Nuclear Matrix Elements for the $\frac{7}{2}^- \to \frac{7}{2}^+$ Beta Transition in ^{141}Ce (*).

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As the properties of the nuclear weak interaction are well understood (**), the study of the nonunique first forbidden beta transitions is mostly oriented at the present time to the investigation of the nuclear structure. Within this line of thought, the $\frac{7}{2}^- \to \frac{7}{2}^+$ beta transition from the decay of ^{141}Ce is investigated in this paper.

The available experimental information on this transition is scarce, consequently no attempt has been made so far to extract the nuclear matrix elements. In order to handle more and accurate experimental data, further measurements of the spectrum shape factor and beta-gamma angular-correlation anisotropy coefficient are performed in this work. Taking into account the values of these observables, together with the ones reported by DEUTSCH *et al.* (5) on the beta-gamma circular polarization, and by HOPPES (6) on the distribution of electrons from oriented nuclei—being characterized by the coefficients ω and N_1, respectively—a search for the nuclear matrix elements was undertaken.

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(**) Terms induced in the weak Hamiltonian by the strong interaction (pseudoscalar and pseudotensor) appear as higher-order corrections. They are only relevant in allowed (1,2), $0^- \to 0^+$ (cf. ref. (3)), and unique first forbidden (4) beta transitions.

(1) B. EMAN and D. TADIČ: *Nucl. Phys.*, **38**, 453 (1962).
(2) D. H. WILKINSON: *Phys. Lett.*, **31** B, 447 (1970); D. E. ALBURGER and D. H. WILKINSON: *Phys. Lett.*, **32** B, 190 (1970); D. H. WILKINSON and D. E. ALBURGER: *Phys. Rev. Lett.*, **24**, 1134 (1970).
(3) F. KRMPOTIČ and D. TADIČ: *Phys. Lett.*, **21**, 680 (1966); F. KRMPOTIČ and D. TADIČ: *Phys. Rev.*, **178**, 1084 (1969).
(4) B. EMAN, F. KRMPOTIČ, D. TADIČ and A. NIELSEN: *Nucl. Phys.*, **104** A, 386 (1967); S. M. ABECASIS and F. KRMPOTIČ: *Nucl. Phys.*, **151** A, 641 (1970).
(5) J. P. DEUTSCH, L. GRENACS and F. LIPNIK: *Journ. Phys. Rad.*, **22**, 662 (1961).
(6) D. D. HOPPES: *Proceedings of the VII International Conference on Low-Temperature Physics* (Amsterdam, 1961), p. 186.

The theoretical formalism used for the expressions of the different observables is described in terms of the lepton wave functions, as is mentioned in the treatment of both $0^- \to 0^+$ and unique first forbidden transitions [3,4]. A complete description of the formulation for nonunique first forbidden beta transitions as well as the details of the experimental procedures will be given in a forthcoming paper [7].

The observables such as the beta spectrum shape factor C_β, anisotropy ε, etc., are given in terms of the nuclear matrix elements defined as follows (cf. ref. [8,9]):

$$(1)\qquad \begin{cases} \eta w = g_A\int i\boldsymbol{\sigma}\cdot\boldsymbol{r}\,, \\ \eta v = -g_A\int \gamma_5\,, \\ \eta x = -g_V\int i\boldsymbol{r}\,, \\ \eta u = -g_A\int \boldsymbol{\sigma}\times\boldsymbol{r}\,, \\ \eta y = -g_V\int \boldsymbol{\alpha}\,, \\ \eta \;\;= g_A\int iB_{ij}\,, \end{cases}$$

where g_V and g_A are the effective vector and axial vector coupling constants, respectively.

The extraction of the nuclear matrix elements is based on the minimization of the χ^2-function—as expressed in ref. [8] (Chap. 10)—by using the MINUIT routine [*].

The parameters taken as free are w, x, u, Λ and Λ_0, where

$$(2)\qquad \begin{cases} \xi\Lambda = \int\boldsymbol{\alpha}\Big/\int i\boldsymbol{r} = y/x\,, \\ \xi\Lambda_0 = \int\gamma_5\Big/\int i\boldsymbol{\sigma}\cdot\boldsymbol{r} = -v/w\,, \\ \xi = \alpha\; Z/2R \end{cases}$$

(R being the nuclear radius and α the fine-structure constant).

