The MACBETH Approach to Evaluate Strategies for the Conservation of Architectural Heritage

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Abstract

The critical appraisal of alternative strategies in conservation projects of historical buildings is often a complex task that requires balancing a variety of conflicting criteria related to multiple stakeholders and requirements. The main purpose of this study is to make the case for the use of multi-criteria decision analysis in supporting a consistent and transparent, inter-disciplinary decision-making process, using the case study of repair and strengthening techniques of timber frame walls. The MACBETH approach is particularly suitable in this field of research since the judgment procedures allow for an appropriate analysis of intangible values such as the ones at stake in architectural heritage.

1. Introduction

Architectural heritage encompasses complex challenges in contemporary societies and economies since diverse values are at stake. On the one hand, historic buildings bear cultural values that may have different degrees of significance but are always part of the common heritage of a society, a culture, a country or even humanity. On the other hand, the best strategy to preserve the historic fabric is through its use by the community. Additionally, historic buildings can generate material or immaterial benefits. Therefore, when addressing alternative strategies within a conservation project, contemporary requirements need to be considered, such as structural safety, energy efficiency, and economic viability. The decision of what to do and how to do it is therefore a complex problem and may have a relevant impact on the cultural value of the building.

The conservation of buildings in historic areas has received a significant scholarly and policy interest during the last decade. Architectural heritage demands an inter-disciplinary approach and involves multifaceted cultural and economic values. Therefore, dealing with such types of challenges requires a decision-support tool with the ability of taking into account conflicting objectives and requirements in an integrated and transparent approach. Moreover, all interventions on architectural heritage encompass the consideration of intangible values that cannot be appraised by a deterministic numerical process. Even though Multi-criteria Decision Analysis (MCDA) is an effective tool for dealing with multifaceted contexts, any of these methods is seldom used in the domain of architecture and engineering (Giove et al., 2011) (Ferretti et al., 2014).

This study aims at making the case for the use of MCDA, specifically the MACBETH approach, to support the selection of the best practice in building conservation by considering non-numeric evaluations of alternative options within the scope of conflicting criteria.

2. Methodological Challenges in the Conservation of Architectural Heritage

In addressing the conservation of architectural heritage, the sphere of interest gravitates around three key factors: (i) the historic building itself; (ii) investors and users; (iii) technicians and experts in traditional building structures (e.g. architects, timber engineers, chemists) (Figure 1). Conflicts may occur within a single project due to the coexistence of different and contrasting objectives. For instance, the best options for a good investment turnover can also be the worst solutions if the priority is the safeguarding of the cultural value of the historical building.

The current practice is largely determined by the requirements or preferences of relatively few decision-makers. In fact, owners and investors heavily impact the decision-making process. As an alternative, a well-informed, interactive and transparent procedure is called for with the objective of assuring the best possible compromise between those types of conflicting objectives. The best short (or long)-term intervention strategy can be fostered by a joint team, which comprehensively analyses all intertwined priorities and drawbacks of distinct options. Then, this team, coordinated by an expert advisor, can suggest the best solutions to the decision-maker (DM), e.g. building owner and/or user. The advisor should select the most appropriate solution as a compromise between the cultural value of the building, safety conditions and other code-required actions, and the customer's budgetary constraints together with the conditions related to national and local regulations (Figure 1).

