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Abstract

This work is on the influence of the rotational component of earthquake excitations to the response of steel structures. In most studies, seismic input is being modeled only using the translational component of the ground acceleration, while the rotational one is ignored. This was due to the observation that the rotational component had minimal effect on low-rise buildings. Hence, the accelerometers used would not measure it, leading to a lack of records. Nowadays, technology provides such instruments and relative records are made available. Indicative of that is that elastic design response spectra for rotational components are introduced to the design codes. In this paper, the results on structural response and internal forces due to the rotational component of a seismic excitation on the steel structures are examined. Dynamic time history analysis and response spectrum analysis of different steel structures are performed (a) considering the rotational component of the excitation and (b) without it. From the numerical results it is shown that the impact of rotational component in structural response and internal forces of the steel structures is significant and should not be ignored during structural design.

Keywords

Rotational component, seismic design, earthquake engineering, steel structures, EC8 part 6.

1   Introduction

Time history analysis of structure involves exclusively fully description of ground-motion along the three dimensions of space. In order to fully describe the ground motion, translational and rotations also need to be considered, which results in a total of six components, three for translation and three for rotation. The luck of rotational component to the analysis was firstly because they were considered as negligible and secondly rotation sensors were not available to directly measure rotations during an earthquake. The rotational components data history can be measured directly in a free field with special accelerometers or can be extracted from measured translational recordings. Many earthquake and seismology scientists focus on rotational records over the last decades. Droste and Teisseyre [1] derived rotations from an array of seismographs. Rotational motions observed during an earthquake in April 1998 ιν Japan was measured with a gyro-sensor, and an inertial angular displacement sensor, by Takeo [2]. Advances in rotational seismology about instrumentation, theory and observations are presented in the work of Igel et al. [3]. The application of compact and cheaper sensors based on electrochemical magnetohydrodynamic technology, used from Liu et al. [4] and

Wassermann et al. [5].There are a lot of procedures that calculate rotational time series from translational recordings. A Single Station Procedure (SSP), is one of them. A number of researchers such as Lee and Trifunac, [6], Castellani and Boffi [7], Li et al. [8] and Basu et al.[9], presented their work base on SSP. An extension to the SSP method is the use of data from a number of closely spaced, spatially distributed stations, this procedure is called Multiple Station Procedures, denoted as MSP, Niazi [10]. An expansion of MSP is the Geodetic Method, GM, Spudich et al.[11],[12], work on this method. Basu et al. [13], point out some limitations of GM, and propose the Acceleration Gradient Method, AGM, which is capable of extracting the free-field rotational time series from the three-component strong motion data recorded at surface stations in a dense array. However, the AGM fails to capture the frequency content above a limit and this limit reduces as the physical dimension of the array increases. The drawback of AGM is overcoming using an alternative procedure, the Surface Distribution Method, SDM, Basu et al. [14].

Falamarz-Sheikhabadi et al. [15],[16], work on the effects of both time delay and loss of coherency in order to derive simple mathematical expressions for generating the middle-field rocking acceleration component and its corresponding response spectra.

They revised the seismic intensity parameters in order to account for the combined action of horizontal and rocking seismic motion on structures. Links between rotational ground motion and site soil conditions are proposed in the work of Sbaa et al. [17] and Perron et al. [18]. They show that the coupling of translational and rotational measurements appears to be useful, not only for direct applications of engineering seismology, but also to investigate the composition of the wavefield, while avoiding deployment of dense arrays.

The need of considering of rotational components is also imprinted to the regulations. The Eurocode 8, part 6, [19], examining slender and tall structures such as towers, chimneys and masts takes into account special variability of the seismic ground motion including rotational components of the ground accelerations. The EC8, part 6, propose an extended response spectrum analysis which requires response spectra of rotational accelerations to be implemented. Such rotational spectra are defined and calculated on the basis of translational response spectra as well.

