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Multiplicity of solutions in model-based multiobjective optimization of wastewater treatment plants

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Abstract Wastewater treatment process design involves the optimization of multiple conflicting objectives. The detection of different equivalent solutions in terms of objective values is crucial for designers in order to efficiently switch to the new optimal operation policies if changes in the process conditions or new constraints occur. In this work, the dynamic multi-objective optimization of a municipal wastewater treatment plant model is carried out. The aim is to simultaneously optimize an economic cost term and an effluent quality index. The selected process variables for the optimization are i) an aeration factor in the aerated tank previous to the clarifier, and ii) an internal recycle flow

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rate. Their time profiles are approximated using the control vector parameterization (CVP) technique. To solve the multi-objective problem and find the Pareto front, the NSGA-II algorithm has been used. The simulation of different realistic scenarios which impose operational constraints (e.g., maintenance operations) reveals that, indeed, multiple solutions exist at least in some areas of the Pareto front. It is observed that different control profiles can produce nearly identical results in terms of Pareto solutions. The *a priori* knowledge of these equivalent solutions for different scenarios provides the decision makers with alternative choices to be adapted to their organizations policies when events altering decision variables bounds or adding new constraints to the process model occur.

Keywords wastewater treatment plant · multiobjective optimization · dynamic optimization · multiple solutions

1 Introduction

Wastewater treatment plants (WWTPs) are crucial nowadays to process the industrial and/or urban effluents generated in modern societies. Many WWTPs use activated sludge to eliminate organic and nitrogen compounds. Such plants, when designed to treat high volumes of water, usually consist of (i) an aerobic area, in which organic compounds as well as ammonia and nitrites are oxidized, (ii) an anoxic area, in which nitrates are reduced to gaseous nitrogen, and (iii) a clarifier to separate the microbial culture from the water being treated.

The reduction of the WWTP carbon footprint is not just an environmental issue. There are also important economic repercussions, and benchmarking is a powerful tool to help reducing economical costs [37]. For instance, wastewater treatment accounts for about 3% of the U.S. electrical energy load similar to that in other developed countries [35]. Depending on the particular WWTP considered, energy becomes the most important cost factor or the second after personnel costs [38]. Among the energy costs, aeration and recycle costs are the highest.

Due to the strict legal and environmental standards that WWTPs must meet, efficient optimization and control tools are mandatory to achieve an optimal-cost operation when dealing with such systems. Model-based optimization is one the most efficient approaches to carry out this task [41,48]. In particular, dynamic optimization (i.e., optimization considering time-varying variables) is a powerful tool for engineers and practitioners in order to find the optimal operating conditions and/or to infer the optimal design of WWTPs. A key aspect in the design and optimization of WWTPs is that the mathematical models describing the processes are inherently nonlinear and dynamic. This requires the use of robust tools to perform the process optimization. As an additional obstacle to find the optimal operating conditions of such processes, the presence of several conflicting objectives to be optimized at the same time must be considered (e.g. productivity and sustainability), which advises the use of sophisticated formulations to find the Pareto front. Typical objective functions usually include operational costs and product quality measured as the amount of pollutants in the effluent.

Some recent examples of the literature that address the problem of finding the optimal operating conditions in WWTPs are the following: Lukasse and Keesman [32] performed a simulation study using an optimal control methodology and selecting from among the best simulated situations; Samuelsson et al. [42] used operational maps from simulations to choose optimal set points; Yong et al. [52] evaluated different control strategies using the COST Simulation Benchmark Model [10]; Moles et al. [36] tested several global optimization methods for simultaneously optimizing operation and design of a WWTP located in Spain; Schütze et al. [43] proposed an integrated approach for the optimization of control strategies; Egea et al. [15] used surrogate model based optimization to accelerate the solution finding of the computationally expensive model of a WWTP. In single-objective optimization the different authors have usually focused in the aeration energy, which causes the highest economical costs in WWTPs and its optimization can produce important savings [2, 3, 7, 8, 33, 39].

