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A Straightforward Methodology to Obtain the Power Coefficients Matrices for Unbalanced Distribution Networks to be Used in Flexibility Markets

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***Abstract*—The objective of the paper is to address the congestion problem in a specific distribution line by means of sensitivity coefficients seeking an intelligent activation of the available flexibility, based not only on economic aspects but also on the efficient use of flexibility. This paper proposes a straightforward methodology to obtain the sensitivity coefficient matrices for the unbalanced distribution networks, employing the perturb-and-observe (P&O) approach to assure an efficient usage of flexible resources. This means that a small change in the active power value of a particular flexibility provider, either load or generator, is applied to evaluate the power variation, therefore the loading variation, in every line of the system. The paper shows how these coefficients can be implemented in a theoretical energy market by the market operators by validating its application in an unbalanced network case and then compared with a real balanced distribution network. The simulations have been carried out in DIgSILENT PowerFactory through its API in Python.**

***Index Terms*—Energy market, network congestion, sensitivity coefficients, unbalanced networks**

I. INTRODUCTION

The growing trend of local generators or demand response programs in the distribution grid has imposed techno-economic challenges, especially related to congestion management. Hence, market players and policymakers have promoted the idea of flexible resources as a route to coping with such challenges. For example, in [1], a flexibility market named distribution-level flexibility market (Flex-DLM) was developed to solve network congestion regarding feeders overloading and voltage/var limits, minimizing the total cost of acquiring demand-side flexibility by the distribution system operator (DSO). This market framework is run after the day-ahead market clearing and excludes the DSO's need for trading in energy markets. In [2], a two-stage auction-based flexibility market based on the Newsvendor problem is modeled to solve expected network congestions at the DSO level, considering the uncertainty of the demand. Under this framework, the DSO acts as an exclusive buyer on a procurement platform aiming to minimize both the flexibility acquisition costs and penalty payment. The work in [3] introduced a four-step approach (data acquisition, forecasting, decision-making, and flexibility mechanism interfacing) to use flexibility for congestion management in the daily operation of the DSO. Particularly, for the last step, the decision-making model considers the loss of life of the transformer and the financial risk of the DSO if a blackout occurs due to overloading as the monetary value of flexibility. These works have in common that a network balanced case to test the proposed approaches is always considered in order to reduce the complexity of the network model, i.e., the unbalanced operational condition due to the uneven demand distribution across the grid. However, in order to cope with this aspect, sensitivity analysis is a key tool that can be implemented.

Sensitivity analysis in power systems allows assessing the impact on their performance when varying one or a set of electrical parameters. This tool is applied in both the planning and operation of power systems when it is required to foresee the changes of specific variables (e.g., $\partial V/\partial P$, $\partial I/\partial P$, $\partial P_{loss}/\partial P$) produced by variations in load and generation. It is also used for solving different power systems optimization problems, such as voltage regulation, loss reduction, network expansion planning, and optimal placement of reactive sources, DERs, FACTS, and conventional generators [4].

In the literature, there exist three main approaches for performing the sensitivity analysis on an electrical system. The first method [5] uses the submatrices of the inverse Jacobian Matrix by employing the Newton-Raphson algorithm in the load flow computation. The second approach [6] is known as the perturb-and-observe (P&O) method, which performs a small change to an input control variable and registers the effect over the whole network. The third technique [7] employs Tellegen's theorem and the notion of adjoint networks. This approach needs an initial load flow solution to create a particular adjoint network to be solved and get the required sensitivities. According to [8], these methods are based on the local sensitivity analysis (LSA), i.e., how a single input affects

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the output while treating the other inputs as deterministic values. This means that these techniques locally estimate each sensitivity in the vicinity of a nominal value of the selected input.

