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Bootstrap Resampling in Gompertz Growth Model with Levenberg–Marquardt iteration

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ABSTRACT

Soybean plants have limited growth with a planting period of 12 weeks, which causes the observed sample to be very small. A small sample of soybean plant growth observations can be bias causes in the conclusion of prediction results on soybean plant growth. The purpose this study is to apply the bootstrap resampling technique in Gompertz growth model which overcomes residual distribution with small samples, the research data was taken from soybean plant growth in four varieties with four spacing treatments, five replications and twelve weeks (long planting period). Gompertz growth model uses nonlinear least squares method in estimating parameters with Levenberg–Marquardt iteration. The value of the Gompertz model after resampling bootstrap has no significant difference. The adjusted R2 value of 0.96 is close to 1. This means that the total diversity of plant heights can be explained by the Gompertz model of 96 percent. Judging from the graph of predictions of soybean plant growth before resampling and after resampling coincide with each other it can also be seen in the initial growth values before resampling 14, 05 and 14.18, the maximum growth values are 55.13 and 55.60. Bootsrap resampling technique can overcome residual normality in the Gompertz growth model, but does not change the information in the initial data.

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A. INTRODUCTION

Resampling bootstrap is used to estimate a population distribution with a small sample size or replace the assumption of an unknown distribution with an empirical distribution obtained from resampling (Efron & Tibshirani, 1993). The use of resampling technique allows obtaining distributions without being based on certain distribution assumptions (solimun, 2017). The bootstrap approach uses a sampling method with returns. Generally, research that uses bootstrap resampling technique is to overcome deviations from the normality assumption in modeling, because normality assumptions are needed to draw conclusions on the results of statistical tests (Arnastauskaitė et al., 2021). Erroneous assumptions can lead to wrong conclusions (Achcar & Lopes, 2016). The boostrap resampling technique can be used to overcome normality in nonlinear regression (Larasati, 2020). Study (Pradani et al., 2021) used the bootstrap resampling technique in nonlinear regression to analyze the progress of Covid cases in Indonesia. Study (Bagus et al., 2013) apply bootstrap on nonlinear regression neuro network for forecasting crude oil production in Indonesia.

The growth model is a one of nonlinear regression model (Huang et al., 2010). which is widely applied to agriculture. The growth model is used to determine the relationship between product growth and time (Wardhani & Kusumastuti, 2013). The plant growth model that forms the S curve is called the Sigmoid growth model. The Gompertz model is one of the most frequently used curves in growth mathematics (Chakraborty et al., 2014). The Gompertz model has a sigmoid (Nguimkeu, 2014). The Gompertz curve has been widely used to model and describe behavioral patterns in agriculture, because it has a relatively smaller asymptote value and an asymmetrical (Tjørve & Tjørve, 2017). The Gompertz growth model is appropriate for the phenomenon of growth with two characteristics: restricted growth and sigmoidal behavior (Román-Román et al., 2012). The Gompert model has three parameters (Akin et al., 2020). That can be used to predict the maximum growth time (Panik, 2014). Levenberg-Marquardt iteration is used to estimate the parameters of the Gompertz model. Levenberg-Marquardt algorithm is a combination of Gauss-Newton iteration and the Steepest Descent method (Wang et al., 2022) which can produce faster convergence. The algorithm reduces the number of squares of errors between the model functions and data points through a well-chosen update sequence for the model parameter values (Gavin, 2019). The Levenberg-Marquardt method acts more like the gradient descent method when the parameters are far from their optimal values, and acts more like the Gauss-Newton method when the parameters approach their optimal values (Hecke, 2017).

The growth of soybean plants has a limited growing period, so the experimental sample data is small (Bello et al., 2015). Used bootstrap resampling technique for small sample size data design. The application of bootstrap resampling on the growth model is still very rare, so this study applies the bootstrap resampling technique on the Gompertz growth model to get good soybean plant prediction results. The main idea behind the bootstrap is that in some situations, it is better to make inferences about a population parameter using only the data at hand, without making assumptions about underlying distributions (Rousselet et al., 2021).

This research was taken from soybean growth data, soybean crop Soybean is the third most important food commodity after rice and corn which has a strategic position in national food policy. Soybean consumption projections show that total demand continues to increase from year to year. This research was taken from soybean growth data, the study used a randomized block design with four varieties, four treatments, five replications, and a plant age of 12 weeks. There were 80 individual plants, then samples were taken from the average repetition of each individual. Bootstrap on the growth model resampling between individual plants (Ghosh et al., 2011). It is hoped that this research can describe accurate predictions of soybean plant growth.