Design and optimization of WWTPs allows the selection of multiple objectives related to operation, physical design, location and others [13, 16]. However, most of the scientific literature refers to optimization and control of the operational aspects. For instance, Fu et al. [19] considered different objectives mainly based on the effluent quality and pumping energy. Flores-Alsina et al. [18] combined multivariate statistics and life cycle assessment concepts to choose a set of different criteria to be optimized simultaneously. Zhang et al. [53] proposed a multi-objective optimization problem where multiple effluent quality indexes as well as the treatment costs where optimized with the help of a surrogate model. Beraud et al. [6] solved a multi-objective optimization problem similar to the one presented in this work. They considered the simultaneous optimization of the effluent quality and the energy consumption. More recently, Hreiz et al. [27] studied the influence of different time-varying variables over two conflicting objectives, namely the mean nitrogen concentration in the effluent and the net electrical consumption in a small size WWTP. In this work, the authors included the idea of excess sludge incineration to produce energy. Chen et al. [9] tested different control strategies in an activated sludge plant using the SA^2/OCM process to simultaneously optimize the effluent quality and the operational costs. A recent contribution [40] analyzed the dynamic set-point controller profiles in a WWTP by multi-objective optimization. More examples about multi-objective and/or dynamic optimization in WWTPs can be found in the review by Hreiz et al. [26].

The most popular optimization algorithm to solve multi-objective optimization problems, which has been used in many of the references cited above, is NSGA-II [12]. This evolutionary algorithm has been modified and combined with other optimization approaches (e.g., [17]), becoming one of the most important references for multi-objective optimization, with implementations in many programming languages. Other evolutionary methods or metaheuristics have also been used for solving multi-objective problems in WWTPs [23].

WWPTs model- based design and optimization are computationally expensive tasks. For this reason, different researchers have used surrogate model-based optimization methods alone or in combination with evolutionary algorithms. For instance, Fu et al. [20] compared the results of the optimization or urban wastewater systems using NSGA-II and ParEGO, a surrogate model-based multi-objective optimization algorithm [30]. More recently, Hartikainen et al. [24] implemented the approximation method PAINT within an interactive optimization platform to construct computationally inexpensive surrogate problems for the original wastewater treatment problem.

The aim of this work is to find and analyze the optimal control profiles of a WWTP model that uses the activated sludge process in a multicriteria approach. Both the aeration and recycle rate policies are investigated in order to simultaneously optimize an economic term and the effluent quality. Preliminary optimization results suggest that different control profiles can lead to equivalent solutions in terms of objective values. These equivalent solutions can be calculated by different procedures. Here we have implemented two different (possible) operational scenarios in which the control variables are forced to change their values in a period of time to simulate maintenance operations or even a failure. Knowing these (alternative) equivalent solutions can be of great importance for WWTP plant operators to know which operational conditions must be applied in case of certain events to maintain the desired standards as much as possible. This approach is related to the concepts introduced by Lewis et al. [31] that explore the idea of dynamic s-Pareto frontiers and preferences, or by Vallerio et al. [47], which consider operational risks and uncertainties as additional objectives to solve multi-objective optimization problems of non-linear dynamic processes. The idea of simulating possible realistic scenarios in a multiobjective formulation could be compatible with the interactive optimization platforms to analyze WWTP optimization problems proposed in recent years [22,24].

This work is organized as follows: section 2.1 presents a description of the WWTP model under study; in section 2.2 the multi-objective dynamic optimization problem is formulated, and the obtained results considering an undisturbed formulation and two possible scenarios are presented, compared and discussed in section 3. The final section depicts the main conclusions of the study.

2 Methods

2.1 WWTP model description

The WWTP which is the object of this study is modelled by the Benchmark Simulation Model No. 1 (BSM1) which can be defined as *a simulation protocol defining a plant layout, a process model, influent data, test procedures and evaluation criteria* [10,29]). It includes a pre-denitrification system consisting of 5 main units, the first two being anoxic and the rest aerobic. The scheme of the plant also includes a secondary clarifier that separates the microbial culture from the effluent treated. Figure 1 shows the plant layout.