From the above-mentioned approaches, the inverse Jacobian Matrix is the most common approach for the sensitivity analysis, particularly for three-phase balanced systems at high- and medium-voltage levels [9], [10]. However, for unbalanced networks, its application is not straightforward. For example, commercial software packages like PowerFactory [11] compute the sensitivity values in terms of the sequence components, which results in an impractical way to handle the information of the network. Thus, to overcome this limitation, this paper presents a methodology to obtain the matrices of power sensitivity coefficients for both balanced and unbalanced networks with flexible resources based on the P&O approach. The calculation of the sensitivity coefficients is performed by taking advantage of the embedded power flow tool in PowerFactory and its ability to automate simulation-related tasks through Python [12].

The paper is structured as follows. Section II is devoted to presenting the sensitivity coefficients algorithm for unbalanced networks. Section III describes the case studies used to test the proposed methodology in a theoretical flexibility market, as well as the obtained results. In Section IV, conclusions are drawn.

II. SENSITIVITY COEFFICIENTS ALGORITHM

The power sensitivity coefficients, named in this paper as impact factors (IFs), are expressed as power variations due to changes in the active power, as shown in (1).

$$S_P = \frac{\partial P_k}{\partial P_j} \tag{1}$$

In order to reduce the complexity of using the analytical derivation per phase in (1), the P&O method is employed to evaluate it through numerical derivatives. This means that a small change in the active power value of a particular flexibility provider, either load or generator, is applied to evaluate the power and loading level variation in each power line of the grid. The aim is to determine which flexibility provider has a greater impact in addressing a congestion issue on a particular power line. Hence, an intelligent activation of the available flexibility is sought, based not only on economic aspects but also on the efficient use of flexibility providers.

The steps of the proposed methodology for computing the IFs, S_{Pkj}, due to changes in the active power of the flexibility provider are detailed below.

1) Execute an unbalanced load flow and store the single-phase active and reactive power of each power line $P_{k,i}$ and $Q_{k,i}$, and flexibility provider, $P_{j,i}$ and $Q_{j,i}$.
2) For each flexibility provider *j*, the following procedure is performed.
a) Vary the active power using (2), where $P_{j,f}$ is the final power of the flexibility provider *j*, $P_{j,i}$ is its initial power, and *n* is an exponential parameter that increases one unit per iteration. Note that as the value of *n* increases, the value of $P_{j,f}$ exponentially decreases up to get close to $P_{j,i}$. This means that at every new iteration, the power variation is modeled by an asymptotic function that tends to 1 p.u.

$$P_{j,f} = P_{j,i} \cdot \left(1 + \frac{1}{1.5^n}\right) \tag{2}$$

b) Execute an unbalanced load flow again and store the new active power values $P_{k,f}$ for every power line *k*.
c) Compute the initial sensitivity using (3). This equation defines that if the active power decreases due to an increase in the demanded power, there is a positive sensitivity value.

$$S_{P(k,i)} = -(P_{k,f} - P_{k,i})/(P_{j,f} - P_{j,i}) \tag{3}$$

d) Increase one unit the value of *n* in (2) and execute a new unbalanced load flow. The active power values of the power lines are overwritten in $P_{k,f}$, and the new value of $P_{j,f}$ replaces the value calculated in *2.a.*
e) Sensitivity is calculated with the new active power values for flexibility provider *j* as stated in (4).

$$S_{P(k,f)} = -(P_{k,f} - P_{k,i})/(P_{j,f} - P_{j,i}) \tag{4}$$

f) The tolerance of the IF is evaluated by using (5).

$$\left|\left(S_{P(k,f)} - S_{P(k,i)}\right)/S_{P(k,f)}\right| \times 100 < 0.01\% \tag{5}$$

g) Replace the value of $S_{P(j,i)}$ by $S_{P(j,f)}$, and continue to step *2.d* until the tolerance condition is met.
3) When the tolerance condition is met in *2.f*, the value of $S_{P(k,f)}$ is computed using (4).

Once the algorithm is evaluated for every flexibility provider *j* in the network, a rectangular matrix per phase is obtained, whose rows represent the affected power lines and their columns, the flexibility provider that provokes the perturbation. This means that the element of row *k* and column *j* symbolizes the power sensitivity that power line *k* experiences when a power variation occurs in flexibility provider *j*. Note that the initial values of $P_{j,i}$, and $P_{k,i}$ are restored after evaluating each flexibility provider *j*.

