# REAL TIME RATE OF PENETRATION, PREDICTION AND OPTIMIZATION DURING DRILLING OPERATIONS

By

**Pervez Ahmed Sheriff Nasser**

Dissertation submitted in partial fulfilment of
the requirement for the
MSc. Petroleum Engineering

JULY 2012

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CERTIFICATE OF APPROVAL

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Approved by,

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**Dr.Reza Ettehadi Osgouei**

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TRONOH, PERAK

JULY 2012

CERTIFICATION OF ORIGINALITY

This is to certify that I was responsible for the work submitted in this project, that the original work is my own except as specified in the reference and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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Pervez Ahmed Sheriff Nasser

# ABSTRACT

Real time rate of penetration, prediction, and optimization during drilling operations targets to optimize weight on bit, bit rotation speed and flow rate for achieving maximum drilling rate and minimizing the drilling cost. Drilling optimization process is done by multiple linear regression technique which is a statistical method. From the actual field data which are collected from the data recording systems and modern well monitoring, a model is developed to predict the rate of penetration as a function of available parameters. At each data point, for effective functions general rate of penetration equation is optimized. Computer network is needed to be established to optimize the parameters in the field, which directly from the data source keep the piped data, and also be collecting the new data continuously to be fed. Central computer has a database that will be calculating the developed model parameters by means of multiple regression technique and inform the team at the field. Real-time optimization process is carried out when the field engineer transmits the present drilling parameters to the central computer, the headquarters will determine the new model parameters as well as optimum drilling parameters with recently received information and send back to central computer in real time.

In real-time environment, drilling rate of penetration can be modeled as a function of independent drilling variables such Weight on bit, rotation speed of the string, flow rate, and formation characteristics. The process is formation specific, and in real-time capability to have rate of penetration plotted against the depth with certain parameters can give a new vision to the nature drilling optimization studies. Also in real-time, any relevant difference between the actual rate of penetration trend and predicted rate of penetration can give valuable hint which will be recognized beforehand. From this study optimized independent drilling parameters are determined using non-linear regression analysis. This process is considered to use in future drilling activities on wide range as will be reducing the drilling costs and minimize of probability of encountering problems.

# ACKNOWLEDGEMENT

This satisfaction and euphoria that accompany the successful completion of any task would be incomplete without the mention of the people who made it possible, whose constant guidance and encouragement crowned my effort with success.

I have immense pleasure in expressing my deep sense of gratitude and indebtedness to my supervisor **Dr.Reza Ettehadi Osgouei,** for his wholehearted support and encouragement throughout the course of the work and also for his inspiring guidance and advice.

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Words are not enough to express my feelings towards my **Parents**, as they have always been the constant support of moral encouragement throughout my course. I express my heartfelt gratitude to my friends and to each and every person who was directly or indirectly associated with my project work.

**Pervez Ahmed Sheriff Nasser**

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# ABBREVIATIONS

| | |
|---|---|
| IADC | International Association of Drilling Contractors |
| MLU | Mud logging unit |
| MW | Mud weight |
| Opt | Optimum |
| PDC | Polycrystalline diamond cutter |
| ROP | Rate of penetration |
| RPM | Revolution per minute |
| WITS | Wellsite information transfer specification |
| WITSML | Wellsite information transfer standard markup language |
| WOB | Weight on bit |
| Q | Flow rate |
| TVD | True vertical depth |

## NOMENCLATURES

| Symbol | | Units |
|---|---|---|
| $a_1$ | formation strength parameter | |
| $a_2$ | exponent of the normal compaction trend | |
| $a_3$ | under-compaction exponent | |
| $a_4$ | pressure differential exponent | |
| $a_5$ | bit weight exponent | |
| $a_6$ | rotary speed exponent | |
| $a_7$ | tooth wear exponent | |
| $a_8$ | hydraulic exponent | |
| $B_c$ | Bit cost | $ |
| $C_f$ | Cost per drilled interval | $ |
| $C_r$ | Daily rig rate | $ |
| D | depth of borehole | ft (m) |
| db | diameter of the bit | in (mm) |
| dn | equivalent bit nozzle diameter | in (mm) |
| $f_1$ | formation strength function | |
| $f_2$ | formation normal compaction function | |
| $f_3$ | formation compaction function | |
| $f_4$ | pressure differential of hole bottom function | |
| $f_5$ | bit diameter and weight function | |
| $f_6$ | rotary speed function | |
| $f_7$ | tooth wear function | |
| $f_8$ | hydraulic function | |
| gp | pore pressure gradient of the formation, | ppg (sg) |
| h | bit tooth dullness, fractional tooth height worn away | |
| H1, H2 | constants for tooth geometry of bit types | |
| J1 | composite drilling parameter representing all but tooth wear | |
| J2 | tooth wear composite function used to calculate fractional tooth wear | |

| | | |
|---|---|---|
| N | rotary speed | rpm (-) |
| Q ,q | volumetric flow rate | gpm (l/m) |
| R | rate of penetration | |
| r | residual error in the drilling ROP equation | |
| t | time (usually bit rotating time) | hours |
| tb | bit drilling time | hours |
| tc | drill pipe connection time | hours |
| x | drilling rate of penetration independent parameter | |
| $x_2$ | normal compaction drilling parameter | |
| $x_3$ | under-compaction drilling parameter | |
| $x_4$ | pressure differential drilling parameter | |
| $x_5$ | bit weight drilling parameter | |
| $x_6$ | rotary speed drilling parameter | |
| $x_7$ | tooth wear drilling parameter | |
| $x_8$ | bit hydraulics drilling parameter | |
| W | weight on bit | 1000 lbf (N) |
| We | vertical weight on bit component | 1000 lbf (N) |
| W/d | weight on bit per inch of bit diameter | 1000 lbf(N/m) |
| (W/d)max | bit weight per diameter where teeth fails Instantaneously | 1000lbf/in(N/m) |
| (W/d)t | threshold bit weight at which the bit starts to Drill | 1000 lbf/in(N/m) |

**Greek**

| **Symbol Unit,** | | **Field (SI)** |
|---|---|---|
| α | hole section inclination from vertical degree | |
| ν | drilling fluid kinematic viscosity | cp (Pa s) |
| $\Delta p$ | differential pressure | psi (Pa) |
| ρ | drilling fluid's density | ppg (kg/m3) |
| $\rho c$ | equivalent circulating mud density at the hole | |

| | | |
|---|---|---|
| | bottom | ppg (sg) |
| $\mu$ | apparent viscosity at 10,000 sec-1 | cp (Pa s) |
| $\tau_H$ | formation abrasiveness constant | hours |

**Subscripts**

**Symbol**

| | |
|---|---|
| f | Fraction |
| i | Index number for $i^{th}$ data point |
| j | Index number for $j^{th}$ drilling rate of penetration equation coefficient |
| min | Minimum |
| max | Maximum |
| N | Nozzle |

# CHAPTER 1

# INTRODUCTION

In today's era, drilling industry looks up in reducing drilling cost and to maintain optimized drilling program and also techniques, which can help them to reduce drilling time. In drilling industry, to carry out save and environmental friendly drilling operations with cost effective well construction various methods are used from different disciplines. Most important disciplines are communication and computer technologies, as it contributes in drilling optimization. Computer technologies and programs has propelled that they can solve complex problems in few minutes.

At present, operators always emphasis on two most common and important features on the rig sites that is drilling expenditure and trouble free drilling operations. Directional drilling has also helped in reducing expenditures by drilling multiple wells from one location, as a result well is located close to each other that can help in collecting past data and use them in a productive manner. One of the valuable parameter which can give the evidence on the drilling cost and time is ROP. Drilling variables can affect the rate of penetration (ROP) and because of this reason it's difficult to create a model. Many complex mathematical models were generated to optimize the drilling operations and to minimize cost by selection of correct bit weight and rotary speed. Drilling performance is determined by formation properties which are uncontrollable and properties like hydraulics, weight on bit and rotary speed are controllable parameters.

This study would be performed in real-time basis as the data will be received and processed using high technology communication and computers. The role of a driller is very critical for implementation of optimum drilling parameters.

## 1.1 BACKGROUND

Optimizing of drilling parameters in real-time is a technique which considers past drilling data and predicts drilling trend that recommends optimum drilling parameters. The optimum drilling parameters benefits in saving drilling costs and also helps in reducing the probability of encountering problems. A productive literature survey was carried out and from the previous research work approach for drilling optimization was developed in real-time environment to have effective drilling optimization. The linear drilling rate of penetration model previously introduced by Bourgoyne and Young (1974) that are based on multiple regression analysis has been utilized in real-time to:

- Achieve coefficients of multiple regression specific to formation,
- Have a rate of penetration vs. Depth prediction as a function of certain drilling parameters,
- Determine optimum drilling parameters specific to the formation being drilled,

The following assumptions given by (Miska1988) are considered so that the equations used in this study are functioning properly:

- Bottom hole cleaning is effectively achieved
- The bit and Bottom hole Assembly (BHA) assembly combination in use is one properly selected for the formation being drilled
- The formation interval being drilled is considered to be homogenous
- The rig and auxiliary equipment are efficiently functioning. [1, 21]

## 1.2 PROBLEM STATEMENT

In real-time to determine the rate of penetration, non-linear regression analysis technique is used. Firstly data is collected from drilling rig site and then it’s stored in the central computer for non-linear regression analysis. The process is done in a homogenous formation; weight on bit corrected applied depending on the well inclination and additional bit rotation because of motor incase of deviated wellbore will tend to help the data to give more proper and correct results. For application estimated optimum drilling parameter are piped back to rig site, optimization of drilling parameters when applied will decrease the drilling cost.

Non-Linear regression is utilized to estimate the rate of penetration as a function of independent drilling parameters. From analysis of drilling parameters for specific formation,

- Relation between the drilling parameters and ROP trend will be found out.
- Predetermined ROP performance will be agreed between the Operator and Contractor concurrently with actual drilling activity.
- ROP as function of controllable parameters will be plotted versus the depth and monitored
- If any departure from the predetermined ROP trend, actions will be taken.

## 1.3 OBJECTIVES OF THIS STUDY

The objective of this project is to achieve the following in real time basis

- To find exponents of multiple regression (non-linear regression) analysis that is essential for rate of penetration equation.
- As function of certain drilling parameters, to have rate of penetration vs. depth plot.
- For specific formation, to estimate the optimum drilling parameters for maximizing the rate of penetration (ROP) and minimizing the drilling costs with safe and environmental friendly drilling operations. [1]
- To determine the profit when applied for the next 1000 feet depth.

## 1.4 SCOPE OF THIS STUDY

- Mud Logging Units (MLU) is one of the sources for data to drilling engineer on the rig sites.
- Wellsite Information Transfer Specifications (WITS) is useful mode of communication that is reliable and helps in transfer of wellsite data from one

computer to the other. Near the exploration and production areas, operators and service companies transfer data through batch transfer mode or to online. The latest generation tool is the Wellsite Information Transfer Standard Markup Language (WITSML) which transmits data on a consistent form.

- New generation technologies such as advanced computer systems and innovative computer technologies are utilized to piped the data in real-time, processed them and to interpret advising optimum drilling parameters back in real-time.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 STUDIES ON DRILLING OPTIMIZATION

Real time optimization of drilling parameters during drilling operations aims to optimize weight on bit, bit rotation speed for obtaining maximum drilling rate as well as minimizing the drilling cost by Tuna Eren[1]. He used statistical method namely multiple linear regression technique for drilling optimization, he developed a model using actual field data which will predict the rate of penetration as a functions of available parameters. To optimize the parameters in the field, computer network is developed which will keep the piped data directly from the data source and collecting new data to be piped. Database present in the central computer will be calculating the developed model parameters continuously by means of multiple regression technique and inform the team at the field. The field engineer will transmit the current drilling parameters back to the central computer and the headquarters will determine optimum drilling parameters by received information. This process is real time optimization process, and it could reduce drilling costs and minimize the probability of encountering problems. Optimizing drilling parameters in real-time is to arrive to a methodology that considers past drilling data and predicts drilling trend advising optimum drilling parameters. Quality of data is important for real-time optimization, as to input for multiple regression analysis data should be accurate. During real time operations the data should be sampled and reporting methodology of directional drilling information. He introduced consideration of wellbore inclination in the analysis, use of the normalized weight on bit resulted in higher accuracy of rate of penetration, prediction and consequently more accurate results for the optimum drilling parameters. Optimum weight on bit and bit rotation speed determined in order to achieve minimum cost drilling.

E.Bjornsson and B.Hucik[2] developed a rule based drill bit selection expert software system and rate of penetration prediction algorithm which is been applied in Western Canada. Post well analysis showed that there was increase in ROP and run length experienced in each hole. Bit life increased up to 33% with TCI bits which contributed to the savings in drilling time below AFE of 15days and comparison with actual drilling performance showed close agreement in trend to the predicted ROP. They developed ROP algorithm as a drilling optimization tool and input used were detailed lithological descriptions of anticipated formation, bit type, mud weight, predicted pore pressure and anticipated operating parameters to calculate an accurate meter based on ROP prediction. ROP algorithm improves drilling decisions and can be applied in the planning phase to develop time curves based on expected performance and contrast, compare BHA types based on performance predictions. They created an analytical approach to allow the system to make decisions on bit selection and drillability in both homogenous and non-homogenous drilling applications. They also introduced ROP model using the concept of mechanical specific energy (MSE) which is capable of making reasonable ROP predictions in wide variety of drilling environments.

$$ROP=2{,}538*W/(MSEmin * Dia \wedge 2)$$

System presented by them can not only reduce the risks but shows the impact of investment and gives estimate of drilling operations. They presented case history on how drilling optimization can be achieved and complex variables can be better understood.

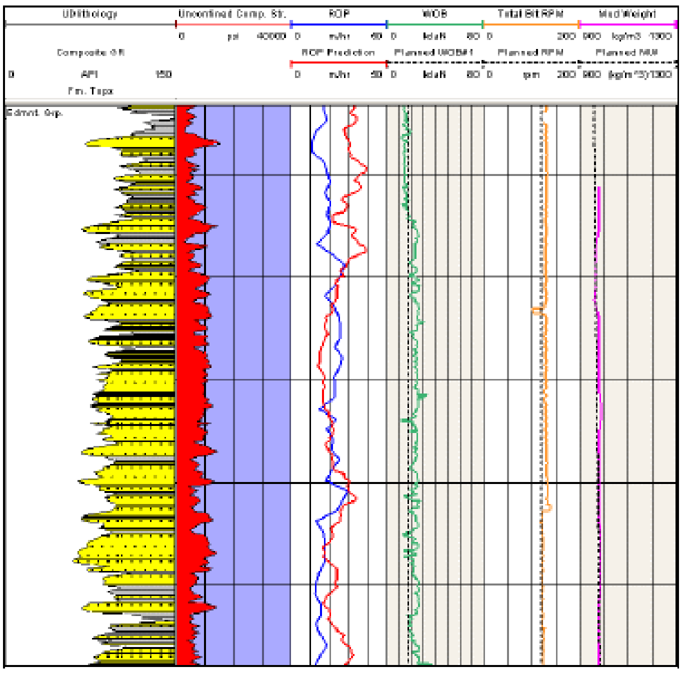


Figure 1: ROP predicted versus the Actual in the U.Cretaceous

Portillio and Jahwari[3] discussed to reach the target in a optimized way using continuous improvement technique and focuses on drilling optimization in the 12 ¼ -in vertical and 8 3/8 -in buildup section, on project in Oman which required drilling optimization for twelve horizontal wells that had many challenge and low rates of penetration. Drilling practices and implementation were made in 12 ¼ and 8 3/8 in vertical sections, 8 3/8 in curve section. They introduced closed-loop vertical drilling system with motors which increased ROP in vertical sections as it optimized the bit and BHA design improved run length, for 8 3/8 in curve section with 15 degree inclination point-the-bit rotary steerable system (RSS) was used and drilling performance enhanced with a power section to drill in a single run as a result 100% success rate in sidetracking. This optimization approach lead to 7.5 drilling days saved in the AFE and improved the drilling performance of horizontal wells to attain 100% delivery in smooth borehole. They gave fit-for-purpose approach to deliver wells in a cost-effective manner. Introduction of oil-base mud reduced borehole instability through reactive shales and due to optimization of bit design remarkable performance of RSS with steer-ability and durability. They served the reference similar horizontal for drilling operations locally and worldwide.

Loo[4] studied about the Sihil field in Mexico which is considered to be one of the most challenging drilling environments. One of the major challenge was directional drilling which include total lost circulation and drilling through rock with a high-strength compressibility, risk was high for borehole instability, stuck of BHA and pack-off conditions as it leads to high cost, and non-productive time. By applying drilling optimization in real time has helped to reduce time and number of drilling incidents. Rotary steerable system was a must in all sections to improve the ROP and avoid differential sticking, his proposal for latest-generation Geo-Pilot XLTM rotary steerable system with ABGTM and a special setup for P4M proved a key factor in selecting them as directional-drilling service provider. Most significant risk for sihil field was with the total-loss of circulation, caused by multiple interconnected failures. His drilling optimization philosophy was using the Remote Operations Center and roadmaps focus on anticipation of these events with advising the customer on decision making to avoid major failure event. His team found out that Torque, drag and torsional vibration was information for total loss circulation. His team objective is to save the customers, quicker return of investment and maintenance of high service delivery standards.

