Large-Scale Preventive Security-Constrained Unit Commitment Considering *N-k* Line Outages and Transmission Losses

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Abstract—This paper presents a new formulation for the preventive security-constrained unit commitment problem modeling N-kline outages and transmission losses. The pre- and post-contingency transmission constraints, representing N-k line outages, are explicitly included by using generalized generation distribution factors. To account for security, a contingency selection procedure based on line outage distribution factors finds a list of worst-case contingencies. Transmission losses are incorporated using piecewise linear expressions. The proposed model is formulated as an instance of mixed-integer linear programming. The effectiveness of the proposed approach is illustrated with the IEEE 57-bus system and the 1,354-bus portion of the European transmission system. As empirically evidenced, the explicit consideration of N-k line outages and transmission losses leads to different decisions in the generation scheduling and dispatch, ensuring secure power system operation.

Index Terms—Generalized generation distribution factors, *N*-*k* line outages, preventive security-constrained unit commitment, transmission losses.

NOMENCLATURE

Parameters

 α_i

No-load cost coefficient of unit *i*,

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 $\xi_{g,i}$ $\zeta_{l,m}^{N-0}$

 $\zeta_{l,m}^{N-k}$

 $C_{j,t}^{ENS}$

 F_i

k

 K^{Max}

 $\begin{array}{c} Pd_{j,t} \\ P_i^{Max}, P_i^{Min} \end{array}$

 RD_i, RU_i

 L_i

 R_t

 $S_{i,0}$

 DT_i, UT_i

 $F_m^{Max,N-0}$

 $F_m^{Max,N-k}$

 $\begin{array}{c} G_m\\ GGDF_{m,i,t}^{N-0} \end{array}$

Slope of block g of the piecewise linear production cost function of unit i,

Parameter of the lth block used in the linearization of the loss function of line m in the pre-contingency state,

Parameter of the *l*th block used in the linearization of the loss function of line *m* in the post-contingency state,

 $B_m \qquad \qquad \text{Susceptance of line } m, \\ C_{i,t}^D, C_{i,t}^U \qquad \qquad \text{Shutdown and start-up}$

Shutdown and start-up cost coefficients of unit *i* in period *t*,

Penalty cost coefficient for the load shedding at bus *j* in period *t*,

Minimum down and up times of unit *i*,

Number of periods unit *i* must be initially online due to its minimum up time constraint,

Pre-contingency active power capacity of line *m*,

Post-contingency active power capacity of line *m*,

Conductance of line *m*,

Pre-contingency generalized generation distribution factor for line *m* and a shift in generation of unit *i* in period *t*,

 $GGDF_{m,i,t}^{N-k}$ Post-contingency generalized generation distribution factor for line *m* and a shift in generation of unit *i* in period *t*,

Number of line outages: k = 0 denotes the pre-contingency state, k > 0 denotes a post-contingency state,

- Maximum number of line outages,
- Number of periods unit *i* must be initially offline due to its minimum down time constraint,

Net demand of bus *j* in period *t*,

Maximum and minimum production limits of unit *i*,

System spinning reserve in period t,

Ramp-down and ramp-up rate limits of unit *i*,

Number of periods unit *i* has been offline prior to the first period of the time span,

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SD_i, SU_i	Shutdown and start-up ramp rate limits of unit <i>i</i> ,
$U_{i,0}$	Number of periods unit i has been online prior to the first period of the time energy
$V_{i,0}$	Initial commitment state of unit i (1 if it is online, 0 otherwise).
Variables	
$\Delta f_{l,m,t}^{N-0}$	Contribution of block l to the pre- contingency active power flow on line m in period t
$\Delta f_{l,m,t}^{N-k}$	Contribution of block l to the post- contingency active power flow on line m in period t .
$\Delta p_{g,i,t}$	Active power produced in block <i>g</i> of the piecewise linear production cost function of unit <i>i</i> in period <i>t</i> .
cd. +	Shutdown cost of unit i in period t .
$cu_{i,t}$	Start-up cost of unit <i>i</i> in period <i>t</i> ,
$ens_{j,t}$	Load shedding at bus <i>j</i> in period <i>t</i> ,
$f_{m,t}^{N=0}$	Pre-contingency active power flow on line m
$f_{m,t}^{N-k}$	in period <i>t</i> , Post-contingency active power flow on line
$f_{m,t}^{+,N-0}, f_{m,t}^{-,N-0}$	<i>m</i> in period <i>t</i> , Auxiliary variables used to model the pre- contingency active power flow on line <i>m</i> in
$f_{m,t}^{+,N-k}, f_{m,t}^{-,N-k}$	period t , Auxiliary variables used to model the post- contingency active power flow on line m in
$f_{m,t}^{loss,N-0}$	period <i>t</i> , Pre-contingency active power loss on line <i>m</i>
$f_{m,t}^{loss,N-k}$	Post-contingency active power loss on line <i>m</i> in period <i>t</i> .
$p_{i,t}$	Active power generation of unit <i>i</i> in period
$ar{p}_{i,t}$	<i>t</i> , Maximum available active power generation
$v_{i,t}$	Binary variable equal to 1 if unit i is online in period t and 0 otherwise.
Sets	
Lg	Set of indexes of blocks of the piecewise linear production costs
Ls	Set of indexes of blocks of the piecewise lin- ear approximations of transmission losses.
Nb	Set of bus indexes,
Ng	Set of generating unit indexes,
Nĺ	Set of indexes of transmission lines in ser-
	vice in the pre-contingency state,
Nlc	Set of indexes of transmission lines in ser- vice under contingency,
Т	Set of time period indexes.

