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Determination of the equivalent stiffness of thick-walled composite beams with an inter-circular cross-section using the semi-analytical method

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1. Introduction

The aim of this work is the search for a new analytical method of calculating the stiffness of the wound composite beams with the circular cross-section. The FE models and several analytical calculations were performed for chosen geometry of the composite beam and all results are compared to experimental data. The experiment of three-point bending on fibre-reinforced composite beams with two different composite lay-ups and two different diameters of the tubes was done. The stiffness of all specimens was also calculated. The comparison of a new semi-analytical method with the well-known analytical methods [1] of stiffness calculation and FE models are introduced in this paper.

Composite beams are very variable not only in their shape or cross-section, but also in the layup of the composite material from which they are made. This creates several variables that we must consider when calculating their deformation. It is well known that the available methods for calculating the deformation of composite materials do not provide relevant results for all possible types and shapes of composite beams. It turns out that these methods differ in the results for the same case of a composite beam or are too complex for the initial design of the part. Also, the results of these methods differ from a possible experiment. Analytical, semi-analytical, and numerical methods are known for calculating the deformation of composite beams. The comparison of Timoshenko's and Bernoulli's method of bending calculation, the method of calculation using ABD matrices and the numerical FE method were chosen as the basis for this work. All these methods were applied to an embedded composite beam with an inter-circular cross-section. The aim of this work is to find out in which specific cases the mentioned methods of calculating the effective stiffness of composite beams are valid for the general composition of the composite material.

2. Analytical methods for calculation the equivalent stiffness modulus of the composite tubes

2.1. The stiffness matrix and the compliance matrix

The Hooke's law contains the stiffness matrix $\boldsymbol{S}$,

$$\boldsymbol{\sigma} = \boldsymbol{S} \cdot \boldsymbol{\varepsilon}\,. \tag{1}$$

The modulus of elasticity is expressed for each layer separately by means of the stiffness matrix $\boldsymbol{S}$ in the main coordinate system of the composite material $O(L, T, T')$. An orthotropic material is considered. [1] To express the equivalent modulus of elasticity in the main coordinate system of the whole beam $O(x, y, z)$, it is possible to use the stiffness matrix $\boldsymbol{S_{xy}}$ or an inverse matrix the compliance matrix $\boldsymbol{C_{xy}}$. The stiffness matrix $\boldsymbol{S_{xy}}$ is expressed by the following transformation (2) to the coordinate system $O(x, y, z)$ and the compliance matrix $\boldsymbol{C_{xy}}$ is inverse to it (3),

$$\boldsymbol{S'}_{xy} = \boldsymbol{T}_{xy} \cdot \boldsymbol{S} \cdot \boldsymbol{T}'_{xy}\,, \tag{2}$$

$$\boldsymbol{C'}_{xy} = \boldsymbol{S'}^{-1}_{xy}\,. \tag{3}$$

The modulus of elasticity in the direction of the beam axis E_x can be obtained from the compliance matrix $\boldsymbol{C'}_{xy}$, and also from the stiffness matrix $\boldsymbol{S'}_{xy}$. The element S'_{11} is an element of the stiffness matrix $\boldsymbol{S'}_{xy}$ and element C'_{11} is an element of the compliance matrix $\boldsymbol{C'}_{xy}$. The usage of the stiffness matrix $\boldsymbol{S'}_{xy}$ represents the upper estimate of the equivalent stiffness, the use of the compliance matrix $\boldsymbol{C'}_{xy}$ the lower estimate of the stiffness of the composite beam. In the results section, their arithmetic mean is also used.

2.2. Calculation of equivalent elasticity modulus E_{eq} by Classical Laminate Theory

To calculate the deflection of a composite beam with a circular cross-section using method from [1] the following equation is used

$$[N \;\cdots\; M\,] = [A \;\vdots\; B \;\cdots\; \vdots\; \cdots\; B \;\vdots\; D\,][\varepsilon^{\circ}{}_m \;\cdots\; k\,]\,. \tag{4}$$

The force loading of the beam can be expressed using the elements of the matrix $\boldsymbol{A}$ [2]. The stress of a composite material using Hooke's law is expressed. To obtain the stress relationship, it is necessary to divide this expression by the total thickness of the composite material t.