Eltayeb and Heydari[5] did a case study which aimed for optimum well profile and BHA for its application so as to define the best drilling parameters to achieve required build up with highest ROP along with risk free drilling environment . Study was on horizontal drilling in Oman to address the challenges in the 8.5-in buildup section that includes high dogleg requirement with dogleg severity (DLS) of 9 deg/100ft to 11 deg/100 ft, poor wellbore quality, long reactive shale intervals, trajectory control, hole cleaning, differential sticking, and difficulties running the liner. By effort of operator and directional drilling service provider solution was given to overcome this challenge faced from conventional steerable motor assemblies. 12.25-in was drilled vertically with the motor and 8.5-in was drilled using new fully rotating steerable system (RSS) to overcome the challenges with faster penetration rates and still delivering the planned 11 deg/100ft DLS. They showed operator was able drill safely and efficiently in 8.5-in without issue, enabling 3days from the planned AFE. Rate of penetration is also enhanced as no sliding orientations are required and because of hybrid RSS with DLS capabilities up to 17 deg/100 ft which is fit-for-purpose technology. They optimized standard BHA design, torque and drag analysis, and hydraulics design for better results. They proved that RSS design without any modifications to the motor can drill12.25-in vertically in 1 day avoiding any nudge and tool deliver performance with 60% steering setting in 8.5-in. Finally section was finished in less time as less footage was drilled and higher build rate shortened the radius by 50%.

Gorek and Derek[6] described the large-scale trial of Real-Time optimization which was conducted on the Integrated Gas Production System in Sarawak, Malaysia implementing models for real-time monitoring and optimization of wells. They highlighted how Digital Oil Field (DOF) practice enables field-based data turned into information and leads to production optimization. They applied technology to achieve consistent gas supply to meet demand, maximizing revenue on producing assets, striking balance between short-term revenue and long-term value. Benefits were increased condensate production at expected ultimate recovery, whilst maintaining a stable gas supply. The optimization is data-driven and solution has been proved significantly better than prior physical model-based solutions. They brought first successful attempts to optimize, in real-time production of big size and complexity which replaced the traditional, daily and monthly optimization. Closed loop control is possible with remotely operated chokes which allows optimization of entire system to be fully automated. Increasing computing capabilities, increased attention on non-CAPEX ways to provide small improvement to the asset, production system optimization (PSO) falls into this category. They described the more successful and pragmatic approach that features easier to apply and sustain data driven models in a software package that is a standard and the objective was to provide a solution that is fast enough to be applied in real time domain and simple enough to be used. The calculated results are physically realistic, stable, and be skewed to prefer either long term or short tern objective, real time optimization is fast, adequate and user friendly to be deployed into the daily operations environment. They expect gains to be marginal but enough to be pursued.

Rommerveit and bjorkevoll[7] developed a system for drilling automation and simulation and named the project to be Drilltronics. They used drilling data in real time to optimize the drilling process and if suppose sensor fails they had advanced models mirroring the drilling process which will calculate the missing parameter. There system consists of software model with algorithm that reflects wellbore behaviour which is driven in real time by drilling data that are logged at high acquisition rate. They used model prediction and measured data comparison for real time diagnosis. The modules are linked together forming Integrated drilling Simulator and model of ROP is combined. Because of this reason, post well analysis, pre-planning and sensitivity during drilling can be achieved. They showed drillers role will evolve from one of basics drilling mechanics into that of real time drilling supervisor. They also showed during Pressure while drilling (PWD) analyze of annular pressure downhole is invaluable to drilling engineers. They identified that measured and analysis of ECD with PWD tool is critical for success in ERD wells. EKD is based on real time analysis of drilling data obtained from mud-logging system in Rig, stick-slip motion is time which bit is non-rotating and drillpipe section is twisted by the rotary table. They also revealed when the torque of the drillstring reaches certain level, BHA breaks free and speeds up to more than twice with nominal speed before it slows down and again comes to a complete stop. This lead to fatigue problems and stick-slip motion leads to reduction in penetration rate. They analyzed and extract friction data from the torque modules by combining measured data for torque with the torque/drag model. As results of this, increase of torque in ERW and prevent damage to equipment. They showed bit optimization module will determine the optimum rotary speed and WOB, and observed how small changes in these parameters will affect the rate of penetration. They specified that gain factors will tell drillers how increase in WOB and RPM affect rate of penetration. Automatic mode will adjust RPM and WOB automatically based in gain factors, they mentioned system will provide early detection of upcoming unwanted events.

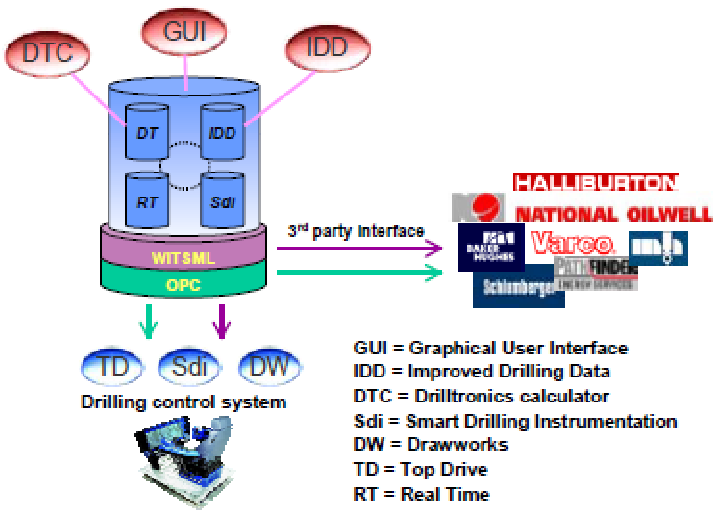


Figure 2: Schematic overview of drilltronics real-time setup.

Cayeux and Dvergsnes[8] developed a drilling control system for real time optimization and automation control which was performed in the North Sea. This system was based on advanced real time process models for calculation of both hydraulic and mechanical forces and it required extensive set of input data which are correct and of high quality. They showed applying automation to the drilling process is complicated and compound challenge, they believed that resolving the generic challenges experienced in industrialization of the described system constitutes an important step towards extended automation of the drilling process. Through applications of model simulation, effects of fluid and pipe movements in the wellbore can be known. They proved extended systems reduces the non-productive time, damaging of well and downhole equipment. Precise description of the drillstring components is required for extended system and necessary required information must be quality controlled and made available at correct time, as it will be helpful in decreasing potentially serious incident and to achieve full potential drilling process. They characterized the drilling automation system by a very short reaction time, and constructed new drilling automated systems to assist drilling operations under more difficult conditions. This depends on adequate dataflow management including both real-time data acquisition systems and accurate description of the wellbore and its components. They also explained 24/7 maintenance and monitoring of drilling parameters is of paramount importance for achieving safe drilling operations. In terms of parameters necessary for performing real time hydraulic, thermal, and mechanical calculations, standards need to adapt depending on additional requirements.

Thomson and Vulgamore[9] showed how a service company uses business intelligence methods to acquire relevant drilling data and import it into knowledge management systems prior to performing data analysis to benchmark performance and identify drilling hazards and opportunities for improvement. They mentioned companies should invest in new technologies to succeed in demanding drilling environment and also implement drilling techniques to overcome drilling challenge. They analyze data to understand challenges combine with design of solutions based on findings is referred business intelligence. MTBF (Mean time before drilling) is the industry standard measure for statistical reliability of the tool. They described statistical analysis is most source of non-productive time. Ng (1989) Breme and Travis (2008) provided the operator to focus on the whole system instead on individual tool. They described each LWD/MWD company collects the drilling data and has database containing information such as distance drilled, circulating hours and drilling hours. They said database lacked information about the loads acting on the tools, drill bit utilized, drilling fluids and well type. They also mentioned about the operator companies having on interest on the system reliability but have interest in system performance and efficiency. Industry strives for HS&E, reliability and efficiency. Common problem in oil industry is to focus on reliability of KPI like MTBF rather than understanding reliability indexes.

Iversen and Dvergsnes[10] conducted field test to make the drilling process more reliable, increase efficiency and improve safety by incorporating with real time calibration. Optimization of operational parameters is done by calibrated dynamic process model, automatically optimization is done by forward model simulation. Results from the test are evaluated by field operator and drilling contractors. They explained automated procedures and test will provide improved control of well conditions. They developed the system to overcome the drilling challenge such as unstable formations, risk of fracture during drilling, hydraulics management and depletion in producing formations. In current drilling process, stress-cage technology using $CaC0_3$ helps in achieving large pressure and to deal challenges. Crew performance is important for operational success which is supported by drillers to maintain process control, and this system will deal with existing challenges through enforcement of process constraints and automatically maintaining operation parameters with safe operation. The test conducted by them was successful with regards to signal transfer, although downhole sensors are poor, measures were taken to overcome this challenge. Challenges related to real-time data are both with regards to accuracy and validity. Measures were taken by them to resolve uncertainty in calculation of operational boundaries and optimization. Results indicated new system may alleviate some of these challenged related to real time optimization. Extended functionalities are assisted to avoid problems in drilling operations.

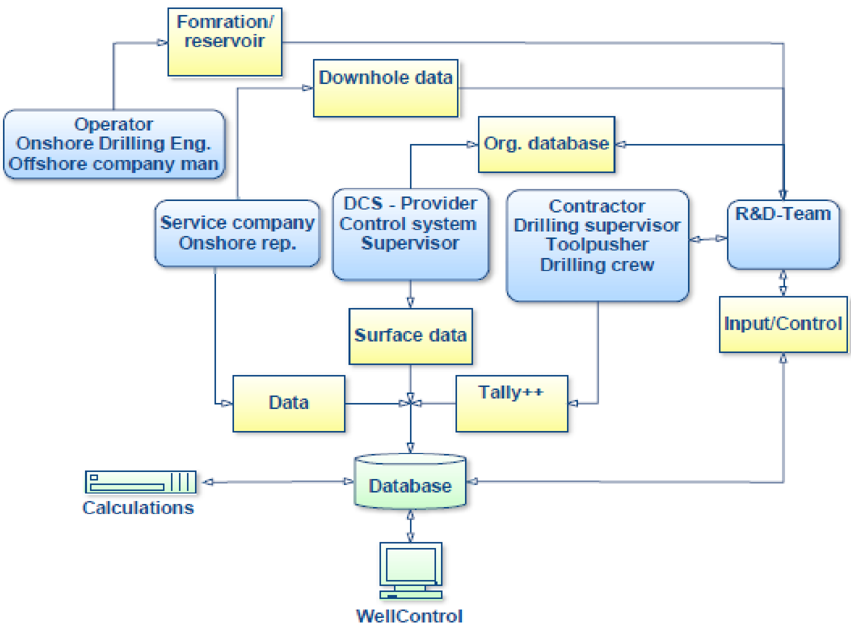


Figure 3: Complex system of data acquisition

Real time tools and technique are used in drilling operations to minimize the cost and time, thus improving drilling performances Rashidi and Hareland[11]. They described the real-time application of a developed model for bit wear analysis. Drilling efficiency is affected by associated drill bit problems; bit wear while the bit is still in the hole can affect the operation. They developed a model based on the difference between rock energy models, mechanical specific energy (MSE) and rock drillability from the rate of penetration models. It has been further modified to develop new software Intelligent Drilling Advisory System (IDA‘s) which can be used for real time wear for both rollercone and PDC bits. This software retrieve the data (drilling), data is quality controlled before bit wear while the bit is still in the hole and will identify necessary tripping which reduce the time and cost. ROP is the one of the key elements affecting drilling costs. Hareland(1994) developed a wear function for both rollercone and PDC bits as function of fractional bit wear. They identified fractional bit wear is function of WOB, rotary speed (RPM) and mechanical specific energy is used to know the drilling efficiency changes and in real time to indicate drilling problems. Both MSE and ROP are used to analyze the drilling performance, ROP can be utilized for pre-planning, post analysis, and well design. They mentioned real time performance affect the overall drilling efficiency and this achieved by bit wear trend against depth. Combination of both ROP and MSE models are be used for real time bit wear estimation. They build software to receive input data from an online server which is sent directly from the rig site. They explained bit wear estimation module can save drilling cost by reducing tripping time.

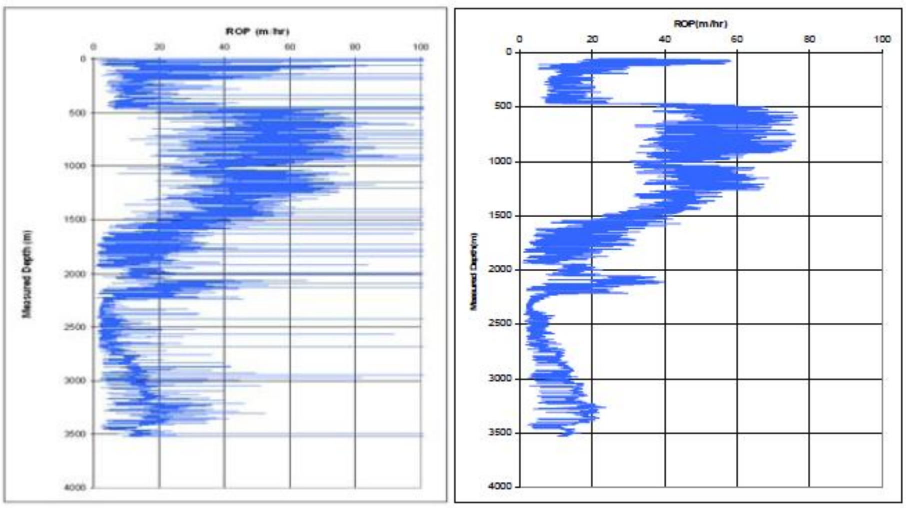


Figure 4: Quality Control on ROP data (Left: Before QC and Right: After QC)

Koederitz and Johnson[12] described about the development and field testing of autonomous drilling system, the optimizer, and provides insight into its future potential. They showed with recent advent of the computer system and controls real time optimization is done to improve the profitability and affect the economics. Development of drilling optimization is described by system architecture, optimization concept and software components. System architecture is divided into advisory, autonomous, semi-autonomous, optimization mimic human research and software system which evaluate and quantify the drilling performance. They explained that optimizer's test process and search method automatically adapt the behaviour of the drilling system including formation changes, current bit, and BHA, also used drilling-specific to know enhance optimization logic will most likely increase its effectiveness. Greatest potential for success operationally are in the areas of fully autonomous automated control and fully electronic control systems. They discussed on requirement of significant resources during optimization, and how autonomous drilling without drilling specific knowledge will have cost-effective application.

Perez[13] did on a case study for crater field to show an optimized well design implemented on it and aimed to reach the final depth in a short time, safely, and by applying lessons learned and best operational practices. He explained design optimization provided by major service company allowed the operator to finish Crater-52 well in 95 days and 155 days ahead of the average duration for previous wells, by this operating cost was reduced. It also facilitates an anticipated production of 285000 barrels of oil. As a result, $3.5 MMUSD was saved in opex, reduced non-productive time from 30 to 5.5% and also raised ROP from 1 to 5 m/hr. It also minimized the lost circulation risks, and improved hydraulics helped increase hole-cleaning effectiveness. During the application of this design, there were no major operational problems, nor were any directional mishaps registered, NPT were reduced by 83% compared to the field average. His work and analysis would not be possible without the contribution made by multidisciplinary team of geoscientists and drilling engineers collaborating to study drilling parameters, lithology, and pore pressure through a stratigraphic column of this field.

Gang Li[14] did a study to investigate correlation between density and sonic logs and estimating formation velocity based on density log. He studied that sonic measurements are always used to estimate mechanical properties of the formations by regression type models. By the example of drilling optimization analysis, considering GR as a lithology indicator, he improved the accuracy of the geomechanical analysis. This leads to significant improvements in mechanical behaviour prediction for drilling and completion, he explained rock strength and rock properties are increasingly important in the optimization of drilling and completion. But to achieve this well coring is required which is time consuming and expensive. He showed finite-mixture models are more flexible than the conventional regression type models, as it improves the estimation of formation strength and mechanical properties compared to Gardner's method and Lindseth's method. This leads to significant improvement in mechanical behaviour when density logs exist.

Saeed Al Reda[15] reviewed bit design and optimization process to develop fit-for-purpose PDC bit that helped to improve the drilling performance by reducing rig days. The main challenge in drilling the 12-in curve section is multiple bit trips and reduced rates of penetration (ROP). To overcome this challenge his company created and evolved a polycrystalline diamond compact (PDC) bit design to be steerable systems (PRSS). His objective of the optimization process was to increase the ROP and drillability of existing PDC bits. As a result, number of bit trips will be minimized with PRSS. He implemented specific bit design algorithms incorporated with new cutter technology and using drilling simulation software to optimize the bit cutting structure design in directional drilling environment to overcome the challenge required in the development of a new PDC technology in conjunction with optimized drilling practices and a reliable drive system. Successful development of the PDC bit in conjunction with PRSS system lead to record runs in the 12-in build section. ROP of 125% app was improved in casing to casing sections, to maximize the gas production 12-in section of the deep gas well is typically drilled directionally depending on the target. He described in most cases the azimuth is planned to remain constant through the section and final inclination varies from 60 to 80 deg depending on well objectives. BUR was 4deg/100ft but some well plans demands upto 5deg/100ft. Rotary steerable system tool were introduced by him to improve hole quality and ROP. The combination of PRSS and the optimized PDC bit has delivered

13 record performances considerably reducing the operator costs and reducing the drilling time. PRSS tool improved the hole quality by reducing the hole tortuosity and exposure time of the open hole which has saved time in conditioning trips and running casing.

Sanchez[16] identified CwD as the most cost effective system. Casing while Drilling (CwD) was the most cost-effective mitigation systems against stuck-pipe, continuous backreaming and unplanned casing setting depths. In Northern Oman, CwD is the new conventional approach for drilling large diameter surface sections instead of the drill-pipe method. Trial phase of CwD, drilling efficiency optimization (DEO) was rarely utilized, as the operator's objective was to prove the CwD concept in deep surface sections with minimum well integrity risk. He however used DEO approach in subsequent CwD implementation campaign to improve the overall performance through the most-top formations. Operator identified a few critical areas of opportunity that led to successful operations. Enabling technologies and resources accelerate the well delivery time by upto 58% and reduce cost. He concluded that CwD deployed with DEO is the best in class value created solution through troublesome top formations. Outcome from the final phase or post well analysis contribute towards enhanced operators drilling sequence. CwD is now considered the choice for drilling surface wellbore sections on deep wells in the North Oman and has allowed the wells to slimmed down, thus reducing number of casing strings.