I. INTRODUCTION

G ENERATION scheduling in electricity markets relies on unit commitment (UC) models wherein a cost-related objective function is minimized subject to operational and technical constraints over a specific scheduling horizon [1]. Among other practical features, widely used UC models incorporate the effect of the transmission network, giving rise to the notion of Security-Constrained Unit Commitment (SCUC) [2]. For tractability purposes, system operators in the US typically adopt a dc-based power flow model to characterize the transmission network in the SCUC [3]. To that end, linear sensitivity factors (LSF) are useful to provide a reduced solution space without affecting optimality [4]–[6]. Notwithstanding, it should be mentioned that attaining optimality for large-scale SCUC instances is challenging because of problem size and NP-hardness [7].

The complexity of SCUC is further stressed due to the requirements set by the North American Electric Reliability Corporation (NERC), whereby constraints modeling the loss of system components are also part of the problem formulation. Due to computational and economic reasons, system operators have typically considered the loss of two components at most through the well-known N-1 and N-2 security criteria [1]. However, power systems are critical infrastructures [8] that have recently experienced catastrophic events worldwide involving the loss of more than two components [9]. Thus, alternative, albeit still within NERC standards [10], security criteria have been suggested, thereby giving rise to challenging SCUC models. Relevant examples are the widely studied N-k criterion [11]–[15] and the N-1-1 criterion recently addressed in [16].

NERC requirements can be implemented by either preventive [17]–[19] or corrective [11]–[16] approaches, wherein the values of decision variables under contingency are respectively disallowed or allowed to change with respect to those under the normal state or base case. Corrective approaches frequently require operator actions, for instance to redispatch several generating units in a very short time (10 to 20 minutes), to avoid undesired conditions regarding overcost, overloading, and load shedding, among other extreme actions [20]. Additionally, such changes in the generation dispatch must comply with ramping limitations without affecting power system security. Moreover, the implementation of such corrective activities is a complex task due to the potentially large number of actions involving different entities, which makes the required communication process challenging. Unlike corrective SCUC, preventive SCUC (PSCUC) precludes overloading in the post-contingency steady state while keeping the same values for operational decision variables in the pre- and post-contingency states. Thus, the consideration of a tighter preventive model for power system operation yields a more expensive optimal solution [21]. By contrast, the use of a preventive model gives rise to a simpler SCUC instance as a single set of generation dispatch variables, a single set of transmission sensitivity factors, and a single set of pre- and post-contingency power balance equations are considered per period. Therefore, PSCUC represents a practical trade-off solution between computational burden and cost. This study is focused on PSCUC, which is the standard in industry practice [22].

Some relevant works on PSCUC can be found in [17]–[19]. In [17] and [18], the deterministic PSCUC problem is addressed by an iterative approach involving the solution of a relaxed version

of the original model and a power-flow-based N-1 security analysis. In the relaxed problem, the base case and a reduced set of contingencies are considered. Using line outage distribution factors (LODF) and the pre-contingency power flow solution, an N-1 security analysis is implemented by solving a power flow for every single line contingency. For the resulting N-1critical line contingencies, i.e., those leading to an overloading condition, the corresponding transmission constraints are added to the formulation of the relaxed problem. Therefore, an iterative filtering process is implemented to determine the minimum set of contingency constraints limiting the feasible solution space. Although this sequential algorithm is suitable for N-1 line outages, its extension to tighter security criteria involving N-k contingencies may experience some computational issues [23]. In [19], the authors extend their previous work [18] to solve the stochastic PSCUC problem applying the progressive hedging algorithm. In order to manage the scenarios characterizing uncertainty, the pre- and post-contingency subproblems are relaxed using LSF.

To the best of our knowledge, few literature contributions address the PSCUC problem for large-scale power systems. Existing works [17]–[19] rely on iterative decomposition-based approaches wherein, at each iteration, an N-I security analysis is performed for every contingency state. Hence, the computational effort required by these approaches is proportional to the number of contingency states accounted for. As a result, computational issues may arise when outages of multiple components are handled, as is the case of N-k line outages [21], [23], [24], and there is no guarantee to obtain the optimal solution because of the nonconvex solution space. Moreover, as another major drawback of previous works, transmission network losses are neglected, thereby potentially leading to suboptimal or even infeasible solutions.

As an alternative to existing works, we present a new formulation for the PSCUC problem to explicitly incorporate the pre-contingency state, N-k line outages, and transmission losses. The proposed model is built upon the classical unit commitment formulation described in [25], where both component outages and the effect of the transmission network were disregarded. The *N-k* contingency constraints consider the outage of the most critical transmission lines or a given list of credible contingencies with up to k lines simultaneously or near simultaneously out of service. Thus, no intervening time between consecutive outages is allowed. Pre- and post-contingency transmission constraints are jointly considered by the use of a particular class of LSF, namely generalized generation distribution factors (GGDF), which was developed in [26] and successfully applied to other network-constrained operational [6] and planning [24] models. Similar to other modeling solutions based on different LSF, the use of GGDF allows explicitly representing both preand post-contingency states in the problem formulation while requiring fewer decision variables than the classical dc-based formulation [6]. As a result, the computational burden is reduced without sacrificing optimality. Moreover, as line flows are solely related to generation levels, the analysis of results is facilitated particularly within the generation scheduling context of the unit commitment problem under consideration. More importantly, GGDF matrices are independent of the choice for the reference bus. Hence, although the problem size remains unchanged as compared to the use of other LSF [6], the computational performance does not depend on such a choice, which may be relevant for practical implementation purposes. In addition, as done in [27] for the transmission network expansion planning problem, piecewise linear approximations are used to represent the quadratic functions characterizing transmission losses. It should be noted that the consideration of active power losses, which are usually neglected in the traditional PSCUC problem, may yield changes in the UC decisions. The resulting model is cast as an instance of mixed-integer linear programming (MILP). Therefore, finite convergence to the optimum is guaranteed, a measure of the distance to optimality is provided along the solution process, and effective off-the-shelf software is available.

