0.0090
ANFIS	50.16	0.0997	60.13	0.0852

Analyzing Table 5.3, it can be observed that all the nonlinear models are capable of modeling the dynamic nonlinear system. Neural network trained by gradient decent with momentum and LM shows a very good result. The NSS model is also capable of identifying the dynamic nonlinear system. NSS model shows the best result with the best fit of 97.96% and 0.0162 SSPE for top temperature and 99.38% of best fit and 0.009 SSPE for bottom temperature process. ANFIS identification did not show a better result; this is because that every data point of the plant has to be evaluated from every rule defined in the ANFIS structure. Due to this process the computation takes a longer period and the process output observed is also noisy.

5.5.2 Nonlinear Identification Model Residual Analysis

5.5.2.1 Gradient Decent with Momentum

Figure 5.19 and Figure 5.20 show the predicted error or residual plot of the developed neural network model using GDM as the learning algorithm along with the residual histogram. These plots are based on the validation result of the model.

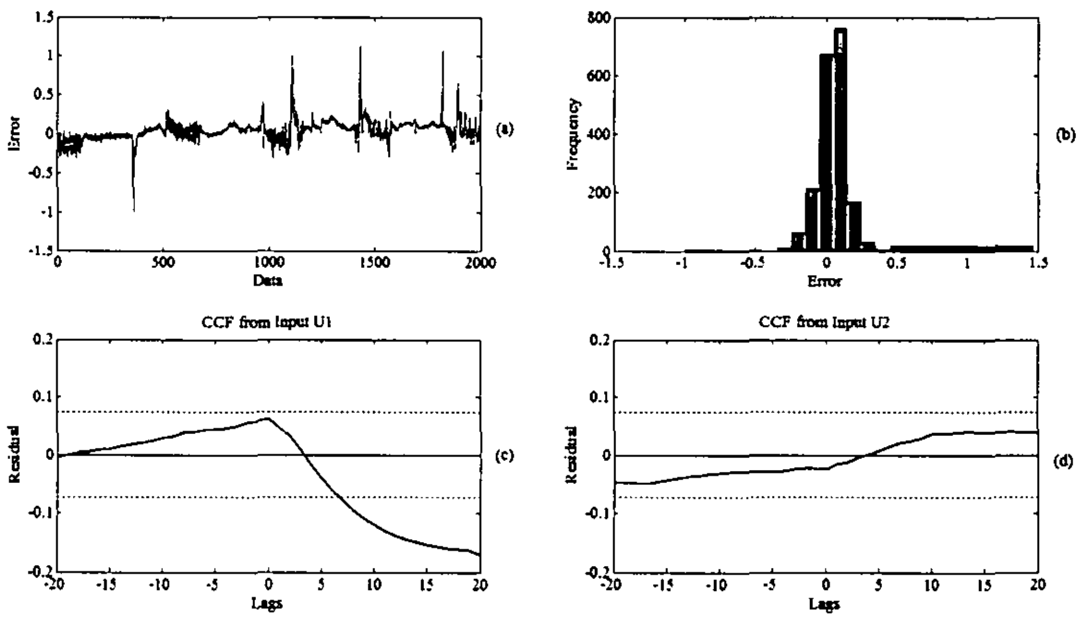


Figure 5.19: Residual Histogram for Top Temperature.

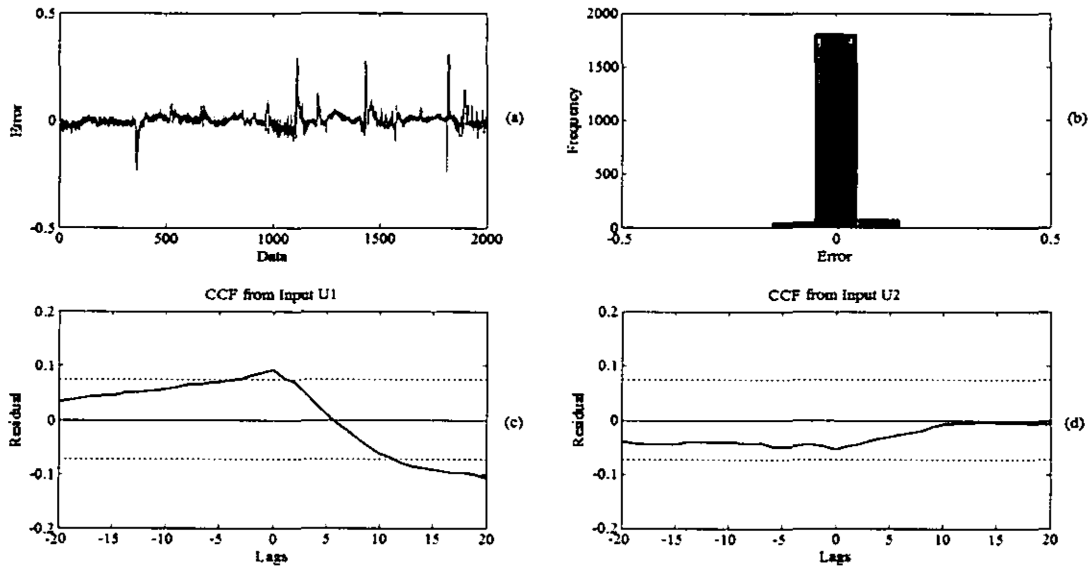


Figure 5.20: Residual Histogram for Bottom Temperature.

Observing Figure 5.19, the model does not show a proper histogram distribution for the validation data. Moreover, the cross-correlation graphs also show that the estimation data used for the development of the model did not completely modelled the top temperature of the process plant since correlation exits between the inputs and the residual. Observing Figure 5.20, the model validation shows a well distribution plot but the cross-correlation of the response shows that the estimation data used did not completely modelled the process plant as the graph has some values out of the 95% confidence interval boundary.