Iqbal[17] presented the algorithm that will be applied to ordinary drilling rig for calculation and optimization procedures with no additional costs which will increase efficiency. He used real time data to re-evaluate the optimum values of parameters (ROP, WOB) by using simple technique. With use of simple program operator change the parameters optimum values and get maximum rate of penetration. This will reduce the drilling costs, he also elaborates how simple program can analyze the ongoing drilling process and suggests recommendations to acquire better results. It is important to know the optimum weight and speeds during the course of drilling as formation drillability and bit life constants are obtained from drilling data. So he used time share computers and teletype terminal to update information which can be developed with 2 or 3mins. He showed simple vigilance procedure can optimize the ongoing drilling process. When drilling an exploratory well with no proven information of lithology, this procedure can improve the rate of penetration and for development well considering such unpredicted alterations and by making drilling program flexible accordingly, an economic, cost effective and efficient drilling can be practiced.

Business operators remain limited in their ability to use the data to develop better procedure for drilling complex wells, to compare data across various vendors working on the same well and from being able to look at the well data in time not depth. Elley and Strathman[18] described business issues to this solution and benefits achieved. Major drilling companies, real time drilling focus on depth not time and don't lend themselves to comparison between wells. They develop and implement time based drilling solution which was used in parallel with the depths based system provided by drilling contractor. Data is captured in real time, stored in historian and made available in vertical trends through time. Solution leverages the data standard of WITSML to ensure compatibility between vendors. They provided system reliability and real time decision making across disciplines are brought to a common collaboration center where all well data is displayed and available. Operators are currently dependent on various solutions offered by service companies to provide real time drilling data. Stat Oil recognized the best solution and commercially available on the market where usage is shared with many different companies creating business processes which improve drilling decisions in real time, making right decisions on real time data are critical for success. Drilling data is normally focussed on depth, Leverage time series data for optimal operations. After the process, there achievement was reduction of cost, shortening time, better well placement in the reservoir and more efficient operations.

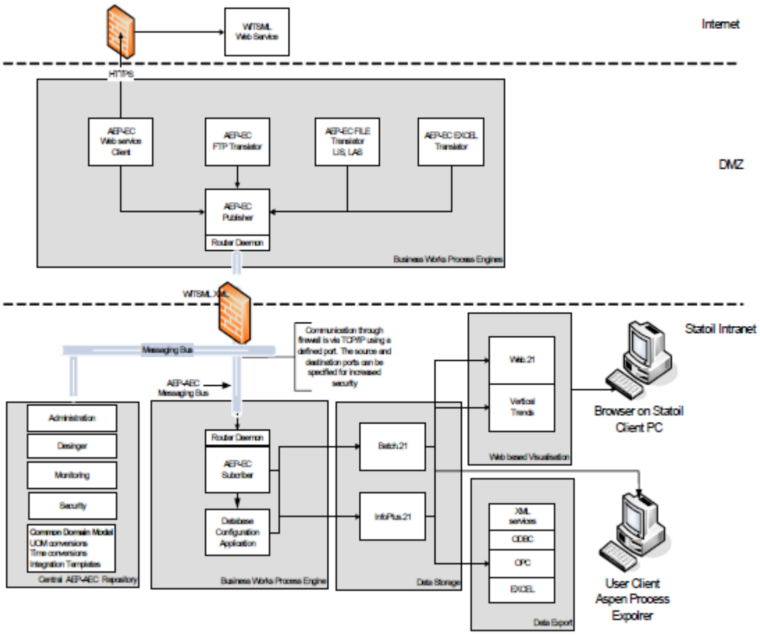


Figure 5: Data flow for time based real-time Drilling solution

Al-Betairi[19] did a case study for optimization of drilling operations in the Arabian Gulf area. He presented a successful application of multiple regression analysis, supported by detailed statistical study to verify the Bourgoyne and Young model. The model estimates the optimum penetration rate (ROP), WOB, rotary speed under the effects of controllable and uncontrollable factors. The drilling model which predicts the effect of various drilling parameters on ROP was used and its reliability and limitations was demonstrated. He accomplished modelling through multi regression analysis of drilling data from the wells in Arabian Gulf area with statistical analysis system package which minimizes the drilling cost with proper selection of optimum WOB and rotary speed. The most comprehensive model was Bourgoyne and Young model which he tried to address the reliability and attached statistical significance to estimated parameters which wasn't included in the model. His objective was to identify correlations between different drilling parameters that cause multicollinearity problem. Eight parameters were included and thoroughly discussed by Bourgoyne and Young. Drilling variables which required multiple regressions was studied by him. Multiple regression was performed with SAS computer program; it was clearly evident from the correlation matrix that multicollinearity would be in expected in regression results and indicates that regression coefficients has become less reliable. In his studies 86% of variable in ROP is explained by the eight drilling parameters and reliability of estimating the coefficients was determined by t-test statistic. Estimates of the model coefficient become sensitive to a particular data set, so he added more points to produce significant changes in some of the coefficients. He described that severity of multicollinearity effect on each drilling parameter is inversely proportional to the influence of that parameter on the ROP. He also mentioned that accuracy of estimating optimum WOB and rotary speed will suffer because of multicollinearity in proposed drilling model.

Dunlop and Isangulov[20] discussed about ROP optimization algorithm. Closed-loop system continually monitors drilling parameters and drilling performance in real time and constantly adjusts weight on bit (WOB) and rotary speed (RPM) maximizes instantaneous rate of penetration (ROP). Field Test conducted by them showed improvement in ROP typically greater than 10% versus non-automated operations, maximum benefits when automation is implemented by means of a closed loop system. The algorithm, the closed-loop system and the performance improvement delivered by this system are currently subject to certain operational constraints. There algorithm did not support mud motors, roller cone bits or under-reamers, but the method could be extended to include them and so optimization is two dimensional in WOB and RPM. PDC bit drilling response is modelled in three operating phases each with linear relationship among WOB, bit torque and Depth of cut per revolution. Depth of cut per revolution= Rate of penetration/RPM. They explained that torque is measured on the surface and bit torque may significantly lower due to drillstring touching the wall. They described that automation will not replace drillers, but should help in dealing with mundane and repetitive tasks. As the algorithm does not currently use any form of offset well data, so setup time is minimal.

## 2.2 LITERATURE SURVEY RESULTS

After an effective literature review, following are the conclusion

- Drilling optimization is performed to decrease the cost.
- Communication and technology have developed during recent years has compared to early era particularly in transfer of data from rig sites. Due to this reason, drilling optimization has become more reliable in real-time.
- WITSML helps in real –time decision making has it is consistent.
- Different algorithm and models were created to adjust WOB, RPM and Flow rate in real-time to increase the ROP and drilling efficiency with environmental and economical friendly drilling operations.
- Automation technique can be used for optimization during drilling operations.
- Directional drilling with Rotary Steerable system can increase the ROP and improve drilling performance.
- Drilling efficiency is affected by drill bit problems; therefore selection of bit is important for optimization.
- Multiple regression technique was used for drilling optimization in real-time basis.
- Multiple regression technique was utilized to linearly model the relation between the dependent rate of penetration and the independent drilling parameters namely WOB, RPM, Formation characteristics and hydraulics.
- For optimum drilling, optimum parameters were found out by relation between the dependent and independent parameters in drilling cost per foot equation.

# CHAPTER 3

# METHODOLOGY

Real-time analysis for rate of penetration optimization, drilling rate of penetration model is utilized which was defined by Bourgoyne and Young. This study targets to optimize weight on bit, rotation speed and flow rate for a specific formation and by using Non-linear Regression analysis in MATLAB. A program is developed to estimate the coefficient of drilling rate of penetration equation and calculate ROP as function of controllable and uncontrollable parameters.

## 3.1 INTRODUCTION

The drilling parameters are collected at MLU and it is piped back to the central computer for analysis. The model used in this study is a function of independent variables such as compaction effect, bit diameter, rotary speed, weight, tooth wear and bit hydraulics. Due to this reason, methodology can function even if mud type, drilling depths are different as there are already considered in the model.

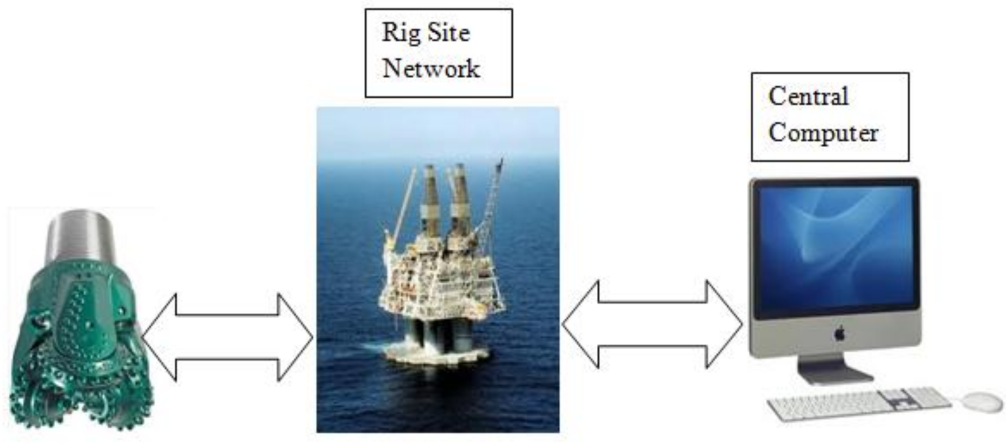


Figure 6: Diagram for drilling optimization theory

Above is the schematic theory application for drilling optimization process. From the rig site network drilling data is collected and the acquired data is piped to the central computer were analysis is done and feed is sent back to the rig site again.

## 3.2 RATE OF PENETRATION MODEL

### 3.2.1 BOURGOYNE AND YOUNG'S METHOD

Bourgoyne and Young's method [21] is statistical synthesis of past drilling parameters which is considered to be essential for drilling optimization. It's regarded as the best method for real-time drilling optimization as linear penetration model is introduced and multiple regression was conducted over this model.

### 3.2.2 BOURGOYNE AND YOUNG'S MODEL

The model [21] proposed by Bourgoyne and Young's is chosen for this study is regarded to be one of the complete mathematical drilling model. It is used in industry for roller-cone bit type and also especially to derive equations which can accomplish ROP estimation using the available input data. Proposed model coefficient is modified depending on the available data, and during non-linear regression analysis the model is modified based on the controllable parameters. Equation 3.1 is general linear rate of penetration equation which is function of controllable and uncontrollable drilling variables.

$$\frac{df}{dt} = e^{\left[a_1 + \sum_{j=2}^{8} a_j x_j\right]} \qquad 3.1$$

In the general ROP equation, as an input to the regression cycle the given normalization constants are modified as a function of data property. Due to utilization of modified constants, perfect and accurate predictions of ROP are given by the coefficients. $a_1$ to $a_8$ are the constants in the above equation and it is found out through non-linear regression analysis using the drilling data. For this study, coefficients are determined on real-time basis which are represented by the effects of formation strength, compaction effect, pressure differential, bit weight, rotary speed, tooth wear and hydraulic exponent. Depending on the formation characteristics, threshold weight and bit diameter value might vary and the fractional tooth height is calculated using formation abrasiveness constants of the field.

The general ROP equation for roller-cone bit types in open form is represented in equation 3.2

$$\frac{df}{dt} = Exp\left( a_1 + a_2(8000 - D) + a_3\left(D^{0.69(g_p - 9)}\right) + a_4 D^{0.69(g_p - \rho_c)} + a_5 Ln\left[\frac{\frac{W}{d_b} - \left(\frac{W}{d_b}\right)_t}{4 - \left(\frac{W}{d_b}\right)_t}\right] + a_6 ln\left(\frac{N}{60}\right) + a_7(-h) + a_8 \frac{\rho q}{350\mu d_n}\right) \quad 3.2$$

Below figure represent general rate of penetration equation for roller-cone bit types with effects of controllable and uncontrollable drilling variables on ROP.

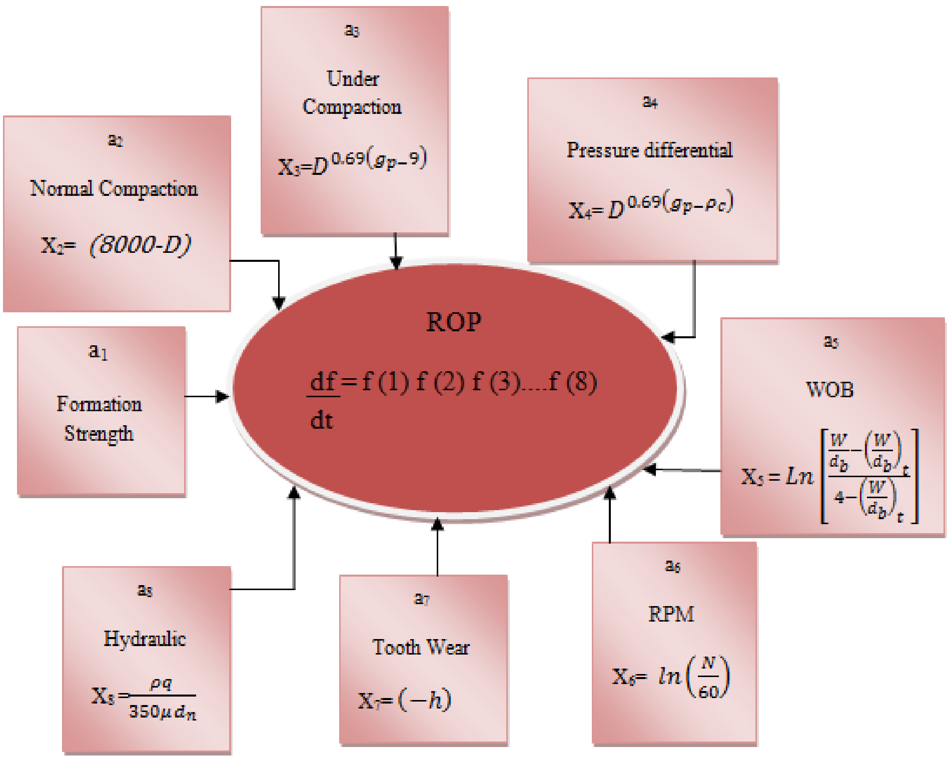


Figure 7: General Rate of Penetration Equation

#### 3.2.2.1 FORMATION STRENGTH FUNCTION

The effect of formation strength coefficient is represented by $a_1$. If the value of this constant is less, then the penetration rate will also be less. Coefficient includes the effect of parameters such as drilled cuttings, drilling fluid, solid content, efficiency of the rig equipment, crew experience and service contractor's efficiency.

Formation strength effects is given by equation 3.3 as described

$$f_1 = e^{a_1} \qquad 3.3$$

$f_1$: Drillability of the formation of interest

#### 3.2.2.2 FORMATION COMPACTION FUNCTION

Formation compaction has two functions over rate of penetration

- Primary function that is effect of normal compaction defined by $a_2$. This effect deals with exponential decrease in penetration rate with increasing depth given in the equation 3.4, other way is this function also takes into the consideration of increase in rock strength with depth due to normal compaction.

$$f_2 = e^{a_2 x_2} = e^{a_2(8000-D)} \qquad 3.4$$

- Additional function of formation compaction over the rate of penetration is the effect of under compaction in abnormally pressured formations defined by $a_3$. The other way is in over-pressured formations, rate of penetration will show increase behavior that is exponential increase in penetration rate with increasing pore pressure gradient equation 3.5.

$$f_3 = e^{a_3 x_3} = e^{a_3\left(D^{0.69}(g_p-9)\right)} \qquad 3.5$$

### 3.2.2.3 PRESSURE DIFFERENTIAL OF HOLE BOTTOM FUNCTION

Pressure differential function is defined by coefficient $a_4$. Penetration rate decreases with decreasing pressure difference, and function will be equal to 1 when pressure differential between the hole and formation is zero as shown in equation 3.6.

$$f_4 = e^{a_4 x_4} = e^{a_4} D^{0.69(g_p - \rho_c)} \qquad 3.6$$

### 3.2.2.4 BIT DIAMETER AND WEIGHT FUNCTION

Weight and bit diameter function is defined by coefficient $a_5$. It has a direct effect on the penetration rate shown in equation 3.7; $\left(\frac{W}{d_b}\right)_t$ threshold bit weight has a value ranging from 0.6 to 2.0. For this study, magnitude of this term is found out depending on the characteristics of the formation. Threshold force is defined as the force at which fracture begins beneath the tooth.

$$f_5 = e^{a_5 x_5} = e^{a_5 Ln\left[\frac{\frac{W}{d_b} - \left(\frac{W}{d_b}\right)_t}{4 - \left(\frac{W}{d_b}\right)_t}\right]} \qquad 3.7$$

Modification

- Weight on bit modification to vertical component was performed. For deviated wells, well trajectories vary with inclinations, so the applied weight can be corrected keeping in mind bottom hole assembly section transferring weight to the bit remain in a constant inclination.
- Applied weight on bit is corrected to its vertical component $W_e$ as described in equation 3.8

$$W_e = \frac{W}{cos \propto} \qquad 3.8$$

#### 3.2.2.5 ROTARY SPEED FUNCTION

Rotary speed function is represented by coefficient $a_6$. The rotary speed also has a direct effect on penetration rate similar to the weight on bit as given in equation 3.9

$$f_6 = e^{a_6 x_6} = e^{a_6 ln\left(\frac{N}{60}\right)} \quad 3.9$$

Modification

- Additional bit rotation by the motor should be applied. Motor efficiency should be taken into account and additional bit rotation count calculation should be done on regular basis.
- Calculation of additional rotation of drilling motor in drilling industry need references which are provided by service providers that contains technical details. [22]

#### 3.2.2.6 TOOTH WEAR FUNCTION

Tooth wear function is defined by coefficient $a_7$. Fractional tooth height is an important phenomenon in calculation of tooth wear function, high the tooth wear less the penetration rate as represented in equation 3.10.

$$f_7 = e^{a_7 x_7} = e^{a_7(-h)} \quad 3.10$$

Bit record plays a vital role in calculation of tooth height; from the similar bit type which has been used in same formation tooth height can be estimated. With absence of this information bit record matching to well that has the similar formation and drilling characteristics are considered to make the model applicable.