The main contributions of this paper are as follows:

- A new PSCUC approach based on GGDF is proposed to effectively consider *N-k* line outage constraints.
- For the first time in the related literature, the effect of preand post-contingency transmission losses on the PSCUC solution is examined and extensively evaluated.
- 3) A study of the benefits of precisely considering pre- and post-contingency *N-k* line outage constraints and transmission losses is presented for two test systems with up to 1,354 buses. It is worth highlighting that the attainment of high-quality near-optimal solutions for such large-scale instances in practical computing times demonstrates the effective performance of the proposed approach in terms of solution quality and computational effort.

The significance of such contributions is backed by (i) the practical interest of SCUC models in current electricity markets, and (ii) the need for solutions effectively addressing the challenges related to the practical applicability of such models. It is worth emphasizing that, unlike previously reported approaches, the proposed tool provides valuable information about the impact of tighter security criteria and a more accurate network model on generation scheduling for large-scale power systems, thereby allowing system operators and regulators to make informed decisions on the practical adoption of these modeling features.

The remainder of the paper is organized as follows. Section II describes the proposed PSCUC model. Numerical results are provided and discussed in Section III. Section IV draws relevant conclusions and suggests future avenues of research. Finally, the conventional SCUC model used for contingency selection is formulated in the Appendix.

II. PROPOSED PSCUC MODEL

The optimization problem addressed in this paper extends conventional PSCUC formulations by incorporating N-k line outages and transmission losses. To that end, pre- and postcontingency GGDF-based network constraints are both considered. This section first describes the process used to select the N-kline outages. Subsequently, the proposed MILP-based problem formulation is presented in detail. Fig. 1 depicts the flowchart for the proposed approach.



Fig. 1. Flowchart for the proposed approach.

A. Contingency Selection

In real power systems, not all line contingencies result in a post-contingency alert condition. Rather, only severe outages produce the overloading of transmission elements eventually requiring load shedding. Thus, a subset of contingencies is often used to obtain a computationally tractable SCUC problem [28]. Using that approach, the proposed PSCUC model includes a set of post-contingency constraints associated with the pre-specified contingency set.

First, based on [6], a conventional SCUC problem is solved using the power generation data, the transmission network constraints, and the hourly profile of the peak net demand, where net demand refers to the demand supplied by conventional generation. The interested reader is referred to the Appendix for the detailed formulation of the conventional SCUC. For the selection of contingencies, line outages are characterized by their time of occurrence and duration. Moreover, we assume that (i) if the system can withstand a line outage for the peak net demand, it will most likely be able to cope with line outages under lower net demand scenarios, and (ii) based on a preventive analysis, both operational data and generation levels do not change when line outages are simulated.

Using the pre-contingency power flows for the peak net demand solution, the post-contingency *N-1* power flows are obtained based on the generation schedule provided by the conventional SCUC and the LODF matrix. Note that radial lines and islanding conditions are not included in this study. For tighter

security levels, i.e., for k > 1, the methodology described in [24] is implemented to obtain the post-contingency *N*-*k* power flow solution, thereby only requiring pre-contingency information. As a result, for each value of *k*, the event causing the largest post-contingency overload is considered as the most severe *N*-*k* contingency.

As sketched in Fig. 1, this iterative process gives rise to a contingency list for the PSCUC problem wherein the contingency state at a given iteration comprises the outages of the lines identified as critical so far.

B. Objective Function and Cost-Related Terms

The aim of the proposed model is the minimization of the sum of production, start-up, shutdown, and load-shedding costs under normal operating conditions (1). Note that the cost of losses is implicitly accounted for through the production costs included in the objective function. Because of the preventive formulation, the pre- and post-contingency costs are the same [6].

$$Min \sum_{t \in T} \sum_{i \in Ng} \left(\alpha_i v_{i,t} + \sum_{g \in Lg} \xi_{g,i} \Delta p_{g,i,t} + c u_{i,t} + c d_{i,t} \right)$$
$$+ \sum_{t \in T} \sum_{j \in Nb} C_{j,t}^{ENS} ens_{j,t}.$$
(1)

Start-up costs are modeled in (2)–(3), whereas expressions (4)–(5) are related to shutdown costs.

$$cu_{i,t} \ge C_{i,t}^U \left(v_{i,t} - v_{i,t-1} \right); \ \forall i \in Ng, \ \forall t \in T$$

$$cu_{i,t} \ge 0; \ \forall i \in Ng, \ \forall t \in T$$
 (3)

$$cd_{i,t} \ge C_{i,t}^D \left(v_{i,t-1} - v_{i,t} \right); \forall i \in Ng, \, \forall t \in T$$

$$(4)$$

$$cd_{i,t} \ge 0; \ \forall i \in Ng, \ \forall t \in T.$$
 (5)

C. Power Balance and Reserve Requirements

System power balance is formulated in (6). Our extensive numerical testing reported in [29] reveals that active power losses are bigger under contingency than in the normal state. Therefore, for the sake of simplicity, post-contingency transmission losses, $f_{m,t}^{loss,N-k}$, have solely been considered in (6). As is customary in the unit commitment literature [1], [2], [4]–[6], [12], [17], [23], [25], system reserve requirements are imposed in (7) so that sufficient generation reserves are ensured to withstand generator outages and fluctuations in nodal net injections.