5.5.2.2 Lavernberg Marquardt

Figure 5.21 and Figure 5.22 show the predicted error or residual plot of the developed neural network model using LM as the learning algorithm along with the residual histogram. These plots are based on the validation result of the model.

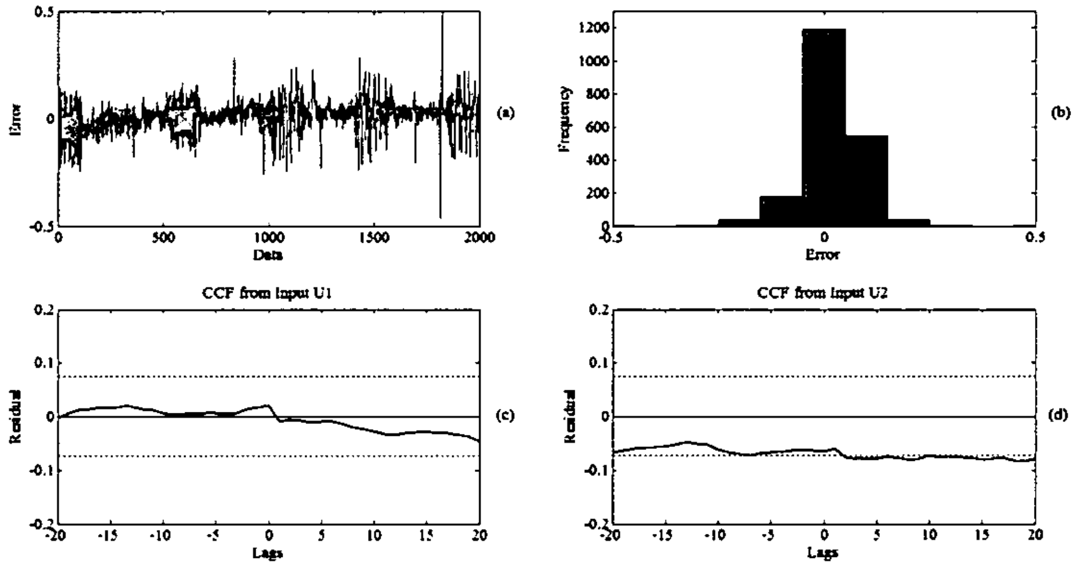


Figure 5.21: Residual Histogram for Top Temperature.

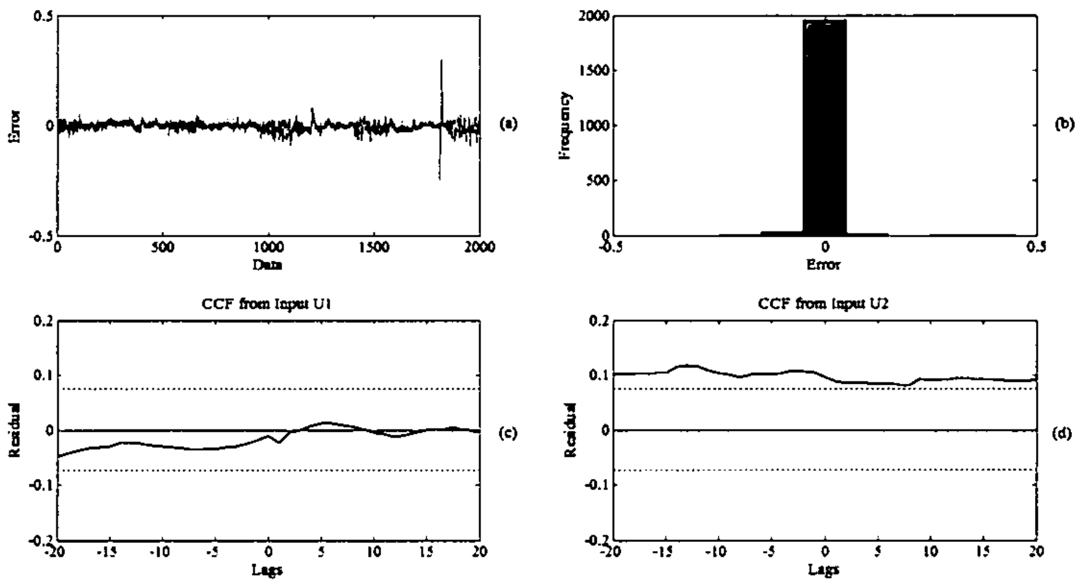


Figure 5.22: Residual Histogram for Bottom Temperature

Observing Figure 5.21, the model shows a proper histogram distribution for the validation data. Moreover, the cross-correlation graphs show that the estimation data used for the development of the model did not completely modelled the top temperature of the process plant since correlation exist between inputs and the residual. Observing Figure 5.22, the model validation shows a well distribution plot of the histogram but the cross-correlation of the response shows that the estimation data used did not completely modelled the process plant as the graph has some values out of the confidence interval line.

5.5.2.3 Nonlinear State Space Model

Figure 5.23 and Figure 5.24 show the predicted error or residual plot of the developed NSS model along with the residual histogram. These plots are based on the validation result of the model.