#### 3.2.2.7 HYDRAULIC FUNCTION

Hydraulic function is represented by coefficient $a_8$. It represents the effect of bit hydraulics and Reynolds number is most suitable which is given in equation 3.11

$$f_8 = e^{a_8 x_8} = e^{a_8 \frac{\rho q}{350 \mu d_n}} \quad 3.11$$

## 3.3 OPTIMIZATION OF DRILLING PARAMETERS

Optimization [23] is selection of best variable from a group of data set. Optimization is solving problems in which minimizing or maximizing a function systemically is necessary. In real world, optimization is an important problem and differentiation plays a key role in solving such problems. Optimization in this study deals with selection of drilling parameters as a function of WOB, RPM and flow rate to decrease drilling expenditure.

Drilling process is said to be optimized when cost of drilling activity is reduced. In drilling industry, for accomplishment of minimizing problems and minimized cost per foot drilled, optimization is done by adjusting the magnitude of two or more independent parameters. Drilling expenditure or cost can be decreased to great extent by employing optimized combination of drilling parameters.

In real-time optimization study, optimization is carried out keeping in mind that the rig equipment, bottom hole assembly (BHA) and the bit to be used is the best optimum selected. The bit should be prevented from damages when run into the hole for minimum cost drilling.

### 3.3.1 WEIGHT ON BIT AND ROTARY SPEED OPTIMIZATION

The mathematical model as a function of drilling parameters like WOB/$d_b$, RPM for penetration rate is shown in equation 3.12

$$\frac{df}{dt} = f\left(\frac{W}{d_b}, N, h\right) \quad 3.12$$

For optimization process, tooth wear is also taken into consideration in the equation 3.12 and vital parameters like WOB, RPM can be manipulated. [1]

Optimized WOB and RPM should lie within the operation as explained in equation 3.13 and 3.14, hole cleaning should be adequate

$$WOB_{min} \leq WOB_{Opt} \leq WOB_{max} \qquad 3.13$$

$$N_{min} \leq N_{Opt} \leq N_{max} \qquad 3.14$$

Drilling cost per foot [24] equation 3.15 is performance of a bit which is judged on the following criteria

- How much footage is drilled(ft)
- How fast it drilled (ROP)
- How much it cost to run (the capital cost of the bit plus the operating costs of running it in hole) per foot of hole drilled

Since the aim of bit selection is to achieve the lowest cost per foot of hole drilled the best method of assessing the bits performance is the last of the above. This method is applied by calculating the cost per foot ratio, using the following equation

$$C_f = \frac{C_b + C_r(t_t + t_c + t_b)}{\Delta F} \qquad 3.15$$

Where

$C_f$ : Cost per drilled interval,

$C_b$ : Bit cost,

$C_r$ : Daily rig rate,

$t_t$ : Round trip time,

$t_c$ : Connection time,

$t_b$ : Bit drilling time,

$\Delta F$ : Footage drilled with the bit in use.

The above equation is solved by necessary calculus the optimum equation for weight on bit for each diameter of the bit size is given by equation 3.16 [1]

$$\left(\frac{W}{d_b}\right)_{Opt} = \frac{a_5\ H_1\left(\frac{W}{d_b}\right)_{max} + a_6\left(\frac{W}{d_b}\right)_t}{a_5\ H_1 + a_6} \qquad 3.16$$

In similar manner the optimum bit speed can be expressed in the following term [1] as given in the equation 3.17

$$(N)_{opt} = 60\left[\frac{\tau_H}{t_b}\ \frac{\left(\frac{W}{d_b}\right)_{max} - \left(\frac{W}{d_b}\right)_{Opt}}{\left(\frac{W}{d_b}\right)_{max} - 4}\right] \qquad 3.17$$

### 3.3.2 FLOW RATE OPTIMIZATION

Significant increase in penetration rate can be achieved through the proper choice of bit nozzles, the penetration rate increase is felt due to improved cleaning action in the hole bottom. Accurate mathematical relations must be developed that defines the effect of the level of hydraulics on penetration rate, operational costs, bit wear, potential hole problems like hole washout and drilling fluid carrying capacity for true optimization of jet bit hydraulics.

The most commonly used hydraulic parameters are

- Bit nozzle velocity

$$V_n = \sqrt{\Delta p_b} \qquad 3.18$$

Nozzle velocity is a maximum when the pressure drop available at the bit is a maximum. The pressure drop available at the bit is maximum, when the pump pressure is a maximum and the frictional pressure loss in the drillstring and annulus is a maximum. The frictional pressure loss is a minimum when the flow rate is a minimum.

- Bit hydraulic horsepower

Speer published a paper reasoning that penetration rate would increase with hydraulic horsepower until the cuttings were removed as fast as they were generated. After his publication, several authors pointed out that due to frictional pressure loss in the drillstring and annulus, the hydraulic power developed at the bottom of the hole was different from the hydraulic power developed by the pump. They concluded that the bit horsepower rather than pump horsepower was the important parameter.

Since the bit hydraulic horsepower $P_{Hb}$ is given by equation 3.19

$$P_{Hb} = \frac{\Delta p_b q}{1{,}714} = \frac{\Delta p_p q - cq^{m+1}}{1{,}714} \qquad 3.19$$

To determine the flow rate at which the bit horsepower is maximum gives

$$\frac{dP_{Hb}}{dq} = \frac{\Delta p_{p-}(m+1)cq^m}{1{,}714} = 0 \qquad 3.20$$

Bit hydraulic horsepower is maximum when the parasitic pressure loss is (1/m+1) times the pump pressure.

- Jet impact force

Eckel working with small bits in the laboratory, found that the penetration rate could be corrected related to a bit Reynolds number group so that

$$\frac{dD}{dt} \propto \left(\frac{\rho V_n d_n}{\mu_a}\right)^{a_8} \qquad 3.21$$

$\frac{dD}{dt}$ : Penetration rate,

$\rho$ : Fluid density,

$V_n$ : Nozzle velocity,

$d_n$ : Nozzle diameter,

$\mu_a$ : Apparent viscosity,

$a_8$ : Constant.

It can be shown that when nozzle sizes are selected so that jet impact force is a maximum, the Reynolds number group defined by Eckel is also a maximum.

The derivation of the proper conditions for maximum jet impact was first published by Kendall and Goins which was given in equation 3.22

$$\Delta p_d = \frac{2\Delta p_p}{(m+2)} \qquad 3.22$$

The jet impact force is maximum when the parasitic pressure loss is (2/(m+2)) times the pump pressure.[25]

Typical optimization criteria are the maximization of the hydraulic energy delivered through the bit nozzles or the maximization of the jet impact force. Hydraulic horsepower spent across the bit nozzles is equal to the product $qP_2$ and jet impact force is obtained by multiplying the flow rate with $\sqrt{P_2}$. [26]

Table 1: Overview of Optimization criteria

| Performance Index | Equation | Criterion | Fraction parasitic pressure loss | Flow rate |
|---|---|---|---|---|
| 1 | $qP_2$ | Max. HP | 1/(m+1) | $P_1$/C(m+1) |
| 2 | $q\sqrt{P_2}$ | Max. Jet impact | 2/(m+2) | $2P_1$/C(m+2) |
| 3 | $q^{\frac{3}{2}}\sqrt{P_2}$ | New A | 3/(m+3) | $3P_1$/C(m+3) |
| 4 | $q^2\sqrt{P_2}$ | New B | 4/(m+4) | $4P_1$/C(m+4) |
| 5 | $q^{\frac{5}{2}}\sqrt{P_2}$ | New C | 5/(m+5) | $5P_1$/C(m+5) |

Where

$P_2$ : Pressure drop across the bit nozzle,

$P_1$ : Pump pressure,

q : Flow rate,

m : Flow rate exponent,

C : Proportionality constant.

Optimization of drilling hydraulics requires calculation of frictional pressure losses in the system. Determining the back pressure and the corresponding gas/liquid injection rates for effective cuttings transport, while achieving maximum drilling rate, are some of the major questions that need to be addressed.[27]

Drilling hydraulics optimization variables includes

- Bit nozzle size,
- Drilling fluid flow rate,
- Drilling rate.

Drilling hydraulics optimization constraints includes

- Drilling cost,
- Wellbore geometry,
- Pump capacity (max pump pressure and flow rate capacity),
- Performance characteristics.

$q_{max}$ is function of the pump horsepower rating (HP) and maximum allowable pump pressure,

$$q_{max} = \frac{1714\ H_p E}{(P_p)_{max}}$$

Constant parasitic pressure loss i.e. intermediate depth when the flow rate is gradually reduced

$$\Delta P_d = CQ^m \qquad 3.23$$

Where $\Delta P_d$ is parasitic pressure drop, C is a function of mud flow properties, hole geometry, pipe geometry etc and increase linearly with depth, m is the flow exponent.

Parasitic pressure loss represented in equation 3.24

$$\Delta P_d = P_p - \Delta P_b \qquad 3.24$$

Where $P_p$ : Pump pressure,

$\Delta P_b$ : Bit pressure drop.

For incompressible drilling fluid bit pressure drop as shown in equation 3.25 [28]

$$\Delta P_b = \frac{K_1 \rho q^2}{Cd^2 At^2} \qquad 3.25$$

$K_1$=8.3311*$10^{-5}$

$Cd$ : Dimensionless discharge coefficient which is equal 0.95.

Calculation of optimum value of parasitic pressure is given by equations 3.26 and 3.27,

$$(\Delta P_d)_{opt} = \frac{2P_p}{m+2} \qquad 3.26$$

Max jet impact force,

$$(\Delta P_d)_{opt} = \frac{2P_p}{m+2} \qquad 3.27$$

Max hydraulic horsepower,

Optimum value of total flow area is calculated by equation 3.28,

$$(A_t)_{opt} = \sqrt{\frac{K\rho (Q_{opt})^2}{(Cd)^2 (\Delta P_d)_{opt}}} \qquad 3.28$$

$A_t$= Total cross section area of the nozzles.

Optimum Flow rate equations are given by

$$(Q)_{opt} = \left(\frac{P_{pump}}{C(m+1)}\right)^{\frac{1}{m}} \quad 3.30$$

$$(Q)_{opt} = \left(\frac{2P_{pump}}{C(m+2)}\right)^{\frac{1}{m}} \quad 3.3$$

#### 3.3.2.1 STEPS FOR FLOW RATE OPTIMIZATION

- Calculate $\Delta P_b$ using the formula given,

  $\Delta P_b = \frac{K_1 \rho q^2}{Cd^2 At^2}$ ,

- Find $\Delta P_d$,

  $\Delta P_d = P_p - \Delta P_b$

- Calculate from the given formula,

  $$m = \left(\frac{\log \frac{\Delta P d_2}{\Delta P d_1}}{\log \frac{q2}{q1}}\right) \quad 3.32$$

- Determine the value of C from the formula given,

  $\Delta P_d = CQ^m$

- Optimization of flow rate using the criteria,

  $P_1$/C (m+1),

  $3P_1$/C (m+3),

  $5P_1$/C (m+5).

Max Jet impact force was not used as it can be used in shallow wells, after the calculation of flow rate optimization is done using the given criteria, these criteria will be averaged to get the optimum flow rate values at different depths.

## 3.4 DATA ACQUISITION

Data collection or processing is phenomenon of transferring the data to a condition that can be process able, after that non-linear (Multiple) regression technique is use to find the coefficient to estimate the drilling rate of penetration. The drilling rate of penetration is determined with certain level of accuracy.

### 3.4.1 DESCRIPTION OF DATA

The available drilling data for this study was from field located in Malaysia, Lithological of the formations is mainly dominated by shale and sandstone.

Given data

- Well,
- Hole section,
- Lithology,
- Depth,
- WOB,
- Rotary speed,
- Stand pipe pressure,
- Mud weight,
- Flow rate,
- ROP,
- TVD,
- Pore pressure gradient.

### 3.4.2 DATA PIPING IN REAL-TIME

Optimization process requires data piping which is regarded as the one of the critical and very important step. Central computer is where the optimization takes place; therefore data should be piped conveniently.

Following is the sequence as how optimization cycle should work,

- From the rig site, data should be piped in real-time environment to a central computer,
- Optimized drilling parameters are estimated through central computer which process the data with the information already existing in its database,
- For application, optimized drilling parameters should be piped back again to the drilling rig site.

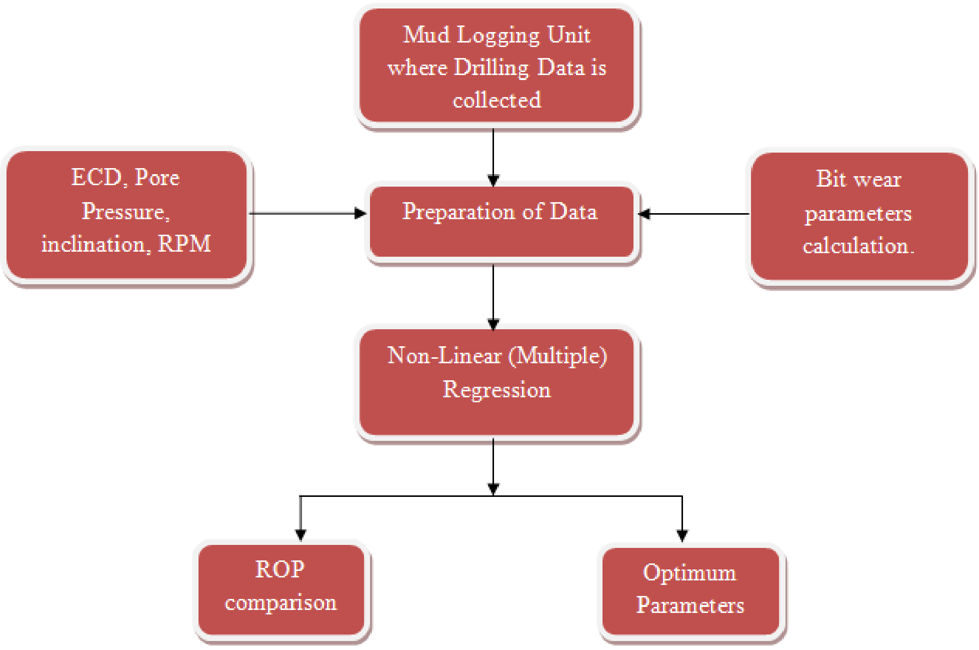


Figure 8: Process flow chart

Figure 8 is the application of process flow. In this study, at the mud logging units drilling parameters data will be collected for optimization process application. On the basis of general rate of penetration equation, data was prepared accordingly for suitable means of non-linear regression application. Wellbore inclination, ECD, Pore pressure, mud rheological properties and additional rotation of the bit due to motor for each data point was determined and added into the database. After the dataset which are needed are stored, non-linear (multiple) regression was executed and regression coefficients was determined. The determined coefficients are used in calculating the predicted rate of penetration and determine the optimum parameters. [1]

## 3.5 NON-LINEAR (MULTIPLE) REGRESSION TECHNIQUE

Non-linear regression is very important for the success of drilling optimization. As there eight main equations, each time the data is regressed, eight unknown coefficient are solved [1]. Non-linear regression process is carried out specially to predict the ROP as a function of other independent variable and also to construct the drilling rate of penetration formula.

Multiple regression model contains more than one regressor variable [29]. Characterization of an observation unit by several variables is known as Multivaraite data analysis [30]. Multivariate analysis methods get affected for the changes in the magnitude of the several properties simultaneously [1]. Non-linear regression will have possible interactions within combinations of variables as well as variable themselves.

To determine the effect of drilling parameters on rate of penetration, the drilling rate of penetration model is given equation 3.1

$$\frac{df}{dt} = e^{\left[a_1 + \sum_{j=2}^{8} a_j x_j\right]} \qquad 3.1$$

From the equation, dependent variable $\frac{df}{dt}$ is exponentially equal to a constant term plus the sum of series of independent variables. Non-linear regression is utilized to find the ‘a’ regression constants.

Regression coefficient and constants are estimated using least squares fit in MATLAB; the general equation is solved by application of least square fit which has capability of constructing a matrix. The matrix will help in determining unknown ‘a’ coefficients which can be inserted in the drilling rate of penetration general equation for ROP prediction and ROP optimization.

MATLAB code using least square fit is written in appendix C. The code is used to solve the constants $a_1$, $a_2$, $a_3$……$a_8$ in the general equation in accurate, fast and error less approach.

Figure 9 is representation of non-linear regression process cycle. Firstly for each data point "X" parameters are calculated. The second step is the collection of calculated "X" parameter for creating a matrix. For this study, 8*8 matrixes are being created. After the calculation of matrix, same are solved and the 'a' coefficients could be estimated. As each time the data point is added, the process is repeated until the final data point. Final data point is regarded to be most accurate.