$$\sum_{i \in Ng} p_{i,t} + \sum_{j \in Nb} ens_{j,t} = \sum_{j \in Nb} Pd_{j,t}$$
$$+ \sum_{m \in Nlc} f_{m,t}^{loss,N-k}; \forall t \in T$$
(6)

$$\sum_{i \in Ng} \bar{p}_{i,t} \ge \sum_{j \in Nb} Pd_{j,t} + R_t; \forall t \in T.$$
(7)

D. Generation Operation

The generation blocks used in the piecewise linear production costs are characterized in (8) and (9). Based on the model for generation operation presented in [25], production limits are set in (10)–(11), ramp rates are modeled in (12)–(14), minimum up times are expressed in (15)–(17), and minimum down times are cast in (18)–(20). Finally, expressions (21) model binary generation scheduling variables $v_{i,t}$.

$$p_{i,t} = \sum_{g \in Lg} \Delta p_{g,i,t}; \forall i \in Ng, \, \forall t \in T$$
(8)

$$0 \le \Delta p_{g,i,t} \le \frac{P_i^{Max}}{|Lg|}; \forall g \in Lg, \, \forall i \in Ng, \, \forall t \in T$$
 (9)

$$P_i^{Min}v_{i,t} \le p_{i,t} \le \bar{p}_{i,t}; \forall i \in Ng, \,\forall t \in T$$

$$\tag{10}$$

$$0 \le \bar{p}_{i,t} \le P_i^{Max} v_{i,t}; \forall i \in Ng, \,\forall t \in T$$
(11)

$$\bar{p}_{i,t} \le p_{i,t-1} + RU_i v_{i,t-1} + SU_i (v_{i,t} - v_{i,t-1}) + P_i^{Max} (1 - v_{i,t}); \forall i \in Ng, \forall t \in T$$
(12)

$$\bar{p}_{i,t} \le P_i^{Max} v_{i,t+1} + SD_i \left(v_{i,t} - v_{i,t+1} \right); \forall i \in Ng,$$

$$t = 1, \dots, |T| - 1$$
(13)

$$p_{i,t-1} - p_{i,t} \le RD_i v_{i,t} + SD_i (v_{i,t-1} - v_{i,t}) + P_i^{Max} (1 - v_{i,t-1}); \forall i \in Ng, \ \forall t \in T$$
(14)

$$\sum_{t=1}^{F_i} (1 - v_{i,t}) = 0; \forall i \in Ng | F_i > 0$$
(15)

$$\sum_{n=t}^{t+UT_{i}-1} v_{i,n} \ge UT_{i} \left(v_{i,t} - v_{i,t-1} \right); \forall i \in Ng,$$

$$t = F_{i} + 1, \dots, |T| - UT_{i} + 1$$
(16)

$$\sum_{n=t}^{|T|} [v_{i,n} - (v_{i,t} - v_{i,t-1})] \ge 0; \forall i \in Ng,$$

$$t = |T| - UT_i + 2, \dots, |T|$$
(17)

$$\sum_{t=1}^{L_i} v_{i,t} = 0; \forall i \in Ng \, | L_i > 0 \tag{18}$$

$$\sum_{n=t}^{t+DT_i-1} (1-v_{i,n}) \ge DT_i (v_{i,t-1}-v_{i,t}); \forall i \in Ng,$$

$$t = L_i + 1, \dots, |T| - DT_i + 1$$
(19)

$$\sum_{n=t}^{|T|} [1 - v_{i,n} - (v_{i,t-1} - v_{i,t})] \ge 0; \forall i \in Ng,$$

$$t = |T| - DT_i + 2, \dots, |T|$$
(20)

$$v_{i,t} \in \{0,1\}; \forall i \in Ng, \ \forall t \in T$$

$$(21)$$

where

$$F_{i} = Min\{|T|, (UT_{i} - U_{i,0}) V_{i,0}\}; \forall i \in Ng$$

$$L_{i} = Min\{|T|, (DT_{i} - S_{i,0})(1 - V_{i,0})\}; \forall i \in Ng.$$

E. Transmission Network Operation

As is customary in generation scheduling, the transmission network effect is characterized using a linear dc power flow model. As described in [6], line flows in a linear power flow model can be expressed using GGDF. For the proposed PSCUC model, pre- and post-contingency GGDF matrices are computed *ex ante* once the contingency list is determined according to the procedure presented in Section II-A. For each contingency state comprising *k* line outages, a different GGDF matrix is rapidly obtained based on the pre-contingency GGDF. For a detailed description of the computation of pre- and post-contingency GGDF, the interested reader is referred to [24].

Using the GGDF-based framework formulated in (22) and (23), pre- and post-contingency line flow limits are imposed in (24) and (25), respectively.

$$f_{m,t}^{N-0} = \sum_{i \in Ng} GGDF_{m,i,t}^{N-0} p_{i,t}; \forall m \in Nl, \, \forall t \in T$$
(22)

$$f_{m,t}^{N-k} = \sum_{i \in Ng} GGDF_{m,i,t}^{N-k} p_{i,t}; \forall m \in Nlc, \, \forall t \in T$$
(23)

$$-F_m^{Max,N-0} \le f_{m,t}^{N-0} + f_{m,t}^{loss,N-0} \le F_m^{Max,N-0};$$

$$\forall m \in Nl, \, \forall t \in T$$
(24)

$$-F_m^{Max,N-k} \le f_{m,t}^{N-k} + f_{m,t}^{loss,N-k} \le F_m^{Max,N-k};$$

$$\forall m \in Nlc, \forall t \in T.$$
 (25)

