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# Japanese Elementary School Students Math Performance:

## A Case of Tottori Prefecture regional "Math Diagnostic Test"

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# Japanese Elementary School Students Math Performance:
## A Case of Tottori Prefecture regional "Math Diagnostic Test" [*1]

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### 1. Introduction

The purpose of teaching mathematics to students is to enhance "students' mathematical power" (NCTM, 1995), that is to extend their mathematical performance to a higher level. Especially in Japan, we set up to develop students' "mathematical way of thinking" as the purpose of mathematics education traditionally (*cf.* Japanese Course of Study). This is the expression including not only the aspects of students' mathematical knowledge or skills but also their attitudes toward mathematics.

On the other hand, the Japanese Course of Study and the textbooks as its realization should be referred to as the "Intended Curriculum" according to the TIMSS curriculum model (Mullis & Martin, 2013), and then the lessons are practiced as the "Implemented Curriculum" the next. Based on these contexts, we need to pursue the evaluation and the assessment of students' performance as part of curriculum research, *i.e.*, the "Attained Curriculum."

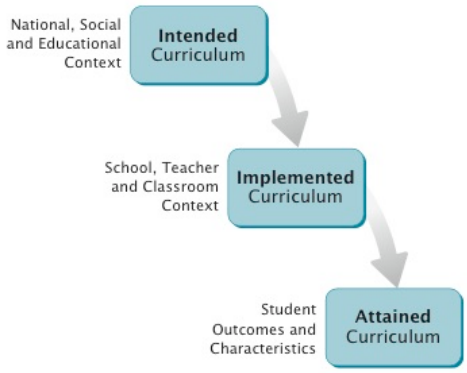



**Fig.1 TIMSS Curriculum Model**

In Japan, "Students Achievement Surveys" of the national level had been implemented intermittently by the initiative of the Ministry of Education in the past. Since 2007, the National Assessment Test has been carried out every year continuously. The purpose of National Assessment Test is to clarify the actual condition of national student, and to improve educational policies and lessons. That is, it is a verification of achievement of the "Intended Curriculum" which is the holistic trends rather than individual performances, and then to infer the global guiding principle for the curriculum development or teaching and learning.

Unlike nationwide survey, each region in Japan has been carried out their own achievement test (the names are different for each region) continuously for many years. Some are the prefecture-scale, other are the city-scale, for depending on the population size. These tests look the same way as the national survey with respect to the surface practice, however those initiatives and purposes differ greatly. Although the scales differ, regional tests may be similar to the national survey in terms of aiming to grasp the holistic trends. However, the intention of regional tests mainly attempts to obtain a direct suggestion for the improvement of "Implemented Curriculum," *i.e.*, the lesson, which differs from the national survey including the verification of curriculum. In addition, most of regional tests are deployed by teachers in the region (belonging to the math subcommittee in the case of elementary schools) under the supervision of university researchers to develop problems and analyze results, instead the national survey is done by the selected expert committee (researchers, policymakers, teachers). Especially, when developing problems, it is the high tendency to develop problems conforming to textbooks that are adopted in the region.

In this paper we verify elementary school students' math performance through Tottori Prefecture regional "Math Diagnostic Test" as a case of regional tests in Japan, in particular, we focus on "problems for examining the thinking process" in this Test using a few years data.

### 2. Tottori Elementary Math Diagnostic Test

*2.1 Organization*

Tottori Prefecture is located in the Chugoku region, western Japan. It is the least populous prefecture in Japan (area is not the least). However, the effort poured into education field is considerable such that top expense item of the prefectural budget is education (although the amount tends to reduce in recent years).

This area has been organized in three broad teachers group organization (eastern, central, western). Developing problems and analysing results of the "Math Diagnostic Test" are deployed by each group in two years replacement. Although all teachers belong to the math subcommittee in their schools, their major were not mathematics necessarily. What subject subcommittee a teacher belongs to is dependent on his/her school condition. Therefore, a teacher whose major is mathematics doesn't belong to math subcommittee, and vice versa.