Once the sensitivity matrices are calculated, the network operator will make them available to the flexibility market operator so that it can take them into account in the market clearing process. The relevant system operator will be in charge of obtaining the IF matrices for its own networks in a transparent, pre-defined, and public manner.

III. CASE STUDIES

With the aim of assessing the effect of the IFs in a theoretical market, i.e., to consider not only the economic offer of a flexibility provider but also its efficiency in providing flexibility for an overloading scenario, different strategies have been defined. The main objective of each proposed strategy is to reduce the congestion problem in a specific power line.

First, an unbalanced power flow is run to identify the power line with the highest loading level, and depending on this value along with the required loading level, e.g., 80%, the theoretical amount of flexibility needed is defined.

Then, the available capacity of each flexibility provider for going up $(P_{\mathrm{up},j})$ or down $(P_{\mathrm{down},j})$, i.e., the available upwards and downwards flexibility, is computed by (6) and (7).

$$P_{\mathrm{up},j} = P_{\max,j} - P_{\mathrm{curr},j} \tag{6}$$

$$P_{\mathrm{down},j} = P_{\mathrm{curr},j} - P_{\min,j} \tag{7}$$

where $P_{\max,j}$ represents the maximum active power capacity of generator k and the total installed capacity of load k; $P_{\mathrm{curr},j}$ is the current value of active power for both the generator and load j; $P_{\min,j}$ defines the minimum active power level of the generator j. For the load case, this value is assumed as 90% of $P_{\mathrm{curr},j}$, i.e., the active power of load j can be reduced up to 10%.

For the theoretical market used in this work, it is considered that each flexibility provider has different activation prices. Thus, based on the IFs of the network and the activation prices, a merit order list (MOL) of flexibility providers for the k^{th} line can be obtained by dividing these two parameters. This means that these three key factors can be defined as congestion management strategies.

So, depending on the selected strategy, i.e., MOL, price-based, or IFs, an ordered list of flexibility providers is defined. With this list and the amount of flexibility required, the flexibility providers are sequentially activated according to their available capacity.

In addition to these three strategies, a fourth strategy named "equitable power distribution" was also evaluated. This strategy seeks to divide the amount of flexibility required among all the flexibility providers depending on their available capacity. For example, 10 MW of available flexibility from 5 flexibility providers (1 MW, 3 MW, 2 MW, 3 MW, and 1 MW) are divided percentage-wise among them to supply 7 MW of required flexibility, i.e., $1\,\mathrm{MW}\ /\ 10\,\mathrm{MW} \times 7\,\mathrm{MW} = 0.7\,\mathrm{MW}$.

For each of the four strategies, two case studies were considered to validate and evaluate the performance of the proposed method; one unbalanced case in a modified benchmark network and another balanced case in a real distribution feeder. Additionally, flexibility prices were randomly generated assuming a mean value of 1 €/kWh with a standard deviation of 0.015 €/kWh in a normal distribution function.

A. *Case 1: Description and Results*

The IEEE 9-node network [13] was modified to introduce an unbalanced operational condition through the demand (see Fig. 1) with the aim of validating and verifying the proposed methodology. The original three loads at buses 5, 8, and 6 were replaced by single-phase loads with different power demands. Loads at bus 5 are considered critical demands that cannot offer flexibility (CrL). Initially, the most loaded power line is Line 4-5 at 111.5% in phase A; labeled in Fig. 1 as the line under congestion (LuC).