B. METHODS

1. Data Sources

This study uses secondary data taken from research data from Agricultural Technology Research Center (BPTP), Indonesia. The object of research on the growth of soybean plants. Soybean plant observation data were collected every week. There are 80 data collected from observations on soybean plants. This research was taken from soybean growth data, the study used a randomized block design with four varieties, four treatments, five replications. The data analysis process is done using of R Studio and microsoft excel software.

2. Analysis Method of Gompertz Model

a. Create a scatter plot between plant height and plant age MST (Week After Planting).

b. Estimating the initial parameters in each growth model (Dreper and Smit, 1998)

1) Determine α (maximum height) of the plant.

2) Determine the value of k based on two arbitrary observations i and j and then enter it into the equation.

$$k = \frac{(Y_2 - Y_1)/(t_2 - t_1)}{\alpha} \quad \textbf{(1)}$$

Where k is growth scale, Y is plant height, t plant age, α is maximum height (so that the iteration process takes faster, in general i and j should be far apart)

3) Calculate β , which is the initial growth value using equation 2.

$$Y_0 = \alpha e^{-\beta} \quad \textbf{(2)}$$

Where e is exponensial, β is the initial growth value

c. Estimating parameters using the nonlinear least squares method with Levenberg-Marquardt iteration (Patmanidis et al., 2017) using equation 3

$$\boldsymbol{\theta}^{(\boldsymbol{n}+\mathbf{1})} = \boldsymbol{\theta}^{(\boldsymbol{n})} - \left[\boldsymbol{Z}\left(\boldsymbol{\theta}^{(\boldsymbol{n})}\right)' \cdot \boldsymbol{Z}\left(\boldsymbol{\theta}^{(\boldsymbol{n})}\right) + \boldsymbol{\lambda I_k}\right]^{-\mathbf{1}} \boldsymbol{Z}\left(\boldsymbol{\theta}^{(n)}\right)' \left(\frac{\partial JKG}{\partial \boldsymbol{\theta}}\right)|_{\boldsymbol{\theta}^{(n)}} \quad \textbf{(3)}$$

Where $\boldsymbol{\theta}^{(\boldsymbol{n}+\mathbf{1})}$ is estimating parameters for each iteration, $\boldsymbol{\theta}^{(\boldsymbol{n})}$ is initial estimator of parameters, $Z\left(\boldsymbol{\theta}^{(\boldsymbol{n})}\right)$ is Matrices derived from parameters functions, $\boldsymbol{\lambda}$ is eigent value, $\boldsymbol{I}$ is Identity matrix.

d. Checking the residual assumptions consisting of the normality assumption of the remainder using the Shapiro-Wilk test (Mohd Razali & Bee Wah, 2011). using equation 4:

$$W = \frac{(\sum_{i=1}^{n} a_i y_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \quad \textbf{(4)}$$

Y_i is plant height, $\bar{y}$ mean sampel, a_i is the expected value of the order statistic and the assumption of homogeneity using the J. Szroeter test"(Kalina & Peštová, 2017).

$$Q = \left[\frac{6n}{n^2 - 1}\right]^{1/2} \left[\frac{\sum_{i=1}^{n} i\varepsilon_i^2}{\sum_{i=1}^{n} \varepsilon_i^2} - \frac{n+1}{2}\right] \quad \textbf{(5)}$$

n is many observations, ε_i is residual

3. Analisis Bootstrap in Growt Model

Bootstrap resampling on the growth model was carried out between individual plants. The steps of the analysis are as follows:

a. Create a Performing bootstrap resampling on soybean plants, each plant item has an observation time of 12 weeks.

b. Estimating the Re-sampling in the form of individual plants.

c. Look for the average sample in each sub sample.

d. Estimating parameters for each sub-sample resulting from bootstrap resampling using nonlinear least squares, with Levenberg-Marquardt iteration

e. Calculate the average parameter of all sub samples (Bose & Chatterjee, 2018). using the formula equation 6.

$$\widehat{\boldsymbol{\theta}}^*(\cdot) = \sum_{b=1}^{B} \widehat{\boldsymbol{\theta}}^*(b) \Big/ B \quad \textbf{(6)}$$