The operation of the transmission network is also characterized by approximating quadratic losses using segments of monotonically increasing slopes. This approximation yields piecewise linear functions, which, for practical purposes, are indistinguishable from the nonlinear models if enough segments are used [27]. Pre-contingency transmission losses are linearized in (26)–(30), whereas post-contingency transmission losses are linearized in (31)–(35).

$$f_{m,t}^{N-0} = f_{m,t}^{+,N-0} - f_{m,t}^{-,N-0}; \forall m \in Nl, \, \forall t \in T$$
(26)

$$\sum_{l \in Ls} \Delta f_{l,m,t}^{N-0} = f_{m,t}^{+,N-0} + f_{m,t}^{-,N-0}; \forall m \in Nl, \, \forall t \in T \quad (27)$$

$$0 \leq \Delta f_{l,m,t}^{N-0} \leq \frac{F_m^{Max,N-0}}{|Ls|}; \forall l \in Ls, \forall m \in Nl,$$

$$\forall t \in T \qquad (28)$$
$$f^{loss,N-0} = (G_m/B^2) \sum \zeta_{l}^{N-0} \Delta f_{l}^{N-0}; \forall m \in Nl.$$

$$\forall t \in T \qquad (29)$$

$$f^{+,N-0} \quad f^{-,N-0} > 0 \quad \forall m \in NL \quad \forall t \in T$$

$$(20)$$

$$f_{m,t}^{N-k} \circ, f_{m,t}^{n,t-k} \ge 0; \forall m \in Nl, \forall t \in T$$

$$(30)$$

$$f_{m,t}^{N-k} \circ f_{m,t}^{-,N-k} \lor_{m} \in Ml_{0} \forall t \in T$$

$$(31)$$

$$\sum_{m,t} f_{m,t}^{(n)} = f_{m,t}^{(n)} - f_{m,t}^{(n)} ; \forall m \in Nlc, \forall t \in T$$

$$(31)$$

$$\sum_{l \in Ls} \Delta f_{l,m,t}^{N-k} = f_{m,t}^{+,N-k} + f_{m,t}^{-,N-k}; \forall m \in Nlc, \, \forall t \in T \text{ (32)}$$



Fig. 2. Hourly levels of net demand in % of the system peak.

$$0 \le \Delta f_{l,m,t}^{N-k} \le \frac{F_m^{Max,N-k}}{|Ls|}; \forall l \in Ls, \forall m \in Nlc,$$

$$\forall t \in T$$
(33)

$$\begin{aligned} f_{m,t}^{loss,N-k} &= \left(G_m / B_m^2 \right) \sum_{l \in Ls} \zeta_{l,m}^{N-k} \Delta f_{l,m,t}^{N-k}; \forall m \in Nlc, \\ \forall t \in T \end{aligned}$$

$$(34)$$

$$f_{m,t}^{+,N-k}, f_{m,t}^{-,N-k} \ge 0; \forall m \in Nlc, \forall t \in T$$

$$(35)$$

where

$$\zeta_{l,m}^{N-0} = (2l-1) \frac{F_m^{Max,N-0}}{|Ls|}; \forall l \in Ls, \forall m \in Nl$$

$$\zeta_{l,m}^{N-k} = (2l-1) \frac{F_m^{Max,N-k}}{|Ls|}; \forall l \in Ls, \forall m \in Nlc.$$

III. CASE STUDIES

This section presents our numerical experience with two case studies both considering a 24-h scheduling horizon. For illustrative purposes, the proposed formulation is first applied to the IEEE 57-bus system. This benchmark is useful to analyze comprehensively the effects of the novel modeling aspects featured by the proposed formulation. Subsequently, we investigate the computational performance on the 1,354-bus portion of the European transmission system. This large-scale system allows us to show the practical applicability of the proposed approach.

For both case studies, we have considered the hourly net demand percentage levels depicted in Fig. 2, which are taken from [30]. Besides, the nodal peak net demand profiles are taken from the MATPOWER database [31]. The number of blocks of the piecewise linear cost functions, i.e., |Lg|, is set at 10. Based on [27], the number of blocks of the piecewise linear transmission loss functions, i.e., |Ls|, is set at 6. For the sake of simplicity, out-of-service lines are assumed to be unavailable along the scheduling horizon.

For expository purposes, the proposed PSCUC model considering transmission losses has been assessed with a simpler version disregarding this modeling aspect. Simulations for both the lossy and lossless models have been performed on a personal

TABLE I IEEE 57-BUS SYSTEM: RESULTS FROM THE CONTINGENCY SELECTION PROCESS

System used for the contingency analysis	Most critical contingency	Most overloaded line
Original system with all 80 lines	7-29	8-9
Original system without line 7-29	3-4	8-9
Original system without lines 3-4 and 7-29	8-9	5-6
Original system without lines 3-4, 7-29, and 8-9	5-6	7-8



Fig. 3. IEEE 57-bus system: Pre- and post-contingency active power flows on line 8-9.

computer running Windows 10 Pro with an Intel Core i7, 2.6 GHz, 8 GB RAM, and 64 bits, using Xpress 8.8 [32] under MATLAB. MATLAB has been used for data manipulation and GGDF matrices calculation.

A. IEEE 57-Bus System

This test system consists of 57 buses, 7 generators, 80 transmission lines, and 42 loads. A 240-MW capacity is considered for every transmission asset, whereas the production cost data are based on those provided in [31]. The remaining system data can be found in [33].

For this case study, no load shedding is allowed and the optimality tolerance is set at 0.0001. In order to account for the performance variability featured by mixed-integer programs [34], all problem instances have been solved 100 times and the average computing times are reported.