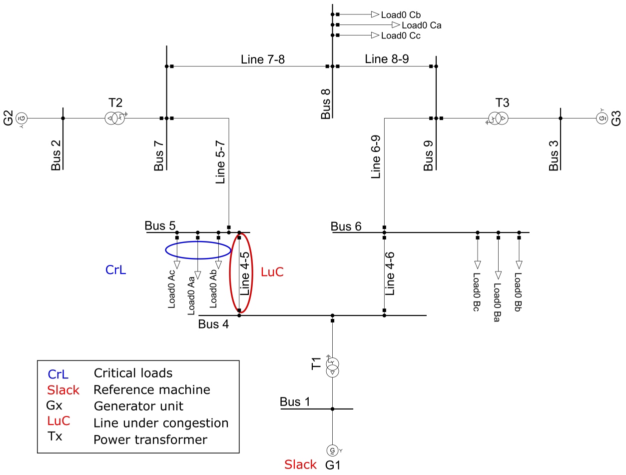

Fig. 1. Modified IEEE 9-node network

Obtained results for total loading level and loading level variation per phase and line for the implemented four strategies are shown in Fig. 2 and Fig. 3, respectively. As it can be observed in Fig. 2 the most loading line is phase A of *Line 4-5*. The amount of required flexibility corresponds to 171.6 MW and was calculated to reduce the loading level of phase A in *Line 4-5* to 80%. Note that in the four implemented strategies, the amount of flexibility required is the same. Based on this flexibility need, the MOL was defined by using the IFs of phase A in *Line 4-5*. As was previously explained in Section II, a rectangular matrix with the IFs is obtained for each phase, where each row corresponds to a line *k*. Even if the loading level of *Line 5-7* and *Line 7-8* increase both for MOL and IF strategies, the final values remain below 50% in all cases.

It can be observed in Fig. 3 that, depending on the implemented strategy, the obtained loading level reduction of phase A in Line 4-5 varies from 24% to 28%. In this case, the highest loading reduction value is due to the MOL and IF strategies. However, the flexibility provided was not enough to obtain the desired loading level, i.e., 80%, due to the reactive power compensation in the system.

As far as the cost of the activated flexibility is concerned (see Table 1), the lowest cost corresponds to the price-based strategy. However, technically, the loading level of the network gets better better using the MOL and IF strategies.

TABLE I
FLEXIBILITY COST BY STRATEGY IN CASE 1

Variable	**MOL**	**Price-based**	**IF**	**Equitable power**
Cost (€/MWh)	995.9	982.9	995.9	994.3

B. *Case 2: Description and Results*

A medium voltage feeder was extracted from one real network in Murcia, Spain, to investigate the applicability of the proposed method in a balanced scenario. It is assumed that the feeder is operated in a combination of a radial and meshed topology to supply power to 75 three-phase loads