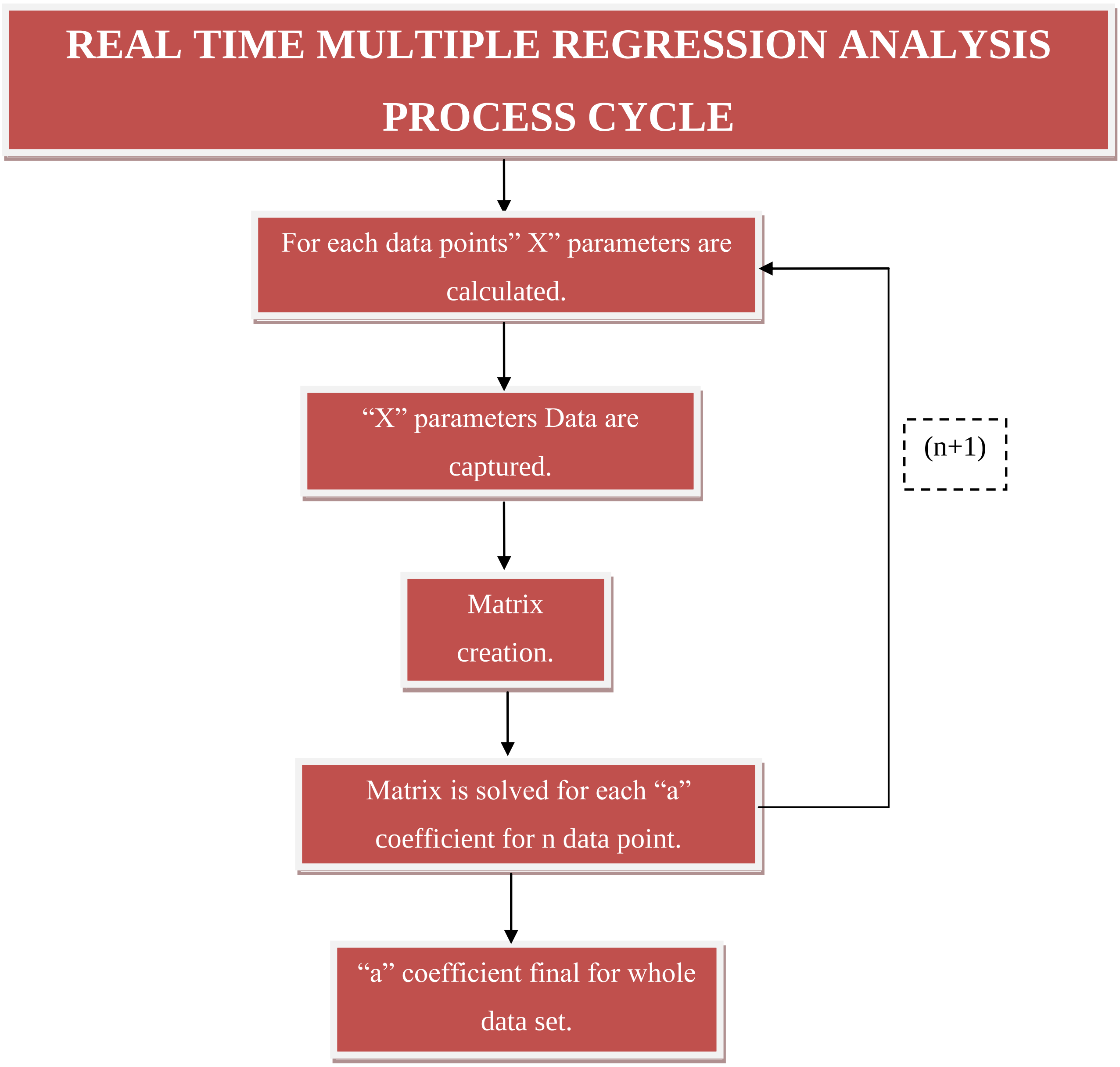


Figure 9**:** Non-linear regression process.

Non-linear regression technique will provide accurate and meaningful results. An example solution of non-linear regression through least square fit in MATLAB is given in appendix D.

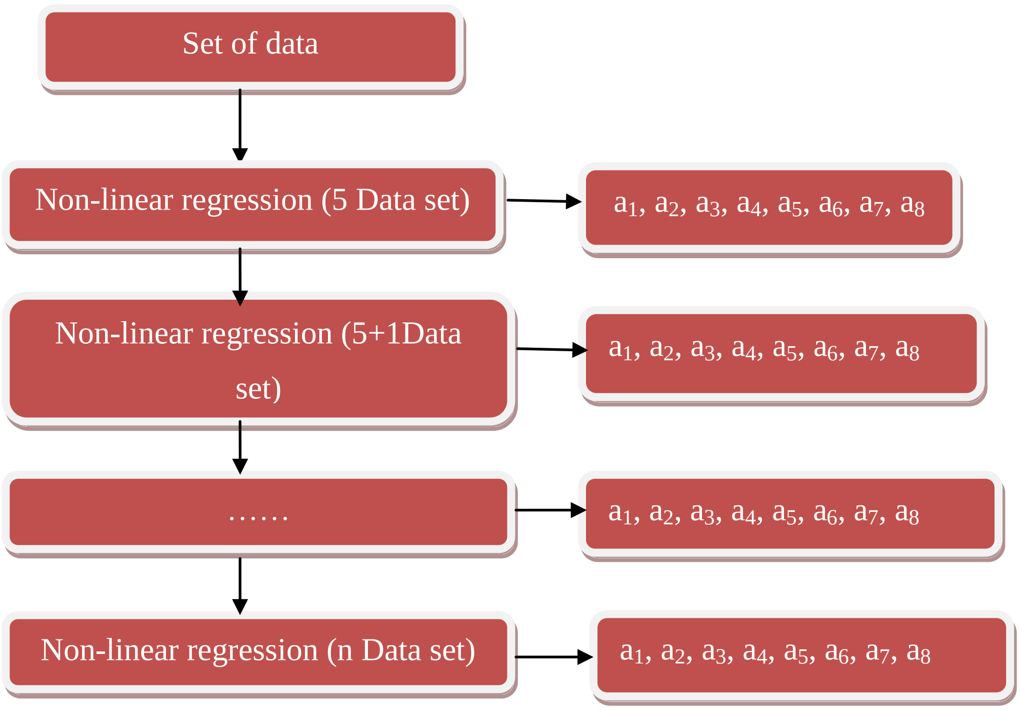


Figure 10: Non-linear regression work flow

This technique is mainly done to predict and then optimized the ROP values. Figure 10 is illustration of non-linear regression workflow. Non-linear regression minimum number of dataset to solve 8*8 matrixes is 5. First step is to solve coefficient of first 5 dataset and the loop is continuously repeated with one more data set in order to solve coefficients.

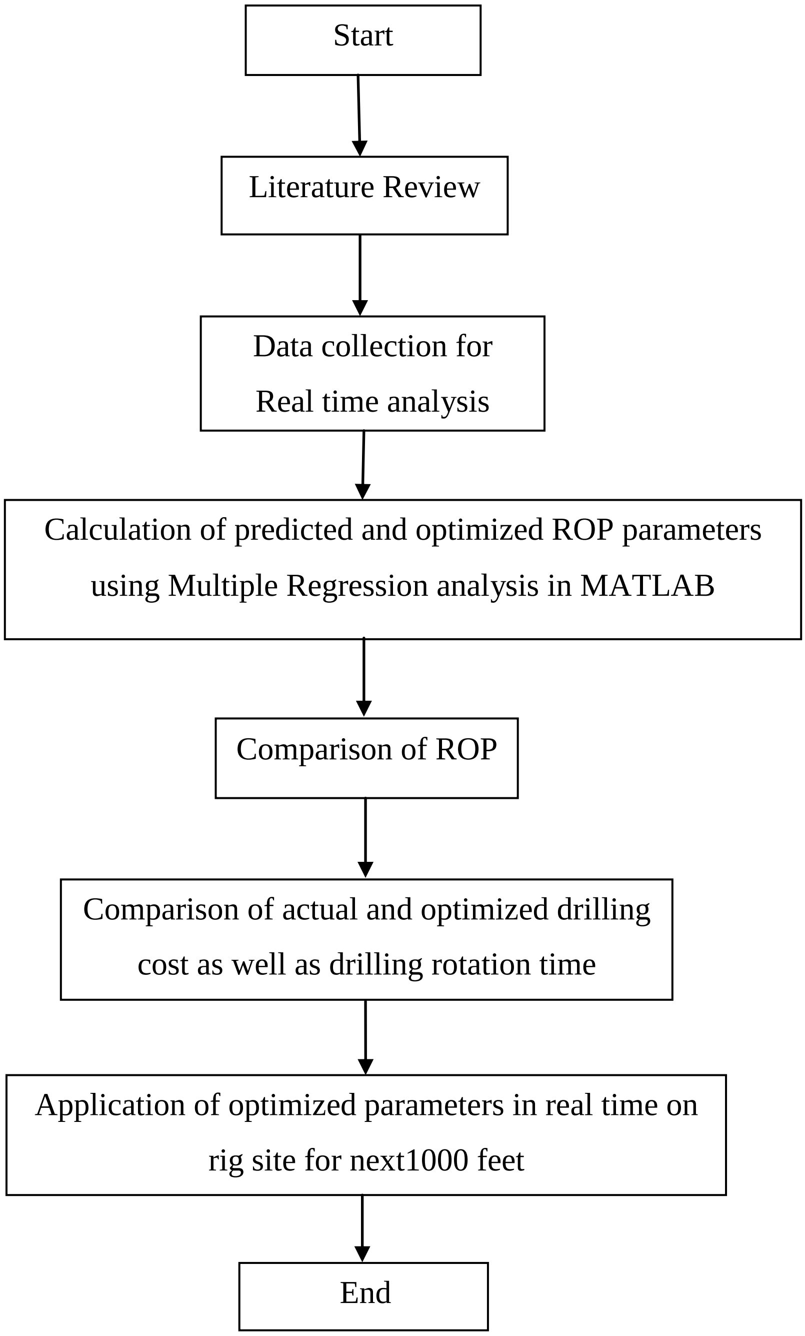


Figure 11: Flow chart for this project.

# CHAPTER 4

# RESULTS AND DISCUSSION

Non-linear regression outputs with respect to specific formation and optimization of three parameters are summarized. The comparison of ROP values between the actual, predicted, and optimized are also represented and summarized in this section.

## 4.1 RESULTS PRESENTATION

From given data, the comparison of ROP actual and predicted was determined first, and then coefficients ($a_1$ to $a_8$) which was found through non-linear regression technique were utilized to find the optimized WOB, RPM, and flow rate using formula and steps as mentioned is chapter 3. Optimized WOB, RPM, and flow rate were plotted against the depth and compared to the actual. After the determination of three optimized parameters, ROP actual, ROP predicted and ROP optimized were also plotted against the depth.

### 4.1.1 RESULTS OF ROP PERFORMANCE MONITORING AGAINST DEPTH

First step was the collection of data and to maintain unit consistency for each parameter regardless of whether it's dependent or independent parameters throughout this study. The given data was sufficient to carry out analysis, run the technique and for comparison which are essential and important part of the study.

Using the drilling rate of penetration theory as given in equation 3.2 and with the help of non-linear regression technique i.e. least square fit in MATLAB, constants ($a_1$ to $a_8$) and coefficient ($X_1$ to $X_8$) for each data point were determined. The determined constants ($a_1$ to $a_8$) were used to estimate the predicted rate of penetration.

$$\frac{df}{dt} = Exp\left( a_1 + a_2(8000 - D) + a_3\left(D^{0.69(g_p - 9)}\right) + a_4 D^{0.69(g_p - \rho_c)} + a_5 Ln\left[\frac{\frac{W}{d_b} - \left(\frac{W}{d_b}\right)_t}{4 - \left(\frac{W}{d_b}\right)_t}\right] + a_6 ln\left(\frac{N}{60}\right) + a_7(-h) + a_8 \frac{\rho q}{350 \mu d_n}\right) \quad 3.2$$

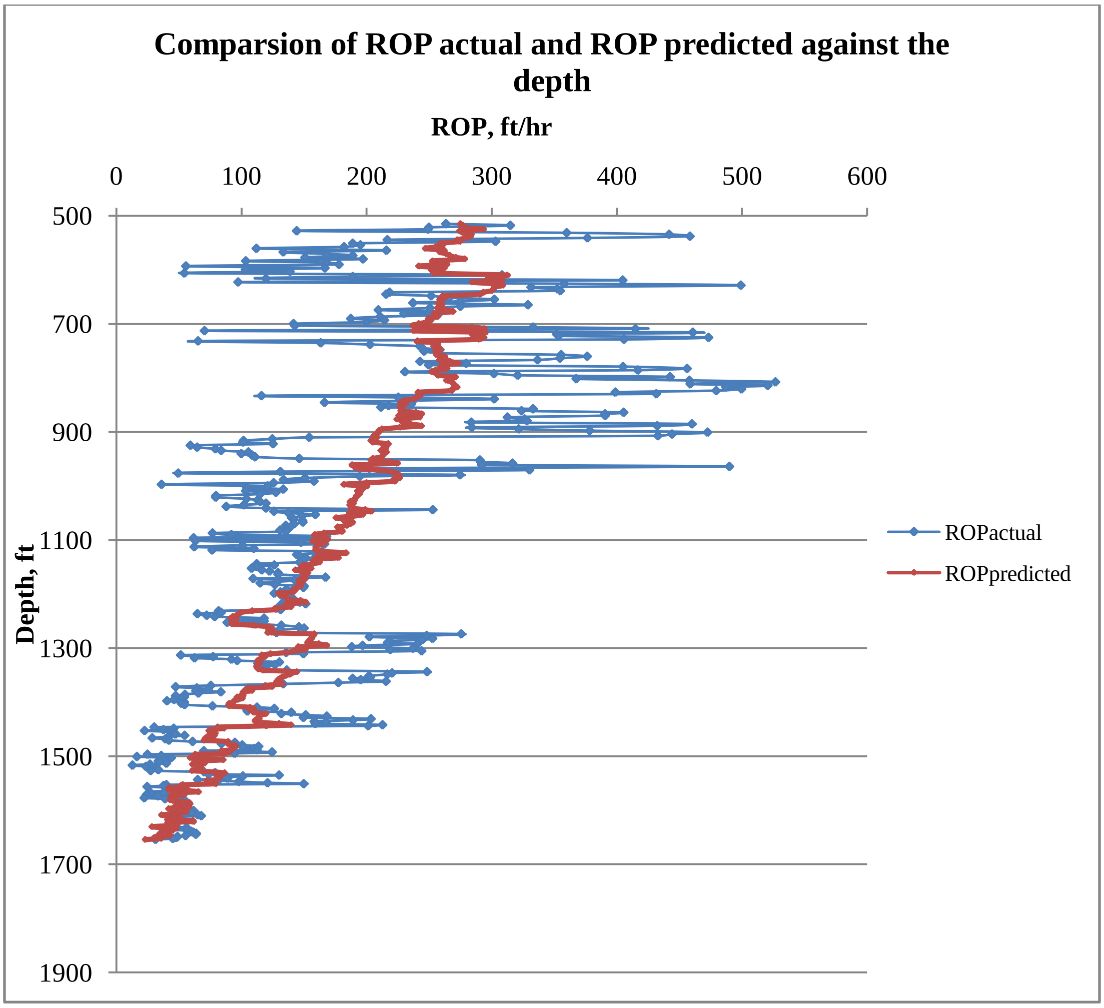


Figure 12: Comparison of ROP actual and ROP predicted against the depth.

Figure 12 is represents the rate of penetration of the actual occurrence and the predicted ones against the depth. To compare the performance of both actual and predicted it is important to have error band, as it helps to identify whether the actual ones lie within the defined range.

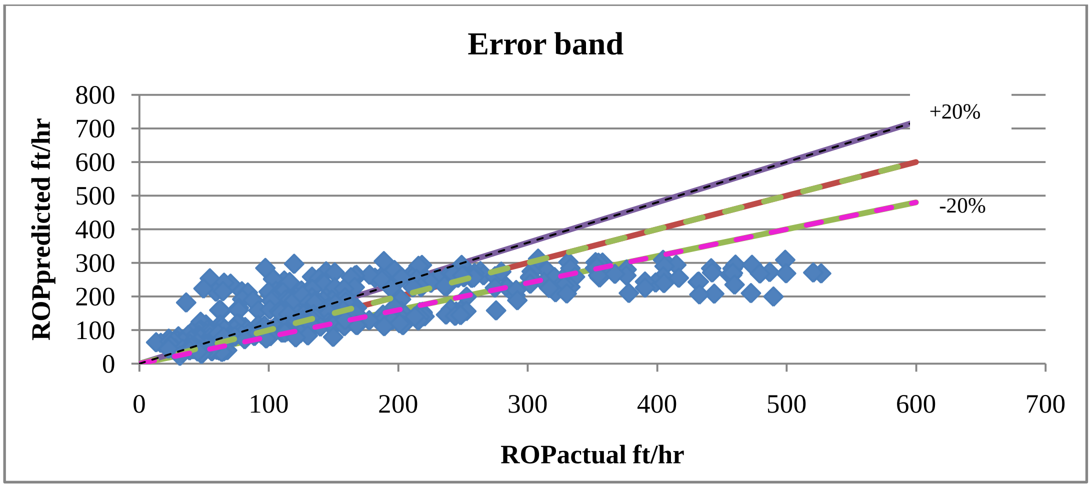


Figure 13: Error Band between ROP actual and ROP predicted

Error performance band (±20%) between ROP actual and ROP predicted is given in figure 13; the magnitude band that is slightly above and below in reference (±20%) to the predicted range is associated with the ROP predicted performance. During the monitoring of ROP actual performance, it was noticed that the actual performance tends to stay in the defined range.

ROP monitoring methodology used in this study can be adapted in field for real-time applications.