The matrix element η is determined by the ft-value ($\log ft = 7.0$).

As is well known, when the ξ-approximation is valid [8-10], the linear combinations

$$(3)\qquad V = v + \xi w \qquad \text{and} \qquad Y = y - \xi(u + x)$$

satisfy the condition $|V| \simeq |Y| \simeq \xi$.

As the single-particle shell-model estimates for the nuclear matrix elements w, x and u are

$$(4)\qquad w = -1.25\,, \qquad x = -0.132\,, \qquad u = -1.26\,,$$

the values of these parameters are varied into the interval $(-10, 10)$. The intervals

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(7) F. KRMPOTIČ, H. E. BOSCH, M. BEHAR, M. C. CAMBIAGGIO, G. GARCÍA BERMUDEZ and L. SZYBISZ: to be published.
(8) H. F. SCHOPPER: *Weak Interaction and Nuclear Beta Decay* (Amsterdam, 1966).
(9) M. MORITA and R. S. MORITA: *Phys. Rev.*, **109**, 2048 (1959).
(*) Kindly provided to us by Drs. F. JAMES and M. ROOS, from CERN.
(10) H. A. WEIDENMÜLLER: *Rev. Mod. Phys.*, **33**, 574 (1961).

chosen for the variation of Λ and Λ_0 are from 1 to 3, and from 0 to 3, respectively, based upon previous theoretical estimates of their values [10-15].

Starting from 50 different initial sets of values for the five mentioned parameters, the corresponding minimization procedures gave rise to many final sets which fulfil the condition $\chi^2 < n = 4$ (where n is the number of observables). Due to this fact, it is preferred to give intervals of variation, as indicated in Table I, rather than to

TABLE I. – *Nuclear matrix elements and ratios for the* $\frac{7}{2}^- \to \frac{7}{2}^+$ *transition in the decay of* ^{141}Ce.

Nuclear matrix elements and ratios	Bounded intervals	Bounded intervals with $\Lambda = 2.4$
w	$-2.20 \div 0.10$	$-1.50 \div -0.30$
x	$0.42 \div 1.10$	$0.60 \div 1.00$
u	$0.93 \div 2.50$	$1.00 \div 1.70$
$\int \boldsymbol{\sigma} \cdot \boldsymbol{r}/R$	$-2.20 \div -0.10$	$-0.80 \div -0.14$
$\int i\boldsymbol{r}/R$	$0.20 \div 0.46$	$0.31 \div 0.41$
$\int \boldsymbol{\sigma} \times \boldsymbol{r}/R$	$0.50 \div 0.77$	$0.53 \div 0.70$
$\int iB_{ij}/R$	$0.31 \div 0.58$	$0.40 \div 0.54$
Λ	$2.10 \div 2.90$	2.4
Λ_0	$0.90 \div 2.30$	$1.00 \div 1.50$
$V = v + \xi w$	$-2.4 \div 3.1$	$-0.9 \div 4.5$
$Y = y - \xi(u + x)$ [a]	$-5.8 \div -1.7$	$-4.2 \div -2.1$

(a) The value of the parameter ξ was fixed to $\xi = 13.2$. In addition the matrix element $\int iB_{ij}$ is defined with an arbitrary phase, and only the relative signs between matrix elements are significant. The limits of variation for Λ and Λ_0 were fixed between 1 and 3 and 0 and 3, respectively.

quote errors for the values of the parameters. In Table I reference is also made to the corresponding nuclear matrix elements given in formula (1) with $g_V = 1.0$ and $g_A = -1.19$, for which the intervals of variation are smaller than in the former cases. In this Table are included finally the linear combinations V and Y, defined in eq. (3).

(11) D. L. PURSEY: *Phil. Mag.*, **42**, 1193 (1951).
(12) T. AHRENS and E. FEENBERG: *Phys. Rev.*, **86**, 64 (1952).
(13) J. EICHLER: *Zeits. Phys.*, **171**, 463 (1962).
(14) J. I. FUJITA: *Progr. Theor. Phys.*, **28**, 338 (1962).
(15) J. DAMGAARD and A. WINTHER: *Phys. Lett.*, **23**, 345 (1966).