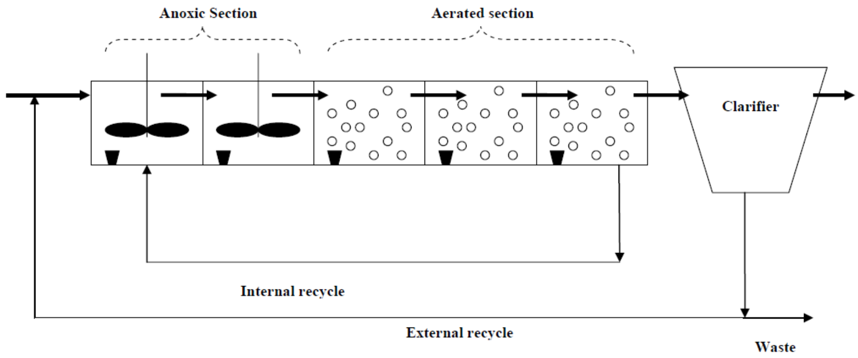


Fig. 1: COST Benchmark WWTP model layout

There are two recycle loops in the plant: internal and external. The internal one recycles nitrates from the last (aerated) reactor to the first (anoxic) reactor. The external one recycles activated sludge and connects the bottom of the clarifier with the plant entrance.

The BSM1 arose to test different control strategies for the operation of this type of plants regarding carbon and nitrogen removal. It has been used in hundreds of applications regarding WWT Plants [28]. The system dynamics are described by algebraic mass balance equations, ordinary differential equations for the biological processes in the bioreactors, as defined by the ASM1-model [25], and the double-exponential settling velocity function [46], for a total number of around 100 differential algebraic equations [1]. The volumes of the reactors are, respectively, 1000 m^3 for the anoxic units and 1333 m^3 for the aerated ones. The secondary settler has 10 layers with a total area of 1500 m^2 and a depth of 4 m.

The influent dynamics are also defined in BSM1 and three different weather conditions can be chosen: dry, rain and storm weather. These are introduced as input files and can be used as standard and realistic representation of influents in the mentioned weather conditions, although there are several different approaches to generate such influent dynamics [34]. The files contain influent information every 15 minutes for a total period of 14 days. Evaluation functions comprise a 100-day initialization period until steady state is achieved, followed by a period of 14 days of a type of weather defined by the corresponding input file. Calculations on the plant performance are based on the data obtained from these last 14 days.

Given the physical design of the plant, there is a number of candidate control variables to optimize different possible objectives. The BSM1 defines two control variables by default: nitrate concentration in reactor 2 and dissolved oxygen in reactor 5. In the original implementation two controllers are modeled to control the mentioned variables by manipulating the internal recycle flow rate (Q_{intr}) and the oxygen transfer coefficient in reactor 5 (K_La_5). In this work we have used the "open-loop" implementation of the BSM1 and approximated the mentioned manipulated variables using zero-order polynomials according to the control vector parametrization approach (CVP, see section 2.2.1) [50, 51]. The aim is to find the manipulated variables dynamic profiles to simultaneously optimize two performance indexes: one related to the process economy and another one related to the process sustainability. The problem formulation and further details on the solving approach are given in the following section.

2.2 Problem formulation

Different criteria can be defined in BSM1 in order to find efficient and sustainable operating conditions. The most usual criteria are related to economical costs, often as a weighted sum of aeration and pumping energy costs (which represent the highest energetic cost in WWTPs) plus the cost of wasted sludge treatment, and the effluent quality considering all the possible remaining pollutants and their concentrations in the outlet stream. These two criteria counter each other, allowing multiobjective formulations to be made. The economical cost term has been defined in this work as follows.

$$C = AE + PE + 3P_{sludge} \tag{1}$$

where AE stands for the aeration energy needed in the aerated tanks in $kWhd^{-1}$, PE is the pumping energy needed in the recycles, also in $kWhd^{-1}$, and P_{sludge} is the wasted sludge that must be treated in kgd^{-1}. Those terms are weighted according to [49]. The aeration energy is given by:

$$AE = \frac{24}{T} \int_{t_0}^{t_{14\text{days}}} \sum_{i=3}^{5} \left(0.0007K_La_i(t)^2 + 0.3267K_La_i(t)\right) dt \tag{2}$$

where $K_La_i(t)$ is the mass transfer coefficient in the i-th aerated reactor at time t (in units of h^{-1}).