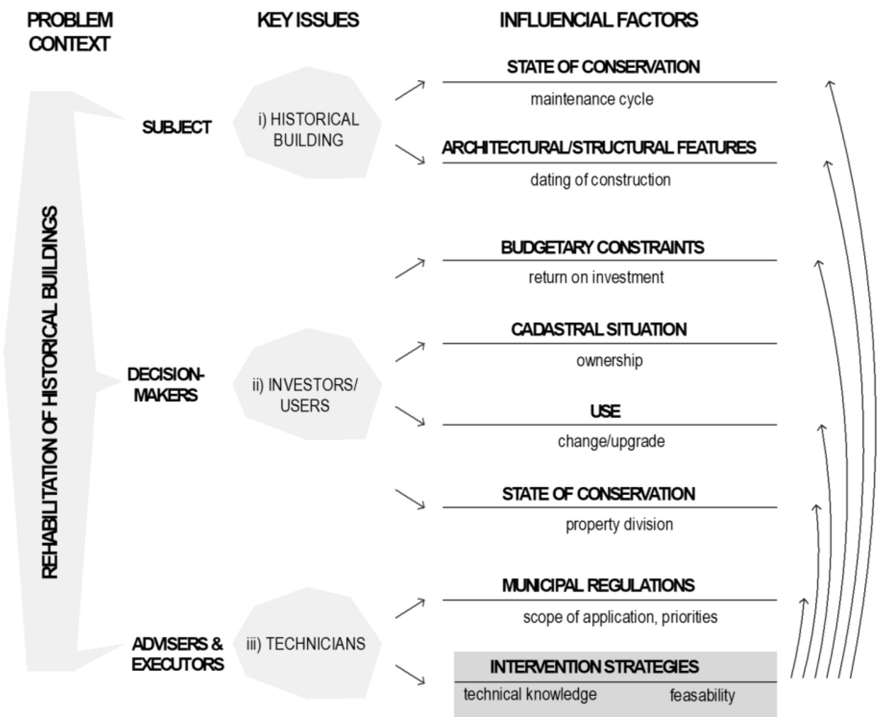

Figure 1. Flow diagram of key factors in the rehabilitation of historical buildings

Depending on the main purpose of each intervention, the team may also define a set of design-related scenarios intended at representing alternative sets of priorities for the decision-making process. The strategies and interventions are case-specific and depend on the cultural value and the state of conservation of the building. For instance, if an historic building has severe structural damage, then it may be acceptable that a part of the cultural value is sacrificed by a more intrusive intervention that assures the structural reliability and, therefore, the very existence of the building itself. On the contrary, a relevant cultural and architectural value (e.g. buildings included in national or regional protected areas, or listed as UNESCO World Heritage) can hinder code-required interventions, like for instance for upgrading thermal comfort conditions, if this is done to the detriment of unique historic materials or integrated artwork. The judgments associated with such interrelated factors and conditions benefit from the support of integrated decision-making processes that include the complexity of the context, and that accurately represent the problem and efficiently allow for variations and alternative scenarios.

3. The MACBETH approach

MACBETH - *Measuring Attractiveness by a Category Based Evaluation Technique* is a decision method developed in the context of multi-criteria decision analysis (MCDA) approaches (Bana e Costa, 1992; Bana e Costa and Vansnick, 1994; Bana e Costa et al., 2015). This method allows the evaluation of several options by including multiple conflicting decision criteria (Bana e Costa and Oliveira, 2012a). The evaluation result is a hierarchy of options, which express a compromise solution. The evaluation reflects the fundamental standpoints and experience of the DM, either an individual or a group (Demesouka et al., 2016). The approach bases the weighting of the criteria and the evaluation of the options in qualitative judgments on differences in attractiveness, instead of attractiveness itself. This feature distinguishes MACBETH from other multi-criteria methods (Mustajoki, 2013). It uses a simple question-answer protocol that involves just two elements under evaluation in each question, requiring only qualitative (semantic) statements to elicit value preferences. According to Fasolo and Bana e Costa (2014), experts feel more comfortable in making comparisons for expressing the importance (or attractiveness) of preferences between every element of evaluation through semantic judgments rather than by numerical values. The MACBETH approach is supported by a dedicated software, M-MACBETH (Bana e Costa and Vansnick, 1999).

As the name suggests, the MACBETH method evaluates attractiveness based on semantic categories that measure the difference in attractiveness. Given two elements x and y under evaluation, it is not enough to know that x is better than y; it is still necessary to measure the difference in attractiveness between them. Is it very weak, weak, moderate, strong, very strong or extreme? Which of these semantic categories define the difference in attractiveness between x and y? The DM is asked to pairwise compare x and y by giving a qualitative judgment of the difference in attractiveness between these two options.

