

# LQR Control of Input LC filter of a Tram with On-board Energy Storage System

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**Abstract**—The main traction power converter of a trams is connected to DC trolley wire by input LC filter. The tram input LC filter is almost undamped resonant circuit, which is often loaded by the motor constant torque command. Sources of the input LC filter oscillation may be present on the vehicle, typically drive control influence. The source of LC filter excitation except the tram drive could be the tram's trolley voltage caused by fluctuations in power consumption and harmonics from surrounding vehicles, and temporary loss of catenary voltage caused by crossings of insulated sections or icing on catenary. The next source could be the tram ESS, which is connected in parallel to the tram dc-link capacitor. The dynamic ESS control, which is necessary for majority of power management techniques, can also destabilize the LC filter. On the other hand, the proper ESS control could be used for LC-filter stabilization. The LQR based stabilization of LC filter using ESS is presented in this paper.

**Index Terms**—LQR control, LC filter, Tram, Energy storage system

## I. INTRODUCTION

An input LC filter is a standard component used to secure the power transfer from the dc trolley wire to the tram electric systems (main traction drive). The input LC filter has relatively low resonance frequency and it is sensitive to oscillations, which may be excited as a consequence of following two main reasons: I. fast change of the trolley wire voltage or II. fast change of the drive power request[1], [2], [3], [4]. This phenomenon can be addressed either by design of the LC filter using passive components (passive LC filter stabilization)[5], [6], or it could be considered when designing the control algorithm (active LC filter stabilization) [7], [8]. The active LC filter stabilization is commonly based on the optimizing the overall control algorithm, where the control action is adjusted to respect the dynamic properties of the filter. Nevertheless by redesigning the requested power taken from the tram DC link, the quality of the control represent a compromise between maintaining the LC filter stability and requested power/force tracking.

In our case, the tram is extended by the supercapacitor energy storage system [9], [10]. The supercapacitors are used to store some amount of the energy which is produced by the traction drive during regenerative braking. This energy is then released if needed and it can be even used to stabilize the input LC filter with no traction drive control action adjustments [11], [12], [13].

This paper deals with the design of a control algorithm for the supercapacitor storage system which extends the peak-shaving power management strategy of the tram, by input LC filter stabilization. This has been achieved by linear quadratic

regulator using linear model of tram input LC filter. The proposed control algorithm has been tested in a simulations.

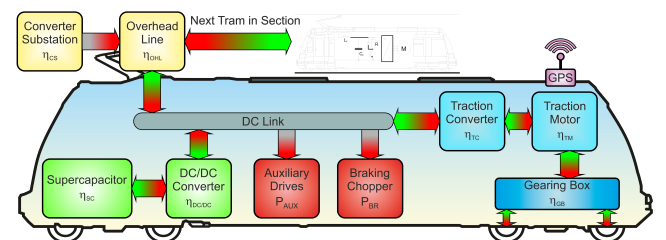


Figure 1. Tram with the ESS

## II. DESCRIPTION OF THE TRAM POWER SYSTEM

The tram input LC filter is almost undamped resonant circuit, which is often loaded by the motor constant torque command. Sources of the input LC filter oscillation may be present on the vehicle, typically drive control influence. From short time point of view, the drive is operated at approximately constant power when a constant torque is required at high moment of inertia (slow speed changes). Then the DC link voltage drop causes an increase in the traction current which causes another voltage drop, so positive feedback occurs in the system.. The source of LC filter excitation except the tram drive could be the tram's trolley voltage caused by fluctuations in power consumption and harmonics from surrounding vehicles, and temporary loss of catenary voltage caused by crossings of insulated sections or icing on catenary. The next source could be the tram ESS, which is connected in parallel to the tram dc-link capacitor. The dynamic ESS control, which is necessary for majority of power management techniques, can also destabilize the LC filter. On the other hand, the proper ESS control could be used for LC-filter stabilization. This stabilization can extend the power management of the ESS (fig. 3) . The ESS power management contains internal control loop, which typically control the ESS current  $I_{ESS}$ , which is equivalent to required ESS power. The ESS current is not directly measured in our case with the step-up ESS converter. The required ESS current is transformed by power equality equation  $U_{SC}I_{SC} = U_C I_{ESS}$  to required supercapacitor's current  $I_{SC}^*$ . This current is driven by the voltage drop ( $U_{LSC}^*$ ) of supercapacitor filtering inductance  $L_{SC}$ . This voltage  $U_{LSC}^*$  is added to the supercapacitor voltage  $U_{SC}$  to form required step-up converter voltage. To calculate duty cycles the required step-

up converter voltage must be normalized by actual LC filter capacitor voltage  $U_C$ .