$\widehat{\boldsymbol{\theta}}^*(\cdot)$ is mean of parameter sample, B is total of Bootstrap Resampling

f. Estimating the standard error using the standard deviation for the replicated bootstrap B using equation 7

$$se(\widehat{\boldsymbol{\theta}}_{bs}) = \left[\sum_{b=1}^{B} \left(\widehat{\boldsymbol{\theta}}^*(b) - \widehat{\boldsymbol{\theta}}^*(\cdot)\right)^2 \Big/ (B-1) \right]^{1/2} \quad \textbf{(7)}$$

$Se(\widehat{\boldsymbol{\theta}}_{bs})$ is standar eror of Bootstrap Resampling

g. Testing the significance of the bootstrap model with the t-test of equation (Ibrahim et al., 2009). Using equation 8

$$t = \frac{\hat{\beta}^*(\cdot)}{\hat{s}e(\hat{\beta}^*)} \quad \textbf{(8)}$$

t is t-tes of Bootstrap Resampling

h. Comparing the estimation results before resampling and after resampling

i. "Model Goodness Test"(Lenart & Missov, 2016).

C. RESULT AND DISCUSSION

1. Scatter Plot

The scatter diagram of soybean plant height against age is described as shown in Figure 1.

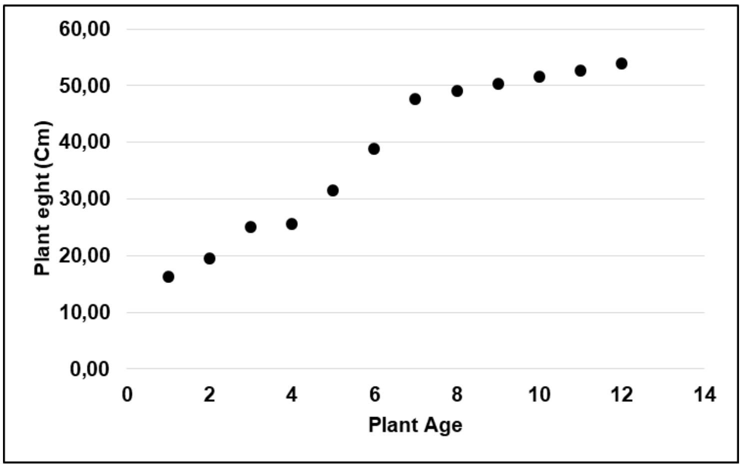

Figure 1. Figure 1 Scatter plot of soybean growth

Based on (Figure 1) the growth of soybean plant height formed a sigmoid pattern so it was hoped that the Gompertz growth model could describe the growth pattern of soybean plants well. The growth of soybean plant height before resampling was done at the age of 3-4 WAP and 8-12 WAP tended to be slow but at the age of 0-3 and 4-7 WAP the growth was relatively fast. Based on Figure 1 the data pattern of soybean plant growth is not linear so the growth model is the right suggestion to find the effect between variables (Hipkins & Cowie, 2016).

2. Initial parameters of the growth model

Estimating the nonlinear regression parameters using nonlinear least square (NLS) with Levenberg Marquardt iteration. The initial estimated value of the parameter must be determined before carrying out the iteration process. As shown in Table 1.

Table 1. Initial Estimation Value of the Parameter

Initial Estimation of Parameter		
α	β	k
53,97	1,20	0,06

Based on Table 1 are the initial parameters obtained using equation 1 and equation 2, where is the maximum growth height, is the initial growth height, and k is the growth scale. The initial parameter values in Table 1 will be used to estimate the actual parameters with the Levenberg-Marquardt iteration. Parameter values α, β, k in table 1 which will determine Levenberg-Marquardt iteration runs fast or slow.

3. Parameter Estimator with Levenberg-Marquardt iteration

The Levenberg-Marquardt iteration is used to find the true parameter estimator which converges. As shown in Table 2.

Table 2. Values of Levenberg-Marquardt Iteration Results

Levenberg-Marquardt Iteration		
$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{k}$
62,06 (p = 0,00000)*	1,87 (p = 0,00000)*	0,23 (p = 0,00072)*

Based on Table 2, the parameter estimation of the Levenberg-Marquardt iteration results is used to form the Gompertz growth model. The Levenberg-Marquardt iteration reaches convergence in a short time and gets a significant parameter estimate value. The p-value of the parameters ,β,k is smaller than the 0.05 level of significance. This means that the parameters can explain the growth of soybean plants. α Parameter that describes the maximum height, β is the initial growth height, k is the soybean growth scale. The values of α, β, k are the parameters of the Gompertz model used to determine the prediction of soybean plant growth (Conde-Gutiérrez et al., 2021).