After determining the relevant evaluation criteria and expressing them in a value tree, the DM may choose between a direct or indirect comparison, the latter being a differentiating element of MACBETH when compared to other MCDA methods. This means that the options may be compared through their impact as measured by a given criterion, instead of having to be compared through themselves. In an indirect comparison procedure, a set of performance levels is required for each criterion. These will serve as reference to assess the impact of each option in each criterion. Two of those performance levels may be identified as upper and lower reference levels, so that the DM can compare the relative "strength" of performance levels established for this criterion, as an indirect form of evaluating the options (Bana e Costa et al., 2002b).

Let X be a finite set of elements under evaluation, which can be options, criteria or performance levels of a criterion (options hereinafter). To establish a measure of attractiveness in X consists of obtaining a numerical scale $v: X \rightarrow \mathbb{R}$ that associates a real number $v\ (x)$ to each option x, such that:

(i) **Strict preference condition:** $v\ (x) >\ v\ (y)$ if and only if option x is more attractive than option y, $\forall x, y \in X$;

(ii) **Indifference condition:** $v\ (x)\ =\ v\ (y)$ if and only if options x and y are equally attractive, $\forall x, y \in X$.

In each criterion, these two conditions only assure an ordinal scale of values about the relative attractiveness of options of X, that is, a comparative position of the various options along an oriented axis according to their relative attractiveness in this criterion. As in the presence of multiple criteria the goal is to measure the options' relative attractiveness across all the criteria, direct mathematical manipulation of the data is essential. In fact, according to Arrow's Theorem (Arrow, 1951), the mathematical aggregation process of ordinal information (in each criterion) in order to obtain a global ordinal value information is not exempt from any arbitrariness. Through MACBETH it is possible to go beyond ordinal information by considering the quotient

$$\frac{v(x) - v(y)}{v(z) - v(w)} \qquad (1)$$

as the measure of the attractiveness difference between x and y when the difference of attractiveness between z and w is taken as the unit of measure, for any options $x, y, z, w\ \in X$ such that $v\ (x) >\ v\ (y)$ (x is more attractive than y) and $v\ (z) >\ v\ (w)$ (z is more attractive than w). This additional condition (1) allows obtaining a proportionate cardinal scale of values v in each criterion, which is expressed in numbers and can be manipulated with the support of basic mathematical operators, ensuring a correct aggregation step. Referring back to the oriented axis, now the relative distances between the options of X along it already reflect the relative differences in attractiveness between them.

The transition from ordinal scale to cardinal scale involves the construction of judgment matrices, where the options are pairwise compared in qualitative terms, choosing one of the seven categories of the MACBETH semantic scale (no, very weak, weak, moderate, strong, very strong or extreme). This comparison defines the difference in attractiveness between options. By designating C_k, with $k =\ 0, 1, \ldots, 6$, the categories of the semantic scale, their significance is shown in Table 1.

Table 1. Semantic scale of MACBETH: categories, their importance and equivalent values

Semantic categories	Difference in attractiveness	Numerical values
C_0	No	0
C_1	Very weak	1
C_2	Weak	2
C_3	Moderate	3
C_4	Strong	4
C_5	Very strong	5
C_6	Extreme	6

If the DM makes the qualitative judgment $(x, y) \in C_k$ for options x and y, it means that the difference in attractiveness between x and y is such that

$$v(x) - v(y) = k\alpha \qquad (2)$$

where α is a coefficient needed to satisfy the condition that $0 \le v(x), v(y) \le 100$.

The options are arranged in the matrix in descending order of their importance from left to right and top to bottom and, for a set X of n options, the number of pairwise comparisons can vary from a maximum of $n(n-1)/2$ qualitative judgments to a minimum of $n-1$ qualitative judgments, comparing only each two consecutive ordered options or one option with all of the other ones.

For a matrix of judgments to be consistent, it must lead to scores such that (Bana e Costa et al., 2012b):

(i) Options that are equally attractive should have the same score.

(ii) If one option is more attractive than another one, then it should have a higher score.

(iii) If the difference in attractiveness between one pair of options is greater than the difference in attractiveness between another pair of options, then the options should obtain scores such that the difference between the scores of the first pair is greater than the difference between the scores of the second pair; this requisite is called ordinal consistency condition.

When each judgment is entered in the matrix, its consistency with the previous judgments is checked and possible inconsistencies are detected. Thereafter, the consistent judgments are transformed into proportional cardinal scales. From a consistent judgments' matrix, the method proposes a score for each option. These scores form the MACBETH cardinal scale.