The pumping energy term is defined as:

$$PE = \frac{0.04}{T} \int_{t_0}^{t_{14\text{days}}} \left(Q_{intr}(t) + Q_r(t) + Q_w(t)\right) dt \tag{3}$$

where $Q_{intr}(t)$ is the internal recycle flow rate, $Q_r(t)$ is the return sludge recycle flow rate and $Q_w(t)$ is the wasted sludge flow rate, all of them at time t with units m^3d^{-1}.

The wasted sludge to be treated, P_{sludge}, is calculated as:

$$P_{sludge} = TSS_w \cdot Q_w(t) \tag{4}$$

where TSS_w is the total suspended solids in the flow wastage.

Regarding the second criterion, the effluent quality in kg pollution units d^{-1} is defined as follows:

$$EQ = \frac{1}{T \cdot 1000} \int_{t_0}^{t_{14\text{days}}} \begin{pmatrix} \beta_{SS} \cdot SS_e(t) + \beta_{COD} \cdot COD_e(t) + \\ +\beta_{BOD} \cdot BOD_e(t) + \beta_{Nkj} \cdot S_{Nkj,e}(t) + \\ +\beta_{NO} \cdot S_{NO,e}(t) \end{pmatrix} Q_e(t)dt \tag{5}$$

where T is the time horizon (i.e. 14 days), $SS_e, COD_e, BOD_e, S_{Nkj,e}$ and $S_{NO,e}$ are the total suspended solids, chemical oxygen demand, biological oxygen demand, total Kjeldahl nitrogen and nitrites/nitrates nitrogen, respectively, all of them measured in the effluent. Q_e is the effluent flow rate. The weighting coefficients β_i are taken from [48].

Once the objectives have been defined the general multiobjective dynamic optimization problem is formulated, which aims to find the time varying control profiles ($\mathbf{u(t)}$) in order to optimize a given set of objectives represented as cost functions ($\mathbf{F}$) subject to the system dynamics and possible algebraic constraints [5]. Mathematically:

$$\min_{\mathbf{u}(t)} \mathbf{F}\left(\mathbf{x}(t), \mathbf{u}(t)\right) \tag{6}$$

subject to:

$$\frac{d\mathbf{x}}{dt} = \mathbf{\Psi}\left(\mathbf{x}(t), \mathbf{u}(t), t\right) \tag{7}$$

$$\mathbf{x}(t_0) = \mathbf{x}_0 \tag{8}$$

$$\mathbf{h}\left(\mathbf{x}(t), \mathbf{u}(t)\right) = \mathbf{0} \tag{9}$$

$$\mathbf{g}\left(\mathbf{x}(t), \mathbf{u}(t)\right) \leq \mathbf{0} \tag{10}$$

$$\mathbf{u}^L \leq \mathbf{u}(t) \leq \mathbf{u}^U \tag{11}$$

where the vector of objective functions, $\mathbf{F}$ (Equation 6), contains all the objectives considered in the problem. In our case, the objectives were already defined as f_1 = operational costs (Equation 1) and f_2 = effluent quality (Equation 5). $\mathbf{x}$ is the vector of state variables (i.e. those variables that change with time and that can not be controlled, such as pollutants concentrations). Copp described a total number of 13 variables for this model [10]. $\mathbf{u}$ is the vector of control variables (the aeration factor in the last aerated reactor and the internal recycle flow rate in our case) whose variation with time need to be found to optimize the objective functions. Equation 7 represents the system dynamics (dynamic mathematical model that defines the BSM1). Equation 8 represents the values of the state variables at the beginning of the process ($t = 0$). Equations 9 and 10 represent, respectively, equality and inequality constraints, which can be considered at the end of the process or at intermediate times (e.g. a maximum pollutant concentration in the effluent). In our formulation no additional constraints have been imposed apart from the process dynamics. Finally, Equation