Power system security has been implemented in the form of N-k line outages with k ranging between 0 and 4. Such line outages are determined using the procedure described in Section II-A, the results of which are listed in Table I. Thus, in the proposed PSCUC model, system operation is represented for the base case or pre-contingency state, indexed by N-0, with all lines available, and four post-contingency states, namely N-1, N-2, N-3, and N-4. According to Table I, the N-1 state is associated with the loss of line 7-29, the N-2 state corresponds to the loss of lines 3-4 and 7-29, the N-3 state is related to the loss of lines 3-4, 7-29, and 8-9, and the N-4 state includes the loss of lines 3-4, 5-6, 7-29, and 8-9.

Fig. 3 displays the hourly active power flows for line 8-9 under the pre-contingency state. This figure also shows the

 TABLE II

 IEEE 57-BUS SYSTEM: PROBLEM DIMENSION

		Los	sless mo	iel		Lossy model								
	k=0	k=1	<i>k</i> =2	<i>k</i> =3	<i>k</i> =4	k=0	k=1	<i>k</i> =2	<i>k</i> =3	<i>k</i> =4				
Equality constraints	206	206	206	206	206	11,726	11,726	11,726	11,726	11,726				
Inequality constraints	9,909	9,861	9,813	9,765	9,717	63,669	63,621	63,573	63,525	63,477				
Total constraints	10,115	10,067	10,019	9,971	9,923	75,395	75,347	75,299	75,251	75,203				
Binary variables	168	168	168	168	168	168	168	168	168	168				
Continuous variables	10,032	9,984	9,936	9,888	9,840	36,912	36,696	36,480	36,264	36,048				
Total variables	10,200	10,152	10,104	10,056	10,008	37,080	36,864	36,648	36,432	36,216				

TABLE III
IEEE 57-BUS SYSTEM: ON/OFF STATUSES OF GENERATING UNITS

	Lossless model								Lossy model																																									
Unit	Hour																			Н	oui	•	_	-																										
	1	2	3	4	5	6	7	8	9	10)1:	11	21	314	41	51	61	71	8	19	20	21	22	23	24	1	2	2 3	4	1 :	5 6	5 7	8	9	10	11	12	13	14	11:	516	51	[7]	18	19	20	21	. 22	223	۶ <u>2</u> 4
		1	1	1																k	= () (8	ull I	line	es a	ava	ila	ble)					_											_					
1	1	1	1	1	1	1	1	1	1	1	1	_ 1	1	. 1		1 1		[]]	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	1	1	1	1]	1	1		1 1		[]	1	1	1	1	1	0	0	0	0	0 0	() () () (1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	0	0
3	1	1	1	1	1	1	1	1	1	1	1]	1	1		1 1		[]	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1]	1	1		1 1		l]	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	. 1	1	1		1 1		1	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	1	1	1	1]	1	1		1 1		l 1	1	1	1	1	1	0	0	0	0) ()	() () () (1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1 1		1	1	1	1	1	1	1	1	1	1	. 1]]	[]	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
		-	-	-				-	-	-	_	_	-	-	_		-	-	<i>k</i> =	= 1	(1	ine	-7-	-29	οι	it o	f s	erv	ice	<u>)</u>				-	-	-	-	1		_	T	_	- 1			r				
1	1	1	1	1	1	1	1	1	1	1	1	.]	1	. 1	1	1 1		1	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	1	1	1	1	. 1	1	. 1		1 1		[]	1	1	1	1	1	0	0	0	0) 0	() () () ()	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	0	0
3	1	1	1	1	1	1	1	1	1	1	1]	1	1		1 1		[]	1	1	1	1	1	1	1	1	1	. 1]	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1]	1	1		1 1		1	1	1	1	1	1	1	1	1	1	. 1]	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1]	1	1		1 1		[]	1	1	1	1	1	1	1	1	1	. 1]	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	1	1	1	1	.]]	1	. 1		1 1		[]]	1	1	1	1	1	0	0	0	0) 0	() () () ()	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	0	0
7	1	1	1	1	1	1	1	1	1	1	1]]	1	1		1 1		[]]	1	1	1	1	1	1	1	1	1	. 1]	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
k = 2 (lines 3-4 and 7-29 out of service)																																																		
1	1	1	1	1	1	1	1	1	1	1	1	.]]	1	. 1		1 1		[]]	1	1	1	1	1	1	1	1	1	. 1]	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	1	1	1	1	1	1	1	. 1		1 1		[]]	1	1	1	1	1	0	0	0	0) ()	() () () 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	0	0
3	1	1	1	1	1	1	1	1	1	1	1]	1	1		1 1		[]	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	. 1	1	. 1	1	1 1		1	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	.]]	1	. 1		1 1		l]	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	1	1	1	1	. 1	1	. 1	1	1 1		1	1	1	1	1	1	0	0	0	0) 0	() () () ()	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1 1		1	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
	r	-	-	-	r	r		-	1	-	_	_	-	-	_	k	= :	3 (1	in	es	3-∕	1, 7	-2	9, a	anc	18-	9 (out	of	se	rvic	e)		_	-	-	-	r		_	-	_	r			r –			—	
1	1	1	1	1	1	1	1	1	1	1	1	. 1	1	. 1		1 1		1	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	1	1	1	1	1	. 1	1	. 1		1 1		1	1	1	1	1	1	1	0	0	0) ()	() () () 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1	1	1	. 1	1	. 1		1 1		[]	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	. 1	1	. 1		1 1		[]	1	1	1	1	1	1	1	1	1	. 1]	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1]	1	. 1		1 1		[]]	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	1	1	1	1	1	.]]	1	. 1		1 1		l]	1	1	1	1	1	1	0	0	0) 0	() () () 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	1	1	1	.]]	1	1		1 1		[]]	1	1	1	1	1	1	1	1	1	. 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
		1	1	1											1	k =	4 (lin	es	3-4	4, :	5-6	, 7	-29), a	ınd	8-	9 0	ut	of	ser	vice	e)	_											_					
1	1	1	1	1	1	1	1	1	1	1	1	.]]	1	. 1		1 1		l]	1	1	1	1	1	1	1	1	1	. 1]	1	1 1	. 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	1	1	1	1	1]]	1	1		1 1		[]]	1	1	1	1	1	1	0	0	0) ()	() () () 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1	1	1]]	1	1	1	1 1		[]]	1	1	1	1	1	1	1	1	1	1	1	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	. 1	1	1	1	1 1		1	1	1	1	1	1	1	1	1	1	1	1	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1]]	1	1	1	1 1]	1	1	1	1	1	1	1	1	1	1	1	1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1
6	1	0	0	0	0	0	1	1	1	1	1	. 1	1	1	1	1 1		1	1	1	1	1	1	1	0	1	0) 0	() () () 1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1 1	1	L 1	1	1	1	1	1	1	1	1	1	1	1	t 1	1 1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1