Let x^+ and x^- be the options that are, respectively, more and less attractive (or equally attractive) than any other option in X. The MACBETH cardinal scale results from the following problem of linear programming, where $v\ (x)$ represents the score of option $x \in X$:

$$Min\ [v(x^+) - \ v(x^-)] \qquad (3)$$

subject to the constraints

$$v(x^-) = 0$$

$$v(x) - v(y) = 0, \qquad \forall (x, y) \in C_0$$

$$v(x) - v(y) \ge k, \qquad \forall (x, y) \in C_k\ , \qquad k = 1, \dots, 6$$

$$[v(x) - v(y)] - [v(z) - v(w)] \ge k - K, \qquad \forall (x, y) \in C_k\ , \qquad \forall (z, w) \in C_K, \\ k, K = 1, \dots, 6, \qquad K < k$$

The objective function $v(x^+) - \ v(x^-)$ in (3) is the largest score difference between two options. The sum of all the score differences is thus minimized, allowing qualitative judgments of each category to be as close to each other as possible. When this problem has no solution, the judgment matrix is inconsistent. As such, it is not possible to associate a numerical value to each judgment. If this linear programming problem has multiple optimal solutions, the average of solutions is taken to ensure the uniqueness of the MACBETH scale (Bana e Costa et al., 2005). The constraints guarantee that the numerical values $0, 1, \dots, 6$ are the corresponding scores for, respectively, the qualitative differences in attractiveness expressed by no, very weak,..., extreme.

In case of hesitation or divergence during the judgment process, several consecutive semantic categories can be chosen by the DM in a qualitative judgment. Given options x and y, with x more attractive than y, the DM uses two or more consecutive categories, from C_i to C_j ($i, j = 1, \dots, 6$ with $i < j$), that is, $(x, y) \in C_i \cup \dots \cup C_j$, admitting that he/she hesitates among all categories of attractiveness difference between C_i and C_j. If the DM does not provide any preference, the difference in attractiveness is noted by positive in the corresponding cell of the matrix, indicating that the difference in attractiveness may be any one of the semantic categories.

$$Min\,[v(x^+) - v(x^-)] \tag{3}$$

subject to the constraints

$$v(x^-) = 0$$

$$v(x) - v(y) = 0, \qquad \forall (x, y) \in C_0$$

$$v(x) - v(y) \geq i, \qquad \forall (x, y) \in C_i \cup \dots \cup C_j\,, \qquad i, j = 1, \dots, 6\,, \quad i \leq j$$

$$\begin{aligned}[v(x) - v(y)] - [v(z) - v(w)] \geq i - j', \qquad & \forall (x, y) \in C_i \cup \dots \cup C_j\,, \\ & \forall (z, w) \in C_{i'} \cup \dots \cup C_{j'}, \qquad i, j, i', j' = 1, \dots, 6, \qquad j' < i, \qquad i \leq j, \\ & i' \leq j'\end{aligned}$$

The generalization of the previous problem to determine the MACBETH numerical scale in these cases is the LP-MACBETH problem. It ensures that the lowest possible category is always associated with each judgment involving more than one category (Bana e Costa et al., 2012a).

The MACBETH scale gives place to a numerical scale of intervals (the precardinal scale) that requires however validation by the DM, who may manually change a given score (of a given option), keeping the other scores and still maintaining compatibility with the matrix of judgments (Bana e Costa et al., 2002a). This refinement is accomplished by adjusting the proportion of intervals between the options' scores without violating the ordinal information and within the boundaries that assure judgments' consistency. By default, all scores are displayed on a scale anchored at reference scores 100 and 0. The final adjusted set of judgments originates the cardinal scale. The method will then generate the quantified scores for each option in each criterion through the resolution of a system of type-(2) equations resulting from all the qualitative judgments.