4. Gompertz Growth Model

The Gompertz Model Equation is formed from the convergent Levenberg-Marquardt iteration results, as shown in Table 3.

Table 3. Gompertz Growth Model

Gompertz Growth Model
$Y_t = 62{,}06\exp^{-1{,}87\exp^{-0{,}23t}}$

Based on Table 3, the Gompertz Growth Model with Levenberg-Marquardt iteration. The estimation of the parameters of the Gompertz model is a unit that cannot be interpreted individually as in multiple linear regression. This value must be processed first in order to get a forecast of soybean plant height at age t. Processing by substituting the t-value in the model.

5. Residual Normality Test

Normality test the residual normality test was carried out using the Shapiro-Wilk test based on the following hypothesis:

H_0: the remainder is normally distributed

H_1: the remainder is not normally distributed

Accepted if $nilai\ p > \alpha$ (0,05), it means the residual is normally distributed if $nilai\ p < \alpha$ (0,05) it means the residual is not normally distributed, as shown in Table 4.

Table 4. Value of Normality Test Results.

P-value	Decision	Conclution
0,99	Terima H_0	Normal

From the results of the residual normality test in Table 4, it was obtained that the p-value was greater than the soybean plant growth data before resampling was carried out. This means that the residuals are normally distributed.

6. Homogeneity Test

The test of homogeneity residual variance in the model can be determined by performing the Szroeter test based on the following hypothesis.

H_0: residual is homogen

H_1: residual is not homogeny

At a significant level of if $|Q| < |Z_{\alpha/2}|$, then H_0 it is accepted which means the variance is homogeneous, as shown in Table 5.

Table 5. Value of Homogeneity Test Results

$\|Q\|$	$Z_{\alpha/2}$	Decision	Conclution
0,41	1,96	H_0 accepted	homogeneous.

From the results of the homogeneity test of residual variance in Table 5, the value of $|Q|$ $0,41 < Z_{\alpha/2} = 1,960$. is obtained smaller than then accept which means the residual is homogeneous.

7. Bootstrap resampling

Bootstrap resampling is used to overcome residual distribution in small sample size data. Bootstrap resampling was carried out one hundred times on 16 sample data with 12 planting periods on soybean growth. With the hope that the estimated value of the parameter will be better.

8. Parameter Estimation after resampling with bootstrap

The average value of parameter estimation after resampling bootstrap with B=100, as shown in Table 6.

Table 6. Values of Levenberg-Marquardt Iteration Results after Bootstrap resampling

Bootstrap Parameter		
α	β	k
62,36 (p=0,00000)*	1,86 (p=0,00000)*	0,23 (p= 0,00000)*

Table 6 describes the estimated parameter values after resampling. The average value of the Levenberg-Marquardt iteration parameter estimates in Table 6 will be used to form the Gompertz growth model. The p-value of the parameters , β, k is smaller than the 0.05 level of significance. This means that the parameters can explain the growth of soybean plants.

9. Gompertz Growth Model after Bootstrap resampling

Table 7 shows the Gompertz growth model with Levenberg-Marquardt iteration. The estimation of the parameters of the Gompertz model is a unit that cannot be interpreted individually. This value must be processed first in order to get the height of soybean plants at age t. Processing by substituting the t-value in the model.

Table 7. Gompertz Growth Model after Bootstrap Resampling

Model After Bootstrap Resampling
$Y_t = 62{,}36\exp^{-1{,}87\exp^{-0{,}23t}}$

10. Plant Height Comparison Graph

The Graph below illustrates the predictions of soybean plant height before sampling and after sampling using bootstrap, as shown in Figure 2.

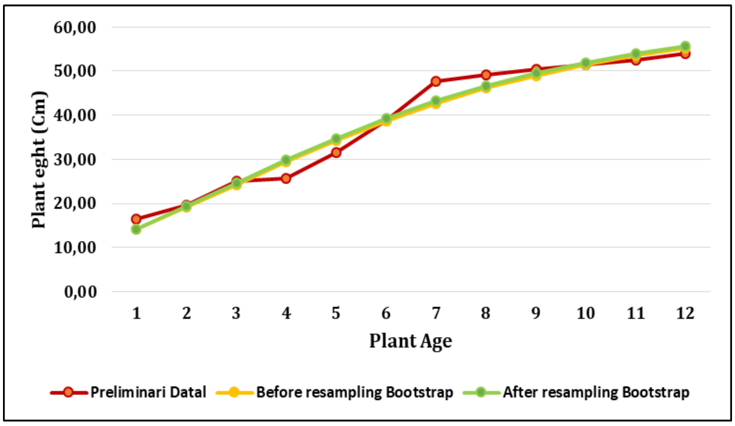

Figure 2. Soybean plant height against plant age, and predictions of growth before sampling and after sampling using bootstrap.