A lot of works investigated the effect of rotational component of ground motion on structural response. In base isolated structures Wolf et al. [20], discussed the effect of rocking excitation on a base-isolated nuclear power plant. Politopoulos, [21], identified the excitation of the rocking mode in a base-isolated building due to rocking excitations. Bozev et al. [22], performed analysis accounting the rotational component of seismic action on towers, masts and chimneys according to EC8, Part 6. Zembaty, [23], work on rotational seismic code definition in Eurocode 8, Part 6, for slender tower-shaped structures while Zembaty and Boffi [24], identified the contribution of rocking motion to bending moments along the height of a tall tower using horizontal and rocking spectra computed based on Eurocode 8. In a recently work, [25], rotational ground-motion records from induced seismic events are examined. A simplified relation for the application of rotational components to seismic design codes and calculation of response of multiple-support structures subjected to horizontal and rocking components are presented in the work [26], [27]. Basu et al., [28], suggests an equivalent accidental eccentricity to account for the effects of torsional ground motion on structures. Torsion in building due to base rotational excitation was investigated by De-La-Llera and Chopra, [29]. Yin et al performed earthquake engineering analysis of measured rotational ground motions at structure, [30].

In this paper the influence of rotational component of earthquake excitation to the response of steel structure is examined. The response is calculated solving directly the equations of motions accounting for the rotational component applied at the base of structure. Directly time history geometrical non-linear analysis is performing in order to calculate the response of structure.

2 Theoretical background

2.1 Dynamic Analysis

Rotational components of earthquake consist of one torsional component which is the rotation about vertical axis and the other two rocking components which is the rotation about the two horizontal axes. The secondary horizontal wave, SH wave, and the surface wave, Love wave, contribute to the torsional motion. Rocking motion is due to the primary waves, P waves, secondary vertical wave, SV waves, and Rayleigh waves. Even though that rotational components of earthquake ground motion have been studied in literature in seismic analysis and design of structures is not taken into account. May this come from the fact that such data are not recorded by the accelerographs. In Eurocode 8, EC8, Part 6,

rotational components of the ground accelerations taking into account is seismic analysis. Response spectrum analysis or direct time history analysis can be used in order to evaluate the response of structure subjected to rotational component. The response spectrum analysis requires response spectra of rotational accelerations to be implemented. In EC8, Part 6 rotational spectra are defined in order to use for the analysis. Time history analysis requires time history rotational motion records in order to apply in the base of structures. Structures that are influence of rotational component are slender and tall structures such as towers, chimneys and masts. Those structures are special mentioned in EC8, Part 6, however, other structures such as long in plan structures, bridges, dams, high-rise buildings or specially buildings such as nuclear power stations, are also expected to be influenced by rotational components of seismic action.

In the earthquake engineering community saying ground acceleration and velocity directly means translational component of acceleration and velocity. Term like translational acceleration and translational velocity is used to clearly distinguish from rotational acceleration and rotational velocity respectively. The term rotational rate is also often used in the rotational seismology literature. Thus, Peak Ground Translational Acceleration, PGTA, is used instead of PGA and Peak Ground Rotational Acceleration, PGRA, is defined as the maximum in the time domain of the absolute value of the rotational acceleration along the three components.

A total movement due to rotational and translational component of earthquake excitation of a single degree of freedom system, SDOF, is shown in Figure 1.
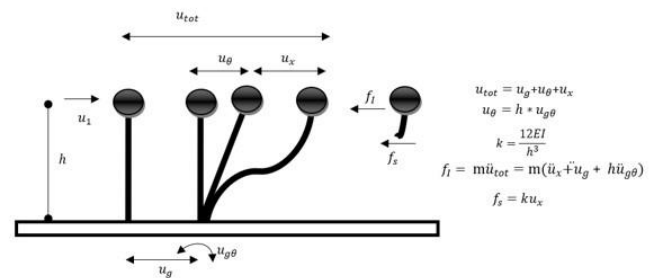

Figure 1 A total movement due to rotational and translational component of earthquake excitation of a single degree of freedom system, SDOF

Applying a dynamic equilibrium to SDOF the equation of motion is obtained as follows:

$$m\ddot{u}_x + c\dot{u}_x + ku_x = -(m\ \ddot{u}_g + m\ h\ddot{u}_{g\theta}) \tag{1}$$