Finally, the global scores representing the options of global attractiveness are computed using an additive aggregation model to rank them, based on the weighting of criteria. If the decision problem involves m criteria, the global score of each option x corresponds to the sum of the products of each criterion weight w_i and the (partial) score $v_i(x)$ of the option on criterion i, with $i = 1, \dots, m$, as indicated above.

$$V(x) = \sum_{i=1}^{m} w_i \cdot v_i\,(x) \tag{4}$$

with $\sum_1^m w_i = 1$, $v_i(x_i^+) = 100$ and $v_i(x_i^-) = 0$.

4. Case study: assessment of intervention techniques on timber frame walls

This section addresses the application of the MACBETH approach for the selection of the best repair and/or strengthening techniques for a traditional timber frame wall (TFW) in a building in Lisbon historic neighbourhood, *Baixa Pombalina*, Portugal (Penn et al., 1996).

Results of this research have been previously published (Stellacci et al., 2018) with a focus on an overview of repair and/or strengthening techniques, and on the possibility of finding a compromise between cultural values and functional requirements (code-required actions). However, a detailed explanation of the MACBETH model structuring was not discussed before. This is thus the main goal of this section.

As above mentioned, the MACBETH approach was chosen for this study due to its ability to encompass tangible and intangible evaluation of options throughout a set of criteria. This feature is particularly significant when cultural values are at stake.

Once the problem was contextualized (Figure 1), the MACBETH analysis was structured into four steps: (i) selection of the repair and/or strengthening techniques (the options of MACBETH model); (ii) definition of a set of criteria, their performance evaluation levels and their respective descriptors; (iii) appraisal the impact of implementing the options using each criterion; (iv) implementation of diverse overall evaluations according to two scenarios, which are common in this domain.

The timber frame wall (TFW) is a traditional load-bearing wall that includes four sub-components: the timber framework (F) and its infill (I); the joints (J), which fasten vertical, horizontal and diagonal bracing members; and the surface finish (S) (Figure 2). The options evaluated in this research are 131 repair and/or strengthening techniques of TFW, selected after an extensive review of scientific literature and from the current practices. The global impact of each of these 131 intervention options on the overall performance of the TFW is the result of its interrelated influence on each of the four sub-components. In this research, the sub-components F and I were evaluated as a single component, since the impact of repair or strengthening technique is similar in these two components.

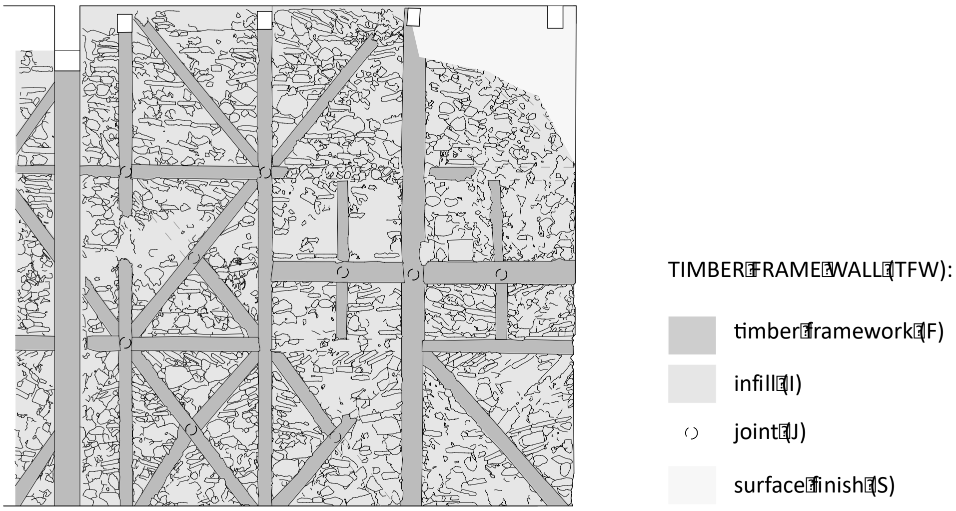

Figure 2. Sub-components of traditional timber frame wall (TFW): timber framework (F); infill (I); joint (J); surface finish (S).

