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Post-processing the results of the topology optimization with the level-set technique

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1. Introduction

Topology optimization (TO) is a popular method for designing innovative components. Its goal is to find the optimal material distribution in the design space based on predefined boundary conditions and constraints. TO is most commonly used in the conceptual phase of the design process. However, with the development of additive technology and post-processing methods, the possibilities of using TO results directly for manufacturing are beginning to emerge.

The seminal paper that laid the foundations for the development of topology optimization is considered to be the work of Bendsøe *et al.* [1]. Since then, the development of TO has taken several directions, and nowadays, many different methods can be found in the literature. An overview of known TO methods can be found, e.g., in the review article [2]. Nevertheless, the most common TO approaches are density-based, especially the SIMP method, which stands for Solid Isotropic Material with Penalization. This method's results are unsuitable directly for manufacturing, so post-processing is required.

In this paper, the optimization problem of minimizing compliance with a constraint on the volume fraction will be formulated first. Then the SIMP method will be introduced. This paper's main contribution lies in using the level-set principle combined with RBF for post-processing the results obtained by the SIMP method. In the final part, the proposed method for post-processing of the topologically optimized gripper of an industrial robot will be applied.

2. Compliance minimization problem

A typical topology optimization problem is to find a maximally stiff structure with respect to its loads and supports. The constraint is the volume fraction, i.e., a certain ratio between the volume of the structure and the volume of the design domain in which the optimization is performed. Such a problem can be described by minimizing an objective function c (twice the strain energy) with a constraint on the volume fraction f, i.e.,

$$\begin{aligned} \min_{\rho}: \quad & c(\rho) = \mathbf{U}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} E_e(\rho_e)\mathbf{u}_e^T\mathbf{k}_0\mathbf{u}_e \\ \text{subjected to}: \quad & V(\rho)/V_0 = f, \\ & 0 \le \rho \le 1, \end{aligned} \tag{1}$$

where $\mathbf{U}$ is the global displacement vector, $\mathbf{F}$ is the global stiffness matrix, $\mathbf{u}_e$ is the displacement vector of the element, $\mathbf{k}_0$ is the stiffness matrix of the element with the corresponding

Young's modulus E_e, N indicates the total number of elements, $V(\rho)$ is the current volume of the material, and V_0 is the volume of the design domain.

2.1 Modified SIMP method

The SIMP method belongs to the class of *Density-based* methods and is one of the most used in TO. It employs finite elements to describe the shape, assigning each element a kind of importance known as fictitious density. This fictitious density (design variable) is used to calculate Young's modulus $E_e(\rho_e)$ of each element.

The result of the SIMP method is a scalar field of fictitious densities ρ, where each element is assigned a constant fictitious density ρ_e. This result can be used by the designer to get a basic idea of the shape of the part but cannot be used directly for manufacturing. Post-processing is required to obtain the shape from the TO results. In the case of 2D objects, post-processing can be done by simple sketching in CAD software. For more complex solids, it is necessary to use automated techniques.

3. Post-processing the results with the level-set technique

TO has become integral to many finite element systems and even some CAD programs. These systems usually use *Density-based* methods to obtain the optimal material distribution. The main advantage of these methods is their robustness. However, the results are highly dependent on the quality of the mesh, the filter parameters, and the post-processing type. Conventional post-processing methods average the element densities into nodal values and consequently interpolate them, most frequently with linear polynomials. It yields only C^0 continuous geometry, which is usually unsuitable for downstream processing. Some software uses B-spline curve segments to smooth the geometry, but here the results are of much higher quality. Unfortunately, all available software offers little or no user customization in terms of post-processing.

This section proposes two approaches to construct the level-set (LS) function. Both approaches use radial basis functions (RBFs) to construct the level-set function. RBFs are suitable for post-processing due to their ability to smooth the geometry but still preserve the shape complexity of the optimized shape [3].

3.1 Density in nodes approach

This post-processing approach uses RBFs to construct a LS function. RBFs can only be used in nodes of a regular grid. If the optimization has been performed on an irregular mesh, the entire design space must be packed into a so-called bounding box and this space discretized with a regular grid. The procedure for computing the node densities of a regular grid is as follows. First, the density at the nodes of the original grid has to be computed. This can be done by taking the continuous density in the elements to the geometric center of the elements and then using the least squares method to compute the density at the nodes. Subsequently, the density at the nodes of the regular grid can be calculated using shape functions. By placing the RBFs (2) in the nodes of the regular network with appropriate weights so that the sum of these functions at the nodes of the regular network corresponds to the nodal values, we obtain an implicit LS function. The zero level of this function corresponds to the geometry of the shape.