$$\sigma_1 = \frac{N_1}{t} = \frac{1}{t}(A_{11} - [A_{12}\; A_{16}\,] \cdot [A_{22}\; A_{26}\; A_{62}\; A_{66}\,]^{-1} \cdot [A_{21}\; A_{61}\,]) \cdot \varepsilon^{\circ}{}_1\,. \tag{5}$$

The equivalent modulus of elasticity is expressed by the following relation.

$$E_{eq} = (A_{11} - [A_{12}\; A_{13}\,] \cdot [A_{22}\; A_{23}\; A_{32}\; A_{33}\,]^{-1} \cdot [A_{21}\; A_{31}\,]) \cdot \frac{1}{t}\,. \tag{6}$$

2.3. A new semi-analytical method

A new semi-analytical approach is based also on the Classical Laminate Theory and tries to calculate the equivalent stiffness of the beam with the combination of the tensile and bending stiffness matrix elements. The assumption for this theory is that the geometry of the composite beam with circular cross-section combines the tensile and bending loading of the material of the composite beam. The combination of the elements of matrix $\boldsymbol{A}$ and matrix $\boldsymbol{D}$ (4) is used in a superposition of the stiffnesses.

$$(EJ)_{equivalent} = (EJ)_{A_{11}} + (EJ)_{D_{11}}. \tag{7}$$

3. Experiment

Composite wound tubes (T700/epoxy matrix) with an inner diameters of 26 mm and 50 mm with the 3 mm (thick) and 1 mm (thin) wall thickness were selected as specimens. The geometrical characteristics of the specimens were chosen to compare the results from the analytical and FE methods in cases of thick and thin-walled beams. The tests were performed for three composite layups. Three-point bending tests were performed on an FPZ 100/1 loading machine. Supports with a span of 400 mm and 750 mm were used for the tests. The beams were loaded with force through the strap with a width of 25 mm. The deflection was measured with a laser extensometer OptoNCDT 1320 and strain with a strain gauge. The sensors were placed in the centre of the beam under the loaded place and one extensometer was placed in the quarter of the span length. Groups of six pieces from each combination of fibres, composite layup, and support span were tested. The average value of the equivalent stiffness EJ_{eq} was evaluated. All specimen types were modelled by available FE methods (in Abaqus) and the equivalent stiffness was calculated. All the mentioned analytical methods were also used to calculate the equivalent stiffness. A comparison of these values is shown in the following section.

4. Results

The results show the deviations of all calculation methods from the experimental data in percentages. The equivalent stiffness EJ_{eq} of beams is compared. The results of two lay-ups are presented. These are results of beams with inner diameter 26 mm, 400 mm span and Diagonal lay-up [90 °, ± 45°] in Fig. 1. The second results are for beams with inner diameter 50 mm, 750 mm span and Typical lay-up [90°, 0°, ± 30°] (Fig. 2) Both results are for the thick and thin variant of composite beams. The average stiffness (EJ_mean in the figures) obtained from the matrix S' and C' shows a good agreement with the experiment, but this method usually predicts higher stiffness compared with experimental data. The new semi-analytical approach shows the constant deviations of less than 25% for the thin variant and less than 7% for the thick variant from experimental data in both cases of specimens. These results are on the safety side of the calculation in most cases compared to the experimental data.