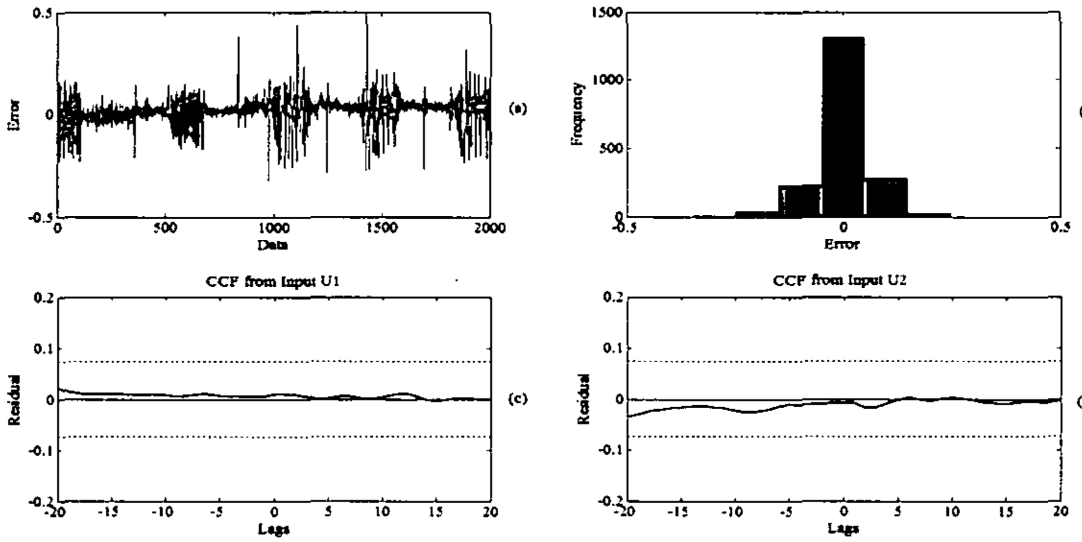


Figure 5.23: Residual Histogram for Top Temperature.

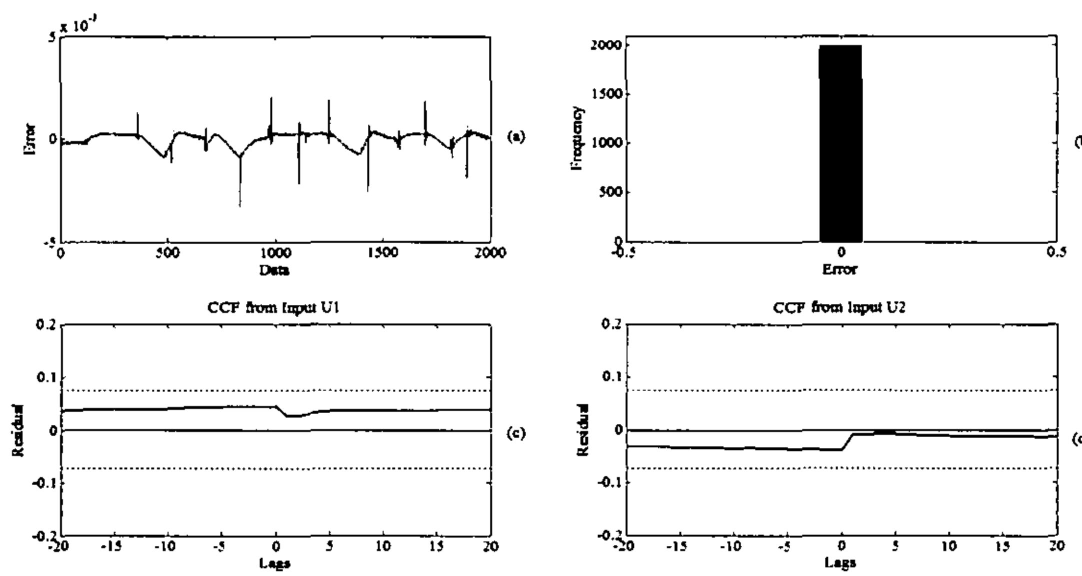


Figure 5.24: Residual Histogram for Bottom Temperature

Observing Figure 5.23, the model shows a well histogram distribution plot for the validation data. Moreover, the cross-correlation graphs also show that the estimation data used for the development of the model completely modelled the top temperature of the process plant since no correlation exist between the inputs and the residual. Observing Figure 5.24, the model validation shows a well distribution plot and also the cross-correlation of the response shows that the estimation data used completely modelled the process plant as the graph lies within the 95% confidence interval boundary.

5.5.2.4 ANFIS Model

Figure 5.25 and Figure 5.26 show the predicted error or residual plot of the developed ANFIS model using along with the residual histogram. These plots are based on the validation result of the model.

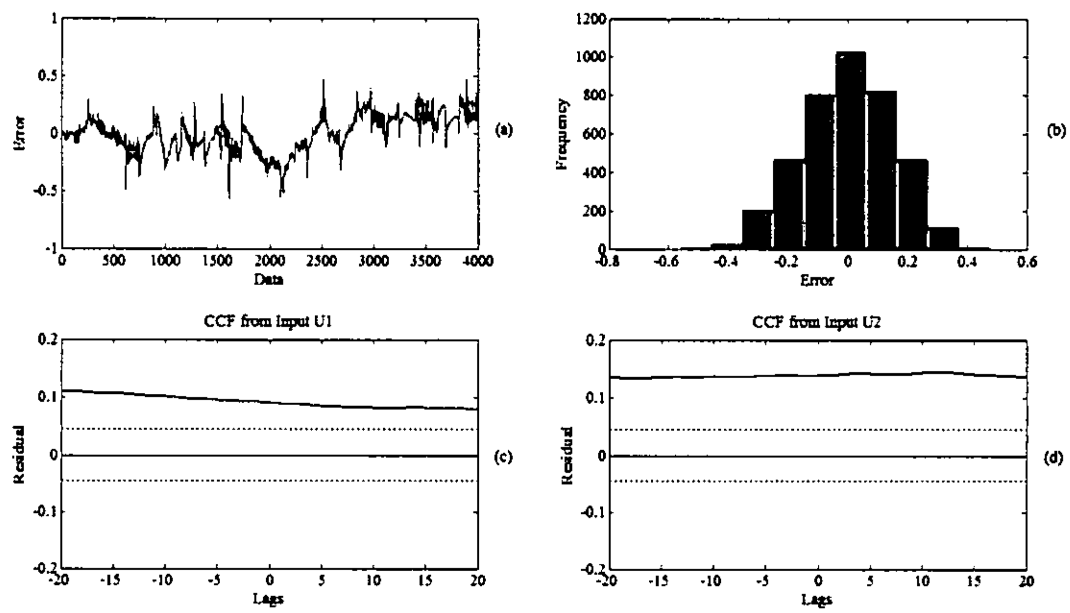


Figure 5.25: Residual Histogram for Top Temperature.