Based on Figure 2 the pattern of soybean plant height growth before resampling and after resampling was close to the actual soybean plant height growth. This shows that the Gompertz model can explain the growth of soybean plant height well. The growth graph of soybean plant height before resampling and after resampling coincided, meaning that resampling did not change the information, and was only an attempt to fulfill the assumption of normality.

11. Comparison of Gompertz Parameters

Table 8. P-value Bootstrap Resampling

Parameter	Before Resampling	After Resampling Bootstrap	*SE* Bootstrap	P-value Bootstrap
α	62,06	62,36	1,61	0,000
β	1,87	1,86	0,05	0,000
k	0,23	0,23	0,00	0,000

Based on Table 8, the P-value of the Gompertz parameter after bootstrap resampling < than =0.05, which means the Gompertz growth model after bootstrap resampling can explain soybean plant growth well. Residual-based bootstrapping in regression is able to explain the parameters in the model estimation without worrying about the assumption error (Li & Dimitris, 2016).

12. Gompertz Model Comparison

Gompertz model before resampling and after resampling using bootstrap, Comparison of the Gompertz model was measured using the goodness test of the adjusted R^2 model. The measurement results, as shown in Table 9.

Table 9. Comparison of R^2adjusted value

R^2 adjusted Before Bootstrap Resampling	R^2 adjusted After Bootstrap Resampling
0,96	0,96

Based on Table 9, the Gompertz model before resampling and after resampling has an adjusted R^2 value of 0.96, which means that the total plant height diversity can be explained by the Gompertz model of 96 percent, meaning that the Gompertz model with three parameters can describe the effect of planting time on plant growth Soya Bean. There is no significant difference in the Gompertz model after resampling with B=100. As shown in Table 10.

Tabel 10. Soybean Plant Height Prediction

Age (week)	Soybean Plant Height Prediction Before Resampling (cm)	Soybean Plant Height Prediction After Resampling (cm)
1	14,05	14,18
2	19,06	19,28
3	24,29	24,60
4	29,46	29,83
5	34,33	34,77
6	38,77	39,25
7	42,70	43,20
8	46,11	46,62
9	49,02	49,52
10	51,45	51,95

Age (week)	Soybean Plant Height Prediction Before Resampling (cm)	Soybean Plant Height Prediction After Resampling (cm)
11	53,47	53,95
12	55,13	55,60

Soybean crop prediction in Table 10 was determined from the substitution of the t-value in the Gompertz model. The predicted results of the Gompertz model plant height in the first week after planting on the data before resampling were 14.05 cm, in the data after resampling it was 14.18 cm. and the maximum limit for the prediction of plant height growth in the Gompertz model before resampling was 55.13 cm, the data after resampling was 55.60 cm. in table 10 the difference in soybean plant growth between before resampling and after resampling is not much different. Bootstrap resampling is used to overcome assumption errors in small data regardless of assumptions (Rahman et al., 2013). In this study, the Gompertz growth model with Levenberg-Marquardt iteration can predict plant growth without resampling. The Levenberg-Marquardt iteration which can considerably speed up the convergence rate by reducing the iteration process and then produce more accurate data the LM algorithm have been shown to achieve excellent results, especially when applied to evaluation and prediction (Zhou et al., 2018) The Levenberg-Marquardt algorithm on the gompertz model can achieve convergence with a short and efficient number of iterations (Conde-Gutiérrez et al., 2021).

D. CONCLUSION AND SUGGESTIONS

The Gompertz model equation before resampling and after resampling using bootstrap has a value of 0.96 close to 1 , which means the model can predict well and the total plant height diversity can be explained by the Gompertz model of 96 percent. It can be seen from the graph that predictions of soybean growth before resampling and after resampling coincide with each other it can also be seen in the initial growth values before resampling 14, 05 and 14.18, the maximum growth values are 55.13 and 55.60. This means that the prediction results of soybean plant height before resampling and after resampling have close values. The resampling process in the growth model does not change the information, and only attempts to fulfill the assumption of residual normality.

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