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Inverse dynamics approximation for controlling mechanisms with flexible elements

J. Lieskovský[a], P. Beneš[a], Z. Šika[a]

[a]*Faculty of Mechanical Engineering, CTU in Prague, Technická 4,160 00 Praha 6, Czech Republic*

Cable-driven mechanisms are usually a typical example of over-actuated systems, at least when the cables are considered rigid. If we include cable compliance, the systems generally become under-actuated. Although such a system remains controllable if kept under tension the end-effector’s acceleration is not directly dependent on the input torques on cable winches which means that the inverse dynamics problem is unsolvable. It is usually desirable to make the cables as stiff as possible, which allows us to approximate the cables as rigid at each individual step of calculating the input torques, invalidating the equations of motion. We can restore equality by introducing fictitious cable tension (in the form of internal joint torques) that corresponds to the expected deformation. This expected deformation is used to maintain a certain degree of pre-tension within the cables and therefore controllability of the entire system.

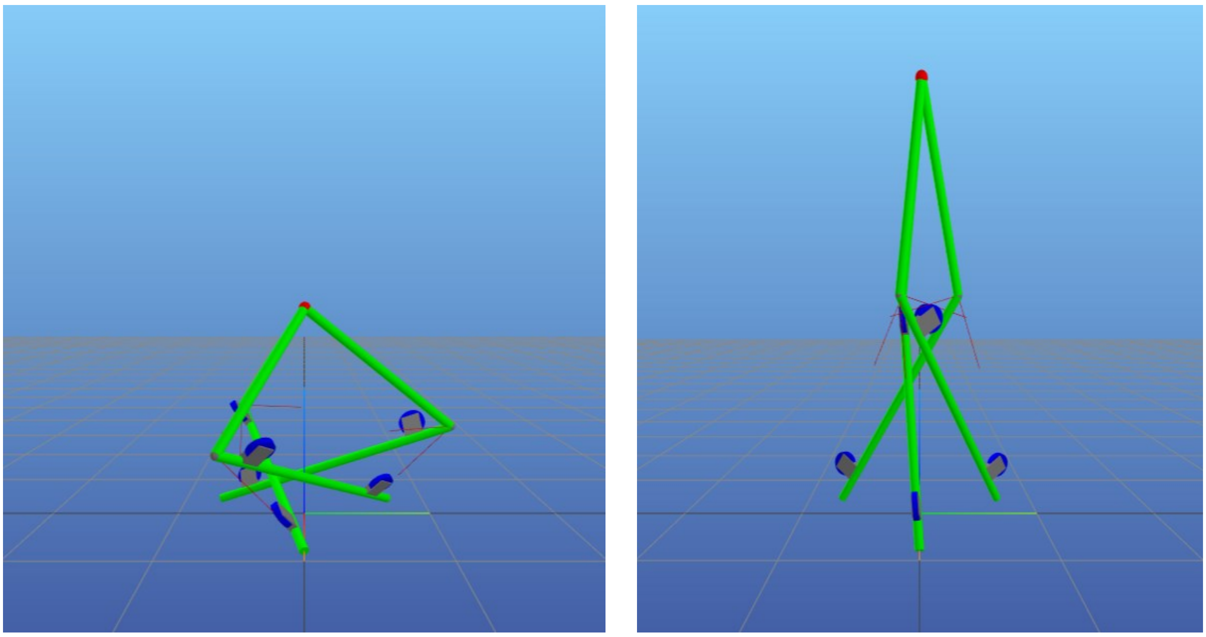

Fig. 1. Control of the tensegrity structure along the vertical axis – position in $t = 5$ s (left) and $t = 15$ s (right), [1]