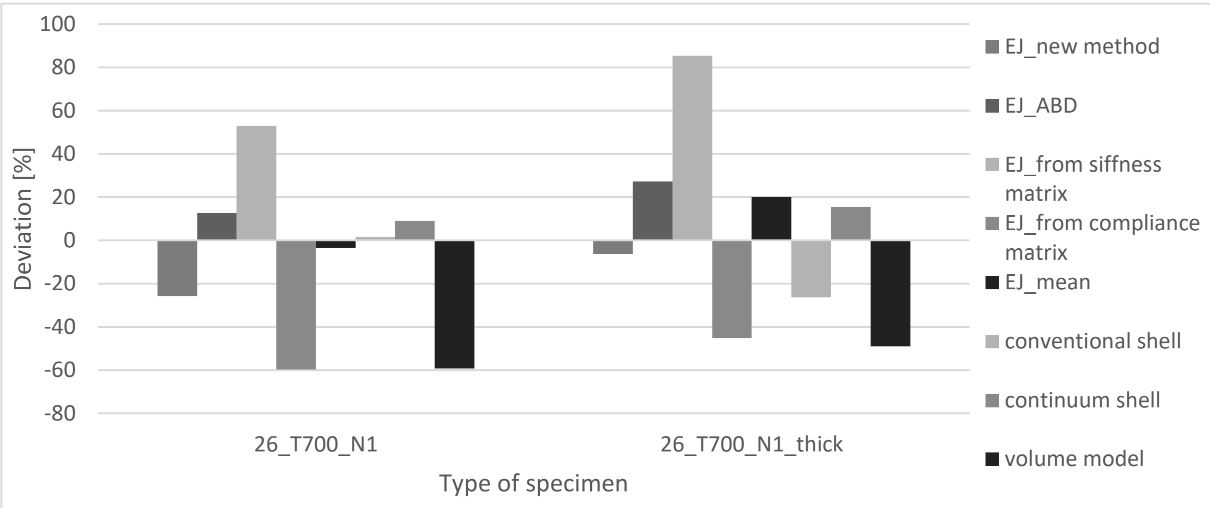

Fig. 1. The deviation from the experiment of equivalent stiffness for beams with ID 26 mm and Diagonal 1 [90°, ±45°]

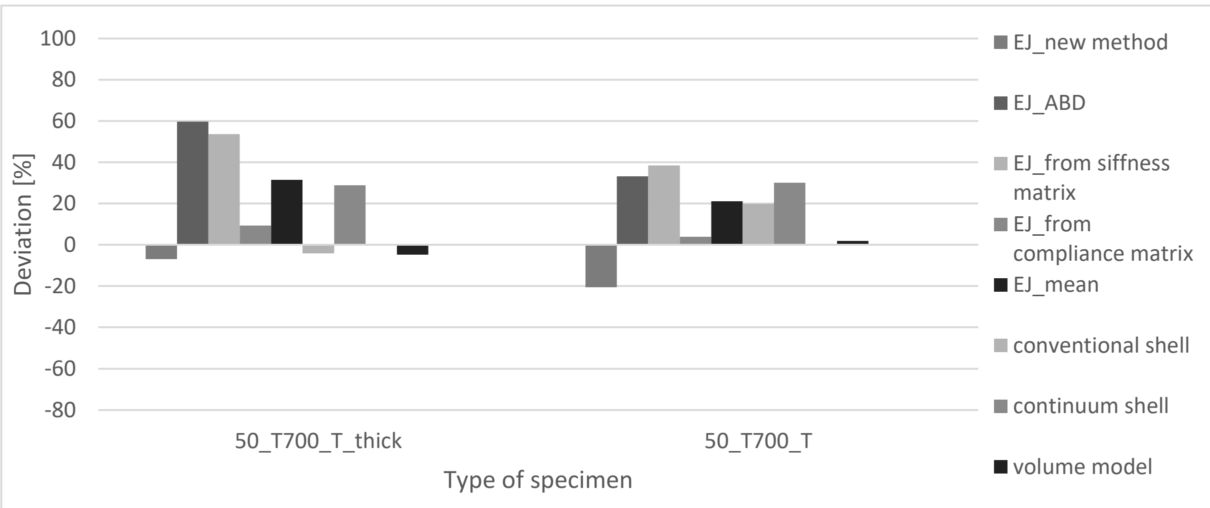

Fig. 2. The deviation from the experiment of equivalent stiffness for beams with ID 50mm and Typical layup [90°, 0°, ±30°]

5. Conclusion

None of the methods described above gives sufficiently accurate predictions of the stiffness of experimentally tested beams. There is still a noticeable problem where the results of different methods show different outcomes of the beam equivalent stiffness EJeq. From the performed comparison, methods with the best results are commented on.

The new semi-analytical approach reached a good agreement with experimental data in both composite lay-ups. A method based on the mean of an upper and lower estimate of the stiffness of the composite lay-up seems to be almost equally suitable, but with the deviation that predicts the greater stiffness than the experiment. In terms of computational complexity, the proposed approaches are less demanding than the FE method and are therefore suitable for fast usage for the preliminary design. The numerical optimization of these approaches is also possible.

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References

[1] Laš, V., Mechanika kompozitních materiálů. 2. vyd., Západočeská univerzita, Plzeň, 2008. (in Czech)
[2] Zavřelová, T., Analysis of composite beam bending, Diploma thesis, Czech Technical University in Prague, Prague, 2015.