A model of structure with concentrated mass and stiffness at each floor with only horizontal degrees of freedom, ignoring the rotation of plate, subjected to ground motion with translational and rotational components is shown in Figure 2.
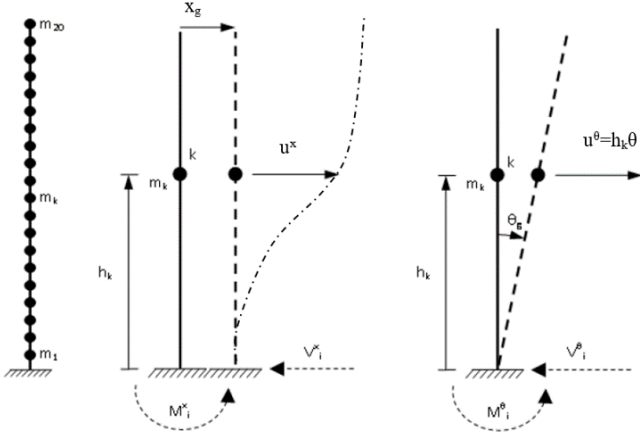
Figure 2 A total movement due to rotational and translational component of earthquake excitation of a single degree of freedom system, SDOF

If [**M**], is the mass matrix, [**K**], the stiffness matrix and [**C**], the damping matrix the equation of motion for structure considering a translational ground acceleration along horizontal direction x together with a rotation acceleration in the vertical plane x-z are given by:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -(\{m\}\,\ddot{x} + \{m\,h\}\,\ddot{\theta}) \qquad (2)$$

Where:

$\{\ddot{u}\}$: is the vector comprising the accelerations of the degrees of freedom of the structure relative to the base,

$\{\dot{u}\}$: is the vector comprising the velocities of the degrees of freedom of the structure,

$\{u\}$: is the vector comprising the displacements of the degrees of freedom relative to the base,

$\{m\}$: is the vector comprising the translational masses in the horizontal direction of the translational excitation. This vector coincides with the main diagonal of the mass matrix [**M**], if the vector $\{u\}$ includes only the translational displacements in the horizontal direction of the excitation,

$\ddot{x}_g(t)$: is the translational ground acceleration,

$\ddot{\theta}_g(t)$: is the rotational acceleration of the base.

The above equation can be extended to three dimensions. In that case the excitation motion consists of three translational acceleration, two rotational accelerations (rotation about the two horizontal axes, i.e., rocking) and one torsional acceleration (rotation about the vertical axis).

Time history analysis can be used in order to calculate the response of structure subjected to translational and rotational component of ground motion. With this analysis the response is calculated solving directly with a numerical procedure the above Equation 2. Time history analysis can be linear or non-linear. Non linearity refers to material (change of stiffness matrix in every time step) or to geometry (solving the equation in deformable position at each time step).

2.2   Response Spectrum Analysis

Response spectrum analysis, EC8 part 6, [19], Bonev et al. [22], can also be perform to calculate the response of structure subjected to translational and rotational component of ground motion. The procedure of response spectrum analysis considering the rotational component is as follows:

Knowing the mass and initial stiffness of the structure the eigenmodes Φ_i, eigenperiods, T_i, or eigenfrequencies, f_i, and the corresponding damping ratios ξ_i of the structure are obtained from the solution of the following eigenvalue problem

$$[K - \omega^2 M]_{nxn}\Phi = 0 \Rightarrow |K - \omega^2 M| = 0 \Rightarrow \begin{cases} \Phi_1, \Phi_2, \dots, \Phi_n \\ \omega_1, \omega_2, \dots \omega_n \end{cases} \Rightarrow T_i = \frac{2\pi}{\omega_i}, f_i = \frac{\omega_i}{2\pi}, \text{i=0,...,n-1} \qquad (3)$$

$$C_n = \Phi_n^T C \Phi_n, \qquad M_n = \Phi_n^T M \Phi_n, \qquad \xi_i = 2C_n M_n \omega_n \qquad (4)$$

The participation factor ψ_i, and seismic forces $F_{q,i}$ for the ith eigenmode for translational and rotational component are given as:

$$\psi_{i,T} = \frac{\Phi_i^T M E^T}{\Phi_i^T M \Phi_i}, \text{i=1,...,n}, \qquad \psi_{i,\theta} = \frac{\Phi_i^T M E^\theta}{\Phi_i^T M \Phi_i}, \text{i=1,...,n} \qquad (5)$$

$$F_{q,i,T} = M\Phi_i \psi_{i,T} S_{d,i}(T_i, \xi_i), \quad \text{i=1,...,n}, \qquad (6)$$

$$F_{q,i,\theta} = M(\Phi_i * h)\psi_{i,\theta} R_{d,i}^\theta(T_i, \xi_i), \quad \text{i=1,...,n} \qquad (7)$$

where E^T and E^θ is the direction matrix for the earthquake for translational and rotational component:

$$E^T = \{1 \ 1 \ \dots 1\}^T, \qquad E^\theta = \{0 \ 0 \ \dots 0 \ 1\}^T \qquad (8)$$