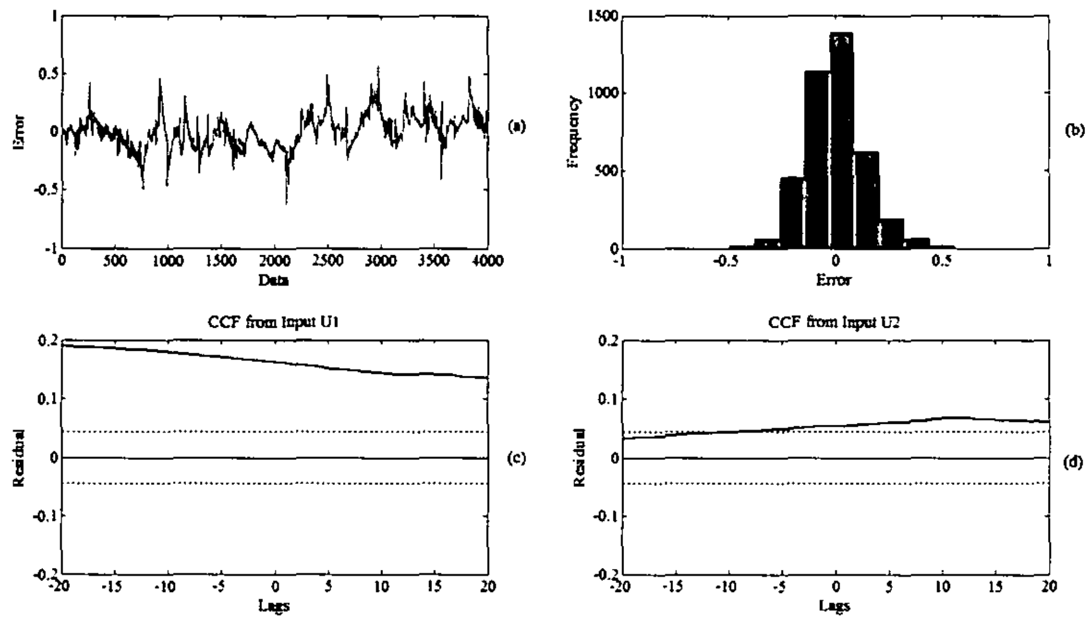


Figure 5.26: Residual Histogram for Bottom Temperature.

The ANFIS model shows a well distributed histogram response for both top and bottom temperature model of the process plant but the cross-correlation performance from input U_1 and input U_2 did not performed well since correlation exist with the residual.

Table 5.5 shows the mean and variance of the prediction errors for all the nonlinear models.

Table 5.4: Nonlinear Models Residual Histogram Performance Measurement

Model	Top Temperature		Bottom Temperature	
	Mean	**Variance**	**Mean**	**Variance**
GDM	0.0408	0.0020	0.0003	0.0002
LM	0.0021	0.0001	-1.917e-004	5.126e-004
NSS	2.3e-005	5.3e-005	4.61e-005	2.01e-005
ANFIS	0.8540	0.8670	0.2210	0.1850

Analysing the residual statistic of the developed nonlinear models from Table 5.5, it shows that Nonlinear State Space model shows a well compatible result to the process output with the minimum mean value of 2.3e-5 and variance of 5.3e-5 for top temperature process and minimum mean 0f 4.61e-5 and 2.01e-5 for bottom temperature process. Neural network models also show a good compatible result. Analysing the residual histogram of these models, the graph is well distributed and centred at the origin. For ANFIS model, the residual is not well distribution at the centre for both top and bottom temperature models.

5.6 Summary

In this chapter, results has been shown and evaluated for different types of nonlinear models. The NN trained by GDM algorithm and LM, NSS model which is the combination of LSS models and NN trained by LM algorithm. All these models show a very significant result compared to the ANFIS. The simulation results shows that the identifier performance for estimating the model output is acceptable to some extend since the models are able to capture the changes in the dynamics of the process. Analyses of the result are given in Table 5.1, Table 5.2 and Table 5.3 and the best model results are highlighted. Further Analysis was performed by observing the prediction error and the residual histogram. Cross correlation testing is also observed from the two inputs U_1 and U_2. NSS model shows the best performance compared to all the other nonlinear models.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

This study has discussed different types of system identification models for modelling a nonlinear process plant. SI mainly focuses on developing empirical models of systems where no prior knowledge is known about it. Injecting an input signal and observing the output response of the system, based on this input-output measurement of the system linear models are developed. By the help of these mathematical models, the dynamic behaviour of the system can be observed and further can be used for control purpose or any other purpose.

Initially, the work started by a brief literature over distillation columns. The concept of distillation column was carefully understood. An experimental work was carried out over the plant. Selection of the input output parameters of the distillation column was chosen based on the purpose of controlling the plant. Distillation Columns are highly nonlinear multivariable systems. Therefore, the Reflux Flow rate of the plant was selected as the first input and the Steam Flow rate as the second input. The outputs of the plant are the distillations process columns Top and Bottom Temperature. Generally, the output of the plant should be considered as the concentration of the top and bottom product from the distillation column, but since as a common practice in the industry is to reduce cost from applying expensive analyzers for the measurement of the product concentration, therefore, the nearest process variable to concentration is the temperature of the top and bottom of the process column.

System Identification technique was applied over the nonlinear plant in order to collect data. Approximately 4000 data were collected from the plant. 2000 data were used for model estimation and another 2000 data for model validation. Modelling approach was performed on multivariable based. Considering the distillation column a MIMO system, the structure was divided into two MISO systems. Linear and nonlinear modelling approach was than performed.

