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Online identification using linear neural unit with guaranteed weights convergence

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For many modern control strategies, a model of the controlled plant must be known. This model does not need to be known a priori and can be found online using an adaptive process. Online identification is very useful, especially if the plant's properties change during the run, for example, in context of vibration testing. This model can be a high order neural unit (HONU), such as a linear neural unit (LNU) with structure

$$y_m = \sum_{i=0}^{n} w_i x_i = \mathbf{w}^T \mathbf{x}, \tag{1}$$

where $\mathbf{x}$ is the input vector and $\mathbf{w}$ is the neural weights vector [1].

The general HONU output formula is

$$y_m = \mathbf{w}^T \mathrm{col}\mathbf{x}, \tag{2}$$

where $\mathbf{w}$ and $\mathrm{col}\mathbf{x}$ are both column vectors. The neural weights $\mathbf{w}$ can be adapted using a gradient descent algorithm, for example, normalised least mean squares (NLMS), which is derived by optimising a criterion

$$J = \frac{1}{2} e(k)^2, \tag{3}$$

where the error e is the difference between the desired and the current output

$$e(k) = y(k) - y_m(k). \tag{4}$$

The gradient of the criterion J with respect to $\mathbf{w}$, which is the steepest direction, is

$$\frac{\partial J}{\partial \mathbf{w}} = e(k)\left(\frac{\partial J}{\partial \mathbf{w}} y(k) - \frac{\partial J}{\partial \mathbf{w}} y_m(k)\right) = e(k)\left(0 - \mathrm{col}\mathbf{x}\right)) = -e(k)\mathrm{col}\mathbf{x}. \tag{5}$$

The weights are then adjusted toward the minimum of the criterion

$$w(k+1) = w(k) - \mu_k \frac{\partial J}{\partial \mathbf{w}} = w(k) + \mu_n e(k)\mathrm{col}\mathbf{x}(k), \tag{6}$$

using a learning rate $\mu_n = \mu \frac{1}{\varepsilon + ||\mathrm{col}\mathbf{x}(k)||}$, where $\varepsilon \approx 1e^{-5}$ and the normalisation improves convergence and $\mu \in (0, 2)$.

To assess the performance of HONU, the sum of squared errors (SSE) over a certain horizon N_e might be used

$$\mathrm{SSE} = \sum_{i=k-N_e}^{k} e(i)^2. \tag{7}$$

To ensure convergence of the weights [2], the learning rate μ might be chosen iteratively such that

$$||\mathbb{A}(k)|| = ||\mathbb{I} - \mu_n \mathrm{col}\mathbf{x}(k)\mathrm{col}\mathbf{x}(k)^T|| \leq 1, \tag{8}$$

where $\mathbb{I}$ is the identity matrix and $\mathbb{A}(k)$ is the matrix of update dynamics

$$\mathbf{w}(k+1) = \mathbb{A}(k)\mathbf{w}(k) + \mathbb{B}(k)\sqcap(k) = \left(\mathbb{I} - \mu_n \mathrm{col}\mathbf{x}(k)\mathrm{col}\mathbf{x}(k)^T\right)\mathbf{w}(k) + \mu_n y(k)\mathrm{col}\mathbf{x}(k). \tag{9}$$

At the start of the adaptation process the value $\mu = 2$ is chosen and the decreased $\mu(k+1) = 0.9\mu(k)$, until the condition (8) is satisfied. Then the weights are updated using the update rule (6). If the condition is violated during the run, then the learning rate μ is lowered until a value that satisfies the condition is reached again.

A plant with a transfer function

$$Y(s) = \frac{30530s + 3.765e06}{s^4 + 204.2s^3 + 15278s^2 + 415450s + 588660} U(s) \tag{10}$$

was used as a test system with square wave input signal. The input signal and the system output are shown in Fig. 1 with time step $\Delta T = 0.01$ s.

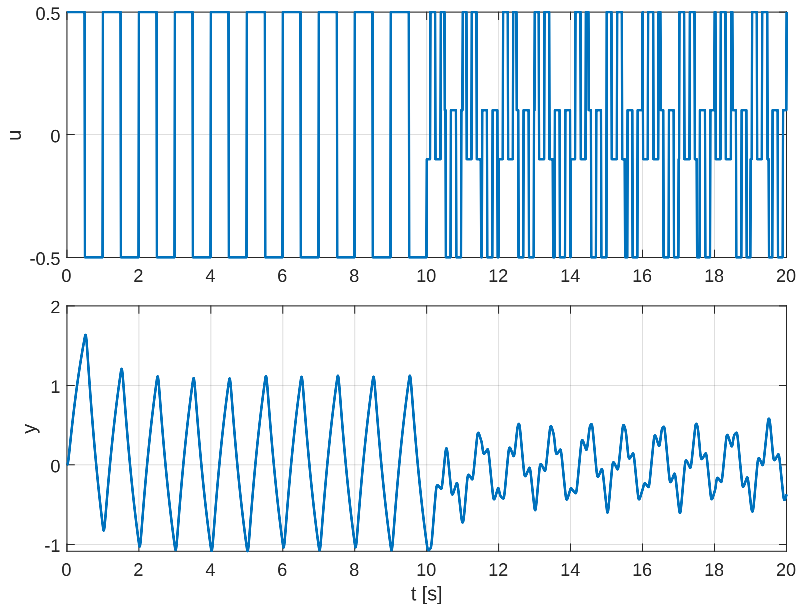

Fig. 1. The input and output of the testing plant

The input vector $\mathbf{x}$ consist of bias and a number of samples of the input and output of the plant