11 corresponds to the lower and upper bounds for the control variables (e.g., the minimum and maximum aeration and internal recycle flow rate allowed for the operation). In our problem those bounds are defined as $[0, 360]$ h^{-1} for K_La_5 and $[0, 70000]$ m^3d^{-1} for Q_{intr}. The values of the operational variables not considered as control variables (e.g., aeration rates in tanks 1 to 4 as well as influent, wastage and external recycle flow rates) are those defined in [10] and remain constant during the optimization procedure. The accurate solution of the differential-algebraic equation (DAE) system defined in Equation 7 often requires the use of an implicit ordinary differential equation (ODE) solver. In this work we have used the ode45 and ode15 included in Matlab-Simulink, where the BSM1 was implemented. The integral terms included in the objective functions are numerically solved by discretization, using the same time step size as in the ODE solution.

2.2.1 CVP for approximating the control variables

A number of solution methods can be used for solving the general dynamic optimization problem [45]. For the problem formulated above, a control vector parameterization approach (CVP) is employed. CVP is a direct method which transforms the original problem into a non-linear programming (NLP) problem, which must be solved by a (global) optimization solver [5]. This method enables the discretization of the control problem by dividing the time horizon into a number of time intervals so that nonlinear programming (NLP) techniques can be applied to the resulting finite-dimensional optimization problem. According to this method, basis functions, usually low order polynomials, are used to approximate the control variables within the time intervals. This parameterization method transforms the infinite-dimensional optimization problem into a nonlinear programming problem. Thus, the differential equality constraints describing the system dynamics are integrated for each evaluation of the performance index of interest. The CVP method has also been used in other applications involving anoxic / aerated systems [4]. In this work we have used zero order polynomials (i.e. steps) to approximate our control variables. We have considered 20 fixed-length time intervals for each control variable, which results in a non-linear optimization problem with 40 decision variables

2.3 Considered scenarios

The analysis of some adjacent solutions in the Pareto front of the problem formulated in Equations 6-11 suggests that control profiles with different shapes can lead to very similar solutions in terms of objective values. This can be observed in the Supplementary Information where sweeps of the control profiles corresponding to all points (200) in the Pareto fronts of the solved problems are shown as figures. An example is given by the adjacent solutions #33 and #34 of the undisturbed problem, where differences between control profiles can be observed whereas the values of the objectives are almost identical. To check whether this can be found in other parts of the Pareto front, we propose a procedure in which extra constraints to the optimization problem are added so that the shape of some control variables is intentionally changed with respect to the undisturbed case. From a practical point of view, these constraints should reflect realistic situations or events that can occur during practical operations of WWTPs like unexpected failures, maintenance operations or punctual changes in environmental requirements or energy consumption. The Pareto fronts of the new optimization problems are then compared with the one of the undisturbed problem to check if there is any kind of overlapping. In this study we propose two very simple scenarios. In the first one we simulate that aeration in tank 5 (corresponding to our first control variable) does not work for some time at the beginning of the process due to a failure. In the second one recirculation is not allowed for some days (also at the beginning of the process) simulating maintenance operations. In the considered scenarios the modification of the optimization problem formulated above is straightforward: the number of decision variables is reduced. In particular, we consider only 36 decision variables from the initial set of 40 since we choose 4 time intervals in which the incumbent control variables are forced to be zero. Other more complex scenarios that involve the formulation of new constraints, changes in the bounds, etc. can be conceived, but, for illustrating the idea of multiplicity of solutions, the proposed scenarios are suitable.

2.4 Optimization method

The whole formulation in Equations 6-11 is a non linear programming problem that must be solved with specific optimization solvers. In the context of WWTP optimization, Egea et al. [14,15] showed that the associated problems are multimodal. Further, problems resulting from the application of CVP are also frequently multimodal. Thus, global optimization solvers must be used. For problems with multiple (conflicting) objectives like the presented here, the aim is to find the optimal trade-offs between such objectives. This trade-off is represented in the Pareto front. All solutions in the Pareto front are optimal in the sense that it is not possible to improve one of the objectives without worsening one or more of the rest.