It is a suitable opportunity for teachers to study mathematical and pedagogical content knowledge and recognize what are emphasis/important contents through the curriculum:

- What kinds of problems are essential/appropriate to verify students' math performance?
- What kind of performance do students show to such problems?

**Fig.2 Tottori Pref.**

**Fig.3 Three Districts in Tottori**

*2.2 Procedure*

The group organization in charge advances the effort of "Math Diagnostic Test" as follows.

1) In response to the determination of the subcommittee in affiliation schools in April, the members in charge of "Math Diagnostic Test" are appointed in math subcommittee of the district. (In math subcommittee, there are also efforts other than "Math Diagnostic Test".)
2) Staffs in charge of each grade are determined among members. Based on the analysis of past implementation and results, they discuss about improvement of the problems and/or development of the new problems of the year.
3) They improve such problems and develop new problems actually.
4) A problem list of each grade is made via work of 3).
5) All members in charge including editorial supervisors (university researchers and a Supervisor of Prefectural Board of Education) participate the workshop to discuss about verification of problem(s) and balance *in* and *between* grades (adjustment of a system/connection of contents, appropriateness of the degree of difficulty, and anticipated time for students solving problems).
6) The organization orders printing Test booklets for a printer.
7) Almost all schools in the Prefecture participate the Test (not mandatory).
8) Scoring the Tests is done by homeroom teachers of each school.
9) Each school reports results (scores) of the Test to a teacher in charge of the district.
10) Staffs aggregate and analyze the results. A report of the analysis will be completed in the next June-July.

*2.3 Examinee students*

The number of examinee students of Math Diagnostic Test implemented in February 2014 is shown in Table 1.

**Table 1. Number of students by each grade**

| Grade | 1st | 2nd | 3rd | 4th | 5th | 6th |
|---|---|---|---|---|---|---|
| Number of students | 4908 | 4867 | 4968 | 5138 | 5057 | 5241 |

*For reference*, the results of the National Assessment Test that the 6th grade students took exam in that year (2013) are shown in Table 2. The score of students of Tottori Prefecture was 11th place of 47 prefectures in Japan.

**Table 2. National Assessment Test 2013**

| | Tottori Prefecture (%) | National Average(%) |
|---|---|---|
| Elementary Math A | 78.1 | 77.3 |
| Elementary Math B | 60.2 | 58.6 |

## 3. Problems for examining the thinking process

### *3.1 Nature of the problem*

Although each Test problem requires only the answer basically, "problems for examining the thinking process" require describing the thinking process by an indication: "*Let's leave your writing/trajectory of thought*." It is the reason why we focus on "problems for examining the thinking process." However, we cannot see students' writings directly. The authors can only verify the trends of students' solving indirectly to be shown in the report of the analysis (*procedure 10* above mentioned).

There are 1-3 "problems for examining the thinking process" in each grade. (In all grades, 25 problems are provided by the Test generally.) In the following, we verify the Test problems and the trends for each grade.

### *3.2 Problem of 1st Grade*

| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(8)** ④ | When distributing 12 pencils by one for one person, 5 pencils are left. When 10 pencils, how many pencils are left? | 39 | 39 | **38** | ・7 (29%)<br>・5 (29%)<br>・2 (14%)<br>・others (14%)<br>・no answer (14%) |

Intention of the Problem

- To find the unknown number from the given condition;
- To identify the number of pencils and persons;
- To represent a problem situation with a diagram or a picture.

Expected Thinking

- Finding the distributed number of the pencils by subtracting the number of remainders from the number of the beginning. This number is the number of people. Then, subtracting the number of pencils to distribute to seven persons from ten: $12-5=7$, $10-7=3$.
- Finding the difference of the number of pencils between the first and the second situation. Since the two small pencil in the second situation, the number of remaining pencils also become two small: $12-10=2$, $5-2=3$

Error Analysis

- Some students can't do two-step thinking, they finish problem solving by finding the number of distributed pencils: $12-5=7$.