N-1 and *N-2* power flow profiles for this line that would result from running the proposed PSCUC model without enforcing any transmission line capacities. As can be seen, the 240-MW capacity, represented by a black horizontal line, is exceeded under both post-contingency states.

Table II provides information on the size of the resulting MILP problem with and without transmission losses for different values of k. Note that k = 0 represents the PSCUC model without considering any contingencies. Analogously, values of k > 0 designate the corresponding PSCUC formulation with

	Lossless	model		Lossy model								
k	Total cost	CPU time	Total cost	CPUtime	Losses							
	(\$)	(s)	(\$)	(s)	(MW)							
0	777,711.65	0.46	780,274.30	1.12	61.25							
1	778,112.02	0.48	781,097.98	1.43	71.88							
2	778,330.65	0.46	781,303.01	1.46	71.01							
3	778,537.86	0.46	793,765.66	1.47	312.63							
4	780,326.43	0.50	795,207.69	1.50	333.50							

 TABLE IV

 IEEE 57-Bus System: Total Costs, Computing Times, and Losses

 TABLE V

 1,354-Bus System: N-k Line Outages

k	Out-of-service lines
1	960-3,279
2	960-3,279, 1,305-6,982
3	960-3,279, 1,305-6,982, 4,550-6,857
4	960-3,239, 960-3,279, 1,305-6,982, 4,550-6,857
5	960-1,754, 960-3,239, 960-3,279, 1,305-6,982, 4,550-6,857
6	960-1,754, 960-3,239, 960-3,279, 1,305-6,982, 4,550-6,857, 6,857-7,513
7	131-255, 960-1,754, 960-3,239, 960-3,279, 1,305-6,982, 4,550-6,857, 6,857-7,513

k simultaneous line outages. As can be observed, for each model, the number of binary variables remains unchanged for all instances. Note, however, that the consideration of losses drastically increases the problem dimension due to the considerable growth in the number of constraints and continuous variables. On the other hand, increasing the value of k barely impacts the problem size as it gives rise to slight reductions in the number of inequality constraints and continuous variables.

Table III presents the resulting generation schedules. Bold typeface in shadowed cells is used to highlight the differences associated with the tightening of the security criterion for each model. In all solutions, generators 1, 3–5, and 7 constitute the base-load units. Note, however, that different unit commitment decisions were required to prevent post-contingency constraint violations, i.e., line overloads, such as those shown in Fig. 3. As expected, increasing the value of k gives rise in general to additional commitments for both the lossy and lossless models.

Table IV lists the total costs and computing times for both models as well as the power losses obtained by the proposed formulation. As compared with the contingency-unconstrained case, the consideration of N-k line outages yields moderate cost increases ranging between 0.05% and 0.34% for the lossless model and between 0.11% and 1.91% for the proposed lossy model. As for the economic impact of losses, accounting for this practical aspect gives rise to cost increase factors between 0.33% for k = 0 and 1.96% for k = 3. Regarding the computational burden, running times remain almost unaltered when parameter k changes. By contrast, considering losses takes around three times as much time due to the larger size of the optimization problem (Table II). Notwithstanding, the effective performance of the proposed approach is corroborated as all instances are solved within 1.5 s. The proposed model also provides an estimation of transmission losses, which represent up to 1.88% of system net demand. Moreover, it should be noted that, in general, as k grows so do losses.

B. 1,354-Bus Portion of the European Transmission System

In order to assess the influence of problem size on the computational performance, the proposed model has been applied to a 1,354-bus system. This benchmark, with 260 generators and 1,991 branches, accurately represents the size and complexity of part of the European high-voltage transmission system [28], [31]. For reproducibility purposes, system data can be downloaded from [33].

Using the contingency selection procedure presented in Section II-A for values of k up to 7, the N-k line outages reported in Table V are considered.

Table VI summarizes the results for both the lossless and lossy versions of the proposed formulation with three different values of the relative optimality tolerance, referred to as GAP, namely 0.01, 0.001, and 0.0001. The computing times reported in Table VI are the average values over a set of 10 simulations.