This paper presents the LC filter stabilization by LQR control. It is important to note, that the LC filter stabilization is not a control of a linear system with constant parameters. This is caused by tram movement and dependency of the equivalent LC circuit on the tram distance from the converter substation. It requires sufficiently robust LC filter control.

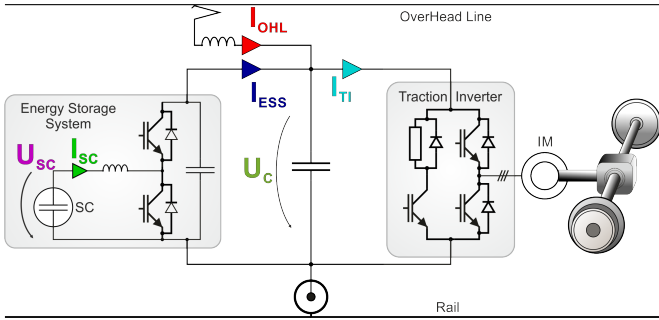


Figure 2. The ESS power circuit

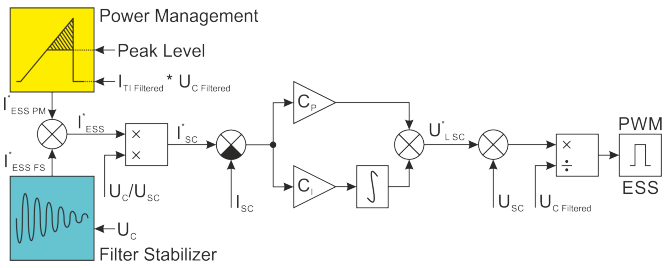


Figure 3. Inner control loop of ESS

#### A. Filter Stabilizer by Linear Quadratic Regulator LQR

The basic principle of stabilization is to minimize criteria:

$$g_{u_c} = (u_c - u_{cf})^2 \quad (1)$$

where,  $u_{cf}$  is the filtered value of  $u_c$ .

The LC filter can be described by the simple scheme in fig :4. The  $U_0$  is OHL voltage,  $R_f$  and  $L_f$  is equivalent OHL resistance/inductance of input tram filter,  $C_f$  and  $U_c$  are filter capacity and its voltage,  $I_{TI}$  and  $I_{ESS}$  are traction inverter current and energy storage system current.

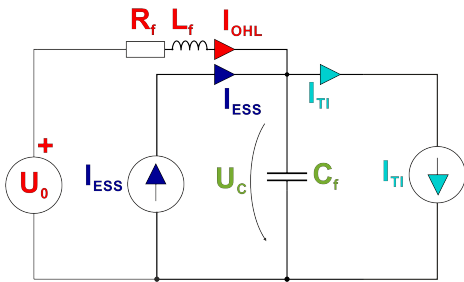


Figure 4. Simplified scheme of the tram with ESS

The state space equations describing this kind of circuit could be:

$$\frac{di_{OHL}}{dt} = -\frac{R_f}{L_f} i_{OHL} + \frac{u_o - u_c}{L_f}, \quad (2)$$

$$\frac{du_c}{dt} = \frac{i_{OHL} + i_{ESS} - i_{TI}}{C_f}, \quad (3)$$

Lets assume that  $\frac{du_o}{dt} \approx 0$ ,  $\frac{di_{ESS}}{dt} \approx 0$ ,  $\frac{di_{TI}}{dt} \approx 0$ , than we can write:

$$i_c = i_{OHL} + i_{ESS} - i_{TI}, \quad (4)$$

$$u_{LF} = u_c - u_0, \quad (5)$$

so:

$$\frac{di_c}{dt} = \frac{di_{OHL}}{dt} = -\frac{R_f}{L_f} i_c - \frac{u_{LF}}{L_f} + \frac{R_f}{L_f} (i_{ESS} - i_{TI}), \quad (6)$$

$$\frac{du_{LF}}{dt} = \frac{du_c}{dt} = \frac{i_c}{C_f}, \quad (7)$$

$$A_C = \begin{bmatrix} -\frac{R_f}{L_f} & \frac{-1}{L_f} \\ \frac{1}{C_f} & 0 \end{bmatrix}, x = \begin{bmatrix} i_c \\ u_{LF} \end{bmatrix} \quad (8)$$

$$B_C = \begin{bmatrix} \frac{R_f}{L_f} & 0 \end{bmatrix}^T, u = (i_{ESS} - i_{TI}) \quad (9)$$