- Some students subtract five remainders from ten pencils: $10-5=5$.
- Some students misread the problem as "how many pencils remain when distributing 10 from 12?": $12-10=2$.

*3.3 Problems of 2nd Grade*

| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(9)** | There are two numbers which one is 3 larger than 1000 and another is 5 less than 1000. How many is the difference between two numbers? | 47 | 51 | **51** | • 2 (19%)<br>• 9 (13%)<br>• others (47%)<br>• no answer (21%) |

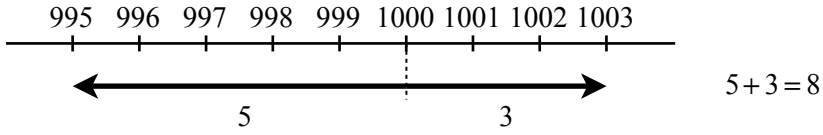
Intention of the Problem

- Appreciating to be able to find the number by using variables/differences based on the reference number.
- Using a number line.

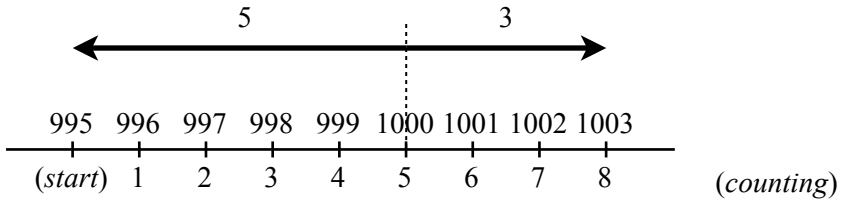
Expected Thinking

- Finding the number by using variables/differences based on the reference number: $3+5=8$.
- Finding the number by using a number line (two cases):

*a*)




*b*)



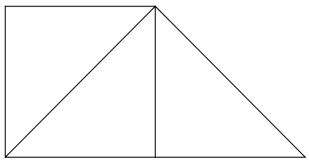

- Finding the number by calculation after obtaining the two numbers:

$1000+3=1003$, $1000-5=995$,

$$\begin{array}{r} 1003 \\ -\ 995 \\ \hline 8 \end{array}.$$

Error Analysis

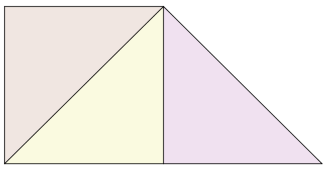
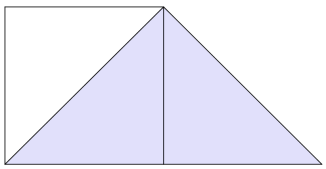
- Some students use the difference without recognizing number sequence: $5-3=2$.
- Some students include also the reference number in counted number: $5+1+3=9$.

| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(18)** | There is a pattern arranged by right triangles as shown in the figure. How many triangles both large and small are in this pattern?  | 42 | 43 | **47** | ・3 (63%)<br>・2 (10%)<br>・others (23%)<br>・no answer (4%) |

Intention of the Problem

・Recognizing geometric figures by composing and decomposing.

Expected Thinking

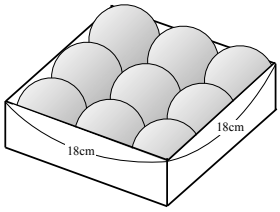



Error Analysis

・Some students count three small triangles only.

・Some students answer that there are two kinds of triangles, big and small.

*3.4 Problems of 3rd Grade*

| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(14)** | Balls are packed in a box accurately as shown in the figure. How long (cm) is the radius of a ball?   | 54 | 54 | **57** | ・6 cm (46%)<br>・9 cm (13%)<br>・2 cm (13%)<br>・others (23%)<br>・no answer (5%) |

Intention of the Problem

・Understanding spherical diameter and radius in relation to the length of vertical and horizontal of the box.