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 \\ u(k+1) \\ \vdots \\ u(k-n_u+1) \\ y(k) \\ \vdots \\ y(k-n_y) \end{bmatrix}, \tag{11}$$

where n_u and n_y are chosen such that the best performance is achieved. The performance of various choices is shown in Fig. 2. Performance is measured with $N_e = 100$.

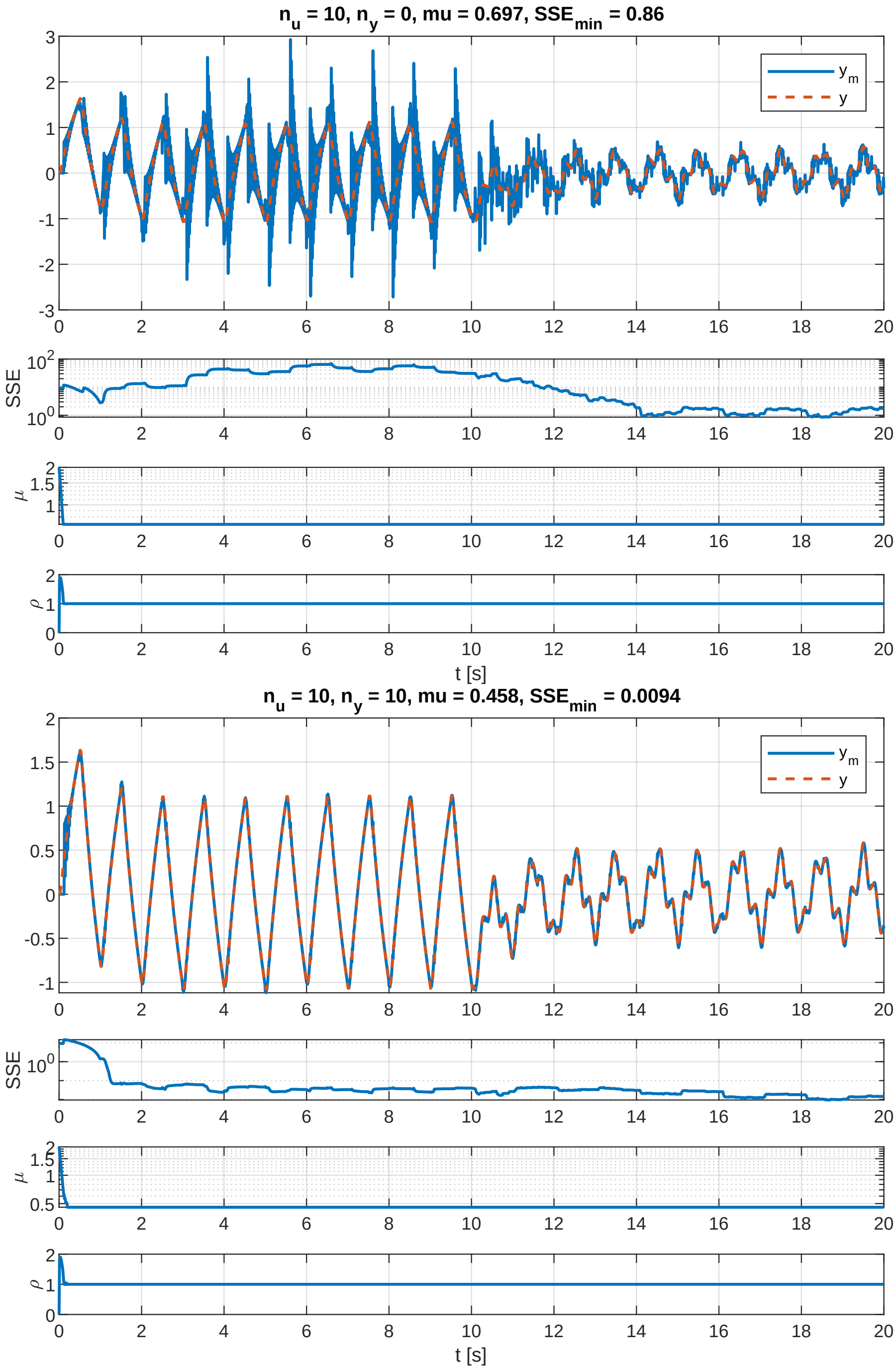

Fig. 2. Performance of the LNU for identification with various choices of n_u and n_u. The best performance is achieved with $n_u = 10$ and $n_y = 2$

The linear neural unit shows great performance in the identification of the plant. The plant is sufficiently approximated for a wide selection of n_u and n_y, which is enabled by the selection of μ_k that guarantees the convergence of the weights. In future work, this approach will be used to automatically identify a model for an adaptive controller.