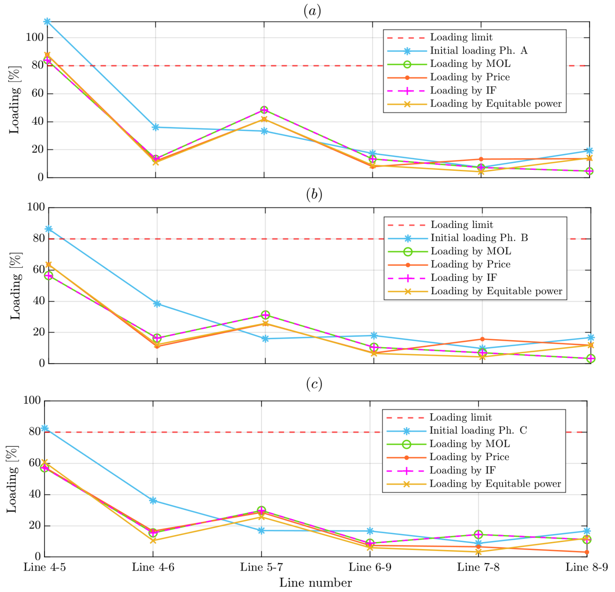

Fig. 2. Loading level per phase and by flexibility strategy

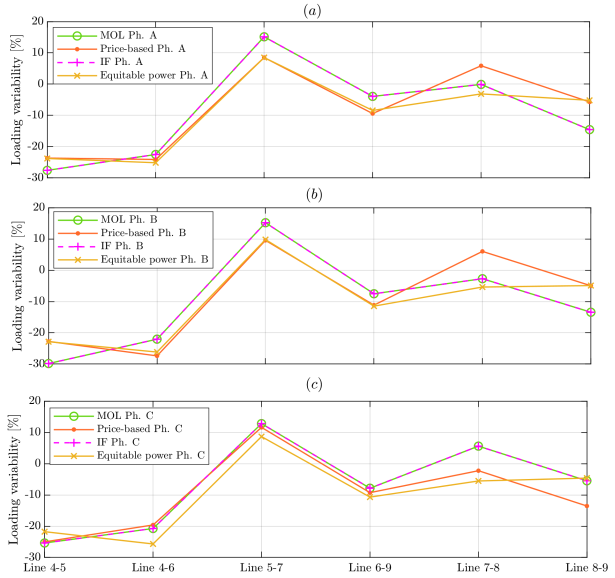

Fig. 3. Loading level variation per phase and by flexibility strategy

through 55.98 km of cables. Besides, there are five small three-phase synchronous generators connected along the feeder to feed private customers. The feeder is connected to a 20 kV network through a 40 MVA synchronous generator. In order to depict the topology of the feeder, its single-line diagram was simplified using three equivalents network sections, as shown in Fig. 6.

At the initial state, the line highlighted in red is under congestion at 115.9%. This operational state enables requesting flexibility from the generator units and the loads, except for those loads that cannot vary their power for being a critical demand (blue), e.g., a hospital. In this case, to decrease the loading level of the LuC to 80%, it was necessary to supply 3.1 MW of flexibility.

Fig. 4 and Fig. 5 show the flexibility supplied by each flexibility provider and the resultant loading level of the network, respectively, after applying the four proposed strategies to the initial operational state. In Fig. 4, it can be observed that each strategy requests different combinations of flexibility providers, but in some cases, the flexibility provider and its capacity are the same for the MOL, price-based, and IF strategies due to the effect of the IFs and activation prices. Based on the provided flexibility, it can be noted in Fig. 5 that all four strategies allow reducing the loading level close to 80%. This is due to the influence of the network topology, the balanced operational condition, and the variability of the activation prices. Besides, note that the equitable power distribution strategy is the most expensive because it activates all the flexibility providers regardless of the cost of their flexibility, as shown in Table 2.

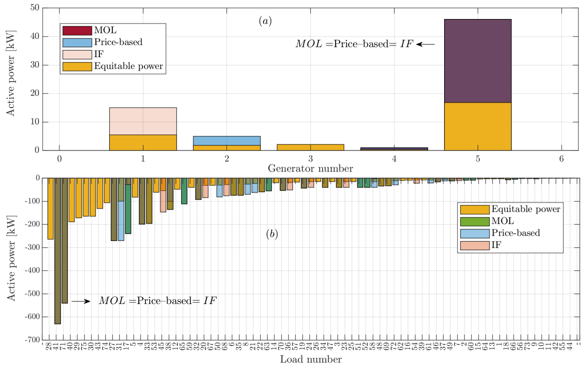

Fig. 4. Flexibility required per flexibility provider

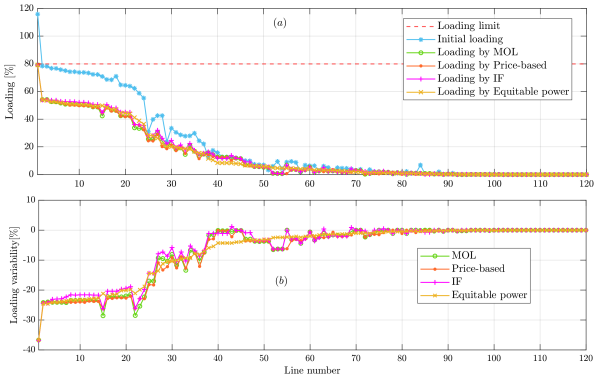

Fig. 5. (a) Loading level and (b) Loading level variation by flexibility strategy

As the provision of flexibility comes from flexibility providers connected to the same feeder, it allows reducing the loading level across all the distribution lines, as shown in Fig. 5. However, on a broader network, the expected loading level across the same feeder will not be entirely modified if the IF or MOL strategies are applied, as several flexible resources from