Basic system identification models ARX, ARMAX, Linear State Space model and Continuous time Process Model were evaluated. Simulation results showed that the identifiers performance for estimating the dynamic system output are acceptable to certain extend. All the linear models are able to capture the dynamics of the nonlinear plant. Performance measurements of these linear models are tabulated.

Extending the work towards nonlinear modelling, three different approaches are used. Neural network is one of the most known and suitable approach for nonlinear modelling. Neural Network has the capability of updating its computations which enables itself to estimate the dynamics of a nonlinear system with minimum presence of error. Multilayer Perceptron (MLP) is used as one approach for modelling the nonlinear system. Two different training algorithms are used to train the network. The first algorithm is Gradient Decent with Momentum and the second is Lavenberg Marquardt. Both algorithms had given a very much significant result. Both networks estimated the nonlinear dynamic system.

The second nonlinear model is the Nonlinear State Space (NSS) model. This model is the combination of linear state space and neural network. The training algorithm used for this model is the Lavenberg Marquardt algorithm. This algorithm also showed a very significant result. The model was capable of estimating the nonlinear dynamic system. Third approach used is the Adaptive Neuro Fuzzy Inference System. ANFIS model is sensitive to noisy data. From the response, it is clear that the ANFIS has given an extremely noisy output. This is due to the evaluation of every data point computing against the rules of the ANFIS structure. Since there isn't any fixed method of how to construct the rule for the ANFIS, rule bases of the ANFIS are formed automatically with the number of membership functions.

Further analysis has been performed to verify which model is to be selected. Prediction error or residual analysis has been performed for the models. Observing the residual histogram for the linear models, the ARMAX and State Space model showed much better analysis. Both models show a well distribution plot and the minimum mean and variance value. The cross correlation performance of these two models showed that the estimation data used are completely modelled since no correlation is obtained between the residual and the inputs. Observing the statistics of the nonlinear models, the nonlinear state space model showed a much better distribution of the histogram with the least mean and variance value.

Analysing the residuals of the linear and nonlinear models, it emerges that the selected models ensures satisfactory performance as it is indeed able to correctly identify the dynamics of the distillation column. Therefore the selected models are considered reliable models for describing the dynamic behaviour of the APC plant. Comparing the results between both linear and nonlinear model, nonlinear modelling technique using neural network approach showed a better performance and indicates that the models are well capable to observe and identify the dynamics of the process. Linear models also showed a good performance comparing to the ANFIS modelling technique.

Out of all the linear and nonlinear models discussed in this research work, it can be concluded that the Nonlinear State Space model is considered the best for the APC plant. A significant advantage of this model is that the linear part of the model computes the state and provides the values to the neural network. The result from the neural network indicates that it can successfully identify the input output behaviour of the APC plant. In this approach, two networks are used for identification. If the some of the system states are considered as the output, than only one network is enough to be used. But for the purpose of controlling the plant, the network needs to be simple for computation.

The aim of this work is to identify the process dynamics of the APC plant by means of different types of linear and nonlinear models. By means of experimental measurement, system identification for the system dynamics provides a constructive solution for the prescription of a consistent model. In this case the results showed that the models are able to give satisfactory descriptions of the experimental data.

6.2 Recommendations

The work in this research presented a comparative study of different types of linear and nonlinear models to identify a highly nonlinear process plant. Since the developed models are completely experimental based, therefore these models can easily be used for or can be much more improved for online system identification. The models can be used not only for the implementation over the APC plant but also for the detection and isolation of faults which can occur through the process dynamics. The developed models can be used in advanced control performances such as the Model Predictive Control (MPC) or other advance controllers for the performance improvement of the APC plant. It could be implanted and become part of the control process which is used to provide information of the distillation column process. Hence, the process can be maintained at the desired operating point.

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APPENDIX A: LIST OF PUBLICATIONS

[1] M.A.Baloch, I.Ismail, N.H.M.Hanif and T.M.Baloch,"ANFIS Identifiation Model of an Advanced Process Control Pilot Plant", International Conference on Intelligence and Advanced Systems (ICIAS) 2010, pp.1-5, 15th-17th June 2010, Kuala Lumpur, Malaysia, ISBN: 978-1-4244-6623-8.

APPENDIX B: LINEAR MODEL EQUATIONS

Top Temperature Model Equations

ARX MODEL

1st order ARX model structure: 110

$$A = [0.987];\ B = [-0.366 \quad 0.02978];\ C = [0.25];\ D = [-0.093 \quad 0.00754]$$

2nd order ARX model structure: 220

$$A = \begin{bmatrix} 0 & -0.81 \\ 1 & 1.81 \end{bmatrix};\ B = \begin{bmatrix} 0.7266 & -0.001847 \\ -0.753 & 0.00427 \end{bmatrix};\ C = [0 \quad 0.5];\ D = [-0.45 \quad 0.00114]$$

3rd order ARX model structure: 330

$$A = \begin{bmatrix} 0 & 0 & 0.7732 \\ 0.5 & 0 & -0.984 \\ 0 & 2 & 2.2 \end{bmatrix};\ B = \begin{bmatrix} -1.063 & 0.003 \\ 0.977 & 0.00057 \\ -0.933 & -0.00014 \end{bmatrix};\ C = [0 \quad 0 \quad 1];$$

$$D = [-1.374 \quad 0.003814]$$

ARMAX MODEL

1st order ARMAX model structure: 1110

$$A = [0.9905];\ B = [-0.2638 \quad 0.025];\ C = [0.25];\ D = [-0.0666 \quad 0.006313]$$