As can be observed, for each value of the optimality tolerance, the effects of losses and security on total costs and running times are similar to those described for the case study based on the IEEE 57-bus system. More specifically, for the lossless PSCUC model, considering *N-k* line outages with *k* up to 7 takes 100 s at most and yields total cost increases over the no-contingency instance ranging between 0.4% and 2.6%. Moreover, the incorporation of losses in the PSCUC model increases the computational burden by two orders of magnitude whereas the total costs rise by factors around 10%. In addition, for the lossy model, the total cost increase due to security lies between 0.1% and 2.3%.

As for the impact of the choice for the optimality tolerance, in general, as the value of GAP decreases, slightly lower total costs are attained at the expense of significantly increasing the computational effort. This general trend is particularly relevant for the more computationally challenging lossy model, for which running times increase from values below 2,000 s for GAP equal to 0.001 up to around 6,900 s for GAP equal to 0.0001, whereas

	GAP = 0	0.01	GAP = 0	.001	GAP = 0.0001						
k	Total cost	CPU time	Total cost	CPU time	Total cost	CPU time					
	(\$)	(s)	(\$)	(s)	(\$)	(s)					
			Lossless	model							
0	18,032,986.69	50.12	17,972,515.80	75.91	17,972,515.80	75.91					
1	18,087,493.63	52.73	18,045,815.36	72.20	18,045,815.31	72.20					
2	18,088,002.35	51.83	18,047,094.63	95.96	18,047,094.63	95.96					
3	18,181,523.57	50.55	18,109,069.93	95.06	18,109,069.93	95.06					
4	18,244,205.39	49.96	18,201,392.43	58.05	18,201,392.43	58.05					
5	18,319,335.74	50.67	18,272,923.64	59.53	18,272,923.64	59.53					
6	18,375,210.23	49.42	18,331,033.88	53.96	18,331,033.88	53.20					
7	18,484,237.87	56.80	18,436,598.10	51.37	18,430,203.35	61.90					
			Lossy n	nodel							
0	19,899,261.98	1,780.35	19,878,339.12	1,856.16	19,876,603.25	6,695.82					
1	19,907,066.83	1,845.42	19,890,256.79	1,991.91	19,880,432.76	6,863.74					
2	19,973,289.41	1,524.09	19,913,980.29	1,772.40	19,898,161.76	6,446.54					
3	20,027,131.62	1,927.75	20,027,131.62	1,954.49	20,013,765.12	2,162.80					
4	20,120,510.55	1,596.73	20,049,079.97	1,742.40	20,042,760.13	4,554.42					
5	20,125,029.70	1,713.58	20,125,029.70	1,710.95	20,107,829.75	6,321.19					
6	20,283,404.25	1,515.77	20,216,812.28	1,582.65	20,206,843.33	6,356.33					
7	20,408,023.12	1,752.97	20,339,869.78	1,771.23	20,322,167.03	1,876.02					

 TABLE VI

 1,354-BUS SYSTEM: TOTAL COSTS AND COMPUTING TIMES FOR DIFFERENT OPTIMALITY TOLERANCES

small total cost improvements below 0.09% are attained. Thus, setting GAP equal to 0.001 is a reasonable selection as it represents a practical trade-off between solution quality and computational effort within an operational setting.

Overall, the results reported in Table VI suggest that tighter security levels and transmission losses might be both effectively accounted for in real-sized power systems without drastically increasing the total cost.

IV. CONCLUSION

This paper has presented a new PSCUC formulation that includes N-k line outages and transmission losses, thereby giving rise to a more accurate and realistic operational model. After identifying a list of credible contingencies, the operation under the associated N-k line outages is explicitly incorporated into the PSCUC model using generalized generation distribution factors. As another relevant modeling aspect, pre- and post-contingency transmission losses are characterized through piecewise linear approximations. Two case studies have been used to illustrate the economic benefits and effective computational performance of optimizing the commitment and dispatch of thermal generating units considering N-k line outages and transmission losses. Simulation results show that tighter security criteria and a more accurate operational model can be effectively adopted at the expense of moderate cost increases.

Further work will address the uncertainty associated with demand, renewable generation, and line outages. Other interesting avenues of research are (i) the consideration of quadratic line losses, which would give rise to a computationally challenging instance of mixed-integer quadratically-constrained programming, (ii) the incorporation of an *N-1-1* security criterion whereby an intervening time between consecutive outages is allowed for system adjustments, and (iii) the extension to a corrective setting wherein pre- and post-contingency generation levels may be different.

APPENDIX

Based on [6] and using a notation consistent with that of this paper, the conventional SCUC model used for contingency selection is formulated as follows:

$$Min \sum_{t \in T} \sum_{i \in Ng} \left(\alpha_i v_{i,t} + \sum_{g \in Lg} \xi_{g,i} \Delta p_{g,i,t} + cu_{i,t} + cd_{i,t} \right)$$
$$+ \sum_{t \in T} \sum_{j \in Nb} C_{j,t}^{ENS} ens_{j,t}$$
(36)

subject to:

Constraints
$$(2)-(5)$$
 and $(7)-(22)$ (37)

$$\sum_{i \in Ng} p_{i,t} + \sum_{j \in Nb} ens_{j,t} = \sum_{j \in Nb} Pd_{j,t}; \forall t \in T$$
(38)

$$-F_m^{Max,N-0} \le f_{m,t}^{N-0} \le F_m^{Max,N-0}; \forall m \in Nl,$$

$$\forall t \in T.$$
(39)

Expressions (36)–(37) are identical to (1)–(5) and (7)–(22) whereas expressions (38)–(39) respectively correspond to (6) and (24) wherein transmission losses are dropped. Note that, unlike the proposed PSCUC model (1)–(35), problem (36)–(39) disregards both system operation under contingency and transmission losses.

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