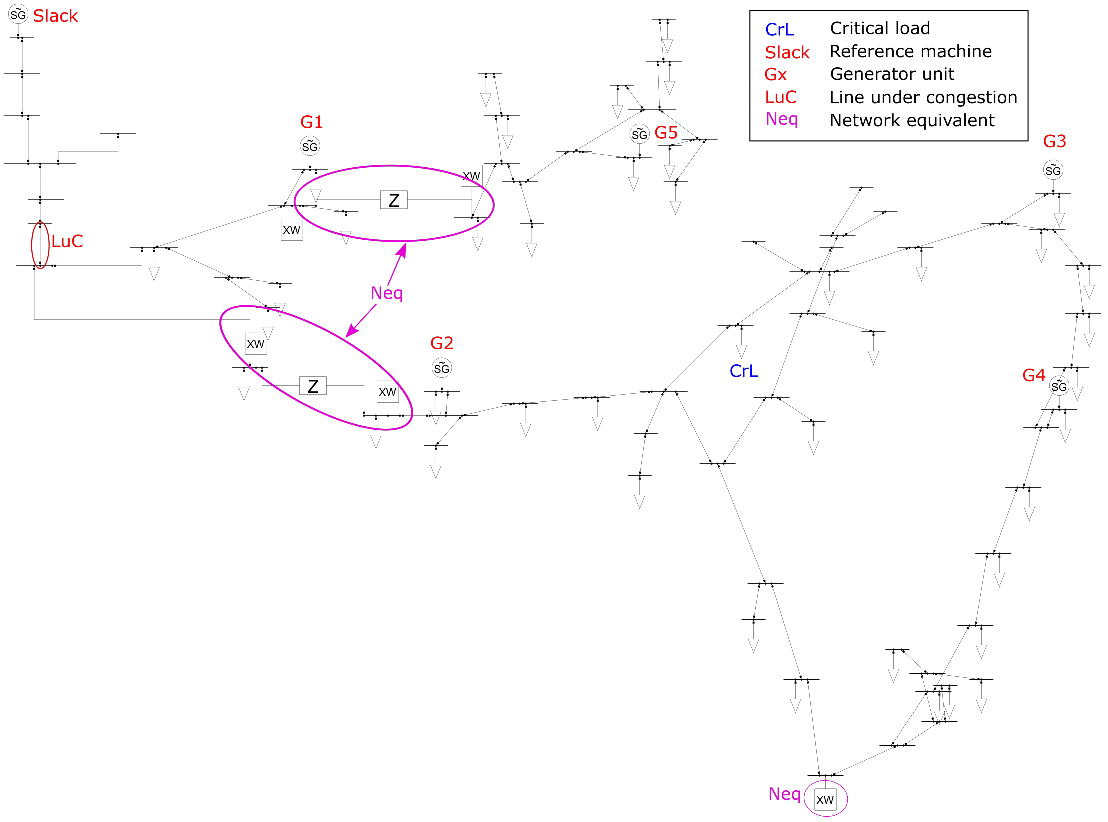

Fig. 6. Real distribution feeder model

other feeders will contribute to the total flexibility to mitigate the congestion in the same distribution line.

TABLE II
FLEXIBILITY COST BY STRATEGY IN CASE 2

Variable	MOL	Price-based	IF	Equitable power
Cost (€/MWh)	982.6	980.5	989.2	996.6

IV. CONCLUSIONS

The objective of the paper was to validate and verify the proposed methodology, which seeks to reduce the loading level at a congested line by means of flexibility provision.

For this purpose, an algorithm for both balanced and unbalanced networks has been developed and implemented in PowerFactory. However, the proposed methodology was designed to be employed in any power system's software package able to perform unbalanced load flows, and that allows the interaction with Python.

The proposed methodology has been tested in an unbalanced and balanced grid and compared with four flexibility strategies: MOL, price-based, IF, and equitable power distribution.

From the obtained results, it can be concluded that IFs are a suitable tool for the efficient use of the available flexibility, especially when both IF and price-based strategies are combined by means of a MOL. Impact factors are of high importance when the analyzed grid is meshed compared to a radial one.

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