$\Phi_i * h$, is a mode vector were every element of Φ_i in each degree of freedom is multiplied with coresponting heigh.

$$\Phi_i * h = \begin{Bmatrix} \Phi_{i,n} h_n \\ \vdots \\ \Phi_{i,1} h_1 \end{Bmatrix} \qquad (9)$$

and $S_{d,i}(T_i, \xi_i)$ is the design spectral acceleration while the $R_{d,i}^\theta(T_i, \xi_i)$ is the rotation response spectra around y axes, in rad/sec^2.

In EC8 part 6 when the rotational components of the ground motion during the earthquake are taken into account, the seismic action may be represented by three elastic response spectra for the translational components and three elastic response spectra for the rotational components. The rotation response spectrum is defined in an analogous way to the response spectrum of the translational components, i.e. by considering the peak response to the rotational motion of a rotational single-degree-of- freedom oscillator, with natural period, T, and critical damping ratio, ξ. R^θ, denotes the ratio between the maximum moment in the oscillator spring and the rotational moment of inertia about its axis of rotation. The diagram of, R^θ, versus the natural period, T, for given values of, ξ, is the rotation response spectrum. When results of a specific investigation or of well-documented field measurements are not available, the rotational response spectra may be determined as:

$$R_x^\theta(T_i) = \frac{1.7\pi S_\varepsilon(T_i, \xi_i)}{v_s T_i}, \quad \text{i=1,...,n}, \qquad (8)$$

$$R_y^\theta(T_i) = \frac{1.7\pi S_\varepsilon(T_i, \xi_i)}{v_s T_i}, \quad \text{i=1,...,n}, \qquad (9)$$

$$R_z^\theta(T_i) = \frac{2.0\pi S_\varepsilon(T_i, \xi_i)}{v_s T_i}, \quad \text{i=1,...,n}, \qquad (10)$$

The maximum seismic forces F_q for each degree of freedom are obtained combining the two-part seismic forces of each eigenmode with Square Root of Sum Squares method (SRSS), thus:

$$F_q = \sqrt{\Sigma_i^n (F_{q,i,T} + F_{q,i,\theta})^2} \qquad (11)$$

The same calculations are done in order to calculate every other response quantity like displacement, internal forces etc.

3   Numerical example and discussion

The influence of the rotational component of earthquake has been numerically approached. A three story, steel structure, with consentrated mass ansd stiffness subjected to an earthquake excitation with translational and rotational component. The story mass is 10 ton and the story stiffness is 10000 kN/m. The layout of the structural system is shown in Figure 3.

The model was subjected to a 6.4 moment magnitude, Mw, earthquake excitation, which took place on 17/11/2015 in Kefalonia island, Greece. The epicentre latitude and longitude were 38.16° and 20.50° respectively. The focus of the earthquake was at 10.7km below surface level. The record history of translational and rotational components were recorded in the framework of the Argonet project, a 3D accelerometric array implemented on the island of Kefalonia in Greece [17], [18], and is shown in figure 4.
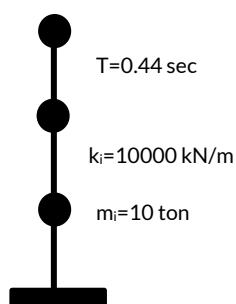
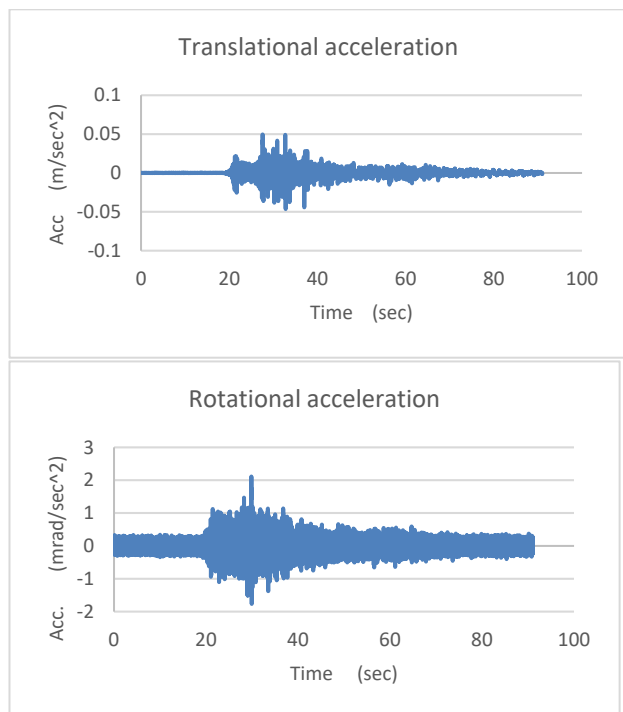**Figure 3** Properties, and layout of the 3 dof model