In this work we have used the popular evolutionary multi objective optimization method NSGA-II [12] already mentioned in Section 1, which is used to capture the Pareto front of the proposed multi-objective model and furthermore, the final optimal control profiles can be selected based on the preference of the decision-maker. NSGA-II is a revised version of the NSGA [44]. The NSGA uses an evolutionary process with surrogates for evolutionary operators including selection, genetic crossover, and genetic mutation. The population is sorted into a hierarchy of sub-populations based on the ordering of Pareto dominance. Similarity between members of each sub-group is evaluated on the Pareto front, and the resulting groups and similarity measures are used to promote a diverse front of non-dominated solutions. NSGA is a very effective algorithm but has been generally criticized for the high computational complexity of non-dominated sorting, the lack of elitism and the need to specify the sharing parameters. Compared to the simple NSGA algorithm, the NSGA-II improves the computational efficiency by reducing the time-complexity from $O(MN^3)$ to $O(MN^2)$, where M is the number of objectives and N is the size of the dataset. Furthermore, it has a better sorting algorithm, incorporates elitism and no sharing parameter needs to be chosen *a priori*. The NSGA-II uses $(\mu + \lambda)$-selection instead of a secondary population as its elitist mechanism. The multi-objective optimization was carried out using the following parameters of the NSGA-II algorithm: binary tournament selection, number of generations (200), population size (200), crossover probability (0.9), mutation probability (0.1). The simulation model was implemented using the software MATLAB & Simulink. Each member of the population was computed using a cluster with 8 nodes. Such nodes are equipped with 2 Intel Xeon E5-2620 at 2 GHz and 32GB of RAM memory.

3 Results and discussion

The dynamic multiobjective optimization problem formulated above was solved for the dry-influent data set. A similar procedure could be performed considering the other weather conditions or a combination of them. The obtained Pareto fronts for the undisturbed, scenario 1 and scenario 2 problems are shown in Figure 2. The results correspond to all the 14 operation days. The shape of the Pareto fronts is similar to that obtained in [11, 21, 27].

As shown in Figure 2, the Pareto fronts indicate that, as expected, the improvement of one objective deteriorates the other, i.e. a lower Effluent Quality index involves increasing the operational costs and vice-versa. To avoid confusion with the nomenclature, it should be recalled that a lower EQ index means a higher effluent quality. Regarding the control profiles, three main areas in the Pareto fronts can be distinguished: a) an area with low operational costs and poor effluent quality (Area 1), b) an area with high operational costs and good effluent quality (Area 2) and, c) an intermediate area (Area 3). Figure 3 shows the control profiles for the representative solutions (undisturbed problem) of each area presented in Table 1.

Table 1: Representative objective values for the 3 main areas of the Pareto front (undisturbed problem)

	(Monetary units d^{-1})	EQ (kg poll units d^{-1})
Area 1	13927	8924
Area 2	16765	6718
Area 3	14992	7584

The combination of the Pareto front and the control profiles associated to each solution are useful decision tools to design the process and possible con-

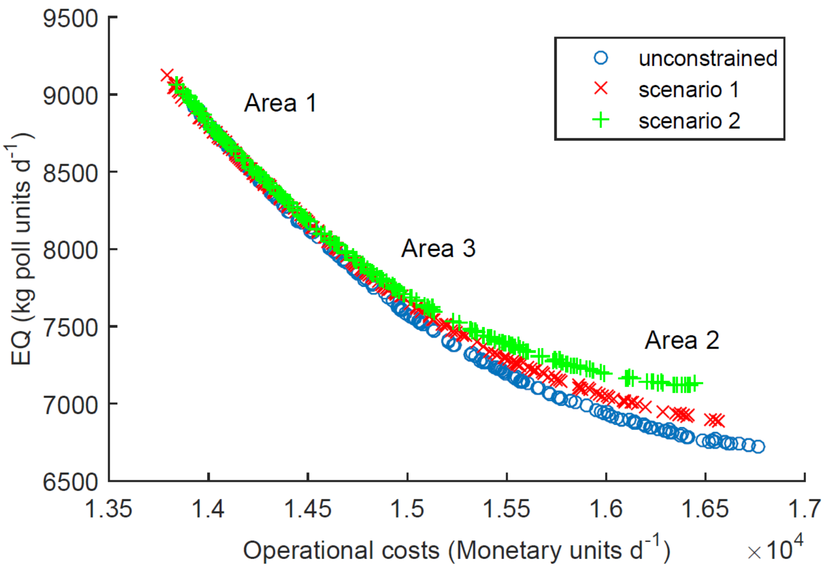
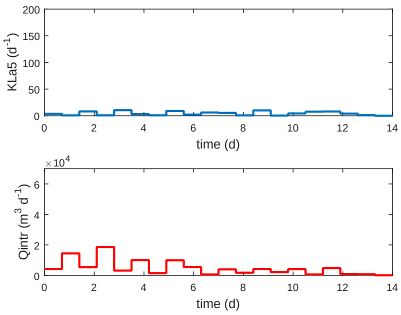