To control capacitor voltage to defined level we can add simple low-pass filter in the form:

$$U_{LFfilt(p)} = \frac{1}{1 + p\tau} U_{LF(p)}, \quad (10)$$

and for the difference of filtered voltage and actual voltage we get the equation:

$$U_{LFdif(p)} = \frac{1}{1 + p\tau} U_{LF(p)} - U_{LF(p)} = \frac{-p\tau}{1 + p\tau} U_{LF(p)}, \quad (11)$$

which leads to differential equation:

$$\frac{du_{LFdif}(t)}{dt} = -\frac{u_{LFfilt}(t)}{\tau} - \frac{du_{LF}(t)}{dt}, \quad (12)$$

$$\frac{du_{LFdif}(t)}{dt} = -\frac{u_{LFfilt}(t)}{\tau} - \frac{i_c}{C_f}, \quad (13)$$

which we want to control to zero.

By using Euler discretization  $A_d = I + A\Delta t$ ,  $B_d = B\Delta t$  we get complete discrete system state space description:

$$A_d = \begin{bmatrix} \left(1 - \frac{R_f}{L_f} \Delta t\right) & \left(\frac{-\Delta t}{L_f}\right) & 0 \\ \frac{\Delta t}{C_f} & 1 & 0 \\ \frac{-\Delta t}{C_f} & 0 & \left(1 - \frac{1}{\tau} \Delta t\right) \end{bmatrix}, \quad (14)$$

, where  $x_{(k)} = [i_{c(k)} \quad u_{LF(k)} \quad u_{LFdif(k)}]^T$  and Bd matrix:

$$B_d = \begin{bmatrix} \frac{R_f \Delta t}{L_f} & 0 & 0 \end{bmatrix}^T \quad (15)$$

An optimal  $u_{(k)}$  can be found solving algebraic Riccati equation (16) for variable  $P$  and using quadratic cost function (17)

$$A_d^T P A_d - P - (A_d^T P B_d)(B_d^T P B_d + R)^{-1}(B_d^T P A_d) + Q = 0, \quad (16)$$

$$g(x, u) = x^T Q x + u^T R u, \quad (17)$$

where  $Q$  is a matrix used to penalize the particular states,  $R$  is the matrix used to penalize controller output variable.

The matrix  $Q$  and  $R$  are diagonal matrices with

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 30 \end{bmatrix},$$

$$R = [1],$$

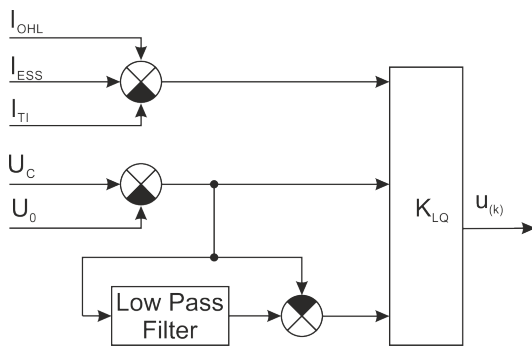


Figure 5. LC filter LQ controller stabilization

### III. SIMULATION TESTS

The simulation test presents the acceleration of the tram (0s - 12s) time-shortened tram drive with constant speed and regenerative tram braking (18s - 27s). The power management with peak-shaving algorithm at 7.5s starts to generate  $I_{ESS}$  current to "shave" the trolley input power (represented by  $I_{OHL}$ ). At 15s the  $U_0$  catenary voltage drop and in 16s the  $U_0$  oscillations are applied to test LQR stabilization.

### IV. CONCLUSION

This paper presents the tram LC filter stabilization by using LQR controller. The LC filter of the tram with on-board ESS can be stabilized by the proper ESS control. The proposed control extends the ESS power management by the LQR based LC filter stabilization. The LC filter stabilization by LQR including low-pass filter and LC filter dynamics show promising results in stabilization of ESS control either oscillations coming from the OHL. Nevertheless, the LQR controller robustness to LC filter parameter variation and its sensitivity to ESS current control dynamic should be investigated in the future.