The optimized WOB, RPM and flow rate are compared with actual occurrences and represented individually as follows

### 4.1.2 WEIGHT ON BIT OPTIMIZATION

WOB optimized was calculated using the equation 3.16, constants $a_5$ and $a_6$ were found out using non-linear regression in MATLAB. Values $\left(\frac{W}{d_b}\right)_{max}$ and $H_1$ for a specific formation as given by the data is taken as 8 and 1.85 respectively.

$$\left(\frac{W}{d_b}\right)_{Opt} = \frac{a_5\, H_1\left(\frac{W}{d_b}\right)_{max} + a_6\left(\frac{W}{d_b}\right)_t}{a_5\, H_1 + a_6} \qquad 3.16$$

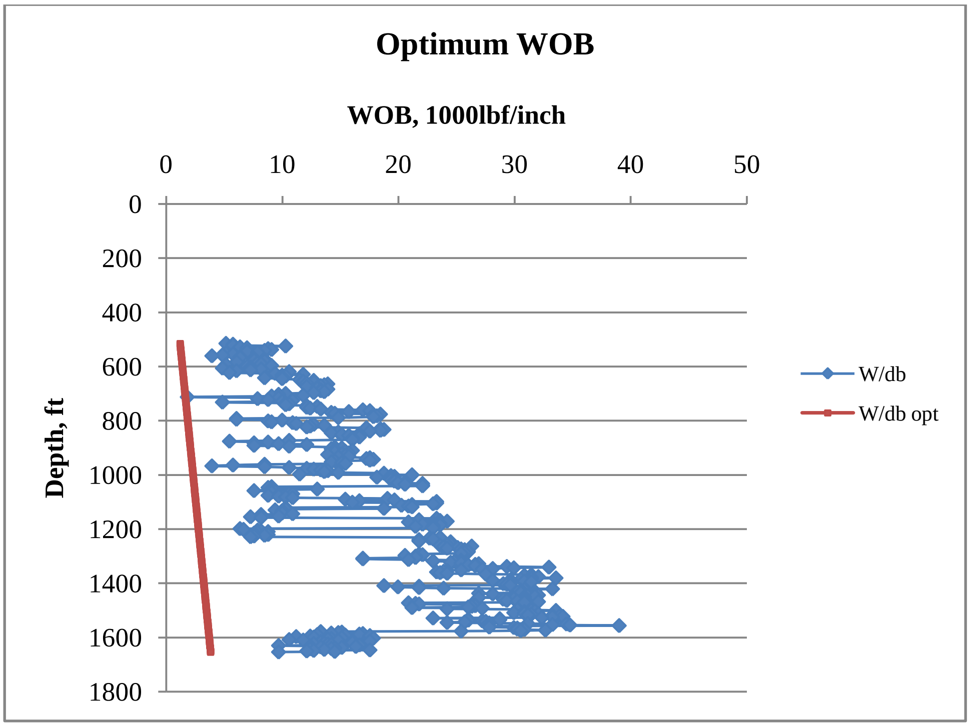


Figure 14: Optimum WOB against Depth

After the calculation was performed using excel, WOB optimized was estimated. Figure 14 is comparison of WOB optimized and WOB actual for a specific formation against depth. From the figure, it is observed that applying lower weight on bit throughout the drilled section could have lead to optimization of drilling process.

### 4.1.3 BIT ROTATION OPTIMIZATION

Equation 3.17 was used to calculate the optimized RPM. $\left(\frac{W}{d_b}\right)_{max}$ and $t_b$ are given as 8 and 20 respectively, $\left(\frac{W}{d_b}\right)_{Opt}$ was calculated for varying depth as shown in WOB optimization, $\tau_H$ and $J_2$ are formation abrasiveness constant and tooth wear parameter which is found using the equations given formation abrasiveness constant in appendix B.

$$(N)_{opt} = 60\left[\frac{\tau_H}{t_b}\,\frac{\left(\frac{W}{d_b}\right)_{max} - \left(\frac{W}{d_b}\right)_{Opt}}{\left(\frac{W}{d_b}\right)_{max} - 4}\right] \qquad 3.17$$

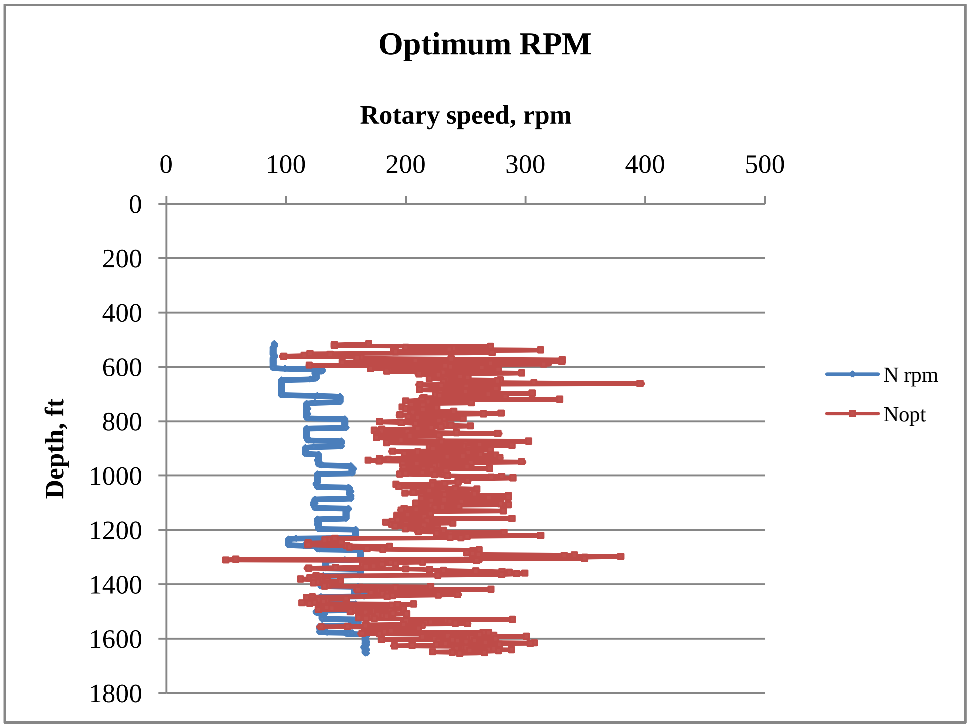


Figure 15: Optimum RPM against Depth

Optimum RPM is compared with actual against depth as shown in figure 15. As the figure indicates that optimum RPM was found to have higher magnitudes in context to what was actually applied. Therefore for optimization of drilling process to take place in the drilled formation, RPM should be higher as compared to the actual ones.

### 4.1.4 FLOW RATE OPTIMIZATION

Steps for flow rate optimization

- Calculate $\Delta P_b$ using the formula given,

  $$\Delta P_b = \frac{K_1 \rho q^2}{Cd^2 At^2}$$

  $K_1$ =8.31E-05 and Cd=0.95 respectively. Using excel, $\Delta P_b$ is calculated, as in the given data $\rho$ and q are known for varying depth.

- Find $\Delta P_d$,

$$\Delta P_d = P_p - \Delta P_b$$

Then next step is to calculate parasitic pressure loss, pump pressure is given in our data and bit pressure drop is calculated, as a result $\Delta P_d$ is determined.

- Calculate from the given formula,

$$m = \left( \frac{\log \frac{\Delta P d_2}{\Delta P d_1}}{\log \frac{q_2}{q_1}} \right) \qquad 3.32$$

To estimate the value of m equation 3.32 is utilized, $\Delta P_d$ and q values are interpolated. The value of m should be a single value; therefore the determined value of m is averaged to get a single value.

- Determine the value of C from the formula given,

$$\Delta P_d = CQ^m$$

As the values of $\Delta P_d$ and $m$ are already calculated in the above steps, value of C is determined using the above formula.

- Optimization of flow rate using the criteria,

$P_1$/C (m+1),

$3P_1$/C (m+3),

$5P_1$/C (m+5).

After the calculation of flow rate optimization is done using the given criteria, these criteria will be averaged to get the optimum flow rate values at different depths. The comparison criteria for flow rate optimization are given in appendix E.

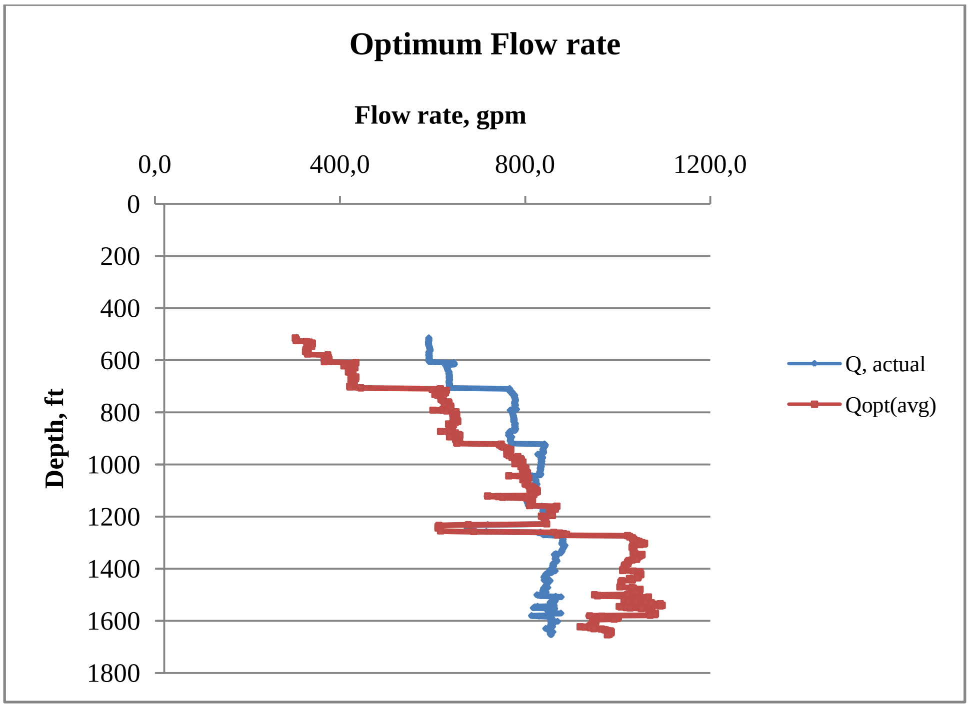

Figure 16: Optimum flow rate

Figure 16 shows the comparison of optimum and actual flow rate. It is observed that in the drilled formation, optimum flow rate should be less till the depth of 1200ft and should be increased gradually afterwards for drilling optimization process.

## 4.2 COST CONSIDERATIONS

The most important aspect of this study is to decrease the expenditure of drilling operations. Rotation times in drilling operations accounts up to 30% of the well cost [31]. During drilling operations, by proper selection of optimized parameters rotation time would be decreased but there is no actual field tests performed supporting it.

The optimized parameters; weight on bit, rotation speed and flow rate are calculated and plotted against depth as explained previously, with utilization of optimum parameter ROP optimized is estimated. The method similar to ROP prediction using non-linear regression technique was considered to determine optimum ROP values. ROP optimized values were plotted against depth comparing the ROP actual and ROP predicted.

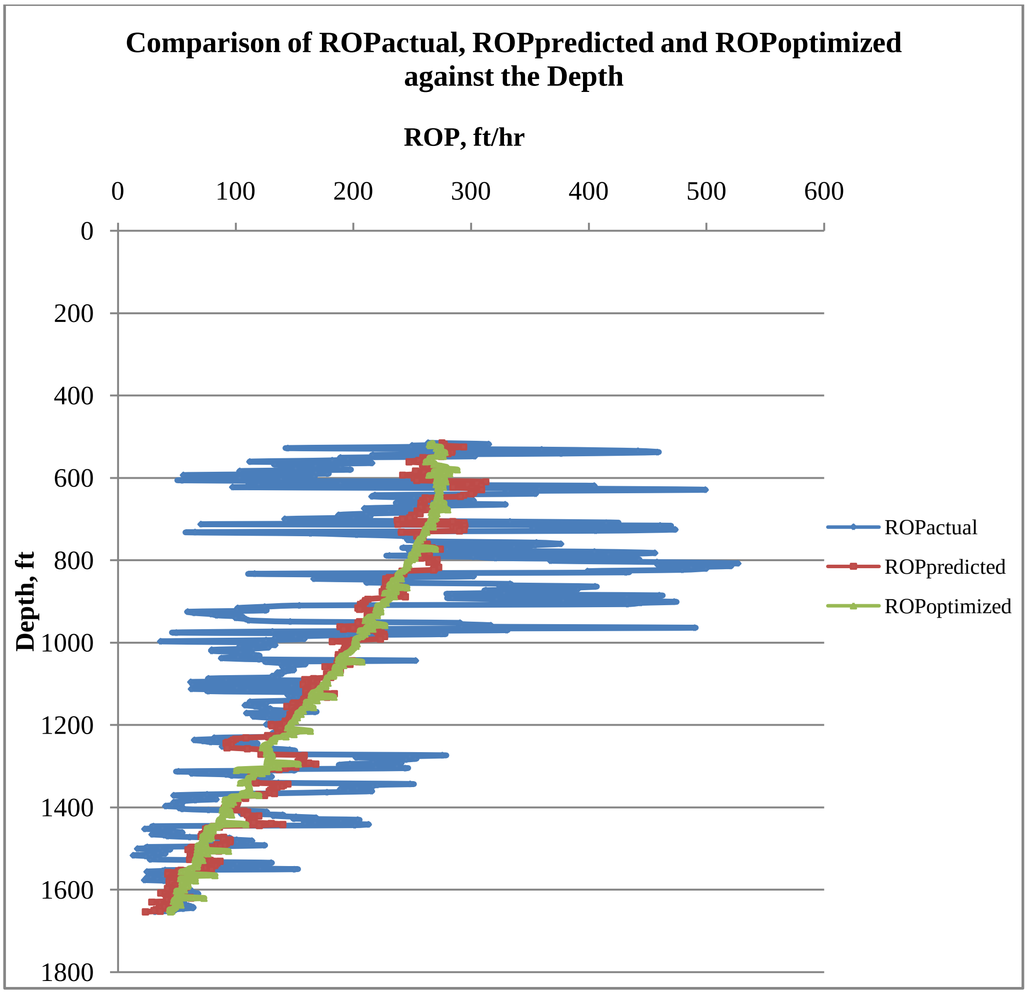


Figure 17: Optimized rate of penetration comparison with actual and predicted

The actual, predicted and the optimized rate of penetration is shown in figure 17, occurrence of three ROP are represented in the same figure. The coefficient "a" determined through predicted rate of penetration at each individual data point have been used, when including optimized weight on bit, flow rate and rotation speed into ROP general equation to estimate the optimized rate of penetration response. The Figure 17 indicates that optimized rate of penetration response is more accomplished and dynamic. Therefore optimized rate of penetration having greater magnitude as compared to actual occurrence will lead to low cost drilling.

Drilling cost is calculated using the equation given below

$$C_f = \frac{C_b + C_r(t_t + t_c + t_b)}{\Delta F}$$

Table 2: Drilling cost comparison required values

| Cb $ | Cr $/hr | Tt hrs (500ft) | Tt hrs (1000ft) |
|---|---|---|---|
| 2500 | 900 | 0.3 | 0.6 |

Above table values are commonly used in drilling industry to calculate the drilling cost. Using the drilling cost equation, actual and optimized cost are determined.

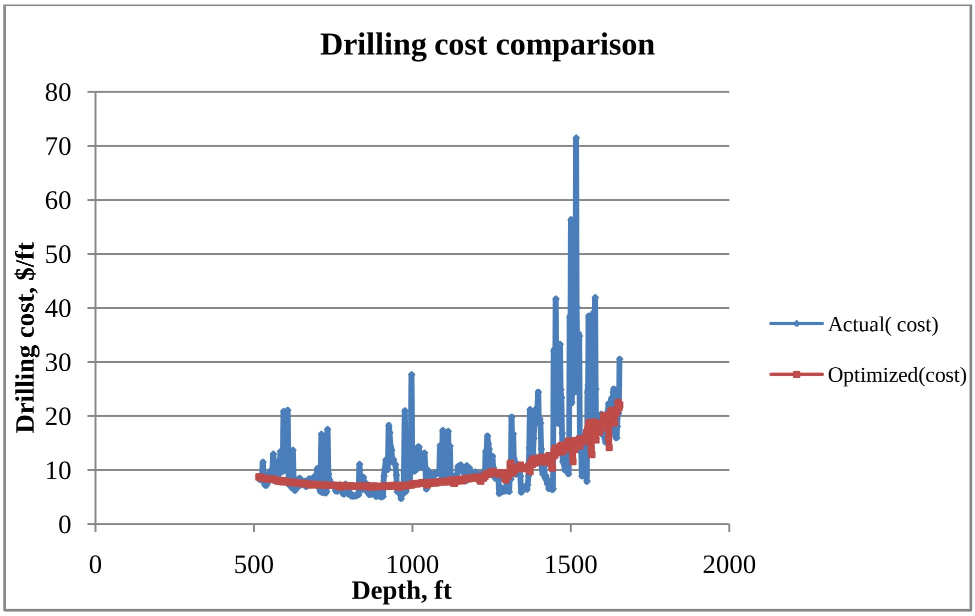


Figure 18: Comparison of drilling cost

Drilling cost comparison between actual and optimized case is given in Figure18. The figure indicates that the optimized drilling cost is comparatively less to that of actual cost. As a result, optimized case would have lead to decrease in drilling cost.

The rotation time is determined by dividing the depth with rate of penetration. Rotation time for both actual and optimized is estimated and compared against depth. Rotation time is critical for drilling optimization process as it contributes majorly on drilling cost and well cost.