2nd order ARMAX model structure: 2220

$$A = \begin{bmatrix} 0 & -0.9561 \\ 1 & 1.956 \end{bmatrix};\ B = \begin{bmatrix} 0.42046 & -0.0217 \\ -0.4214 & 0.0218 \end{bmatrix};\ C = [0 \quad 0.5];$$

$$D = [-0.22 \quad 0.01133]$$

3rd order ARMAX model structure: 3330

$$A = \begin{bmatrix} 0 & 0 & 0.5788 \\ 0.5 & 0 & -1.074 \\ 0 & 2 & 2.57 \end{bmatrix};\ B = \begin{bmatrix} -0.4651 & 0.0002 \\ 0.487 & -0.0015 \\ -0.51 & 0.00321 \end{bmatrix};\ C = [0 \quad 0 \quad 1];$$

$$D = [-0.8035 \quad 0.00033]$$

STATE SPACE MODEL

1st order State Space model

$$A = [0.977];\ B = [-0.2332 \quad 0.0225];\ C = [0.5];\ D = [0 \quad 0]$$

2nd order State Space model

$$A = \begin{bmatrix} 0 & -0.8113 \\ 1 & 1.811 \end{bmatrix};\ B = \begin{bmatrix} 0.789 & -0.043 \\ -0.7903 & 0.0426 \end{bmatrix};\ C = [0 \quad 1];\ D = [0 \quad 0]$$

3rd order State Space model

$$A = \begin{bmatrix} 0 & 0 & 0.4375 \\ 1 & 0 & -1.326 \\ 0 & 1 & 1.88 \end{bmatrix};\ B = \begin{bmatrix} 1.124 & 0.000712 \\ -1.605 & -0.00855 \\ 0.4627 & 0.01122 \end{bmatrix};\ C = [0 \quad 0 \quad 2];\ D = [0 \quad 0]$$

CONTINUOUS TIME PROCESS MODEL

1st order Continuous Time Process model

$$A = \begin{bmatrix} -0.254 & 0 \\ 0 & -0.0133 \end{bmatrix};\ B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.0625 \end{bmatrix};\ C = [-0.3725 \quad 0.1206];\ D = [0 \quad 0]$$

2nd order Continuous Time Process model

$$A = \begin{bmatrix} -0.254 & -0.01 & 0 & 0 \\ 0.00781 & 0 & 0 & 0 \\ 0 & 0 & -0.125 & -0.03437 \\ 0 & 0 & 0.0625 & 0 \end{bmatrix};\ B = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0 \\ 0 & 0.125 \\ 0 & 0 \end{bmatrix};$$

$$C = [0 \quad -0.0002 \quad 0 \quad 0.1373]; \qquad D = [0 \quad 0]$$

3rd order Continuous Time Process model

$$A=\begin{bmatrix} -0.2033 & -0.08364 & -0.04 & 0 & 0 & 0 \\ 0.125 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.03125 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0425 & -0.02545 & -0.00221 \\ 0 & 0 & 0 & 0.01563 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.002 & 0 \end{bmatrix};$$

$$B=\begin{bmatrix} 0.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.125 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad C=\begin{bmatrix}0 & 0 & -0.4235 & 0 & 0 & 0.094\end{bmatrix}; \quad D=\begin{bmatrix}0 & 0\end{bmatrix}$$

Bottom Temperature Model Equations

ARX MODEL

1st order ARX model structure: 110

$$A=\begin{bmatrix}0.9817\end{bmatrix};\ B=\begin{bmatrix}0.01652 & 0.08717\end{bmatrix};\ C=\begin{bmatrix}0.0625\end{bmatrix};\ D=\begin{bmatrix}0.00105 & 0.0056\end{bmatrix}$$

2nd order ARX model structure: 220

$$A=\begin{bmatrix}0 & -0.9101 \\ 1 & 1.91\end{bmatrix};\ B=\begin{bmatrix}0.04445 & -0.05 \\ -0.0451 & 0.0526\end{bmatrix};\ C=\begin{bmatrix}0 & 0.125\end{bmatrix};$$

$$D=\begin{bmatrix}-0.0061 & 0.00674\end{bmatrix}$$

3rd order ARX model structure: 330

$$A=\begin{bmatrix}0 & 0 & 0.5897 \\ 0.5 & 0 & -1.02 \\ 0 & 2 & 2.446\end{bmatrix};\ B=\begin{bmatrix}-0.47 & 0.01162 \\ 0.4 & 0.0174 \\ -0.3 & -0.026\end{bmatrix};\ C=\begin{bmatrix}0 & 0 & 0.25\end{bmatrix};$$

$$D=\begin{bmatrix}-0.1997 & 0.005\end{bmatrix}$$

ARMAX MODEL

1st order ARMAX model structure: 1110

$$A = [0.978];\; B = [-0.00304 \quad 0.097];\; C = [0.0625];\; D = [-0.0002 \quad 0.0062]$$

2nd order ARMAX model structure: 2220

$$A = \begin{bmatrix} 0 & 0.4524 \\ 0.25 & 0.8614 \end{bmatrix};\; B = \begin{bmatrix} -0.06303 & 0.02124 \\ 0.0153 & 0.0504 \end{bmatrix};\; C = [0 \quad 0.125];$$

$$D = [-0.0174 \quad 0.006]$$

3rd order ARMAX model structure: 3330

$$A = \begin{bmatrix} 0 & 0 & 0.5626 \\ 1 & 0 & -1.325 \\ 0 & 1 & 1.745 \end{bmatrix};\; B = \begin{bmatrix} -0.1342 & 0.006046 \\ 0.6368 & -0.00685 \\ -0.5 & 0.0104 \end{bmatrix};\; C = [0 \quad 0 \quad 0.5];$$

$$D = [-0.12 \quad 0.00537]$$

STATE SPACE MODEL

1st order State Space model

$$A = [0.978];\; B = [0.028 \quad 0.05];\; C = [0.125];\; D = [0 \quad 0]$$

2nd order State Space model

$$A = \begin{bmatrix} 0 & -0.526 \\ 1 & 1.515 \end{bmatrix};\; B = \begin{bmatrix} -0.4647 & -0.03 \\ 0.467 & 0.0341 \end{bmatrix};\; C = [0 \quad 0.5];\; D = [0 \quad 0]$$