$$N_i(x) = exp\left[-\left(\frac{\|x - x_i\|}{B}\right)^2\right] \tag{2}$$

The described approach has been tested on a gripper of an industrial robot. From the results shown in Fig. 1, it can be seen that this method gives smooth results, but has the characteristics

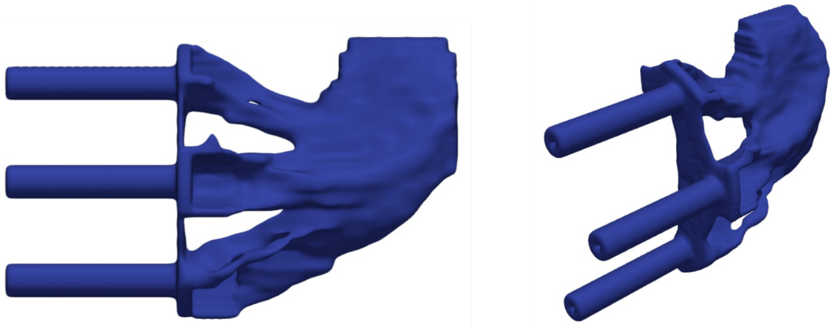

Fig. 1. Post-processing result of the topologically optimized gripper of an industrial robot

of a filter, i.e., it ”blurs” the shape complexity of the component in places where there is a step change in the TO results between empty and full space.

3.2 Approach based on signed distance function

Another approach for post-processing is based on a signed distance function. This function assigns to each point in the control area the shortest distance to the boundary of the solid, including a sign, so it is possible to distinguish whether the point is inside or outside the solid. To construct a discrete form of the distance function, the density field was nested in a regular Cartesian grid, and for each vertex of this grid, the closest point to the boundary of the solid was found. The density isocontour defines the boundary of the solid. This threshold density boundary was computed to respect the volume fraction. The distance field was used as a weighting factor for RBFs to construct the LS function. The zero level of the implicit LS function corresponds to the geometry boundary.

The approach has been tested on a 3D beam. From the results in Fig. 2, it can be seen that a very smooth geometry was achieved, and no complications in the boundary regions.

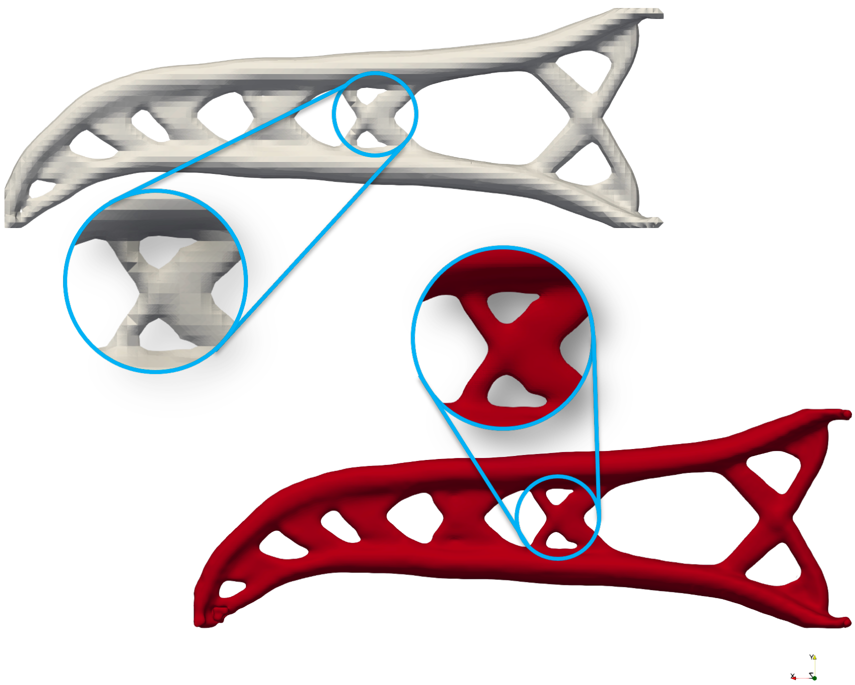

Fig. 2. Example of 3D beam test geometry. Above is the original C^0 continuous geometry and below is the smoothed shape

4. Conclusions

The last part of the paper describes two post-processing procedures using the LS method. The construction of the LS function itself was performed using Radial Basis Functions. The first approach uses the nodal densities as weights of the RBFs, while the second obtain these weights from the distance field. The results from the first approach are very smooth. However, there is a blurring of shape complexity at the points of sharp transition between the elements of the original mesh and the empty space (i.e., high-density elements and empty space). The second post-processing approach seems to be more appropriate because of its ability to describe even shape-complex objects. This capability to describe shape is due to the density of the control region discretization.

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