Theoretical bands of the shape factor C_β and beta-gamma angular-correlation coefficient ε, calculated from the matrix elements given in Table I, are shown in Fig. 1 and 2, respectively. The corresponding results for the coefficients ω and N_1 are quoted in Table II.

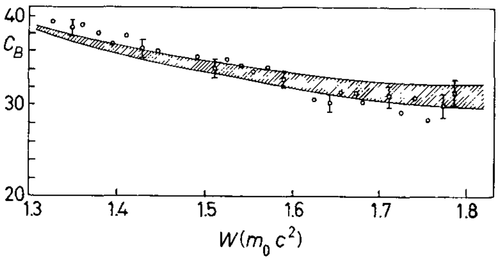

Fig. 1.

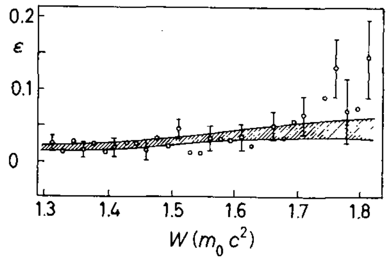

Fig. 2.

Fig. 1. – Theoretical derivation (shaded band) of the observable C_β as a function of energy, fitted with the experimental results obtained in the present work.

Fig. 2. – Theoretical derivation (shaded band) of the angular correlation coefficient ε as a function of energy, fitted with the experimental values obtained in the present work.

Table II. – *Values for the coefficients ω and N_1.*

Coefficients	Experimental value	Ref.	Theoretical values	
			A free	$A = 2.4$
ω	0.12 ± 0.16	(5)	$0.10 \div 0.30$	$0.12 \div 0.26$
N_1	-0.37 ± 0.10	(6)	$0.30 \div 0.39$	$0.33 \div 0.37$

Further trials were performed taking the parameter A as fixed and equal to 2.4 (cf. ref. (13,14)), and again several minima were obtained within somewhat smaller intervals than in the former case (see also Table I).

The theoretical bands for the observables C_β and ε are almost the same as the ones given in Fig. 1 and 2, while the values for ω and N_1 are shown in Table II.

A more recent measurement on beta-gamma circular polarization made by Daniel *et al.* (16) showed a different result ($\omega = -0.16 \pm 0.05$) from the one reported by Deutsch *et al.* (5). Consequently, a further analysis was performed taking into account this new value for ω, but no attempt to fit the data was successful.

As is disclosed from Table I, the ξ-approximation is not valid in this case. Even though this situation holds, it was impossible to univocally determine the nuclear matrix elements, which might indicate that the experimental information is still not sufficient.

Comparing the values of the parameters x and y given by the expressions (4) with the ones reported in Table I, one can conclude that the single-particle shell model does not account for the observables corresponding to the beta transition treated here.

(16) H. Daniel, W. Collin, M. Muntze, S. Margulies, B. Martin, O. Mehling, P. Schmidin and H. Schmitt: *Nucl. Phys.*, **118** A, 689 (1968).

Moreover, neither the shape factor nor the angular correlation results are satisfactorily explained within this approach (*).

These facts would force one to admit a considerable configuration mixing for the states involved in the transition, which is not supported by other experiments, in particular by the determination of the electric-quadrupole moment and magnetic-dipole moment of the ^{141}Pr ground state and magnetic-dipole moment of ^{141}Ce ground state [18].

This conflicting situation might be solved if further measurements on beta-gamma circular polarization and longitudinal polarization—both as a function of energy—are performed. Furthermore, it would be desirable to have a more realistic description of ^{141}Ce and ^{141}Pr that would explain simultaneously the beta-decay observables and the static moments of these nuclei.

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(*) BEEKHUIS and VAN DUINEN [17] derived a similar conclusion on the basis of their measurements on the shape factor for the same beta transition.

(17) H. BEEKHUIS and R. J. VAN DUINEN: *Nucl. Phys.*, **108** A, 382 (1968).

(18) M. LEDERER, J. HOLLANDER and I. PERLMAN: *Table of Isotopes* (New York, 1967).