The modified equations of motion can be written as

$$\boldsymbol{M}_d \ddot{\boldsymbol{y}}_d + \underbrace{\boldsymbol{M}_s \ddot{\boldsymbol{y}}_s}_{\boldsymbol{0}} + \boldsymbol{c} = \boldsymbol{W}\boldsymbol{f} + \boldsymbol{p} + \boldsymbol{A}_s \Delta \boldsymbol{y}_s + \boldsymbol{B}\boldsymbol{u}, \tag{1}$$

where $\ddot{\boldsymbol{y}}_d, \ddot{\boldsymbol{y}}_s$ are desired and superfluous accelerations, $\boldsymbol{M}_d$ and $\boldsymbol{M}_s$ are corresponding mass matrices, $\boldsymbol{c}$ is the vector of bias terms, $\boldsymbol{f}$ is the vector of external forces with wrench Jacobian matrix $\boldsymbol{W}$, $\boldsymbol{p}$ represents passive joint torques, $\boldsymbol{u}$ is the vector of inputs with matrix $\boldsymbol{B}$ and finally $\boldsymbol{A}_s \Delta \boldsymbol{y}_s$ represents the vector of elasticity torques, where $\Delta \boldsymbol{y}_s$ are expected changes in deformation. The solution was formulated as a quadratic programming problem [3] with maintaining tension within cables as optimizing parameters.

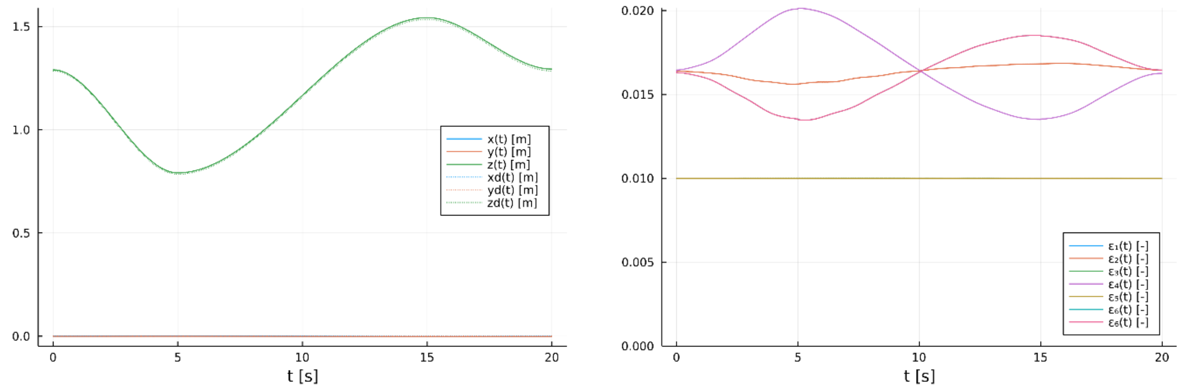

Fig. 2. Control of the tensegrity structure along the vertical axis – position of the end-effector (left), strain in cables (right), [1]

Figs. 1 and 2 show one of the verification experiments, which was the control of the tensegrity manipulator [2] along the vertical axis. The cable-pulley interaction is modelled using compliant involute joint with internal torque model, accounting for elasticity, damping and general uni-directionality of the cable Hermite C^1 continuous splines were used as desired trajectories. Fig 1 shows the manipulator in two different positions along the trajectory. The left part of the Fig. 2 shows that the position of the end-effector corresponds to the desired trajectory. The right part shows the strain in the cables during the manoeuvre.

The described approach can be used, for example, in conjunction with computed torques controllers. For this type of control, a precise inverse dynamics model is the key to proper functioning.

Acknowledgements

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References

[1] Lieskovský, J., Reconfiguration of tensegrity structures, Master thesis, CTU in Prague, 2022.
[2] Skelton, R. E., de Oliveira, M. C., Tensegrity systems, Springer-Verlag, 2009.
[3] Stellato, B., Banjac, G., Goulart, P., Bemporad, A., Boyd. S., OSQP: An operator splitting solver for quadratic programs, Mathematical Programming Computation 12 (4) (2020) 637–672.