### ACKNOWLEDGMENT

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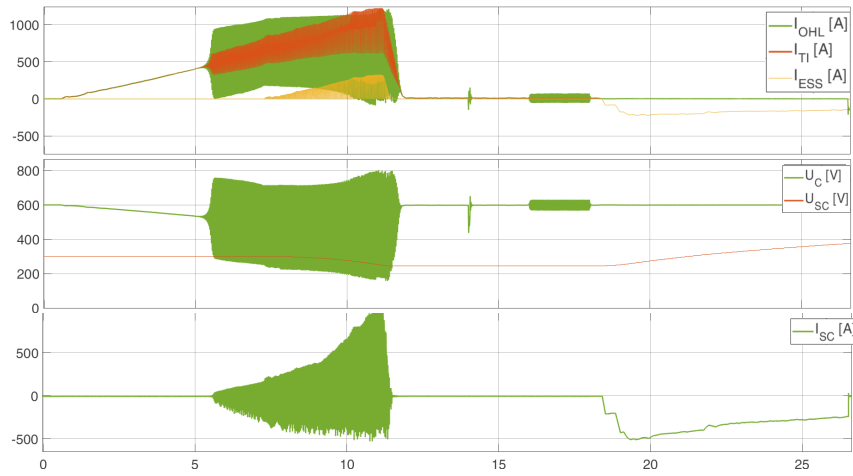


Figure 6. The simulation of the tram with LC filter without stabilization

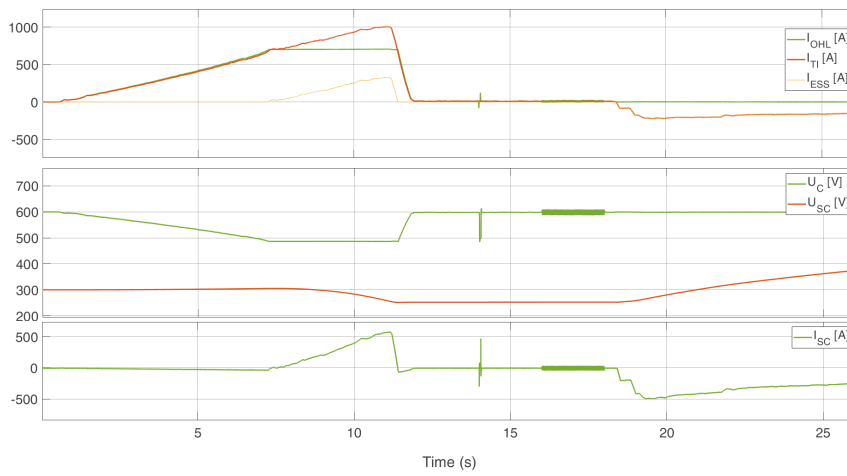


Figure 7. The simulation of the tram with LC filter with stabilization

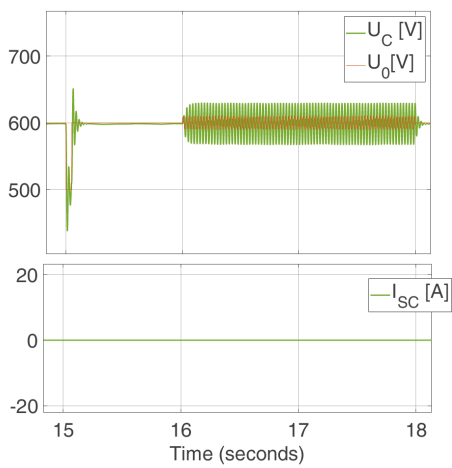


Figure 8. The simulation of the tram with LC filter without stabilization - detail

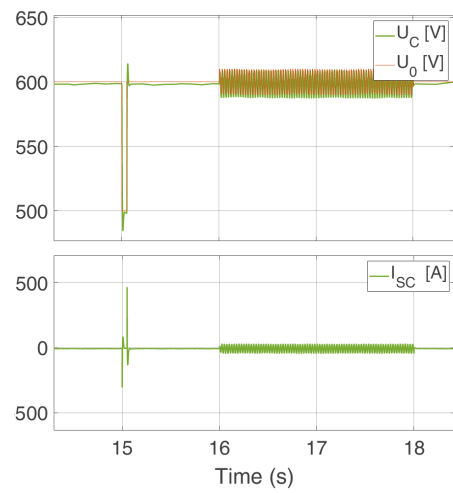


Figure 9. The simulation of the tram with LC filter with stabilization - detail