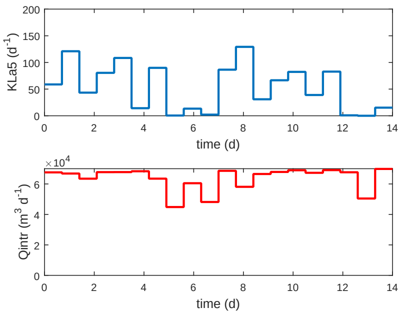
Fig. 2: Pareto fronts for the undisturbed, scenario 1 and scenario 2 problems

trol strategies. Figure 3 shows expectable control profiles from the qualitative point of view regarding the areas they refer to. In Area 1 (low operation costs and poor effluent quality, Figure 3a), almost no aeration and recirculation are applied, which reduces the electricity consumption but also the oxidation and de-nitrification capacity. In Area 2 (high operation costs and good effluent quality, Figure 3b), the aeration and specially the recirculation become significant, which increases notably the electricity consumption but allows a better oxidation and de-nitrification. The profiles in Area 3 (solution balancing both objectives, Figure 3c), seem to represent an intermediate case between the previous ones, with punctual episodes of high aeration rates and an almost continuous intermediate recycling rate.

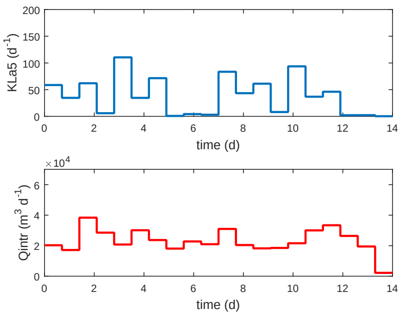
Going back to Figure 2, where the pareto fronts for the considered cases (undisturbed, scenario 1 and scenario 2) are shown, it can be observed that all the three Pareto fronts converge in Area 1. While this is not a general case and the picture could be different when simulating other scenarios, two aspects should be highlighted: i) despite of the constraints imposed in scenarios 1 and 2, the same (or very similar) solutions in terms of objective values regarding the Area 1 of the Pareto front can be achieved, and ii) due to these constraints, the control profiles leading to those equivalent solutions must present different shapes. The identification of such shapes would allow WWTP operators to efficiently change the operating conditions when some of the considered scenarios occur without damaging any of the pursued objectives. An additional conclusion from Figure 2 is that the absence of recirculation has a deeper im-



(a) Profiles for solution in Area 1



(b) Profiles for solution in Area 2



(c) Profiles for solution in Area 3

Fig. 3: Control profiles for representative solutions in the different areas of the Pareto front (undisturbed problem)