A panel of experts validated the set of 131 options and defined the evaluation criteria for this specific decision-making problem. The panel was composed of a chemist, two architects and two engineers. The criteria that were agreed upon stem from the leading principles of architectural conservation science and were selected so that each one satisfies Roy's axioms: exhaustibility, cohesion, and non-redundancy (Roy, 1996). The criteria are Material Compatibility (MC), Material Permanence (MP), Structural Reliability (SR), Structural Authenticity (SA), and Visual-

Tactile Appearance (VTA). In order to optimize the consistency of the assessments, sub-criteria and specific features were defined for each criterion. Afterwards, performance levels and their respective textual descriptors were established. The impact appraisal was carried out by the panel of experts, as follows: the chemist evaluated the 'Material Compatibility' of each option; the architects addressed 'Material Permanence' and 'Visual-Tactile Appearance'; the engineers evaluated 'Structural Reliability' and 'Structural Authenticity'.

With the exception of the criterion VTA, which is solely related to the surface finish (S), the impact of each option in each criterion was analysed separately according to the three wall sub-components: framework + infill (F+I); joints (J); surface finish (S). Each option may indeed be regarded as a cluster of interrelated aspects (Bana e Costa et al., 2009). However, there is still the need to reach a single performance level. The challenge was therefore to translate partial qualitative assessments into aggregated sound judgments. This was done by using a subset of MACBETH models, assuring coherence throughout the whole process. A sub-model was created for each of the four criteria (MC, MP, SR and SA). In these sub-models, the local preference procedure (local difference in attractiveness) was used to evaluate how different qualitative performance levels compare with each other in terms of their impact in each sub-component of the timber frame wall. Then, the relative importance that may be attributed to each sub-component in the final overall impact on the wall was estimated using the MACBETH global preferences procedure. The final agreed cardinal values representing all these judgments are shown in Table 2.

Through this methodology, it was then possible to appraise the impact of each option according to each criterion. For the criteria MC, MP, SR and SA, this resulted in an impact estimation ranging from 0 to 100 resulting from the above-mentioned subset of models. In the case of the criterion VTA, the impact estimation of each option was performed directly in the global MACBETH assessment model using qualitative performance levels. The difference in attractiveness between these levels was judged through defining local preferences. The resulting cardinal values are also shown in Table 2.

Table 2. Cardinal values used to estimate the options' impact in each criterion (based on Stellacci et al., 2018)

Criterion	**Sub-component of TFW**	**Evaluation level (E_l)** (local preferences)					**Weights** (global preferences)
		H	**M**	**L**	**VL**	**0**	
MC	F+I	100	17	-	0	-	0.42
	J	100	17	-	0	-	0.08
	S	100	71	-	0	-	0.50
MP	F+I	100	75	38	0	-	0.56
	J	100	89	78	67	0	0.06
	S	100	73	46	18	0	0.38
SR	F+I	100	71	41	20	0	0.35
	J	100	58	25	8	0	0.55
	S	100	47	15	7	0	0.10
SA	F+I	100	80	35	20	0	0.45
	J	100	70	25	10	0	0.45
	S	100	70	35	10	0	0.10
VTA	-	100	44	22	11	0	-

In the global MACBETH assessment model, the local preferences for the criteria MC, MP, SR and SA were attributed considering a simple linear variation scale since the performance levels are in this case the result of the integrated impact previously assessed by the subset of individual models for each criterion.

The global model structuring was completed with the definition of global preferences, i.e., the difference in attractiveness between the five mentioned criteria. This same approach was also used to represent two design-related scenarios that are common in architectural heritage conservation projects. The scenarios differ in the degree of intrusiveness of the intervention, considering two key factors: the pre-existing conditions of the building and the expected results of the intervention. The pre-existing condition is assessed through the degree of integrity and authenticity, as well as the level of structural safety of the building. According to this assessment, two main types of strategies may be defined for the expected outcome: a conservative approach, where the cultural value is privileged with the objective of safeguarding the original spatial configuration, building components and materials; or an intrusive approach, where the structural safety is privileged with the objective of safeguarding the built artefact (e.g. due to the risk of collapse). These two scenarios were represented by varying the judgments related to the global difference in attractiveness in the MACBETH model, as shown in Table 3. A third scenario embodies a compromise between the two previous design-based scenarios and was introduced in the modelling to serve as a base for comparison in analysing the results.