3rd order State Space model

$$A = \begin{bmatrix} 0 & 0 & 0.67 \\ 0.5 & 0 & -0.887 \\ 0 & 2 & 2.1 \end{bmatrix};\; B = \begin{bmatrix} -0.484 & -0.012 \\ 0.2257 & -0.0033 \\ 0.035 & 0.0255 \end{bmatrix};\; C = [0 \quad 0 \quad 0.5];\; D = [0 \quad 0]$$

CONTINUOUS TIME PROCESS MODEL

1st order Continuous Time Process model

$$A=\begin{bmatrix}-0.254 & 0\\ 0 & -0.0133\end{bmatrix};\ B=\begin{bmatrix}0.5 & 0\\ 0 & 0.0625\end{bmatrix};\ C=\begin{bmatrix}-0.3725 & 0.1206\end{bmatrix};\ D=\begin{bmatrix}0 & 0\end{bmatrix}$$

2nd order Continuous Time Process model

$$A=\begin{bmatrix}-0.254 & -0.01 & 0 & 0\\ 0.00781 & 0 & 0 & 0\\ 0 & 0 & -0.125 & -0.03437\\ 0 & 0 & 0.0625 & 0\end{bmatrix};\ B=\begin{bmatrix}0.0001 & 0\\ 0 & 0\\ 0 & 0.125\\ 0 & 0\end{bmatrix};$$

$$C=\begin{bmatrix}0 & -0.0002 & 0 & 0.1373\end{bmatrix};\qquad D=\begin{bmatrix}0 & 0\end{bmatrix}$$

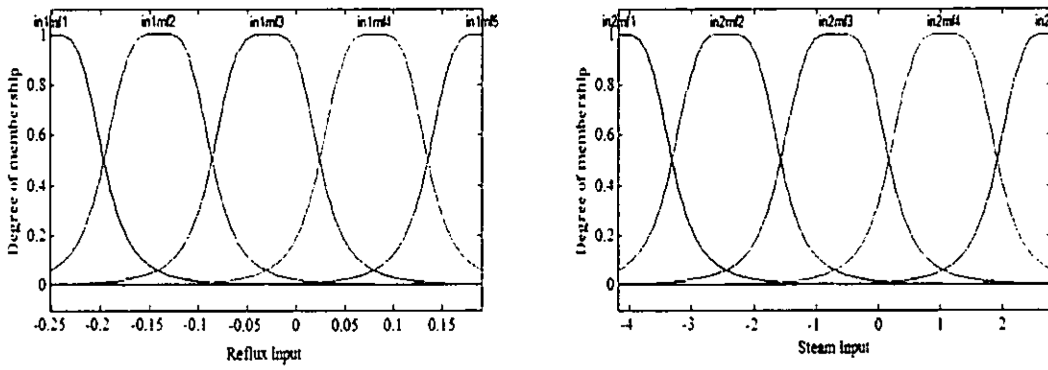
3rd order Continuous Time Process model

$$A=\begin{bmatrix}-2000 & -976 & -1.936 & 0 & 0 & 0\\ 1024 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -976.6 & -976 & -5.896\\ 0 & 0 & 0 & 1024 & 0 & 0\\ 0 & 0 & 0 & 0 & 4 & 0\end{bmatrix};\ B=\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{bmatrix};$$

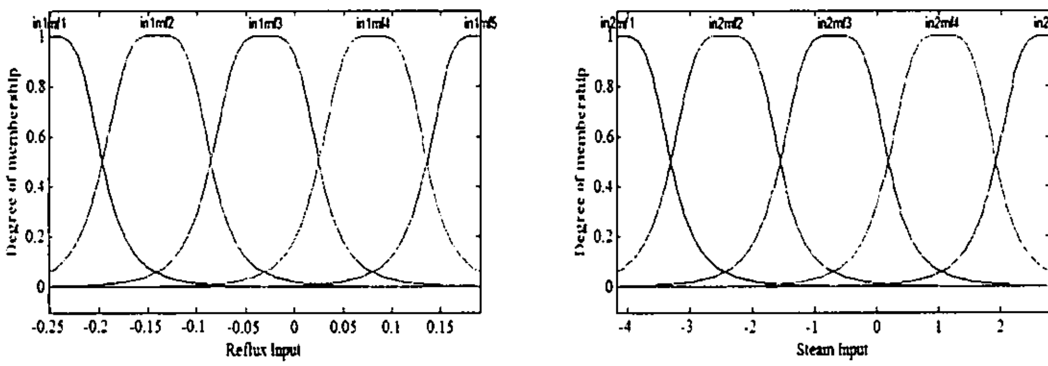
$$C=\begin{bmatrix}0 & 0 & 1.323 & 0 & 0 & 1.65\end{bmatrix};\ D=\begin{bmatrix}0 & 0\end{bmatrix}$$