Figure 4 Translational and rotational component of earthquake excitation

A dynamic response history analysis was applied, with suitable software like SAP 2000, [31], and the response of structure was calculated. Two cases of earthquake excitation were considered. One with application only of translational acceleration at the base of structure and the second with simultaneously translational and rotational acceleration excitation. The results of the analysis are shown in Table 1. In this table the ratio of the response of the structure excited with translational and rotational acceleration to the response of structure excited only with translational acceleration is also shown.

Then a response spectrum analysis was apllied to the structure. The parameters of accelerations spectrum were: a_g=0.24g, Soil type B, damping coefficient ξ=5%, importance factor γi=1. Based on the above parameter the rotational and translation spectrum were constructed and response spectrum analysis was performed. The displacement was calculated for two cases: one using only the translational spectrum and the second using translational and rotational ones. The results are shown in Table 2. In this table the ratio of the response of the structure caculated using translational and rotational spectrum to the response of frame using only translational spectrum is also shown.

Table 1 Analysis results from response history analysis and ratio of response due to translational and rotational component to only translational component

	Top displacement (cm)		
	1st floor	2nd floor	3rd floor
Only translational component, **T**	0.1	2.5	5.05
Translational and rotational component, **R+T**	0.15	4.0	9.6
Ratio, (R+T)/T	**1.5**	**1.6**	**1.9**

Table 2 Analysis results from response spectrum analysis and the ratio of response due to translational and rotational component to only translational component

	Top displacement (cm)		
	1st floor	2nd floor	3rd floor
Only translational component, **T**	0.041	2.6	5.9
Translational and rotational component, **R+T**	0.049	2.9	8.26
Ratio, (R+T)/T	**1.2**	**0.9**	**1.4**

The graphical representation of the ratio of the above tables is shown in Figure 5.
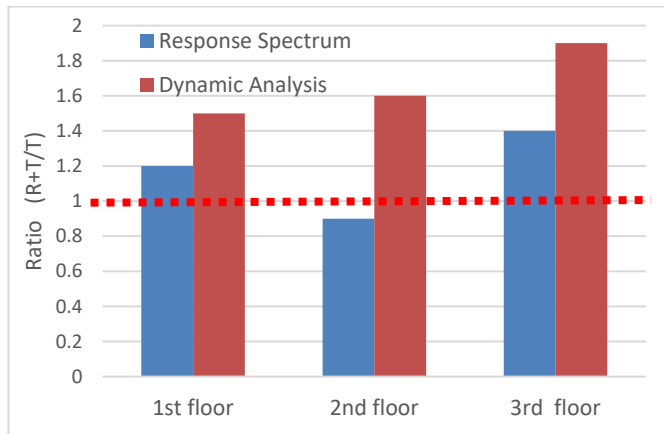
Figure 5 Translational and rotational component of earthquake excitation

From the analysis results it is shown that the displacement of structure subjected to rotational and translational component of earthquake is larger compared to the displacement of the structure subjected only translational excitation. The ratio of the response calculated with accounting rotational and translational component to the response calculated with acounding only translational component ranges from 1.2 to 1.9 depending on the type of analysis. It is worth to note that for the response spectrum case this ratio is smaller than 1.

4   Conclusions

The influence of rotational component of earthquake excitation to the response of structures was examined. From the numerical results it was obtained that the response of structure subjected to rotational and translational component are about double times higher than the response of structures subjected to only translational component. Parametric investigations of strucrural types as steel plane frame with different heights and bays should be done. Furthermore, analysis in 3D space steel structures with application of rotational component in two directions is under author's investigation.

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