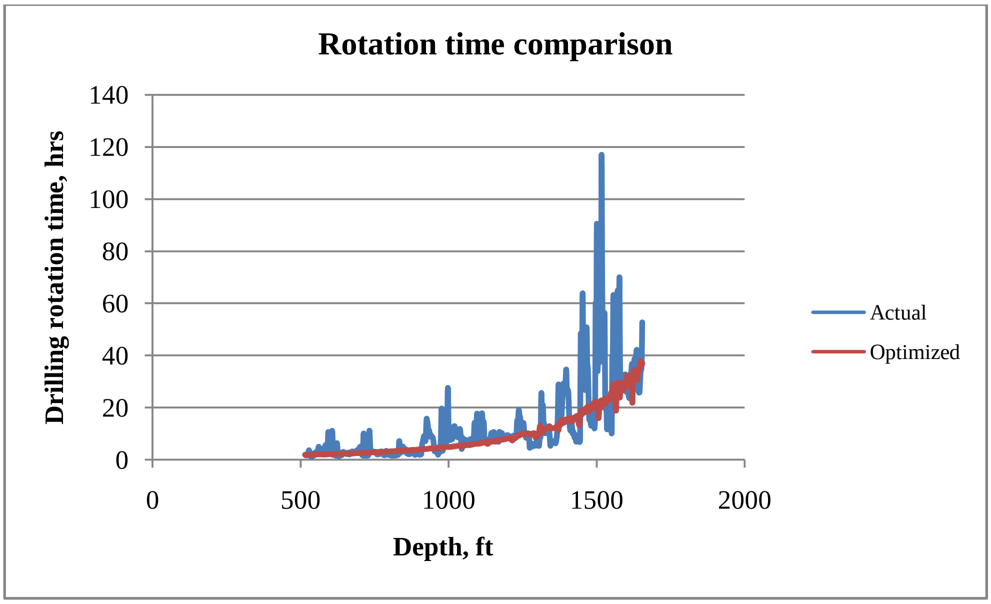


Figure 19: Optimized and actual drilling time comparison

Figure 19 is representation for actual and optimized drilling rotation time, it is noticed from the graph that the optimized drilling rotation time is relatively less as compared to actual rotation time. Therefore, by utilization of optimum drilling parameter the time required to drill the same hole section will be less.

Real-time optimization has shown that when the optimized drilling parameters are used, the drilling cost and drilling rotation time will have high magnitude as compared to the actual drilling.

## 4.3 DISCUSSION

Data quality is most valuable in real-time drilling optimization studies due to the fact that results of optimization process largely depend on it. Data consistency is vital for this study and noisy data should be discarded and filtered before the exercise of non-linear regression. Data transmission engineer should perform task of filtering the data before it's transmitted from the rig site.

ROP prediction will be proper and accurate as data points are more. This is because the regression constants gradually become more perfect and accurate with the increase in data points.

From the results of ROP predicted and ROP observed chart, it was clearly observed that tolerable results were attainable. Keeping in mind dependent variable's function, the predicted value can be estimated by utilizing optimized drilling variables as well as assuming the optimized rate of penetration results without performing drilling activity.

This study consists of eight functions; there is high possibility that this number can be increased. By the increment of these functions, different effects which are believed to influence the drilling operation can be taken into considerations. Additional Functions includes torque, cuttings content, friction factor between the bottom hole assembly and hole section.

In this study, it is observed that the optimized drilling time and drilling cost to drill the same distance was less. There are no prospects to check the actual rate of penetration. An estimation of rate of penetration was given by the determination and adaption of controllable drilling parameters. Estimation was done using most recent coefficient of non-linear regression process.

The known fact is that more the data points, higher are the accuracy of ROP performance predictions. ROP versus depth are plotted to indicate the difference between predicted and actual ROP performance for a specific formation. The ROP predicted showed meaningful magnitude as compared to actual ROP, this was purely due to accuracy and efficiency of non-linear regression technique. Non-linear regression technique utilized general rate of penetration equation for determining rate of penetration.

# CHAPTER 5

# CONCLUSION AND RECOMMENDATION

## 5.1 CONCLUSION

The objective of this study was achieved; the methodology for real-time rate of penetration, prediction and optimization during drilling operation was conducted and explained. The task of achieving optimum controllable drilling parameters was executed; non-linear regression technique was helpful in determining the optimum drilling parameters. From the previous chapter of results and discussion, it was noticeable that drilling cost would reduce if optimum drilling parameters were applied in real-time.

Modern Well monitoring equipment is necessary for collection of data and to conduct non-linear regression process. Non-linear technique which is the least square fit in MATLAB is used to estimate the constants in general rate of penetration equation. It is confirmed that technique would be reliable and efficient in actual drilling activities, well planning and drilling construction. Regression coefficient is better with increase of data quality. In the world of drilling optimization technology, best answer is to predict the rate of penetration and recommend optimum drilling parameter for optimization.

In this study, for a specific formation which is drilled regression coefficients such as $a_1$ to $a_8$ are identified. More the data volume high the accuracy, regression exponents is believed to have weight over the general rate of penetration magnitude as each individually have an effect. Coefficient $a_1$ is an important parameter which has effect of crew efficiency, rig properties, and equipment quality. For future applications in optimization study, these effects should be considered. Finally for a specific formation regression coefficients can be attained and desired drilling rate performance will be compared depending on the coefficients together with along drilling optimization.

For real-time optimization process to occur, operator and contractor should agree on predetermined ROP performance. This study displays that the operator and the contractor will agree on the predetermined ROP performance and monitor drilling activity depending on the actual performance.

The ROP performance predictions chart indicated that proper and accurate prediction of rate of penetration was accomplished. Optimized parameters that are dependent drilling parameters such weight on bit, rotation speed and flow rate chart showed more accuracy due to more volume of data points and regression technique.

Although actual field test wasn't performed, optimized drilling rotation time if drilled in formation given will help in reducing time. Optimized cost found in this study was also less for the same formation; as a result optimization process would lead to reduced drilling expenditure and time.

Therefore from the charts and results performed in this study, there was a clear indication on how dominant the optimum parameters were on ROP performance. After the collection of actual data for the formation and running the analysis, required for the real-time drilling optimization as explained. The optimum parameters such as weight on bit, rotation speed and flow rate are applied to the same formation for further drilling (For example next 1000 feet depth). When these parameters are applied in real time, there are high chances that the drilling cost and drilling time will reduce and also the probability of encountering problems will decrease. This process is known as real time drilling optimization process because drilling cost and time are widely reduced by the application of optimum parameters.

Table 3: Profit obtained due to application of optimized parameters

| Actual cost $/ft | Optimized cost $/ft | Profit $/1000ft |
|---|---|---|
| 12.66199 | 10.47726 | 2189.733 |

Above table 3 shows the profit gained when the optimized parameters are applied for the next 1000 feet depth. It is clearly indicated that when it's applied there is profit of 2189$.

## 5.2 RECOMMENDATIONS

It is highly recommended that

- Cutting transport should included in the general rate of penetration equation
- Torque considerations should be added to Bourgogne and Young's approach.
- There are only eight effects on the rate of penetration model, so for studies adding one or more effects can be taken into consideration for future studies.
- Downhole drag force and detailed geometry of the bottom hole assembly can be considered as this contribute to the results which is found using non-linear regression analysis.
- The methodology used in this study should be modified so that is also applicable to PDC bits.

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# APPENDIX-A

## EQUATIONS DEFINED BY BOURYONE AND YOUNG, 1974

Bouryone and Young [21] model because of its wide approach, it has found greater acceptance in drilling industry. Bouryone and Young equations are summarized in this section; the rate of penetration equation is given by

$$\frac{df}{dt} = e^{\left[a_1 + \sum_{j=2}^{8} a_j x_j\right]} \qquad 3.1$$

$x_1$ is dummy variable, $x_2$ to $x_8$ are determined with respective equation, non-linear regression technique is used to calculate the 'a' coefficients. Both 'x' and 'a' are related to each other as explained in the above equation.

Table 4: Effect on rate of penetration by drilling parameters

| | |
|---|---|
| 1 | Formation strength |
| 2 | Formation depth |
| 3 | Formation compaction |
| 4 | Pressure differential across the hole bottom |
| 5 | Bit diameter and bit weight |
| 6 | Rotary speed |
| 7 | Bit wear |
| 8 | Bit hydraulics |

Each effect in table 3 are individually explained below

Formation strength effect is given in equation A.1

$$x_1 = 1 \qquad A.1$$

This effect is not yet modeled.

Formation depth effect is given in equation A.2

$$x_2 = 8000 - D \qquad \text{A.2}$$

Formation compaction effect is given in A.3

$$x_3 = D^{0.69}(g_p - 9) \qquad \text{A.3}$$

Differential pressure across the hole bottom effect is given in equation A.4

$$x_4 = D^{0.69(g_p - \rho_c)} \qquad \text{A.4}$$

Bit weight and bit diameter effect is given in equation A.5

$$x_5 = Ln\left[\frac{\frac{W}{d_b} - \left(\frac{W}{d_b}\right)_t}{4 - \left(\frac{W}{d_b}\right)_t}\right] \qquad \text{A.5}$$

Rotary speed effect is given in equation A.6

$$x_6 = ln\left(\frac{N}{60}\right) \qquad \text{A.6}$$

Tooth wear effect is given in equation A.7

$$x_7 = (-h) \qquad \text{A.7}$$

Bit Hydraulics effect is given in equation A.8

$$x_8 = \frac{\rho q}{350\mu d_n} \qquad \text{A.8}$$

# APPENDIX-B

# FORMATION ABRASIVENESS CONSTANT CALCULATION

The calculation for formation abrasiveness constant is given in this appendix

Instantaneous tooth wear equation is given in equation B.1

$$\frac{dh}{dt} = \frac{1}{\tau_H}\left(\frac{N}{60}\right)^{H1}\left[\frac{\left(\frac{W}{d_b}\right)_m - 4}{\left(\frac{W}{d_b}\right)_m - \left(\frac{W}{d_b}\right)}\right]\left(\frac{1+\frac{H_2}{2}}{1+H_2 h}\right) \qquad \text{B.1}$$

$\tau_H$ is formation abrasiveness constant, hrs, $H_1$ and $H_2$ are tooth geometry constants

Table 5: Tooth wear parameters for roller cone bits

| Bit Class | $H_1$ | $H_2$ | $\left(\frac{W}{d_b}\right)_{max}$ |
|---|---|---|---|
| 1-1 to 1-2 | 1.90 | 7 | 7 |
| 1-3 to 1-4 | 1.84 | 6 | 8 |
| 2-1 to 2-2 | 1.80 | 5 | 8.5 |
| 2-3 | 1.76 | 4 | 9 |
| 3-1 | 1.70 | 3 | 10 |
| 3-2 | 1.65 | 2 | 10 |
| 3-3 | 1.60 | 2 | 10 |
| 4-1 | 1.50 | 2 | 10 |

The tooth wear parameter is symbolized as $J_2$, equation B.2

$$J_2 = \left[\frac{\left(\frac{W}{d_b}\right)_m - \left(\frac{W}{d_b}\right)}{\left(\frac{W}{d_b}\right)_m - 4}\right]\left(\frac{60}{N}\right)^{H1}\left(\frac{1}{1+\frac{H_2}{2}}\right) \qquad \text{B.2}$$

The formation abrasiveness constants is given in equation B.3

$$\tau_H = \frac{t_b}{J_2\left(h_f + H_2 \frac{{h_f}^2}{2}\right)} \quad \text{B.3}$$

$t_b$ is bit rotation time and it calculated after the formation abrasiveness constants is known as given in equation B.4

$$t_b = J_2 \tau_H \left(h_f + H_2 \frac{{h_f}^2}{2}\right) \quad \text{B.4}$$

The calculation of formation abrasiveness constant and bit rotation time is done through excel using the formulas given for determining optimized rotary speed.

# APPENDIX-C

## NON-LINEAR REGRESSION ANALYSIS CODE

Non-linear regression technique was done using least square fit in MATLAB. Following is the MATLAB code for ROP prediction and ROP optimization.

```
function B=myfun9(x,xdata)
global B;

Beta1=x(1)+x(2).*(8000-
xdata(:,1))+(x(3).*(xdata(:,1).^0.69.*(xdata(:,2)-
9)))+(x(4).*xdata(:,1).*(xdata(:,2)-xdata(:,3)));
Beta2=x(5).*log((xdata(:,4)-0.02)./(4-
0.02))+x(6).*log(xdata(:,5)./60);
Beta3=x(7).*(-
xdata(:,6))+(x(8).*xdata(:,7).*xdata(:,8))./(350.*xdata(:,10).*xdata
(:,9));
B=(Beta1+Beta2+Beta3);
```

- Non-linear regression code for ROP prediction

```
%NONLINEAR REGRESSION
clc
global B;
clear all
format short e
data=xlsread('Data.xls','Sheet3');

D=data(:,2);
gp=data(:,4);
rhoc=data(:,5);
wob=data(:,6);
N=data(:,7);
h=data(:,8);
rho=data(:,9);
q=data(:,10);
dn=data(:,11);
visc=data(:,12);
ROPactual=data(:,14);
ydata=ROPactual;
%xdata=[rs gg tf go pb api visc];
xdata=[D gp rhoc wob N h rho q dn visc];

xo=[1.234 1.234 1.234 1.234 1.234 1.234 1.234 1.234];
% xo=[0.1 0.1 0.1 0.1 0.1];
Options=optimset('Display','iter','TolFun',1e-
25,'MaxFunEvals',1000000,'MaxIter',100000);
% Options=optimset('Display','iter','TolFun',1e-6);
lb=[];
ub=[];
```

```
%[x, resnorm]=lsqcurvefit('myfun',xo,xdata,ydata)
[xg,outputg,lambdag,jac1g]=lsqcurvefit('myfun9',xo,xdata,ydata,lb,ub
,Options);

coeffg=xg'

% B=zeros(1,28);
% for D = 3 : 26



ROP_Predicted=(coeffg(1)+coeffg(2).*(8000-
xdata(:,1))+coeffg(3).*(xdata(:,1).^0.69.*(xdata(:,2)-
9))+coeffg(4).*xdata(:,1).*(xdata(:,2)-xdata(:,3))...
+coeffg(5).*log((xdata(:,4)-0.02)./(4-
0.02))+coeffg(6).*log(xdata(:,5)./60)+coeffg(7).*(-
xdata(:,6))+(coeffg(8).*xdata(:,7).*xdata(:,8))./(350.*xdata(:,10).*
xdata(:,9)));

ROP_Predicted


x2=(8000-xdata(:,1));
x3=(xdata(:,1).^0.69.*(xdata(:,2)-9));
x4=xdata(:,1).*(xdata(:,2)-xdata(:,3));
x5=log((xdata(:,4)-0.02)./(4-0.02));
x6=log(xdata(:,5)./60);
x7=(-xdata(:,6));
x8=(xdata(:,7).*xdata(:,8))./(350.*xdata(:,10).*xdata(:,9));
[x2 x3 x4 x5 x6 x7 x8]

%end

%subplot(121), plot(data(:,11));
%hold on;
%subplot(122),plot(B);
%

plot(ydata,D,'-',ROP_Predicted,D,'-')

set(gca,'YDir','rev')
set(gca, 'xaxislocation','top')
```

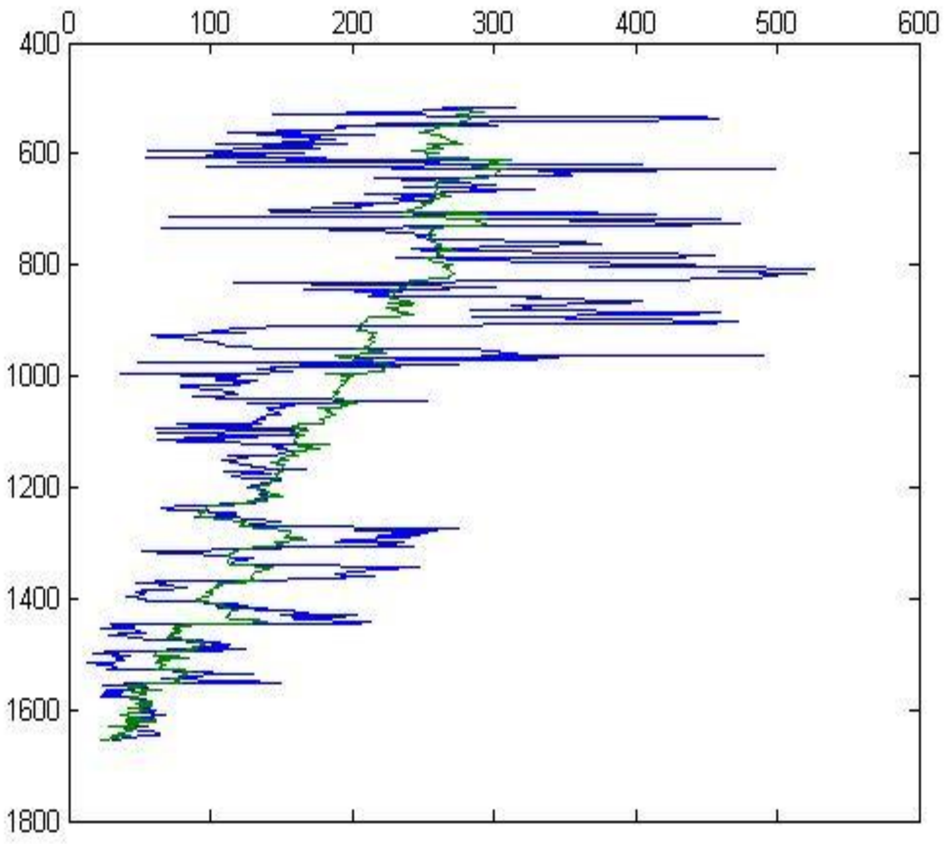


Figure 20: Comparison of ROP actual and ROP predicted using MATLAB

- Non-linear regression code for ROP optimization

```
%ROP Optimization
%NONLINEAR REGRESSION
clc
global B;
clear all
format short e
data=xlsread('Data.xls','Sheet4');

D=data(:,2);
gp=data(:,4);
rhoc=data(:,5);
wob=data(:,6);
N=data(:,7);
h=data(:,8);
rho=data(:,9);
q=data(:,10);
dn=data(:,11);
visc=data(:,12);
ROPactual=data(:,14);
ydata=ROPactual;
%xdata=[rs gg tf go pb api visc];
```

```
xdata=[D gp rhoc wob N h rho q dn visc];

xo=[1.234 1.234 1.234 1.234 1.234 1.234 1.234 1.234];
% xo=[0.1 0.1 0.1 0.1 0.1];
Options=optimset('Display','iter','TolFun',1e-
25,'MaxFunEvals',1000000,'MaxIter',100000);
% Options=optimset('Display','iter','TolFun',1e-6);
lb=[];
ub=[];
%[x, resnorm]=lsqcurvefit('myfun',xo,xdata,ydata)
[xg,outputg,lambdag,jac1g]=lsqcurvefit('myfun9',xo,xdata,ydata,lb,ub
,Options);

coeffg=xg'

% B=zeros(1,28);
% for D = 3 : 26




ROP_Optimized=(coeffg(1)+coeffg(2).*(8000-
xdata(:,1))+coeffg(3).*(xdata(:,1).^0.69.*(xdata(:,2)-
9))+coeffg(4).*xdata(:,1).*(xdata(:,2)-xdata(:,3))...
+coeffg(5).*log((xdata(:,4)-0.02)./(4-
0.02))+coeffg(6).*log(xdata(:,5)./60)+coeffg(7).*(-
xdata(:,6))+(coeffg(8).*xdata(:,7).*xdata(:,8))./(350.*xdata(:,10).*
xdata(:,9)));

ROP_Optimized



x2=(8000-xdata(:,1));
x3=(xdata(:,1).^0.69.*(xdata(:,2)-9));
x4=xdata(:,1).*(xdata(:,2)-xdata(:,3));
x5=log((xdata(:,4)-0.02)./(4-0.02));
x6=log(xdata(:,5)./60);
x7=(-xdata(:,6));
x8=(xdata(:,7).*xdata(:,8))./(350.*xdata(:,10).*xdata(:,9));
[x2 x3 x4 x5 x6 x7 x8]

%end

%subplot(121), plot(data(:,11));
%hold on;
%subplot(122),plot(B);
%

plot(ydata,D,'-',ROP_Optimized,D,'-')

set(gca,'YDir','rev')
set(gca, 'xaxislocation','top')
```