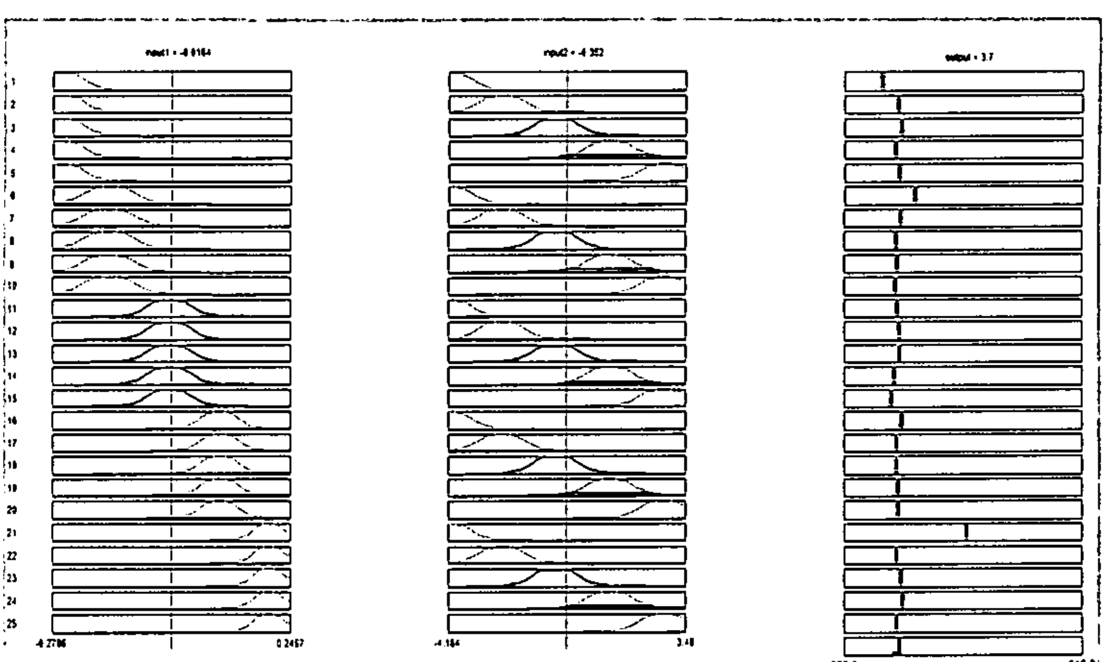
APPENDIX C: ANFIS MEMBERSHIP FUNCTIONS & RULES VIEW

Top Temperature Input MFs



Bottom Temperature Input MFs





APPENDIX D: ANFIS MODEL RULES

Top Temperature Rules

Reflux = U1 ; Reflux Flow rate Input   Steam = U2 ; Steam Input Flow rate Input

Top Temperature = Y1 ; Top Temperature Output   MF = Membership Function

If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF1**) than **(Top Temperature)** is (Y1**MF1**)
If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF2**) than **(Top Temperature)** is (Y1**MF2**)
If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF3**) than **(Top Temperature)** is (Y1**MF3**)
If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF4**) than **(Top Temperature)** is (Y1**MF4**)
If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF5**) than **(Top Temperature)** is (Y1**MF5**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF1**) than **(Top Temperature)** is (Y1**MF6**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF2**) than **(Top Temperature)** is (Y1**MF7**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF3**) than **(Top Temperature)** is (Y1**MF8**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF4**) than **(Top Temperature)** is (Y1**MF9**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF5**) than **(Top Temperature)** is (Y1**MF10**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF1**) than **(Top Temperature)** is (Y1**MF11**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF2**) than **(Top Temperature)** is (Y1**MF12**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF3**) than **(Top Temperature)** is (Y1**MF13**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF4**) than **(Top Temperature)** is (Y1**MF14**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF5**) than **(Top Temperature)** is (Y1**MF15**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF1**) than **(Top Temperature)** is (Y1**MF16**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF2**) than **(Top Temperature)** is (Y1**MF17**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF3**) than **(Top Temperature)** is (Y1**MF18**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF4**) than **(Top Temperature)** is (Y1**MF19**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF5**) than **(Top Temperature)** is (Y1**MF20**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF1**) than **(Top Temperature)** is (Y1**MF21**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF2**) than **(Top Temperature)** is (Y1**MF22**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF3**) than **(Top Temperature)** is (Y1**MF23**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF4**) than **(Top Temperature)** is (Y1**MF24**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF5**) than **(Top Temperature)** is (Y1**MF25**)

Bottom Temperature Rules

Reflux = U1 ; Reflux Flow rate Input Steam = U2 ; Steam Input Flow rate Input

Top Temperature = Y2 ; Bottom Temperature Output MF = Membership Function

If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF1**) than **(Bottom Temperature)** is (Y2**MF1**)
If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF2**) than **(Bottom Temperature)** is (Y2**MF2**)
If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF3**) than **(Bottom Temperature)** is (Y2**MF3**)
If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF4**) than **(Bottom Temperature)** is (Y2**MF4**)
If **(reflux)** is (U1**MF1**) and **(steam)** is (U2**MF5**) than **(Bottom Temperature)** is (Y2**MF5**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF1**) than **(Bottom Temperature)** is (Y2**MF6**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF2**) than **(Bottom Temperature)** is (Y2**MF7**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF3**) than **(Bottom Temperature)** is (Y2**MF8**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF4**) than **(Bottom Temperature)** is (Y2**MF9**)
If **(reflux)** is (U1**MF2**) and **(steam)** is (U2**MF5**) than **(Bottom Temperature)** is (Y2**MF10**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF1**) than **(Bottom Temperature)** is (Y2**MF11**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF2**) than **(Bottom Temperature)** is (Y2**MF12**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF3**) than **(Bottom Temperature)** is (Y2**MF13**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF4**) than **(Bottom Temperature)** is (Y2**MF14**)
If **(reflux)** is (U1**MF3**) and **(steam)** is (U2**MF5**) than **(Bottom Temperature)** is (Y2**MF15**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF1**) than **(Bottom Temperature)** is (Y2**MF16**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF2**) than **(Bottom Temperature)** is (Y2**MF17**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF3**) than **(Bottom Temperature)** is (Y2**MF18**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF4**) than **(Bottom Temperature)** is (Y2**MF19**)
If **(reflux)** is (U1**MF4**) and **(steam)** is (U2**MF5**) than **(Bottom Temperature)** is (Y2**MF20**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF1**) than **(Bottom Temperature)** is (Y2**MF21**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF2**) than **(Bottom Temperature)** is (Y2**MF22**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF3**) than **(Bottom Temperature)** is (Y2**MF23**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF4**) than **(Bottom Temperature)** is (Y2**MF24**)
If **(reflux)** is (U1**MF5**) and **(steam)** is (U2**MF5**) than **(Bottom Temperature)** is (Y2**MF25**)