Expected Thinking

・Finding the radius after obtaining the diameter: $18 \div 3 = 6$, $6 \div 2 = 3$ 3 cm.

・Find the radius on the basis of the number of radii (per side): $2 \times 3 = 6$, $18 \div 6 = 3$ 3 cm.

Error Analysis

・Some students answer the diameter: $18 \div 3 = 6$, 6 cm.

・Some students recognize incorrectly the length of one side of the box as a diameter: $18 \div 2 = 9\,(\text{cm})$.

・Some students divide the length of one side of the box by nine balls: $18 \div 9 = 2\,(\text{cm})$.

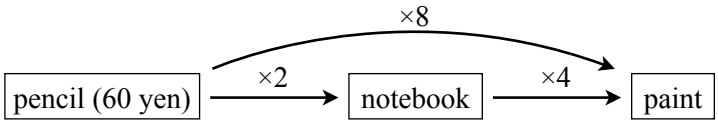
| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(18)** | There are pencils, notebooks, and paints.<br>A pencil is 60 yen.<br>A notebook is twice the price of the pencil.<br>A paint is four times the price of the notebook.<br>How many times the paint is the price of the pencil? | 57 | 57 | **55** | ・6 times (44%)<br>・480 times (10%)<br>・2 times (10%)<br>・4 times (5%)<br>・others (26%)<br>・no answer (5%) |

Intention of the Problem

・Solving a problem by using multiplication operators.

Expected Thinking

・Finding the number by calculating operators: $2\times4=8$ 8 times.




・Comparing the prices of pencil and paint after finding the prices of notebook and paint in turn: $60\times2=120$, $120\times4=480$, $60\times\square=480$, 8 is appropriate for □; or $480\div60=8$.

Error Analysis

・Some students add the multiplication operators: $2+4=6$ (times).

・Some students obtain the price of paint (*their thinking process are correct*).

・Some students answer either of operators.

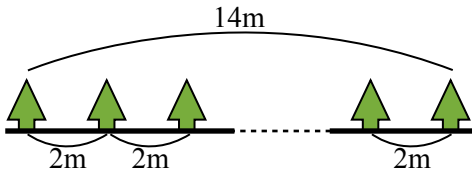
| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(21)** | Trees are lined in a row away by 2m. Trees of both ends are away 14m.<br>How many trees are lined? | 35 | 39 | **43** | ・7 (76%)<br>・28 (7%)<br>・16 (3%)<br>・others (8%) |

Intention of the Problem

・Comprehending the problem situation accurately, and coping with it correctly.

Expected Thinking

・Finding that the number of trees is always one greater than the number of interval of trees, sometimes by representing the problem situation into a figure: $14\div2=7$, $7+1=8$




Error Analysis

・Some students are regarded “the number of interval of trees” just as the "number of trees”.

・Some students calculate inaccurately using the numerical values in the problem sentence: $2\times14=28$, $2+14=16$.

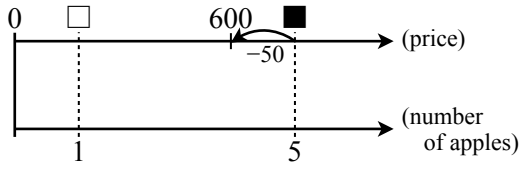
*3.5 Problems of 4th Grade*

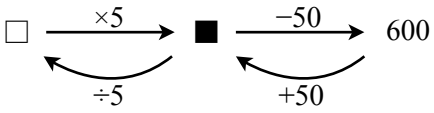
| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(23)** | I bought five apples. I paid 600 yen because the price was cut by 50 yen. How much was an apple? | 58 | 59 | **57** | ・110 yen (24%)<br>・120 yen (15%)<br>・others (49%)<br>・no answer (12%) |

Intention of the Problem

・Thinking back to the order by representing the problem situation into a diagram.