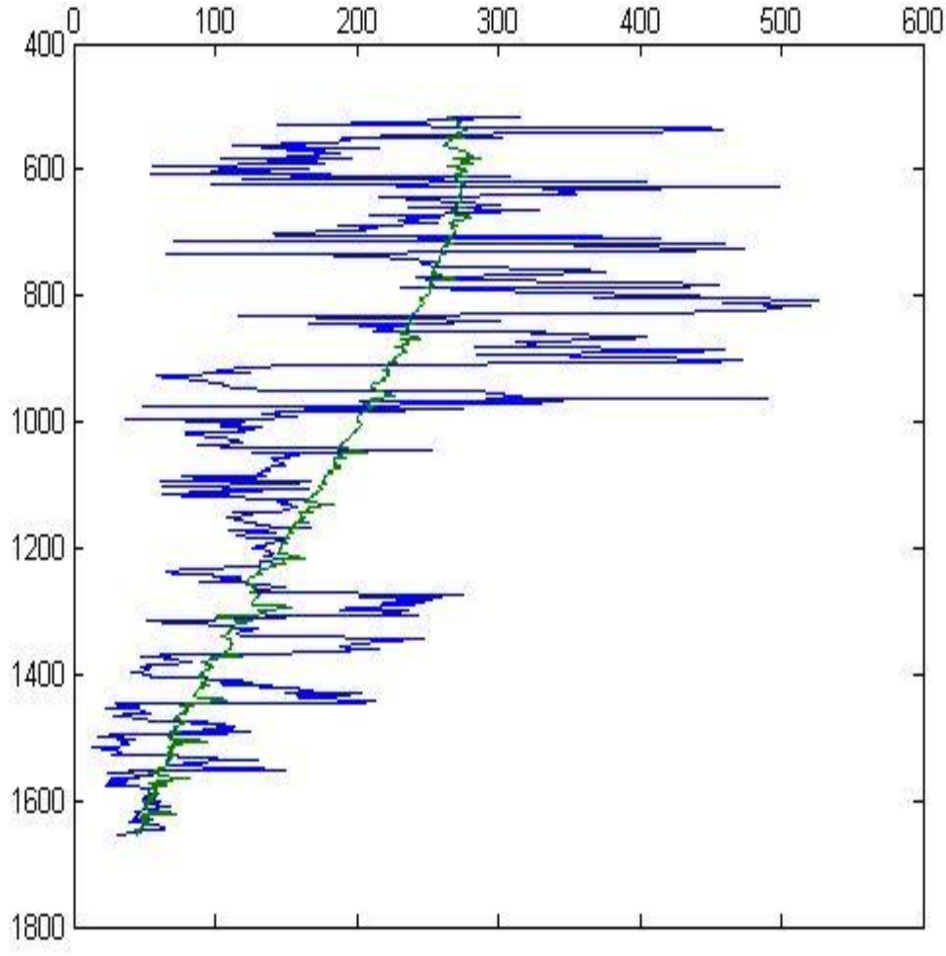


Figure 21: Comparison of ROP predicted and ROP optimized using MATLAB

# APPENDIX-D

## NON-LINEAR REGRESSION TECHNIQUE EXAMPLE USING MATLAB [33]

Not all processes are linear (in fact most real processes are not linear). It is therefore more appropriate in such cases to have nonlinear data fitting methods.

One basic nonlinear function is a polynomial. Consider that we have a set of data {$x_i$, $y_i$} with i = 0, 1, ...M data points, and we would like to fit a polynomial of order n. The general equation for a polynomial is given by

$$P_n(x) = a_0x^0 + a_1x^1 + a_2x^2 + ... + a_nx^n$$

Or more compactly written as

$$P_n(x) = \sum_{k=0}^{n} a_k x^k$$

Let us find once again the least squares approximation of our data using this polynomial fit rather than a linear fit. It should be noted that a linear fit is merely a polynomial of degree one (recall that the degree or order of a polynomial is the number of the highest power to which a variable in the polynomial equation is raised).

To compute the least square polynomial fit we will use the same approach as the linear case. It is important to choose n < M (do not try to fit a polynomial with a higher order than the number of points of data). Let us begin with

$$E = \sum_{i=1}^{M} [y_i - P(x_i)]^2$$

$$E = \sum_{i=1}^{M} y_i^2 - 2\sum_{i=1}^{M} y_i P(x_i) + \sum_{i=1}^{M} P^2(x_i)$$

$$E = \sum_{i=1}^{M} y_i^2 - 2\sum_{i=1}^{M} y_i \left[\sum_{k=0}^{n} a_k x_i^k\right] + \sum_{i=1}^{M} \left[\sum_{k=0}^{n} a_k x_i^k\right]^2$$

Which we can simplify to

$$E = \sum_{i=1}^{M} y_i^2 - 2\sum_{k=0}^{n} a_k \sum_{i=1}^{M} y_i x_i^k + \sum_{h=0}^{n}\sum_{k=0}^{n} a_h a_k \left[\sum_{i=1}^{M} x_i^{h+k}\right]$$

Now similar to before, to find the minimum of the error, E, we must find where the partial derivative with respect to each variable is zero:

$$\frac{\partial E}{\partial a_k} = 0$$

for each k = 0, 1, 2, ...n

$$0 = \frac{\partial E}{\partial a_k} = -2\sum_{i=1}^{M} y_i x_i^k + 2\sum_{h=0}^{n} a_h \sum_{i=1}^{M} x_i^{h+k}$$

We now have n+1 equations in n+1 unknown parameters $a_k$. These are referred to as the normal equations:

$$\sum_{h=0}^{n} a_h \sum_{i=1}^{M} x_i^{h+k} = \sum_{i=1}^{M} y_i x_i^k$$

k = 0, 1, 2, ...n

Written in this form we may not initially recognize this equation as being simple to solve. Let us consider this equation in a less short shorthand notation, then take a step back and look for a pattern.

$$a_0 \sum_{i=1}^{M} x_i^0 + a_1 \sum_{i=1}^{M} x_i^1 + a_2 \sum_{i=1}^{M} x_i^2 + ... + a_n \sum_{i=1}^{M} x_i^n = \sum_{i=1}^{M} y_i x_i^0$$

$$a_0 \sum_{i=1}^{M} x_i^1 + a_1 \sum_{i=1}^{M} x_i^2 + a_2 \sum_{i=1}^{M} x_i^3 + ... + a_n \sum_{i=1}^{M} x_i^{n+1} = \sum_{i=1}^{M} y_i x_i^1$$

$$a_0 \sum_{i=1}^{M} x_i^n + a_1 \sum_{i=1}^{M} x_i^{n+1} + a_2 \sum_{i=1}^{M} x_i^{n+2} + ... + a_n \sum_{i=1}^{M} x_i^{2n} = \sum_{i=1}^{M} y_i x_i^n$$

If we consider this in a matrix algebra perspective, we see that we can put this in the Ax = b canonical form:

let us define

$$x = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

and

$$b = \begin{pmatrix} \sum_{i=1}^{M} y_i \\ \sum_{i=1}^{M} y_i x_i \\ \sum_{i=1}^{M} y_i x_i^2 \\ \vdots \\ \sum_{i=1}^{M} y_i x_i^n \end{pmatrix}$$

And finally

$$A = \begin{bmatrix} \sum_{i=1}^{M} x_i^0 & \sum_{i=1}^{M} x_i^1 & \sum_{i=1}^{M} x_i^2 & \cdots & \sum_{i=1}^{M} x_i^n \\ \sum_{i=1}^{M} x_i^1 & \sum_{i=1}^{M} x_i^2 & \sum_{i=1}^{M} x_i^3 & \cdots & \sum_{i=1}^{M} x_i^{n+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{M} x_i^n & \sum_{i=1}^{M} x_i^{n+1} & \sum_{i=1}^{M} x_i^{n+2} & \cdots & \sum_{i=1}^{M} x_i^{2n} \end{bmatrix}$$

Then we can write this,

$$\begin{bmatrix} \sum_{i=1}^{M} x_i^0 & \sum_{i=1}^{M} x_i^1 & \sum_{i=1}^{M} x_i^2 & \cdots & \sum_{i=1}^{M} x_i^n \\ \sum_{i=1}^{M} x_i^1 & \sum_{i=1}^{M} x_i^2 & \sum_{i=1}^{M} x_i^3 & \cdots & \sum_{i=1}^{M} x_i^{n+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{M} x_i^n & \sum_{i=1}^{M} x_i^{n+1} & \sum_{i=1}^{M} x_i^{n+2} & \cdots & \sum_{i=1}^{M} x_i^{2n} \end{bmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{M} y_i \\ \sum_{i=1}^{M} y_i x_i \\ \sum_{i=1}^{M} y_i x_i^2 \\ \vdots \\ \sum_{i=1}^{M} y_i x_i^n \end{pmatrix}$$

Or more simply

$$Ax = b$$

X is found by

$$x = A^{-1}b$$

Which can be solved, as before by Gaussian elimination, or another (more efficient) algorithm.

MATLAB implementation

The exact same approach can be used as before to implement this in MATLAB using the left matrix divide.

Let’s consider a simple case where you have five points and you want to fit a quadratic polynomial using least squares regression (quadratic is of degree 2).

The points are (1, 2) (3,4) (2,3.5), (4,7.5) and (5, 11.5). The we have x= [1, 3, 2, 4, 5],

Y= [2, 4, 3.5, 7.5, 11.5]. If we substitute the points into the equation for a quadratic polynomial we have

$$a_0 + 1a_1 + (1)^2 a_2 = 2$$
$$a_0 + 3a_1 + (3)^2 a_2 = 4$$
$$a_0 + 2a_1 + (2)^2 a_2 = 3.5$$
$$a_0 + 4a_1 + (4)^2 a_2 = 7.5$$

Which is over-constrained problem since there is more information than there are unknowns. Now we can recognize the canonical linear algebra problem and define

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix}, \ x = \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix},$$

And

$$b = \begin{Bmatrix} 2 \\ 4 \\ 3.5 \\ 7.5 \\ 11.5 \end{Bmatrix}$$

Then after typing these into MATLAB, we can for x by writing x= A\b at the command prompt. MATLAB promptly solves for x, which we assign to a by a=x, ie ($a_0$, $a_1$, and $a_2$). Now to plot your fit over your data, type:

X= [1, 3, 2, 4, 5]; y= [2, 4, 3.5, 3.5, 11.5];

Plot (x,y,'*r');

Then create another set of points by creating the x values, and plugging them into our equation from the least squares fit,

Xf = 0.5: .1:6;

Yf = a(1)+a(2)*xf +a(3)*xf.$^2$ ;

Hold on; plot (xf, yf, 'r').

Your results should look like the following

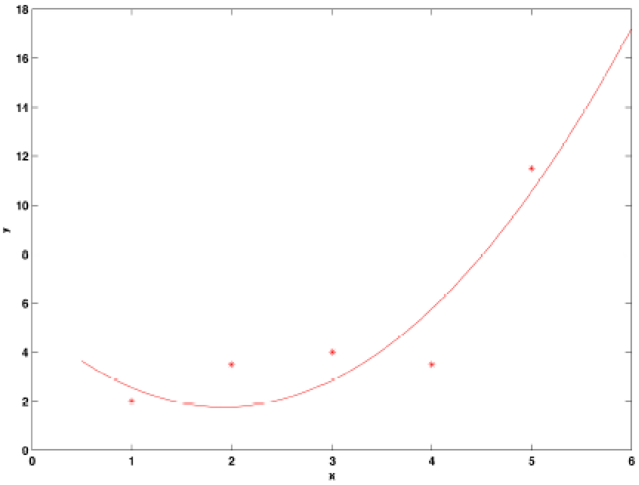


Figure 22: A basic quadratic least squares data fit to four points

- Example MATLAB code using non-linear regression analysis.

```
%x data
x = 0:0.1:5-0.1;

%y data
%the rand command is used here to introduce some error into
%the output because real data is never perfect
y = -x.^3 +5.*x.^2+x-15 + rand([1,50])/2;

%plot the data as a scatter plot
figure
scatter(x,y)
```

```
function output= myPolyCurve (param,input)
a = param(1);
b = param(2);
c=  param(3);
d=  param(4);
% this is the 3rd order polynomial equation here
output = a.*input.^3 + b.*input.^2 + c.*input.^1 + d;
```

```
%the better your initial condition guesses are, the faster
%the lsqcurvefit command will converge onto a solution
initialConditions = [-10 5 10 -15];

%newParameters is an array containing the optimal values that will
%generate a curve that will best fit your data

%error is the sum of the error squared.  the lower this number is,
the better
[newParameters,error] = lsqcurvefit(@myPolyCurve,
initialConditions,x,y);

figure
scatter(x,y) %plot the scatter plot
```

```
hold %hold the figure

%use new parameters to get new output values
y2 = myPolyCurve(newParameters,x);

%plot the new data using the color red
plot(x,y2,'r')
%configure the optimset for use with lsqcurvefit
options = optimset('lsqcurvefit');

%increase the number of function evaluations for more accuracy
options.MaxFunEvals = 500;

lb = [ -20 -20 -20 -20]; %define the lower bound
ub =[20 20 20 20]; %define the upper bound

%re-evaluate the curve fit with new options
[newParameters2,error2] = lsqcurvefit(@myPolyCurve,
initialConditions,x,y,lb,ub,options);

%use new parameters to get new output values
y3 = myPolyCurve(newParameters2,x);

%%% plot all the data on the same figure %%%
figure
scatter(x,y) %plot the scatter plot
hold %hold the figure

%plot the new data using the color red
plot(x,y2,'r')

%plot the new data using the color green
plot(x,y3,'g')
```

# APPENDIX-E

## FLOW RATE OPTIMIZATION

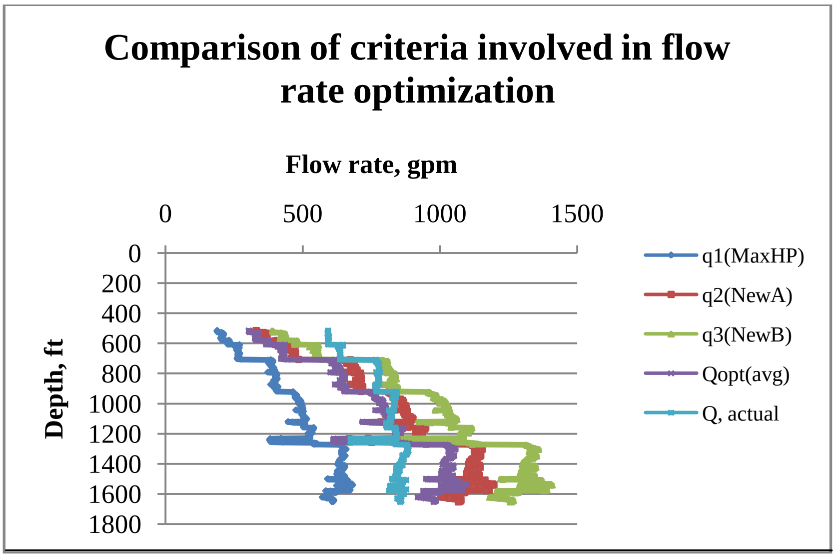


Figure 23: Criteria involved in flow rate optimization

Figure 23 is comparison of all the criteria involved in flow rate optimization, Qopt is the average values of q1, q2 and q3, all the flow rates criteria are compared with Q actual.