Acknowledgement

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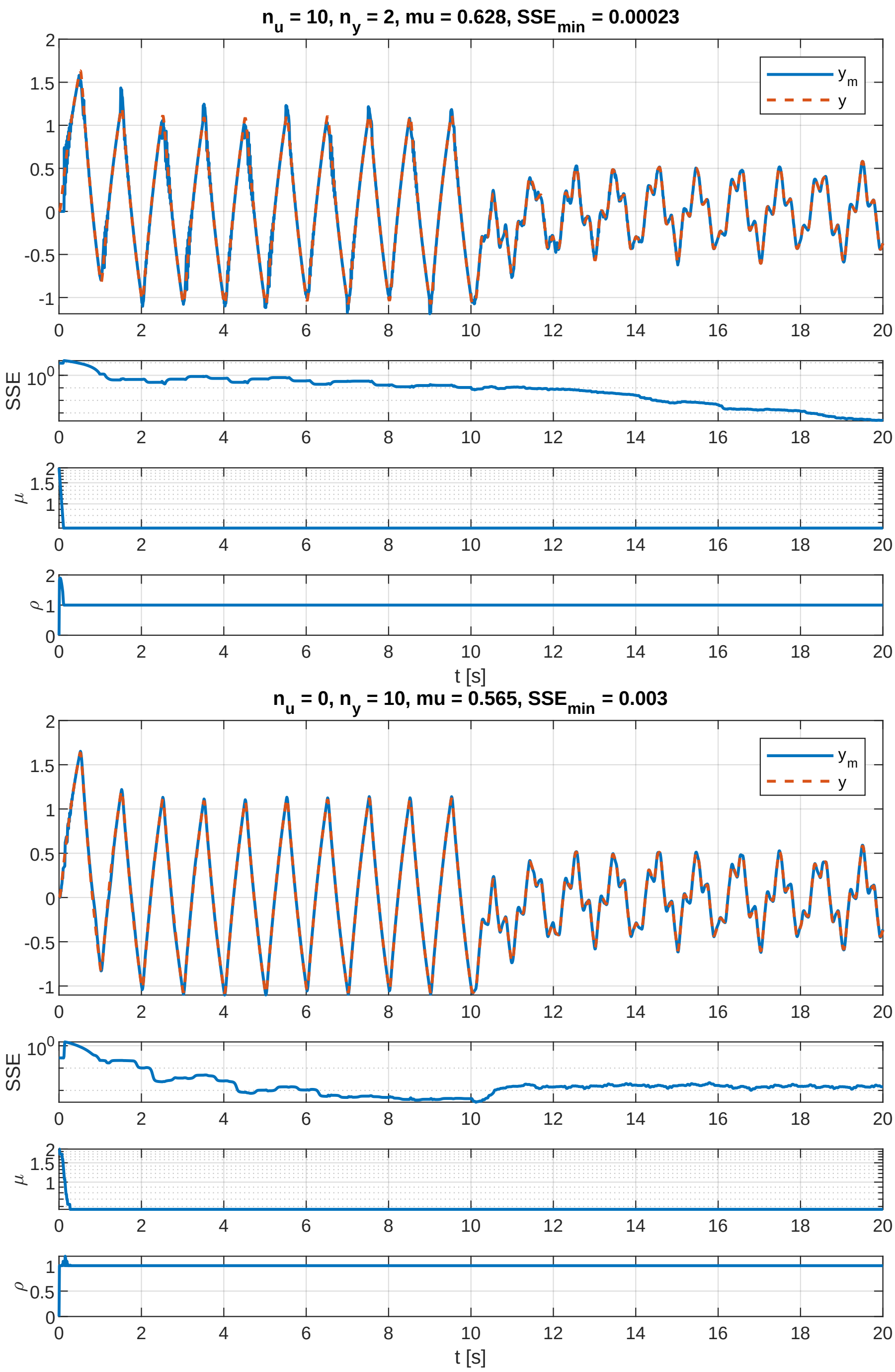

Fig. 2. (*Continued*)

References

[1] Bukovsky, I., Voracek, J., Ichiji, K., Noriyasu, H., Higher order neural units for efficient adaptive control of weakly nonlinear systems, Proceedings of the 9th International Joint Conference on Computational Intelligence, Funchal, Madeira, Portugal, 2017, pp. 149-157.

[2] Bukovsky, I., Dohnal, G., Benes, P. M., Ichiji, K., Homma, N., Letter on convergence of in-parameter-linear nonlinear neural architectures with gradient learnings, IEEE Transactions on Neural Networks and Learning Systems, 2021, doi: 10.1109/TNNLS.2021.3123533. (in press)