Expected Thinking




・$(600+50)\div 5=130$ 130 yen

・$600+50=650$, $650\div 5=130$ 130 yen

Error Analysis

・$(600-50)\div 5=110$ (yen)

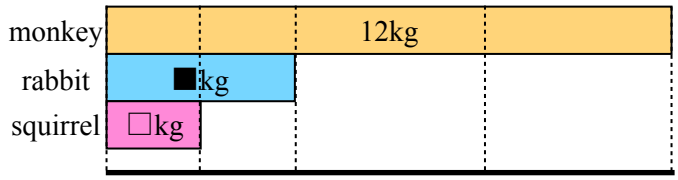
・$600\div 5=120$

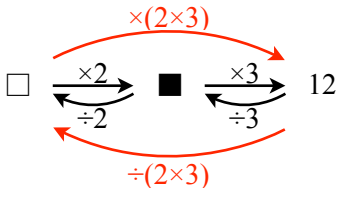
| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(24)** | The monkey weighs 12 kg, and is three times the weight of the rabbit. The weight of the rabbit is twice the weight of the squirrel. How many kg does the squirrel weigh? | 72 | 74 | **72** | ・72 kg (25%)<br>・4 kg (12%)<br>・others (38%)<br>・no answer (25%) |

Intention of the Problem

・Solving a problem by using operators with drawing diagrams.

Expected Thinking



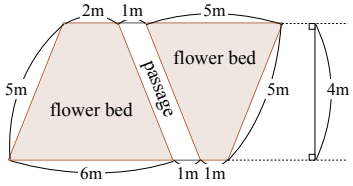

・$12\div 3=4$, $4\div 2=2$ 2 kg

・$12\div(2\times 3)=2$ 2 kg

Error Analysis

・Some students can't represent the problem situation into a diagram or a figure, and consequently they do incorrect calculation variously with using the numerical values of the problem sentence: $12\times 3\times 2=72$, $12\div 3=4$, $12-3-2=7$.

*3.6 Problems of 5th Grade*

| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(14)** | There is a parallelogram-like garden. Find the area of the whole flower beds.   | 55 | 58 | **54** | ・35 $m^2$ (37%)<br>・32 $m^2$ (11%)<br>・40 $m^2$ (3%)<br>・others (31%)<br>・no answer (18%) |

Intention of the Problem

・Finding a way of obtaining the area of flower beds by using quadrature formulas of a parallelogram and a trapezoid, and equivalency transformation.

Expected Thinking

・Subtracting the area of passage part from the area of the whole garden:

$(6+1+1)\times 4-1\times 4=28$ $\underline{28\ m^2}$,

$(2+1+5)\times 4-1\times 4=28$ $\underline{28\ m^2}$.

・Considering flower bed parts to be two trapezoids: $(2+6)\times 4\div 2+(5+1)\times 4\div 2=\underline{28\ (m^2)}$.

・Finding the area as a parallelogram by moving a part of the flower bed:

$(6+1)\times 4=\underline{28\ (m^2)}$, $(2+5)\times 4=\underline{28\ (m^2)}$.

Error Analysis

・Some students calculate the area by using the "hypotenuse" of a parallelogram as the "height": $(5+2)\times 5=35$.

・Some students don't remove the area of passage part: $(6+1+1)\times 4=32$.

・Some students consider that there are two same trapezoids: $(2+6)\times 4\div 2=16$, $16\times 2=32$.

・Some students calculate the area both by using the "hypotenuse" of a parallelogram as the "height" and by considering that there are two same trapezoids: $(6+1+1)\times 5=40$.

| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(16)** | Taro has taken five examinations. The average of exam up to fourth was 88. The fifth score was 98. Find the average of all five exam. | 40 | 42 | **47** | ・93 (34%)<br>・37.2 (6%)<br>・138 (6%)<br>・others (33%)<br>・no answer (21%) |

Intention of the Problem

・Finding overall average by using the temporary average.