Table 3. Cardinal expressions of the judgments representing the design scenarios

Scenario	Criteria weightings (%)				
	MC	**MP**	**SR**	**SA**	**VTA**
1	32	19	5	19	25
2	25	18	39	5	13
3	20	20	20	20	20

Considering that the set of options to be evaluated is very large (131), it was decided to discard those whose global weighted score, for all three scenarios as defined in Table 3, was lower than 50 (in the 0-100 scale). 74 options were thus excluded from the analysis of results. For each scenario, M-Macbeth provides a ranking of the remaining options at stake (57). Within this group, those scoring in the first quarter in at least two scenarios are highlighted in Figure 3: TF01, TF04, TF07, TF10 and TF13.

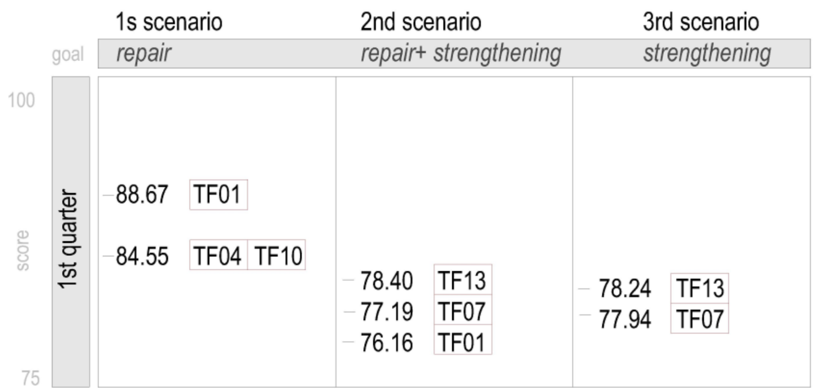

Figure 3. Visual scoring of 1st quarter best-scoring solutions (adapted from Stellacci et al., 2018)

It is important to underline that, among the global group of 131 options considered in this study, just those five repair and/or strengthening techniques for TFW are capable of fulfilling the priorities defined by the group of experts. These best-scoring solutions represent similar interventions on timber framework, infill, and surface, whereas they differ on the type of intervention for the joints (Stellacci et al., 2018).

The goal of this research – finding a compromise solution, supported by a group of experts, whose standpoints are in some cases conflicting – was achieved by circumscribing the priorities in this specific problem context, subdividing the decision process into sub-phases and structuring a MACBETH analysis. Fostering a debate during the attribution of semantic value to the difference between each pair of attributes was crucial to provide a large amount of information gathered by a group of experts over several years.

These experts also performed a qualitative assessment of the consistency of the outputs from the MACBETH model through the different perspectives associated with their field of expertise. It was found that the results accurately express those sensitivities.

5. Conclusions

MACBETH is an appropriate tool for the evaluation of strategies in conservation projects of historic buildings since it allows to build tailor-made parameters for the domain of cultural heritage, i.e. non-numeric, heterogeneous and conflicting criteria. As shown in this study, the application of MACBETH ensures consistency, transparency and legitimacy to the decision-making process related to the ranking of 131 interventions on timber frame walls of historic buildings. Moreover, the effectiveness with which this MCDA approach deals with large sets of inputs is also a relevant observation from the study. Indeed, the outputs reflect the complexity of the decision-making problem in conservation practice, which mostly depends on the coexistence of conflicting judgments of the experts involved in the questioning procedures.

The subjectivity of semantic judgments was overcome in this study by the involvement of a panel of experts in conservation science throughout all phases of the problem structuring. In fact, each one calibrated the evaluation levels of each criterion and judged the options according to his/her own field of expertise. Grounded on a set of data collected through academic and site work research, it is shown in this research that experts' evaluations, although subjective, played a crucial role in the decision-making tool.

The forthcoming applications of this model are extremely challenging. The results encourage further in-depth research with the aim of supporting the evaluation of other complex problems related to the conservation of historic buildings. Furthermore, the way the MACBETH method structures the problem assists by itself in solving conflicting situations as well as in increasing the awareness of the agents involved. This MCDA approach can support the decision-maker in a critical appraisal of a large spectrum of practical solutions.

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