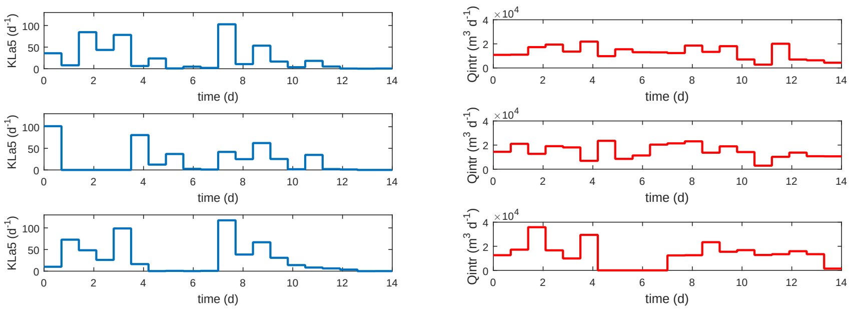
pact on the pareto solutions of area 2 than the absence of aeration in tank 5. This could be provoked because, although no aeration is applied on tank 5, tanks 3 and 4 are also aerated, which produces some oxidation of ammonia to nitrates. However, the lack of recirculation to increase nitrates reduction to nitrogen can not be compensated by any other mechanism.

To illustrate the existence of the mentioned multiple solutions we have selected similar solutions from Area 1 of the three pareto fronts. Table 2 shows the objective function values for each of them and Figure 4 shows their corresponding control profiles.

The maximum differences from the objective values in Table 2 are below 0.1% for EQ and 0.4% for the operational costs, thus we can consider them as equivalent solutions from the point of view of the objectives. However, Figure 4 shows different control policies for each scenario. This would prove the existence of multiplicity of solutions and their previous identification would allow to react efficiently when one of these events occur during WWTPs operation. Figure 4a (middle) shows the constraint imposed in scenario 1: no

Table 2: Equivalent solutions in terms of objective values from the Pareto fronts of the three considered scenarios

	(Monetary units d^{-1})	EQ (kg poll units d^{-1})
Undisturbed	14454	8183
Scenario 1	14493	8175
Scenario 2	14509	8176



(a) KLa_5 control profiles (b) Q_{intr} control profiles

Fig. 4: Control profiles for equivalent solutions from the Pareto fronts. Top: undisturbed; middle: scenario 1; bottom: scenario 2

aeration during the first 1-3 days of the process, while Figure 4b shows the one of scenario 2: no recirculation between days 4 and 7. Interestingly, the optimal aeration profile for this scenario 2 considers almost no aeration within the same period (days 4 to 7). The reason for this could be to avoid an excess of nitrates in the effluent during a certain period of time.

The fact that multiple equivalent solutions can be found for different scenarios in a system is not a general claim of this study. Certain systems can be very sensitive to changes in operational conditions which make very difficult to find such equivalent solutions. But for WWTPs, since typical control variables are usually related to aeration and recirculation and the objectives are related to operational costs and effluent quality, these equivalent solutions may exist. Therefore, by means of dynamic simulation and multiobjective optimization we encourage the simulation of different realistic and possible scenarios to identify such equivalent solutions, if they exist, and anticipate the control actions when these simulated events occur in the real process.

4 Conclusions

WWTPs have a high environmental and economical impact because of the effluent quality returned to the environment and their high energy consumption, respectively. These two objectives are usually simultaneously considered when designing these plants. They are conflicting objectives, and determining their trade-offs is crucial in the decision making process. The non-linear, dynamic and multiobjective nature of the models describing WWTP processes make that the optimization problems formulated for the design are complex and they must be solved with efficient and robust optimization techniques to obtain the Pareto front of optimal solutions.

Once the Pareto front has been obtained the simulation of possible and realistic operational scenarios (e.g., typical failures, maintenance operations, possible changes in legislation, etc.) can be performed to identify equivalent solutions in terms of objectives by comparing the obtained pareto fronts, and use the best control policy adapted to the incumbent event. In this work we have considered two simple realistic scenarios and have detected that this multiplicity exists in some area of the Pareto front. The application of this methodology could result in "alternative" Pareto fronts (or areas of the Pareto front) in terms of control profiles, which would enrich the knowledge of the process and would allow different options for the design. The exploitation of this idea can be quite relevant in the decision making process within current scenarios in which the energy costs are fluctuating hourly, supplying the decision maker a set of possible strategies to follow depending on the actual economical, technical or legal circumstances.

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Conflict of interest

The authors declare that they have no conflict of interest.

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