Expected Thinking

・Finding overall average by calculating the total from temporary average to the fourth and adding the fifth score:

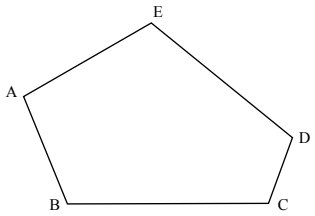
$88\times 4=352$, $352+98=450$, $450\div 5=90$; or $(88\times 4+98)\div 5=90$.

・Finding overall average by dividing the difference between the temporary average to the fourth and the fifth score into five equal parts:

$98-88=10$, $10\div 5=2$, $88+2=90$; or $(98-88)\div 5=2$, $88+2=90$.

Error Analysis

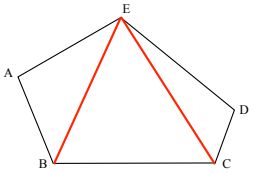
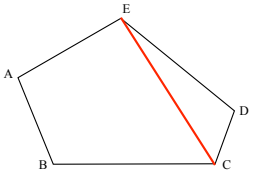
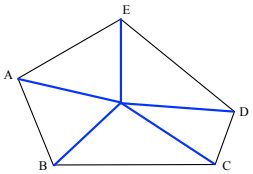
- $(88+98)\div 2=93$.
- $(88+98)\div 5=37.2$
- Some students consider the total of five times to be five times of 98 (average):
  $88\times 4=352$, $98\times 5=490$, $490-352=138$.

| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(21)** | A figure surrounded with five straight lines is called a pentagon. How many degrees is the sum of interior angles of the following pentagon ABCDE?   | 62 | 67 | **67** | • 900° (18%)<br>• 720° (7%)<br>• 520° (7%)<br>• others (27%)<br>• no answer (41%) |

Intention of the Problem

- Finding the sum of a polygonal interior angles based on "the sum of a triangle's interior angles is 180°."

Expected Thinking

-  $180\times 3=540$
-  $360+180=540$
-  $180\times 5-360=540$

- Sum of interior angles of the n-gon is $180\times(n-2)$ in general.
  So, if $n=5$, $180\times(5-2)=540$.

Error Analysis

- $180\times 5=900$.
- $180+540=720$, or $180+540=520$ (miscalculation).

*3.7 Problem of 6th Grade*

| Prob. No. | Problem | Ave. of correct answer (%) 2012 | 2013 | **2014** | Errors (Rate to the whole errors) |
|---|---|---|---|---|---|
| **(20)** | T-shirt of 1200 yen has become to 960 yen at the bargain sale. What percent discounts? | 40 | 45 | **44** | ・80 % (50%)<br>・1.25% (9%)<br>・125% (7%)<br>・5% (5%)<br>・others (28%)<br>・no answer (18%) |

Intention of the Problem

・Understanding the proportion.

Expected Thinking

・$1280-960=240$, $240\div1200=0.2$, $0.2\times100=$20 (%).

・$960\div1200=0.8$, $1-0.8=0.2$, $0.2\times100=$20 (%).

Error Analysis

・$960\div1200=0.8$, $0.8\times100=80$.

・$1200\div960=1.25$; $1.25\times100=125$.

・$1200-960=240$, $1200\div240=5$.

## 4. Concluding Remarks

In this paper we described and analyzed the Tottori Prefecture regional "Math Diagnostic Test", especially focusing on "Problems for examining the thinking process." These efforts by teachers groups contribute to their professional development, especially in terms of "mathematical content knowledge study (*kyozai-kenkyu*, in Japanese)." Furthermore, such a Test is also effective for improvement of teaching in the classroom. In fact, a student's response of a problem shows his/her Attained Curriculum. It leads to find where of the teaching and learning process his/her teacher has a problem by analyzing it.

On the other hand, the teachers groups have difficulty for "the development of new problems." If an international database of math problems is developed, it is hoped that it helps the solution to such difficulties.

### Notes

*1 This paper was presented at the International Conference on the Open Environment for the Worldwide Mathematical Education, 8-11, April, 2015, Moscow, Russia. This version is slightly modified.

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